A TEXT BOOK OF PHYSICS

VOL I

D.P. RAY CHAUDHURI

FOR HIGHER SECONDARY STUDENTS

A Mukherjee & Co Pvt Ltd

Written according to the revised Syllabus introduced by the West Bengal Council of Higher Secondary Education for Classes XI and XII.

A TEXT BOOK OF PHYSICS

[For Higher Secondary Students]

Vol 1

[Mechanics, Properties of Matter, Vibrations and Waves, Heat]

Dr. D. P. Ray Chaudhuri

Formerly Professor of Physics University of Kalyani ;
Head of the Dept. of Physics
Scottish Church College

Assisted by Prof. J. K. Mookerjee, M. Sc., M.A.
Scottish Church College and Prof. D. Ray M.A.
New Alipore College

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PREFACE Old Edition

I took up preparing the book on the basis of the syllabus forwarded by the council of Higher Secondary Education to the West Bengal Publishers and Book Sellers' Association. From time to time the Council forwarded modifications. These have been accommodated. Nevertheless the printed syllabus differed in some minor respects from what was forwarded to the association. Though they did not require any substantial modification of the text, the author regrets the deletion of reference to the SI units. The topmost international scientific bodies strongly advocated the use of one system only (Syste'me International) for all scientific measurements all over the world. West Germany accepted it even for domestic purposes. But in our country school children are still required to learn *systems* of units. This is entirely unnecessary. Incidentally it may be noted that the council for Indian School certificate have issued clear instructions for the exclusive use of SI units for their examinations (Classes XI to XII). I wonder why our syllabus framers are so indifferent about international units and symbols, as well as the way how to use them. With the financial support of the Unesco a booklet entitled 'Symbols, units and Nomenclature in Physics' was circulated around 1967 by the NCERT to all institutions of higher educations in India. Authors, research workers, writers of popular scientific articles from the rest of India are following the International Recommendation contained in the pamphlet. But in our own state of West Bengal we find very little evidence of it.

I have tried in this book to give the student some aquaintance with SI units and the their handling. As there is talk of syllabi being soon revised and lightened, I do fervently hope more attention will be paid to the SI units and obsolete topics dropped from the course. What matters primarily is to determine the purpose of the physics course. Should it be part of a General

Science course or a prepatory course for higher studies ? I fear the two purposes cannot be successfully combined. The present course seems to support my apprehension.

The syllabus might have advised the use of modern nomenclature such as Relative density for specific gravity, specific heat capacity for specific heat, specific latent heat for latent heat. The word specific is at present used in the sense 'per unit mass'. The usage should be encouraged. Since heat is energy, the 'calorie' is dropping out of use and the 'joule' is used as the unit of heat. Such usage has the added advantage of doing away with the 'mechanical equivalent of heat.' Let us hope that the new syllabus will be more uptodate in such matters.

When a page limit is prescribed for a book it is necessary that the maximum number of pages to be allotted to the main topics be at the same time be specified. Otherwise, there will be too much variation from book to book, creating confusion among the students and teachers alike. An approximate page limit is desirable ; but wasting space by printers for profit may alone make a difference of 10% or more in the same text.

Let us hope that the relevant successor authorities will look into these matters.

D. P. R.

PREFACE (New Revised Edition)

The above provides an insight to the depth and sincerity of purpose for improving the teaching of Physics in West Bengal evinced by Dr. D. P. Raychaudhuri in this his last text book for our students. A teacher who became a legend in his life time, characterised by a perceptive and grateful young colleague as a teacher of teachers, seems to have cried in vain for rationalising and modernising our approach to teaching of physics in West Bengal. We seem to remain immovably shackled to tradition that have become out of tune.

Dr. Raychaudhuri had during the last five months of his long and venerated life engaged in revising this very volume when he left us (July 1985) loaded with years, honours and very deep respect. The Publishers requested us. To make the book ready for the Press. That was last October.

From the very first it was deemed proper to do away with the page limitations that constrained the late author and to introduce a large number of worked out sums and exercises culled from various public examinations all over India including those of the current rage, Joint Entrance and I.I.T. Admission examinations. Extensive expansions have been effected, *many from the pen of the late author himself*, to amplify the basic ideas and aquaint the student with modern applications of these ideas. We particularly draw the attention of colleagues to the chapters on units and Dimensions from Dr. Raychaudhri's text on Properties of Matter, and that on Elementary calculus introduced. More emphasis have been laid on relevant experiments that give flesh and blood to physical laws and principles. In mechanics, graphical and calculus methods of deriving various laws have been added to show their elegance and vivid picturisation. However, it must be deplored that the brevity and succintness that was the guinea-stamp of D. P. R. have been lost. Still most of the additions that have been brought

about come from his other works. The discerning teachers would still be aware of jolts and jars as they peruse the book. We crave their indulgence for we have striven to the best of our limited ability to make the book useful to the demands of current students as desired by the new Publishers of the book. In revising thus we have utilised extensively many books, indigenous and foreign that command extensive circulations at present. Our very humble thanks are due to all the authors.

Due to rushing the book through the Press in only five months the Printer's Devil had had a field day for which we can only ask for forgiveness, from colleagues and students.

August, 1986 J. K. M.

D. R.

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West Bengal Council of Higher Secondary Education Syllabus

PHYSICS (*Elective.*)

Full Marks 80 (Theory)

PAPER—I

1. MECHANICS

Particle : Dynamics :

Rest and motion, reference frame, displacement, velocity and acceleration, momentum, kinematical equations (in one dimension), elementary problems.

Scalars and Vectors, Composition and resolution of Vectors. Representation of vector by co-ordinates. Addition of vectors by geometrical and analytical methods. Relative velocity and acceleration.

Newton's laws of motion, inertia, units of force, impulse, and impulsive forces, conservation of linear momentum-elastic collisions of particles moving in the same line, jets and rockets. Friction, static and kinetic friction, co-efficient of friction.

Statics :

Centre of mass, centre of gravity, conditions of equilibrium of a system of particles.

Dynamics of Rotational Motion :

Rotational motion of a particle, angular velocity, angular acceleration, relation between angular velocity and linear velocity, angular momentum, moment of a force about a point and about an axis, torque, relation between angular momentum and torque, (statement only) couples, centripetal force, centrifugal force (as a pseudo-force).

Work, Energy and Power :

Definition of work, relevant units, work done by and against a force, Mechanical energy—kinetic and potential forms. Conservation of energy—with the case of a freely falling body as an example. Power—definition, units.

2. GENERAL PROPERTIES OF MATTER

Gravitation :

Newton's law of universal gravitation. Constant of Gravitation (no experimental details on the determination of the gravitational constant), Gravitational attraction for extended bodies. Gravitational attraction of the earth. Laws of falling bodies. Variation of acceleration due to gravity. Simple pendulum. Motion of planets, satellites, Escape velocity (no deduction) weightlessness in orbiting satellites.

Elastic Properties of Matter :

Stress, strain, elastic limit, Hooke's law, elastic madulli, young's modulus, Bulk modulus, rigidity modulus, Poission's ratio.

Hydrostatics :

Density, Specific Gravity, (methods of determination of Sp. Gr. not required), Archimedes' principle (demonstrations) flotation, pressure in fluids, transmission of fluid pressure, Pascal's law and its applications. Air pressure and its measurement, Siphon, principles of lift pump, compression pump, vacum pump.

3. HEAT

Recapitulation of the basis concepts of heat and temperature. Thermal expansions of solids and liquids simple demonstrations, co-efficient of expansion for solids, relation between them. Applications of expansions of solids.

Real and apparent expansions for liquids, relation between expansion co-efficients, Anomalous expansion of water. Effect on marine life.

Thermal expansion of gases.

Boyle's law, Charles' law, Equation of state of an ideal gas ; volume and pressure co-efficient, Absolute scale of temperature.

Calorimetry :

Preliminary definitions, principle of calorimetry (no questions on measurement to be set). Calorimetric problems.

Change of State :

Latent heat (brief discussions of determination) evaporation and boiling. Effects of pressure on melting point and boiling point.

Vapour pressure, relative humidity. Dew, fog and cloud, Hygrometry, Regnault's hygrometer.

Mechanical equivalent of heat :

Heat as a form of a energy, relation between the Caloric and the erg Determination of mechanical equivalent of heat (paddle method). First law of thermodynamics.

Kinetic Theory of Gases :

Evidence of molecular structure of matter and of random molecular motion. Brownian movement (qualitative description). Basic assumtions of the kinetic theory of ideal gases. Pressure of an ideal gas (mention of the formula) ; derivation not required. Concept of temperature from kinetic theory. Qualitative discussions of limitations of ideal gas laws.

Transmission of Heat :

Conduction of heat, simple demonstrations ; thermal conductivity. Practical applications of thermal conduction. Convection of heat, convection current. Radiation ; radiation as a form of energy.

4. VIBRATION AND WAVES

Vibrations :

Oscillations and its characteristics. Simple harmonic motion, examples. Relation with uniform circular motion. Graphical and mathematical representations. Energy in simple harmonic motion.

Superposition of two simple harmonic motions in the same direction (graphical) (i) in phase (ii) in opposite phases.

Nature of vibrations (transverse and longitudinal). Free and forced vibrations, resonance, damped oscillations (qualitative discussions with examples).

Waves :

Types of waves, characteristic features of propagating waves, preliminary definitions and relations. Reflections and refraction of waves. Supervision of waves, stationary waves ; vibrations of strings and air columns.

Interference, beats, Doppler effect, polarization (qualitative discussions).

Nature of Waves :

(i) Sound waves as elastic waves. Velocity of sound. Laplace's formula (Newton's formula $V = \sqrt{E/\rho}$ to be assumed).

(ii) Light as a wave phenomenon. Finite velocity of light. Interference of light. Polarization (qualitative ideas). Validity of geometrical optics as an approximation.

Part-0

INTRODUCTORY

CHAPTER 0-1

PHYSICS : UNITS AND DIMENSIONS

0-1.1 What is Physics : *Physics* deals with those phenomena in the inanimate world where the type of **matter** *does not change*. Chemistry is essentially concerned with phenomena in which the type *does change*. The distinction does not however, always hold. Primarily also, we may regard the subject matter of physics as dealing with **transformation of energy** from one form to another. This however, includes chemistry in the scope of physics.

Branches of Physics : Physics in fact, is the study of properties of matter and energy. It has several main divisions. Of these, we shall be mostly concerned with *mechanics, properties of matter, vibrations and waves, sound, heat, light, magnetism, electricity and modern physics.*

In modern physics we recognise two branches—*atomic physics* dealing with atoms and *nuclear physics*, with atomic nucleus. Then there is the *solid state physics* on the basis of constituent atoms and molecules. A still later branch is the *plasma physics* concerned with the properties of neutral mixtures of positive and negative ions. It is expected to tell us of what goes on in ide the sun and the stars and how do they manage to produce such huge quantities of energy. The latest branch in the field is *particle physics* to search out, study and correlate the fundamental particles out of which such atomic blocs as the electron, proton etc. are made.

Of all these mechanics is the most fundamental, for it enters into practically all the branches of physics. In fact *mechanics is the grammar of physics*. It deals with such ideas as motion, force, energy and their relations with matter and each other. Heat, sound, light include those phenomena which we recognise with our senses of touch, hearing and sight. We do not possess special senses to detect electricity and magnetism; they are studied through the effects they produce.

0-1.2. Matter and Energy : In introducing physics we have spoken of matter and energy above. What are they ? All the objects

we see around, e.g. books, tables, houses, trees, sun, moon, stars are all called *bodies*. **Matter** is the material of which all these bodies are made of. They can be perceived by our senses. **Energy** is that unseen agency which can bring about internal or external changes in matter. A piece of coal is matter, for it can be seen and touched. It can be crushed into powder (external change) by a hammer or can be burnt in a fire (internal change) into ashes: the mechanical energy of the moving hammer and the heat energy of the fire have brought about these changes in matter. All natural phenomena are effects of the action of energy on matter. These two are the only pair of entities we perceive in the universe around us.

Matter is found to exhibit four properties—(1) It occupies some **space** i.e. it has some **volume** (2) offers resistance to any force tending to displace it or to stop it when moving, thus exhibiting the **property of inertia** (3) can **transfer momentum** i.e. make another body change motion by colliding with the latter and (4) always attracts another piece of matter because of **gravitation.**

We have noted that *energy brings about changes in matter.* Matter however cannot undergo any change by itself. In nature, three classes of changes are found to occur namely (*i*) change in position (*ii*) that in condition (*iii*) that in constitution or type. A moving car shows change in position, boiling off of water change in condition or state, burning of coal, change in type of matter. These changes are brought about by the agency of energy.

During these changes neither matter nor energy suffers any destruction. They are only transformed from one form to another, either of matter or of energy. When coal is burnt we obtain an equivalent amount of ashes and gases. When electricity passes through a wire we obtain heat, in some cases light. The facts that neither mass nor energy can either be created or destroyed, are contained in the **principles of conservation of mass and energy.**

Albert Einstein (1879-1955) postulated that, **just one entity** that is conserved, of which matter and energy are only two manifestations. One may be converted into another. If a quantity of mass (matter) m disappears, it results in the release of mc^2 quantity of energy (E). As c, the speed of light is a very large quantity ($=3\times10^8$ m/s), the amount of energy so released is very large—the fact behind the cataclysmic explosive power of nuclear bombs or the almost unlimited energy of the sun and stars. The converse conversion of energy into mass has also been noticed; if very short γ rays (obtained from radio-activity) of 1.02 Mev energy is made to pass very

close to a *heavy* atomic nucleus, a pair of electron and positron is formed (so-called *pair-production*). Recently, sky-watchers have discovered the so-called **Black Holes** far away in outer space from which no signal appears to come. Some astronomers are of opinion, that these regions suck in any energy passing by, just like giant eddies in the oceans and *probably* condense the swallowed-up energy into mass, following the same relation $E=mc^2$.

0-1.3. Physics and Measurement : The subject-matter of physics rests squarely on *accurate* measurement alone. Lord Kelvin declared that it may be deemed that we have some knowledge about quantities that can be measured and that measure numerically expressed ; but if we cannot measure a thing or the measurement numerically recorded, we have no knowledge concerning it. Since we cannot measure feelings like affection, anger, hunger or fear, they are outside the scope of the subject matter of physics.

The term *physical quantity* or simply 'quantity' is used to mean *any* thing that can be measured. Length, weight, volume, time, speed, temperature, density, electric current, field intensity are only a few you know, amongst innumerable physical quantities. Laws or principles of physics can be usefully presented as relations between different physical quantities. For their unambiguous application, they require clearcut definitions. According to many, **definitions have to be operational i.e. so enunciated as to include a guide to the measurement of the relevant quantity.**

From such definitions we derive after many manipulations, theoretical or experimental, *a physical quantity made up of a number with a unit* : e.g. velocity (a ratio of displacement to time) in cm/s or momentum (mass $\times$ velocity) in g cm/s etc. A measurement involves thus two things, a number and a unit. Mere mention of a number is meaningless ; the *unit must be stated.*

What is a unit ? To measure any quantity we compare it with a quantity of the same kind, *agreed upon by convention*, which we call a unit. As for example, if we find that to measure the length of a bench we have to lay a *meter-stick* thrice successively alongside it, we conclude that the bench is three meters long. Hence a

Physical quantity = Numerical value $\times$ unit

0-1.4. Fundamental Quantities and their Units : Sometimes straightforward operational definitions cannot be provided for some quantities. The quantity of matter in a body is its *mass* or

degree of hotness of a body is its *temperature*, are such unsatisfactory definitions. Such are said to be **fundamental indefinables.** In mechanics we have three such, namely *mass, length and time* as you already know. All quantities in mechanics can be defined in terms of these three. For example area=length × breadth, volume=Area × height, density=mass/volume, velocity=distance/time, force=mass × length/(time)^2. Such units are called **derived units** as against **fundamental units** of mass, length, time. This classification is somewhat artificial. for derived unit in one system may be the fundamental unit in another ; e.g. in engineering practices, force and in hydrostatics, density are often taken as fundamental units. But fundamental units in all systems of measurements are chosen *arbitrarily*. What however is necessary is that they must be internationally accepted. The *desirable characteristics*, that a fundamental unit or any *standard* to measure a physical quantity, should have are (*i*) it should not change with time

(*ii*) must be reproducible and

(*iii*) must be internationally accepted :

Absolute (or coherent) units of a system : Derived units of a system which involve the fundamental units of the system only in unit measure, are known as *absolute units* (or coherent units). The absolute unit of work in the cgs system is 1 erg, which is 1 dyn × 1 cm = 1 g × 1 cm/s^2 × 1 cm. In the erg, all the fundamental units occur in unit measure. The joule (symbol J), which is equal to 10^7 ergs, is not an absolute unit in the cgs system, as it involves a measure 10^7. It is the *practical unit* of work in the cgs system, the erg being too small a work for practical use In the mks system, however, the joule comes out to be an absolute unit as 1 joule = 1 newton × 1 metre = 1 kg × 1 m/s^2 × 1 m.

If in a numerical problem you find that the quantities are in different systems of units, or are not in absolute units of the same system, convert all of them to absolute units of the same system before you apply any formula to calculate the result.

0-1.5. SI Units (Systemème International d'Unite's) or the International System of Units : For the coherent* system based on the six

* A coherent system of units is a system based on a certain set of 'basic units' from which all 'derived units' are obtained by multiplication or division without introducing any numerical factors.

basic units

Name	Symbol	Name	Symbol
metre	m	ampere	A
kilogram	kg	kelvin*	K
second	s	candela	cd

the name *International System of Units* has been recommended by the General Conference on Weights and Measures in 1960. The units of this system are called SI units. They have been accepted by all important international bodies as the *only* system for use in all scientific measurements.†

In developing any system of units, certain quantities are taken as *fundamental* (or *basic*). A very well defined amount of each such quantity is taken as a *fundamental unit.* In the International System of Units, the fundamental quantities are (1) length (2) mass (3) time (4) electric current (5) temperature and (6) luminous intensity as has already been listed. Supplementary units to be used with these are (i) *Radians* for angles (ii) *Steradians* for solid angles and (iii) the *Mole* for amounts of substances.

The International System defines the units of these quantities as accurately as present-day technology permits and you will presently marvel at the precision achieved. Of these, you are already familiar with the meter, the kilogramme and the second, the trio of MKS units. The SI units are only their refined modifications.

0-1.6. The MKS system of units : This was the first scientific system for mechanical units and initiated by a committee under Charles Borda, appointed by the Republic of France, immediately after the

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* Of late, the use of the degree sign (°) on the absolute scale has been dropped. So we write 273K instead of 273°K. But this method of writing has not yet become universal.

† You may be interested to learn that West Germany passed a law in 1970 to the effect that from 1st January, 1978, SI units be used there for all domestic purposes. Of late, the India Government have instructed that clinical (doctor's) thermometers should be graduated in celsius degrees instead of in fahrenheit degrees, as at present. As this comes into effect, a fever of temperature 104°F reads 40°C on our thermometer and 313K on a West German thermometer.

Degree Kelvin (introduced since 1954) is 1/273.16th part of the triple point temperature of pure water under normal atmospheric pressure. Triple point is the temperature at which all the states of aggregation coexist. It is denoted by K and not by °K. In modern usage K° represents not a temperature but a temp. difference.

Candela is the luminous intensity, in the perpendicular direction of a surface 1/600 000 square meter of a black body at the freezing point of platinum under normal atmospheric pressure (101 325 N/m^2).

Thus we have

In the MKS system as also in the SI units

Unit of force$=$1 newton (symbol N)$=1\ kg\times 1\ m/s^2=10^5$ dyn.

Unit of work$=$1 joule (symbol J)$=1\ N\times 1\ m=10^7$ erg.

Unit of power$=$1 watt (symbol W$=1J/1s)=10^7$ erg/s.

To give you enough acquaintance with these quantities we have freely used them. *All derived units of the MKS system are SI units.*

We have mentioned above that the SI system includes supplementary units of plane angles and solid angles, the **radian** and **steradian.**

Unit for angle. For use in scientific measurements, the unit for *plane angle* is the *radian* (symbol, rad). A **radian** is the angle subtended at the centre of a circle by an arc equal in length to the radius. To express an angle in radian measure, also known as *circular measure*, we may describe an arc of a circle with the vertex of the angle as centre. The length of the arc limited by the angle divided by the radius (*i.e.*, *arc AB* : radius *OA*) (fig. 0-1.1) is the value of the angle in radians, as you must have learnt in trigonometry. Thus

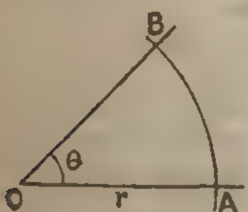

Fig. 0-1.1

$$\text{Angle } (\theta) \text{ in radians} = \frac{\text{arc } (s)}{\text{radius } (r).}$$

This measure is satisfactory for all systems of units ; for, an angle in radian measure is a ratio of two lengths and hence is a pure number. A pure number does not depend for its value on any system of units. So an angle in radians has the same value in all systems.

Now, length of arc$=$radius$\times$angle subtended by arc in radians.

$$\text{or } s=r\theta. \qquad (0\text{-}1.6.1$$

The ratio of the circumference of a circle to its diameter is a constant. It is represented by the Greek letter π. The total angle around a point is 2π radians ($\pi = 3.1416...$). Hence 2π rad $= 360°$ or 1 rad $= 360°/2 \quad = 57°17'45'' = 57.3°$.

The unit for *solid angle* is the **steradian** (symbol, sr). Solid angle is measured as for a plane angle to make it a pure number. It is the ratio of an area (A) and the square of its distance (r^2) from the point where the angle is sought. Note that a plane angle is a ratio of two lengths while the solid angle is the ratio of an area and $(\text{length})^2$ and hence both are dimensionless pure numbers.

A *steradian* is measured by the ratio of an area of r^2 cm on the surface of a sphere of radius r and square of the radius or by the unit area on the surface of a sphere of unit radius. The circumference of a circle, we have seen, subtends an angle of 2π radians at its centre while the solid angle subtended at the centre of a sphere is 4π radians.

CGS System : Till 1954 this mechanical system was in use throughout the scientific world. They are intimately related too, for the cgs was actually derived from, the MKS units. All of you are familiar with them. The system is slowly falling out of use everywhere and being replaced by the MKS system which is identical with SI units so far as mechanical indefinables are concerned. The units in the cgs system are very small and so an extra set of practical units are necessary. That is not the case with the MKS.

FPS System : The British system of mechanical units *foot* for length *pound* for mass and *second* for time were valid in days gone by throughout the British Empire and the United States. They are still in use only for engineering purpose in our country and U.S.A. but no longer for scientific purposes. It has been discarded in India in 1957. However in Britain nowadays **slug** is the *unit of mass* and *pound* **the unit of weight** which generates an acceleration of 32 ft./s^2 on a slug.

The following relations interconnect the three systems—

1 foot	=	30.48 cm	=	0.3048 m
1 lb (or slug)	=	453.6 g	=	0.4536 kg
1 s	=	1 s	=	1 s

0·1.7. International unit-symbols : In writing and using symbols of units of physical quantities, we shall follow the present international recommendations. Some of the symbols and prefixes are given below. The symbols are always printed in *roman (upright) type.* The symbol

of a unit, derived from a proper name, should start with a capital roman letter (such as N for newton, J for joule, Hz for hertz).

CGS		MKS	
Name	Symbol	Name	Symbol
centimetre	cm	metre	m
gram	g	kilogram	kg
second	s	second	s
dyne	dyn	newton	N
erg	erg	joule	J
hertz	Hz or cps	hertz	Hz

Prefixes to indicate decimal fractions or multiples of a unit

deci $(=10^{-1})$	d	deca $(=10)$	D
centi $(=10^{-2})$	c	hecto $(=10^{2})$	H
milli $(=10^{-3})$	m	kilo $(=10^{3})$	K
micro $(=10^{-6})$	μ	mega $(=10^{6})$	M
nano $(=10^{-9})$	n	giga $(=10^{9})$	G
pico $(=10^{-12})$	p or $\mu\mu$	tera $(=10^{12})$	T
femto $(=10^{-15})$	f	peta $=(10^{15})$	P
atto $(=10^{-18})$	a		

0-1.8. International recommendations for the use of unit-symbols :

The symbols for units (given after the names of the units) represent the respective units. They are to be treated as *algebraical quantities*, and not as abbreviations. They should not be used in the plural form. So, do not write 100 cms ; write it as 100 cm. This is the international practice. It applies to derived units also. Note that the international symbol for the 'gram' is 'g', not 'gm'. For the kilogram, it is 'kg', not 'kgm'. The symbol for the second is 's', not 'sec'. All symbols are printed in Roman characters (as we do in this book) and not in italics. No symbol shall be followed by a stop (.) as a sign of abbreviation. Write 'g', 'cm', 'kg', etc. and not 'g.', 'cm.', 'kg.'

We illustrate below the practices stated above :

1 cm × 2 cm × 3 cm = 6 cm³ (just as in algebra you write $a \times 2a \times 3a = 6a^3$).

$5\ \mathrm{g} \times 4\ \mathrm{cm} \div 2\ \mathrm{s} = 10\ \mathrm{g\ cm/s} = 10\ \mathrm{g\ cm/s^{-1}}$

A velocity of 2 metres per second is written as 2 m/s or 2 m s^{-1}. An acceleration of 50 centimetres per second per second is written as 50 cm/s^2 or 50 cm s^{-2}.

In this book we shall follow the international recommendations*.

Remember that a physical quantity consists of a number and a unit together. The unit-symbol is an essential part of it and behaves like a proper algebraic quantity. *Symbols for units cancel or multiply like algebraical quantities.* For example, 25 g cm $s^{-2} \div 5$ g $= 5$ cm s^{-2}.

You will get further examples in the worked out problems in the book. Keep a watch.

0-1.9. Dimensions of Physical Quantities: In physics we are concerned with the measurement of 'quantities', which we call physical quantities. *Physical quantities met with in mechanics can all be expressed in terms of three selected quantities.* We generally select **length** $[L]$, **mass** $[M]$ and **time** $[T]$ for the purpose and call them *the fundamental quantities.* Engineers use length, force and time as fundamental quantities.

Quantities met with in mechanics can be measured in terms of units derived from the units of the fundamental quantities. Such units are called **derived units.** Take, for example, the area of a plane figure. Area is a physical quantity of a nature different from that of length. If we liked we could measure area in terms of the area of a selected rectangle, which, for permanence, might be made of platinum or gold. It is, however, convenient to make use of the unit of length in defining the unit of area. In fact, the unit of area is taken as the area of a square having sides of unit length. A rectangle of sides 2 cm by 3 cm has an area of 2 cm × 3 cm = 6 sq. cm (or 6 cm^2). The physical quantity 'area' is said to have **two dimensions** in respect of the fundamental quantity 'length', as 'length' occurs twice in the calculation of 'area'. This is expressed *symbolically* by writing $[A] = [L]^2$ where the symbol $[A]$ represents the physical quantity 'area', and $[L]$ stands for the physical quantity 'length', without any reference to any particular area

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* In spite of all that has been said above, you may still find in many books and question papers the symbols written variously as 'gm', 'kgm.', 'cm.', 'cms.', 'ft.', 'sec.', etc. Though in the main text we shall follow the internationally recommended way of writing, we shall leave the questions compiled from various sources just as we found them. So you may not be at a loss when you see them.

or length, or the units in which these might be expressed. Similarly, the quantity 'volume' $[V]$ involves length thrice and is said to have **three dimensions** in respect of length, or in symbols $[V]=[L]^3$. These examples show that *by the term 'dimensions' of a physical quantity is meant the powers to which the fundamental quantities must be raised in order to specify that quantity.* Dimensions are always calculated from the definition of the quantity.

Density is $\frac{\text{mass}}{\text{volume}}$. Therefore, the dimensions of the physical quantity 'density' $[D]$ are $\frac{[M]}{[L]^3}$ or, $[M][L]^{-3}$. In words we may say that the dimensions of density are one in mass and minus three in length. The rectangular brackets are used to imply dimensions and in case there is no confusion, they may be dispensed with. Thus if ρ represents density we may write $[\rho]=ML^{-3}$.

If a person walks 4 miles in one hour we say that his speed is 4 miles per hour. The physical quantity 'speed' is expressed in terms of the fundamental quantities 'length' and 'time'. Since speed is $\frac{\text{length}}{\text{time}}$, the dimensions of speed are $\left[\frac{L}{T}\right]$ or, LT^{-1}.

Acceleration is the rate of change of velocity, i.e., $a=\frac{\text{velocity}}{\text{time}}$.

Hence its dimensional equation is $[a]=\frac{[LT^{-1}]}{[T]}=[LT^{-2}]$.

Momentum of a body is the product of the mass of the body and its velocity. Hence the dimensional equation of momentum is obtainable from the relation mv = mass × velocity. [Momentum] $=[M][LT^{-1}]=[MLT^{-1}]$.

Force P on a body is given by the product of its mass and acceleration. We have $P=ma$, whence the dimensional equation.

$$[P]=[ma]=[M][LT^{-2}]=[MLT^{-2}].$$

Pressure is defined as the force per unit area and its dimensional equation is given by $[p]=\left[\frac{\text{force}}{\text{area}}\right]=\frac{[MLT^{-2}]}{[L^2]}=[ML^{-1}T^{-2}]$.

Work or energy is the product of force and displacement.

$\therefore\ [W]=[MLT^{-2}][L]=[ML^2T^{-2}]$.

The principle of homogeneity: On the basis of the above discussion we find that the symbol $[L]^2$, $[L^2]$, or simply L^2 merely

represents the physical quantity 'area' *without any reference to the value of any particular area, or the unit in which the value might be expressed.* $[L]^3$, $[L^3]$ or L^3 similarly represents a 'volume', and ML^{-3} a 'density'.

We know that a quantity can only be equated to the same quantity, *i.e.*, a 'length' to a 'length', an 'area' to an 'area', a 'speed' to a 'speed'. But a 'length' cannot be equated to 'volume', or a 'speed' to a 'mass'. Since any quantity of a given kind has a *fixed set* of dimensions, **the dimensions, of all terms on the two sides of an equation must be identical.** Otherwise the equation will be invalid. This assertion of the equality of dimensions in the terms of an equation is called the **principle of homogeneity.** This useful principle may be applied to check the correctness of an equation.

The method of dimensions has three simple uses, namely, (i) to find the value of a quantity when its units are changed from one system to another, (ii) to derive relations between physical quantities when they are simply related, (iii) to test the correctness of an equation.

(i) Change of units: An application of the idea of dimensions is concerned with the change of the numerical value of a quantity when the unit is changed. The numerical value of a quantity varies inversely as the value of the unit in terms of which it is expressed. This is true also for derived units. The method of changing units with the help of dimensions is shown below.

Example: *A substance has a density of 0.5 when the gram and the centimetre are units of mass and length respectively. Find the density when the pound and the foot are taken as units.*

Solution: We know, density has the dimensions ML^{-3}.

$\therefore$ The value in terms of the gram and the centimetre may be written as $0{\cdot}5 \text{ g cm}^{-3}$.

In the proposed units let the value be x lb ft^{-3}

Then $0{\cdot}5 \text{ g cm}^{-3} = x \text{ lb ft}^{-3}$

$$\therefore \quad x = 0{\cdot}5 \frac{\text{g}}{\text{lb}} \cdot \frac{\text{ft}^3}{\text{cm}^3} = 0{\cdot}5 \times \frac{1}{453{\cdot}6} \times \frac{(30{\cdot}48)^3}{1} = 31{\cdot}21$$

$\therefore$ The required density $= 31{\cdot}21$ lb ft^{-3}

[Note again that the units of mass and length are treated as algebraic quantities.]

The above example of conversion of units is a very simple one and can be solved in other ways. In more complicated cases, it will

be found that the method of dimensions proves very helpful in converting readings from one system of units into another.

(ii) Deriving relations between physical quantities: When a physical quantity is known to depend simply on two or three others, the relation between them can be found by the method of dimensions. This is illustrated below.

(1) Period of a Pendulum: Suppose that the period t of a pendulum depends on (a) the length l of the string, (b) the mass m of the bob, and (c) the acceleration g due to gravity. Then we may write

$$t = k l^x m^y g^z$$

where k is a constant without dimension.

$$\therefore \quad [T] = [L]^x [M]^y [LT^{-2}]^z.$$

Equating the powers of L, M and T, we get

$$y = 0, \quad x + y = 0 \quad \text{and} \quad -2z = 1$$

$$\therefore \quad x = \tfrac{1}{2} \text{ and } z = -\tfrac{1}{2} \text{ or } t = k.\sqrt{l/g}. \qquad \text{(0-I.9.1)}$$

This is the required relation. As $y=0$, the period does not depend on the mass of the bob. The constant k has to be determined experimentally or, where possible, by mathematical analysis. In this case we have found $k=2\pi$.

(2) *Frequency of a stretched string:* Let the frequency of vibration (f) of a stretched string depend on its mass m, its length, l, and the stretching force F. We may then write

$$f = k m^x l^y F^z.$$

$$\therefore \quad [T^{-1}] = [M]^x [L]^y [MLT^{-2}]^z.$$

$$\therefore \quad x + z = 0, \; y + z = 0, \; 2z = 1.$$

$$\therefore \quad z = \tfrac{1}{2}, \; x = -\tfrac{1}{2}, \; y = -\tfrac{1}{2}.$$

$$\therefore \quad f = k\sqrt{\frac{F}{ml}} \qquad \ldots \text{(0-I.9.2)}$$

if μ = mass per unit length of the string $m = \mu l$, and

$$f = \frac{k}{l}\sqrt{\frac{F}{\mu}} \qquad \ldots \text{(0-1.9.2a)}$$

k has a value of $\tfrac{1}{2}$.

(3) the velocity v of a compression wave in a gas depends on the pressure p and density ρ of the medium. To find the relation between these quantities.

In this case $v \propto p^x \rho^y$ or $v = k p^x \rho^y$.

The dimensions of velocity, pressure and density are respectively $[LT^{-1}]$, $[ML^{-1}T^{-2}]$ and $[ML^{-3}]$. Hence we obtain the dimensional equation

$$[LT^{-1}] = [ML^{-1}T^{-2}]^x [ML^{-3}]^y = [M^{x+y}][L^{-x-3y}][T^{-2x}]$$

Equating the powers of $[L]$ $[M]$ and $[T]$ on the two sides, we get

$$x+y=0,\ -x-3y=1,\ \text{and}\ -2x=-1.$$

$$\therefore\ x=\tfrac{1}{2}.\ \text{Since}\ x+y=0,\ y=-\tfrac{1}{2}.$$

$$\therefore\ v = k p^{\frac{1}{2}} \rho^{-\frac{1}{2}} = k\sqrt{\frac{p}{\rho}}. \tag{0·I.9.3}$$

Here k has a value of 1.

Limitations of the method: The method of dimensional analysis is subject to the following limitations: (*i*) If any physical quantity depends upon *four* or more other physical quantities, we fail to find the relationship between them for the following reason. In the method of solving a physical problem by the help of dimensions we obtain three equations depending on the powers of $[L]$, $[M]$ and $[T]$. And from three equations we cannot determine four or more unknown quantities.

(*ii*) Two quantities may have the same dimensions, but differ in nature. For example, consider energy and the moment of a force. The moment of a force is given by the product of the force and its distance from a point or axis. Its dimensions, therefore, are $[MLT^{-2}] \times [L] = [ML^2T^{-2}]$. These are also the dimensions of work or energy. Though the dimensions are the same, it cannot be claimed that energy and the moment of a force are quanties of the same kind. This discrepancy is explained by the fact that while energy is a scalar, the moment of a force, is a vector.

(*iii*) It cannot take pure numbers into consideration.

We conclude with a summary table of units.

TABLE OF UNITS

Quantity	Dimensions	CGS		MKS	
		Unit	Symbol	Unit	Symbol
Fundamental					
Length	L	centimetre	cm	metre	m
Mass	M	gram	g	kilogram	kg
Time	T	second	s	second	s
Derived					
Area	L^2	—	cm^2	—	m^2
Volume	L^3	—	cm^3	—	m^3
Density	ML^{-3}	—	g/cm^3	—	kg/m^3
Velocity	LT^{-1}	—	cm/s	—	m/s
Momentum	MLT^{-1}	—	g cm/s	—	kg cm/s
Acceleration	LT^{-2}	galileo	cm/s^2	—	m/s^2
Force	MLT^{-2}	dyne	dyn $(=g\ cm/s^2)$	newton	N $(=kg\ m/s^2)$
Pressure	$ML^{-1}T^{-2}$	(barye)	dyn/cm^2	(pascal)	N/m^2
Work, Energy	ML^2T^{-2}	erg	dyn cm	joule	J(= N m)
Power	ML^2T^{-3}	—	erg/s	watt	W(=J/s)
Surface tension	MT^{-2}	—	dyn/cm $(=erg/cm^2)$	—	$N/m (=J/m^2)$
Viscosity	$ML^{-1}T^{-1}$	poise	g/cm/s	—	kg/m/s
Moment of inertia	ML^2	—	$g\ cm^2$	—	$kg\ m^2$

EXERCISES

1. What are SI units? Name fundamental units of the International system, and any three derived units. How are those named, connected with the cgs system?

2. What is meant by a *coherent* system of units? Illustrate with two examples.

3. What is the present international system of writing unit symbols?

4. Illustrate the statement that a physical quantity consists of a numerical value and a unit. Explain it.

5. What does physics deal with? On what is it based?

6. Give two examples to show that unit symbols are treated as algebraic quantities.

7. What is meant by the dimensions of a physical quantity? Calculate the dimensions of velocity, acceleration pressure, work and power.

8. What is meant by the principle of homogeneity? What uses can we make of it?

9. The velocity v of a wave in a gas depends on its pressure p and density ρ fo the medium. Find how v depends on p and ρ. [Ans.: $v=k\sqrt{p/\rho}$.]

10. Assume that the frequency f of a stretched string depends only on its mass m, its length l, and the stretching force F. Find how f is related to m, l and F. [Ans.: $f=k\sqrt{F/ml}$.]

*ELEMENTARY CALCULUS

0-2.1. Introduction

In physics, in mathematics, not to speak of even in our daily life, we often come across continuously changing quantities—the most personal example, your own age. A little scrutiny would reveal in every case, a pair of changing quantities—one changing along with the other.

As examples we may cite the cases of the weight of a normal healthy baby which depends on his age, the solar heat on a clear summer morning on the hour, the distance covered by a moving car on its speed, heat developed in a conductor on the current it carries, the heat radiated by a hot source on its temperature, the area of a circle on its radius, the volume of a cube on its side—the list is endless.

Note that in each case, the preceding depends on the one following. These two quantities that are changing, are called **variables.** In each of the above pair cited, the first is the **dependent variable** the second, the **independent** one ; for only when the second one changes, the first is found to do so. The dependent variable, say y, is said to be a *function* of the independent variable x, the relation between them being expressed as $y=f(x)$. The expression just means that *y depends on x.*

Instances where one quantity depends for its value on two other varying quantities are also many ; as for example, the area of a rectangle *depends on* its length and breadth, the volume of a cone on its height and base-diameter, the distance covered by a moving car on its speed and time taken, power of an agent on the applied force and velocity generated. The general functional relation in such cases can be expressed as

$$z=f\ (x,\ y)$$

where z is the dependent and x, y independent variables.

Very often we need to know the *rate of change of the dependent variable with respect to that of the independent variable.* When both these changes are **very, very small** the process of finding this rate (or ratio of the changes) is said to be **differentiation** and the rate,

* The subject-matter of this chapter rightfully belongs to the Second paper of Maths you are going to study. Here we are presenting it in a brief and utilitarian way to show you that simple differentiation and integration processes are actually easy. You may ignore this chapter if you so desire.

the **differential co-efficient.** Let us take an example: the circumference of a circle $C = 2\pi r$ where r is its radius. Differentiation aims at finding how the circumference increases with increase of radius; let the former increase by δC when the latter expands by δr where both δC and δr are *small but finite changes*; let both these quantities decrease continuously till they become vanishingly small; then $\delta C \to dC$ and $\delta r \to dr$ and the ratio dC/dr is said to be the *differential co-efficient*. Let us find it; we have

$$C = 2\pi r \quad \text{or} \quad C + dC = 2\pi(r + dr) \quad \text{or} \quad dC = 2\pi . dr$$

$$\therefore \frac{dC}{dr} = 2\pi$$

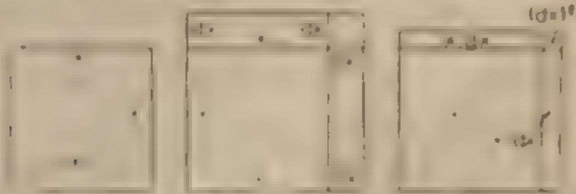
In this way we shall proceed to find *very simply* a number of necessary differential co-efficients. In fact Newton invented infinitesimal (i.e. differential) calculus in his quest for finding accurately the rate of change of motion with time—a basic problem in mechanics. The *reverse process*, of adding up vanishingly small quantities to build up a finite quantity (as adding up innumerable dr's to arrive at C, the circumference) is the twin branch of calculus—the process of **integration.** In both the methods of differentiation and integration a clear concept of small quantities of different orders is imperative. We proceed to [illegible] in the next article.

0.2.2. Quantities of Different orders of Smallness:

Quantities are not small or large by themselves but are regarded as small or comparable with respect to similar quantities. The earth has a diameter of 8000 miles, that of the sun 108 times as much and their separation is 93 million miles. Though the diameters are quite large quantities to man, astronomers consider them, the earth and the sun as points compared to their separation as it is so very large in comparison. Again note that 60 minutes add up to an hour, 24 hours to a day, seven days to a week; so 1440 minutes add up to a day and [illegible] a week. Thus compared to a week how very small a minute is! Consider then how very smaller a time-interval one second must be. One minute is 1/60th of an hour—a quantity of *first order of smallness*; then a second, 1/60th of a minute must be a quantity of *second order* of smallness. Again a rupee being 0.01 part of a 100-rupee note is a quantity of first order of smallness while a paise being 0.01 part of a rupee, or [illegible] 0.0001 part of a 100-rupee note must be a quantity of second order of smallness. So we find

that as a small quantity diminishes progressively, its higher powers becomes progressively vanishing and hence negligible. But you must remember that a small quantity multiplied by a large one (e.g. a few hundred times one paise) may not remain so.

Let us have a square of side 10 m. (Fig. 0-2.1a). Let each of sides be increased by 1 m when the area increases from 100 m² to 121 m²



(a) (b) (c)

Fig. 0-2.1

$(10^2 + 2 \cdot 10 \cdot 1 + 1^2)$. In fig. 0-2.1(b) the increase is depicted: two rectangles (upper and lower) and one small square (upper right) have been added to the original square. Let us next increase the side by 1 cm only; observe that how very small the added square has become.

0-2.3. Differential co-efficients of some Algebraic functions:

(1) Let $y = x$, let x be increased by δx when the increase in y is δy. Then $y + \delta y = x + \delta x$ or $\delta y = (x + \delta x) - y = (x + \delta x) - x = \delta x$. If we now consider δx to become very very small i.e. it tends to its limiting value very close to zero, then we write

$$\mathrm{Lt}_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = 1 \qquad (0\text{-}2.3.1)$$

(2) Let $y = x^2$, then $y + \delta y = (x + \delta x)^2 = x^2 + 2x.\delta x + (\delta x)^2$. If we now make δx very small, its square $(\delta x)^2$ become a quantity of second order of smallness and hence negligible.

So $\delta y = (x + \delta x)^2 - y = x^2 + 2x.\delta x + (\delta x)^2 - x^2 = 2x.\delta x$

$$\mathrm{Lt}_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = 2x \qquad (0\text{-}2.3.2)$$

(3) Let $y = x^3$, then $y + \delta y = (x + \delta x)^3 = x^3 + 3x^2.\delta x + 3x(\delta x)^2 + (\delta x)^3$. Again for very small values of δx, $(\delta x)^2$ and $(\delta x)^3$ become negligible.

So $\delta y = (x + \delta x)^3 - x^3 = 3x^2.\delta x$

$$\therefore \mathrm{Lt}_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = 3x^2 \qquad (0\text{-}2.3.3)$$

(4) Let $y=x^4$; then $y+\delta y=(x+\delta x)^4=(x+\delta x)^2(x+\delta x)^2$

$$=[x^2+2x.\delta x+(\delta x)^2]^2$$

$$[x^2+2x.\delta x+(\delta x)^2]^2=x^4+4x^3.\delta x+6x^2.(\delta x)^2+4x^3.(\delta x)^3+(\delta x)^4$$

As before, neglecting higher powers of (δx) we obtain

$$\delta y=(x+\delta x)^4-x^4=4x^3.\delta x$$

$$\therefore \quad \frac{dy}{dx}=\underset{\delta x\to 0}{\mathrm{Lt}}\ \frac{\delta y}{\delta x}=4x^3 \qquad (0\text{-}2.3.4)$$

(5) Let $y=x^5$.

Then $y+\delta y=(x+\delta x)^5=(x+\delta x)^3(x+\delta x)^2$

$$=x^5+5x^4\delta x+10x^3(\delta x)^2+10x^2(\delta x)^3+5x(\delta x)^4+(\delta x)^5$$

Again neglecting the higher powers of δx when it becomes very small itself, we get as above

$$\frac{dy}{dx}=\underset{\delta x\to 0}{\mathrm{Lt}}\ \frac{\delta y}{\delta x}=5x^4 \qquad (0\text{-}2.3.5)$$

Let us tabulate the results obtained so far and we get

y	x	x^2	x^3	x^4	x^5
$\frac{dy}{dx}$	$1.x$	$2x^1$	$3x^2$	$4x^3$	$5x^4$

From the table we may lay down the procedure for differentiating simple algebraical functions as

(*i*) Bring down the power in front of the quantity and

(*ii*) Reduce the existing power by 1.

That is generalising, if we have $y=x^n$

$$\text{then} \quad \frac{dy}{dx}=nx^{n-1} \qquad (0\text{-}2.3.6)$$

0-2.4. Differential Co-efficients of Algebraic functions with constants :

Constants may be associated with algebraic quantities in two ways—

(*i*) it may be an added (or subtracted) constant e.g. $y=x^2\pm a$
or (*ii*) it may be a co-efficient, as for example $y=Kx^3$.

In the first case *the constants just simply disappear* on d fferentiation as it should, for a constant cannot change and so ca have no place in any rate of change. In the second case howev *the constant retains its position* as the co-efficient in the differenti co-efficient.

(1) Let $y=x^2 \pm a$: then $y+\delta y=(x+\delta x)^2 \pm a$

$\therefore \quad \delta y=[x+\delta x)^2 \pm a]-(x^2 \pm a)$

$$=2x\delta x+(\delta x)^2 \quad \text{or} \quad \frac{\delta y}{\delta x}=2x+\delta x$$

$$\therefore \quad \underset{\delta x\to 0}{\text{Lt}} \frac{\delta y}{\delta x}=\frac{dy}{dx}=2x \qquad \text{(0-2.4}$$

Note that the differential co-efficient of only x^2 and that $(x^2 \pm a)$ are identical i.e. *the constant added to the function d appears on differentiation.*

(2) $y=Kx^3$; then $y+\delta y=K(x+\delta x)^3$

$\therefore \quad \delta y=K(x+\delta x)^3-Kx^3=K[(x+\delta x)^3-x^3]$

$$=K[3x^2.\delta x+3x.(\delta x)^2+(\delta x)^3]$$

$$\therefore \quad \frac{\delta y}{\delta x}=K[3x^2+3x.\delta x+(\delta x)^2]$$

$$\therefore \quad \underset{\delta x\to 0}{\text{Lt}} \frac{\delta y}{\delta x}=\frac{dy}{dx}=K.3x^2 \qquad \text{(0-2.4}$$

Note that if you just multiply the differential coefficient of by K you obtain that of Kx^3. In these two examples constant or K may have any value.

0-2.5. Differentiating Sums, Differences, Products and Quotient

(1) Let $y=u \pm v$ where all the three are variables. To differ tiate, let each of them increase by small but finite amounts. T we get $y+\delta y=(u+\delta u)\pm(v+\delta v)$

Then $\delta y=\delta u \pm \delta v$ and $\dfrac{dy}{dx}=\dfrac{du}{dx}\pm\dfrac{dv}{dx}$ (0-2.

As an example, let $y=(x^2+c)\pm(ax^4+b)$

Then $\dfrac{dy}{dx}=2x \pm 4ax^3$

(2) Let $y=uv$ then $y+\delta y=(u+\delta u)(u+\delta u)=uv+u.\delta v+v.\delta u+\delta u.\delta v$

$\therefore \quad \delta y=u.\delta v+v.\delta u+\delta u.\delta v$

or, $\dfrac{\delta y}{\delta x}=u.\dfrac{\delta v}{\delta x}+v.\dfrac{\delta u}{\delta x}+\dfrac{\delta u.\delta v}{\delta x}$

Now the last term being a product of two *small* quantities, is of second order of smallness and so, negligible. Then

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx} \tag{0-2.5.2}$$

So to differentiate a product

(*i*) take the differential co-efficient of the second and multiply it by the first term as such

(*ii*) differentiate the first term and multiply by the second term as such and

(*iii*) add up the two products.

As an example let $y=(ax^2\pm m)(bx^4\pm n)$

$$\frac{dy}{dx}=(ax^2\pm m).\frac{d}{dx}(bx^4\pm n)+(bx^4\pm n)\frac{d}{dx}(ax^2\pm m)$$

$$=(ax^2\pm m)(4bx^3)+(bx^4\pm n)(2ax)$$

$$=4abx^5\pm 4bmx^3+2abx^5\pm 2anx$$

$$=6abx^5\pm 4bmx^3\pm 2anx.$$

(3) Lastly, let $y=u/v$; to differentiate we follow the procedure laid down above and write

$$dy=u.d\left(\frac{1}{v}\right)+d\,(u)\cdot\frac{1}{v}=u.\left(-\frac{1}{v^2}\cdot dv\right)+\frac{1}{v}\cdot du$$

$$=\frac{du}{v}-\frac{u.dv}{v^2}=\frac{v.du-u.dv}{v^2} \tag{0-2.5.3}$$

i.e. to differentiate a quotient

(*i*) Multiply the denominator by the differential co-efficient of the numerator

(*ii*) next multiply the numerator by the diff. coeff. of the denominator

(*iii*) subtract the second from the first and finally

(*iv*) divide the difference by the square of the denominator.

As an example we consider the function, $y = \dfrac{x^2+c}{ax^4+b}$

$$\frac{dy}{dx} = \frac{(ax^4+b)\frac{d}{dx}(x^2+c) - (x^2+c)\,\frac{d}{dx}(ax^4+b)}{(ax^4+b)^2}$$

$$= \frac{(ax^4+b)\,2x - (x^2+c)\,4ax^3}{(ax^4+b)^2}$$

$$= \frac{(2ax^5+2bx) - (4ax^5+4acx^3)}{(ax^4+b)^2}$$

$$= \frac{2bx - 4acx^3 - 2ax^5}{(ax^4+b)^2}.$$

Exercises : (1) Find from first principles how the following quantities change w. r. t. change in radius—circumference and area of a circle, surface area and volume of a sphere, lateral area and volume of a cone of slant length l and height h.

Verify that the rate of variation of spherical volume equals its surface area and that of a circular area equals its circumference.

[2π, $2\pi r$, $8\pi r$, $4\pi r^2$, $2\pi l$ and $\frac{2}{3}\pi rh$]

(2) The relation between the candle power of an electric lamp and its voltage is $C = aV^b$ where a, b are constants.

Find the rate of variation of C.P. with voltage at 80, 100 and 120 V given that $a = \frac{1}{3}\times 10^{-10}$ and $b=6$. $\left[\dfrac{dC}{dV} = abV^{b-1}\right.$, 0.98, 3.00 and 7.47 cp/volt $\left.\right]$

(3) Differentiate w. r. t. x (i) ax^2+bx+c (ii) $\dfrac{(ax+b)}{(cx+d)}$ (iii) $\dfrac{(x^n+a)}{(x^{-n}+b)}$

(4) EMF to maintain an electric arc of length l with a current strength j is $E = a + bl + (c+kl)/j$

where a, b, c, k are constants. Find rate of variation of emf with l and j.

0-2.6. Exponential and Logarithmic functions and their Differentials

A. **Binomial theorem.** An expression of the form $(ax+b)$ containing a pair of terms is called a *binomial* (bi→two, nomial → pertanining to number) expression. Newton devised a formula by which a binomial raised to any *positive integral* power can be expressed in the form of a series of finite number of terms. This is a very important result and we establish it in a very simple way. We know that

$$(x+y)^2 = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$$
$$(x+y)^4 = (x+y)^2\ (x+y)^2 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$
$$(x+y)^5 = (x+y)^2(x+y)^3 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$$
$$(x+y)^6 = (x+y)^3\ (x+y)^3 = x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6$$
$$(x+y)^7 = (x+y)^4(x+y)^3 = x^7+7x^6y+21x^5y^2+35x^4y^3+35x^3y^4+21x^2y^5+7xy^6+x^7$$

Hence extending this series we may write

$$(x+y)^n = x^n+\frac{nx^{n-1}}{1}y+\frac{n(n-1)}{1.2}x^{n-2}y^2+\frac{n(n-1)(n-2)}{1.2.3}x^{n-3}y^3+\dots\dots+y^n$$
$$=x^n+\frac{n}{\lfloor 1}x^{n-1}.y+\frac{n(n-1)}{\lfloor 2}x^{n-2}y^2+\frac{n(n-1)(n-2)}{\lfloor 3}x^{n-3}y^3+\dots\dots+y^n \quad (0\text{-}2.6.1)$$

[The sign $\lfloor$ is called a *factorial*. It is in fact a short-hand to express the product of successive +ve integers e. g. $\lfloor 5=5.4.3.2.1$, $\lfloor 4=4.3.2.1$ etc.]

The expression above (0-2.6.1) is called the Binomial theorem —one you will have to learn in your Maths. paper I with great emphasis.

B. Exponential Series: Let us investigate the binomial expansion of the expression $(1+{}^1/n)^n$. We have from the binomial theorem

$$\left(1+\frac{1}{n}\right)^n = 1+\frac{n}{\lfloor 1}.1^{n-1}.\frac{1}{n}+\frac{n(n-1)}{\lfloor 2}.1^{n-2}\frac{1}{n^2}+\frac{n(n-1)(n-2)}{\lfloor 3}.1^{n-3}.\frac{1}{n^3}+\dots+\left(\frac{1}{n}\right)^n$$
$$=1+1+\frac{n-1}{\lfloor 2.n}+\frac{(n-1)(n-2)}{\lfloor 3.n^2}+\frac{(n-1)(n-2)(n-3)}{\lfloor 4.n^3}+\dots+\left(\frac{1}{n}\right)^n \quad (0\text{-}2.6.2)$$

Let now n grow to a very, very large number *i.e.* $n\to\infty$; then $(n-1)$, $(n-2)$, $(n-3)$ etc. differ very little from n itself and all of them can be taken as equal. Then we have

$$Lt_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 1+1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\dots+\frac{1}{\infty}$$

Now the sum of this convergent series upto infinity is said to be the *exponential term* and denoted by e.

$$\therefore\ e=1+1+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\ldots\ldots \text{to infinity}=2.718\,281. \qquad (0\text{-}2.6.3)$$

Let us put various values to n and find the value for e. We then have

$$\left(1+\frac{1}{2}\right)^2=2.250,\ \left(1+\frac{1}{5}\right)^5=2.489,\ \left(1+\frac{1}{10}\right)^{10}=2.594,$$

$$\left(1+\frac{1}{20}\right)^{20}=2.653,\ \left(1+\frac{1}{100}\right)^{100}=2.704$$

$$\left(1+\frac{1}{1000}\right)^{1000}=2.7171,\ \left(1+\frac{1}{10\,000}\right)^{10000}=2.7182$$——we are getting an incommensurable quantity as the sum, where $2<e<3$.

The term e carries great significance, greater than the other incommensurable quantity π which you know ; for, e is related to the natural rate of growth or decay of many physical quantities. If the time rate of growth or decay of a variable at any instant is proportional to its value at that instant then its growth or decay rate is said to be **natural, logarithmic or organic.**

Growth of money at compound interest in a bank, normal rise in population in a city, the growth in height of a healthy sapling, *decay* in mass of a radio-active element, discharge of a condenser through a resistance, damping of amplitude and velocity of a vibrator in a resistive medium, in each case *with time*, all occur *exponentially.*

To get at what is meant let us study the growth of money at compound interest. Let 100 rupees increase to 110 at the end of a year at 10%, in two years (Principal 110.00) to 121 (for, interest is 11.00), after three years (Principal 121.00) to 131.10 and so on. After 10 years, 100 rupees grow to nearly 259.40. Note that after a year, a rupee earns 10 paise as interest growing into 1.10 and after 10 years multiplies 259.3 75 times. Now let y_0 be the initial principal which increases annually by $\frac{1}{n}$th fraction for n years, then after this lapse of time it will total up to $y_n=y_0\left(1+\frac{1}{n}\right)^n$

But remember, money is *increasing continuously* and not in annual jumps. To get at a more accurate value, we divide ten years into 100 parts when the

interest will be allowed 100 times and the total will be then

$$y_n = 100\left(1+\frac{1}{100}\right)^{100} \simeq 270.40 \text{ rupees}$$

To get at a more realistic value let the interest be paid 10 000 times in ten years when the total will reach

$$y_n = 100\left(1+\frac{1}{10\,000}\right)^{10\,000} \simeq 271.82 \text{ rupees}$$

Thus the ratio $\left(\frac{y_n}{y}\right)=\left(1+\frac{1}{n}\right)^n$ represents the total increase after n increments. Also we note, however large n may be, the expression $\left(1+\frac{1}{n}\right)^n \to 2.71828$ ...... (Don't you forget this value).

The Exponential Series : We shall have

$$e^x = \underset{n\to\infty}{\mathrm{Lt}}\left[\left(1+\frac{1}{n}\right)^n\right]^x = \underset{n\to\infty}{\mathrm{Lt}}\left(1+\frac{1}{n}\right)^{nx}$$

$$= 1+\frac{nx}{\lfloor 1}\cdot\frac{1}{n}+\frac{nx(nx-1)}{\lfloor 2}\cdot\frac{1}{n^2}+\frac{nx(nx-1)(nx-2)}{\lfloor 3}\cdot\frac{1}{n^3}+\cdots$$

to infinite no. of terms

$$= 1+x+\frac{x(nx-1)}{\lfloor 2 n}+\frac{x(nx-1)(nx-2)}{\lfloor 3 n^2}+\cdots$$

$$= 1+x+\frac{x^2}{\lfloor 2}+\frac{x^3}{\lfloor 3}+\cdots \tag{0-2.6.4}$$

[for, when n is very large $nx=(nx-1)=(nx-2)=\ldots$ etc]

$$\therefore \quad \frac{d}{dx}(e^x) = 1+x+\frac{x^2}{\lfloor 2}+\ldots = e^x \tag{0-2.6.5}$$

i.e. *on differentiating* e^x, *we get the same quantity back.* The importance of the quantity e^x lies, amongst others, on this unique property. Another factor lending importance to e, is the fact that Napier the inventor of logarithms, took this as the base of his system.

Logarithm and its Differentiation : If in the Naperian system, y is taken to be e^x then x will be the logarithm of y to the base e. i.e. if

$$y = e^x \quad \text{we have} \quad x = \log_e y \text{ or } \ln y \tag{0-2.6.6}$$

The term ln refers to natural logarithm (*Logarithm Naturalis*).

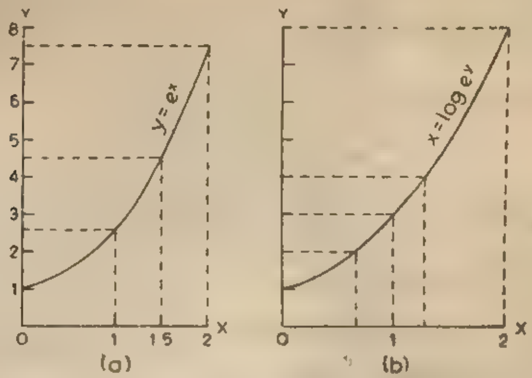


Fig. 0-2.2

The logarithm in general use has 10 as its base. The relation between the two is given by

$$\ln x = \log_e x = 2.303 \log_{10} x \tag{0-2.6.7}$$

Now then $y = \log_e x$ or $x = e^y$

$$\therefore \quad \frac{dx}{dy} = e^y \quad \text{or} \quad \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x} \tag{0-2.6.8}$$

This result is very peculiar for differential co-efficient of $\log_e x$ turns out to be x^{-1} which is not obtained from differentiation of powers as illustrated in (0-2.3.6). If we follow that method we shall find that $\frac{d}{dx}(x^0) = 0 \times x^{-1} = 0$ and not x^{-1}. Fig. 0-2.2(a) & (b) show a quantity increasing exponentially and the corresponding logarithmic curve. They look alike, for basically the equations are identical, though values taken, differ.

Examples: Differentiate e^{-ax}, $e^{\frac{1}{3}x^3}$, $e^{\sqrt{x^2+a^2}}$, $\log_e (a+x^3)^3$

(1) $y = e^{-ax}$. Let $z = -ax$; $\therefore$ $y = e^z$

$$\therefore \quad \frac{dy}{dz} = e^z \text{ and } \frac{dz}{dx} = -a \quad \therefore \quad \frac{dy}{dx} = \frac{dy}{dz}\frac{dz}{dx} = -ae^{-ax}$$

Alternatively, $\log_e y = -ax$; $\therefore$ $\frac{1}{y} \cdot dy = -a.dx$

$$\therefore \quad \frac{dy}{dx} = -a.y = -ae^{-ax}$$

(2) $y=e^{\frac{1}{3}x^2}$. Let $z=\frac{x^2}{3}$ $\therefore$ $\frac{dz}{dx}=\frac{2x}{3}$

$$\therefore \quad \frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}=e^{\frac{1}{3}x^2}\cdot\frac{2x}{3}=\frac{2x}{3}e^{\frac{1}{3}x^2}$$

Alternatively $\log_e y=\frac{x^2}{3}$ or $\frac{1}{y}\, dy=\frac{2x}{3}\cdot dx$

$$\text{or} \quad \frac{dy}{dx}=y\cdot\frac{2x}{3}=\frac{2x}{3}e^{x^2/3}$$

(3) $y=e^{\sqrt{x^2+y^2}}$. $\therefore$ $\log_e y=(x^2+a^2)^{\frac{1}{2}}$

$$\therefore \quad \frac{1}{y}\cdot dy=\tfrac{1}{2}\cdot(x^2+a^2)^{-\frac{1}{2}}\cdot 2x\,dx$$

$$\therefore \quad \frac{dy}{dx}=y\cdot\frac{x}{\sqrt{x^2+a^2}}=\frac{xe^{\sqrt{x^2+a^2}}}{\sqrt{x^2+a^2}}$$

(4) $y=\log_e(a+x^3)$. Let $z=(a+x^3)$

$$\therefore \quad y=\log_e z. \quad \therefore \quad \frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}=\frac{1}{z}\cdot 3x^2=\frac{3x^2}{a+x^3}$$

0-2.7. Trigonometrical functions and their Differentiations :

A. Two Limiting Theorems : Elementary trigonometry tells us that (See fig. 0-2.3) in radian measure

$$\theta=\frac{\text{arc }(l)}{\text{radius }(r)}=\frac{\text{PQ (arc)}}{\text{OQ (radius)}}$$

$$\sin\theta=\frac{\text{Opposite side (PR)}}{\text{Hypotenuse (OP)}}$$

$$\cos\theta=\frac{\text{Adjecant side (OR)}}{\text{Hypotenuse (OP)}}$$

Note that the *semi-arc* PQ is somewhat *longer* than the *semi-chord* PR. Now consider a smaller angle θ'; see that (*i*) the semi-arc P′Q and semi-chord P′R′ have shortened (*ii*) their difference has diminished (*iii*) the adjecant side OR′ has lengthened. These changes mean (*i*) both θ and $\sin\theta$ have diminished (*ii*) their values have come closer (*iii*) $\cos\theta$ has increased in value.

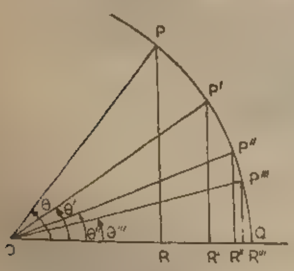


Fig. 0-2.3

For a smaller value of θ (θ'' in the figure) the trend continues. For a very small value of θ (θ''') the semi-arc and the semi-chord have become almost identical and the adjecant side OR''' almost is equal to OQ (i.e. OP). Thus we conclude

$$\text{(i)} \quad \underset{\theta\to 0}{\text{Lt}} \sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \simeq \frac{\text{arc length}}{\text{radius}} \simeq \theta \qquad (0\text{-}2.7.1)$$

$$\text{(ii)} \quad \underset{\theta\to 0}{\text{Lt}} \cos\theta = \frac{\text{adjecant side}}{\text{hypotenuse}} \simeq \frac{\text{radius}}{\text{radius}} \simeq 1 \qquad (0\text{-}2.7.2)$$

$$\text{(iii)} \quad \underset{\theta\to 0}{\text{Lt}} \tan\theta = \frac{\sin\theta}{\cos\theta} \simeq \frac{\theta}{1} \simeq \theta \qquad (0\text{-}2.7.3)$$

In various deductions these results come handy.

B. Differentiation of Sine θ :

Let us have $y=\sin\theta$; then $y+\delta y=\sin(\theta+\delta\theta)=\sin\theta.\cos\delta\theta+\cos\theta.\sin\delta\theta$. Now $\delta\theta\to 0$, $\sin\delta\theta\simeq\delta\theta=d\theta$ and $\cos\delta\theta=1$. Then

$$y+dy=\sin\theta+\cos\theta.d\theta \text{ or } dy=\cos\theta.d\theta$$

$$\therefore \quad \frac{dy}{d\theta}=\cos\theta \qquad (0\text{-}2.7.4)$$

C. Differentiation of Cosine θ :

Let $y=\cos\theta$; then $y+\delta y=\cos\theta.\cos\delta\theta-\sin\theta.\sin\delta\theta$.

When $\delta\theta\to 0$ $\sin\delta\theta\simeq\delta\theta=d\theta$ and $\cos\delta\theta=1$

$\therefore \quad y+dy=\cos\theta-\sin\theta.d\theta \quad \therefore \quad dy=-\sin\theta.d\theta$

$$\therefore \quad \frac{dy}{d\theta}=-\sin\theta \qquad (0\text{-}2.7.5)$$

D. Differentiation of Tan θ :

Let $y=\tan\theta=\dfrac{\sin\theta}{\cos\theta}$. Then by equation (0-2.5.3) we shall have

$$\therefore \quad d(\tan\theta)=\frac{\cos\theta.d(\sin\theta)-\sin\theta.d(\cos\theta)}{\cos^2\theta}$$

$$=\frac{\cos\theta.\cos\theta d\theta-\sin\theta.(-\sin\theta)d\theta}{\cos^2\theta}$$

$$=\frac{(\cos^2\theta+\sin^2\theta)d\theta}{\cos^2\theta}=\frac{d\theta}{\cos^2\theta}$$

$$\therefore \quad \frac{d}{d\theta}(\tan\theta)=\sec^2\theta \qquad (0\text{-}2.7.6)$$

E. Differentiations of Inverse Circular Functions :

Let $y=\sin^{-1} x$; then $x=\sin y$. Then by equation 2-7.4, we have

$$\therefore \frac{dx}{dy}=\cos y=\sqrt{1-\sin^2 y}=\sqrt{1-x^2}$$

$$\therefore \frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}} \qquad (0\text{-}2.7.7)$$

Again let $y=\cos^{-1} x$; then $x=\cos y$

$$\therefore \frac{dx}{dy}=-\sin y=-\sqrt{1-\cos^2 y}=-\sqrt{1-x^2}$$

$$\therefore \frac{dy}{dx}=-\frac{1}{\sqrt{1-x^2}} \qquad (0\text{-}2.7.8)$$

Examples : Differentiate : (*i*) $\cos^3\theta$ (*ii*) $\sin(x+a)$ (*iii*) $y=\log_e \sin\theta$ (*iv*) $\tan 3\theta$

(i) Let $y=\cos^3\theta=(\cos\theta)^3=z^3$ where $z=\cos\theta$

$$\therefore \frac{dy}{dz}=3z^2. \quad \frac{dz}{d\theta}=-\sin\theta$$

$$\therefore \frac{dy}{d\theta}=\frac{dy}{d\theta}\cdot\frac{dz}{d\theta}=3z^2.(-\sin\theta)=-3\cos^2\theta.\sin\theta$$

(ii) Let $z=x+a$ $\therefore$ $y=\sin z$ or $\dfrac{dy}{dz}=\cos z=\cos(x+a)$

Again $\dfrac{dz}{dx}=1$ $\therefore$ $\dfrac{dy}{dx}=\dfrac{dy}{dz}\cdot\dfrac{dz}{dx}=\cos(x+a)$

(iii) $y=\log_e\sin\theta$. Let $\sin\theta=z$ then $y=\log_e z$ $\therefore$ $\dfrac{dy}{dz}=\dfrac{1}{z}$ and $\dfrac{dz}{d\theta}=\cos\theta$

$$\therefore \frac{dy}{d\theta}=\frac{dy}{dz}\cdot\frac{dz}{d\theta}=\frac{1}{z}.\cos\theta=\cot\theta$$

(iv) $y=\tan 3\theta$. Let $3\theta=z$ $\therefore$ $\dfrac{dz}{d\theta}=3$ and $y=\tan z$

$$\therefore \frac{dy}{dz}=\sec^2 z \text{ or } \frac{dy}{d\theta}=\frac{dy}{dz}\cdot\frac{dz}{d\theta}=\sec^2 3\theta.3=3\sec^2 3\theta.$$

0-2.8. Geometrical Interpretation of Differentiation :

Any expression containing a pair of variables x *and* y *can be represented graphically.* In co-ordinate geometry you learn that

the relations $y=mx+c$, $x^2+y^2=r^2$, $\left(\frac{x^2}{a^2}\right)+\left(\frac{y^2}{b^2}\right)=1$, $\left(\frac{x^2}{a^2}\right)-\left(\frac{y^2}{b^2}\right)=1$, $y^2=4ax+b$ represent respectively a straight line, a circle, an ellipse, a hyperbola, a parabola. In each case we may find the ratio $\left(\frac{dy}{dx}\right)$ also we shall find that *this ratio represents the slope of the* **tangent** *at any point on the curve concerned*, **with the x-axis**.

Let PQR (fig. 0-2.4) represent a part of an ellipse on co-ordinate axes OX and OY, and Q a point (x, y) on it. If the point Q is moved along the curve, its y-value will change along with its x-co-ordinate. Suppose x is moved by a small value dx to the right; we observe that on the curve y rises upwards by an amount dy. Then the ratio $\frac{dy}{dx}$ would measure the degree or amount by which the curve is sloping up between Q and T. On close scrutinee, you will recognise that at different small parts of the curve, the *sloping up* is different and so we cannot very well speak of a single slope in the region QT.

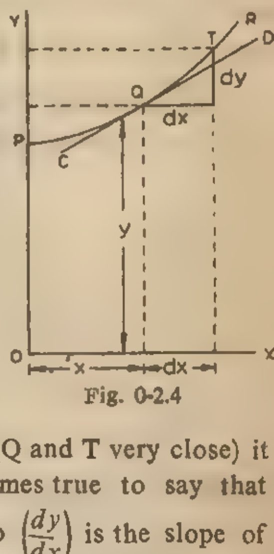


Fig. 0-2.4

If however we take QT to be very small (Q and T very close) it will become *practically straight*; then it becomes true to say that the ratio $\left(\frac{dy}{dx}\right)$ is the slope of the curve along QT. The straight line QT then becomes indistinguishable from the curve QT. [Recall that when the arc subtends a very small angle, the arc and the chord become almost the same as we noticed when discussing $\underset{\theta \to 0}{\text{Lt}} \sin\theta \simeq \theta$ (eq. 0-2.7.1)]. And

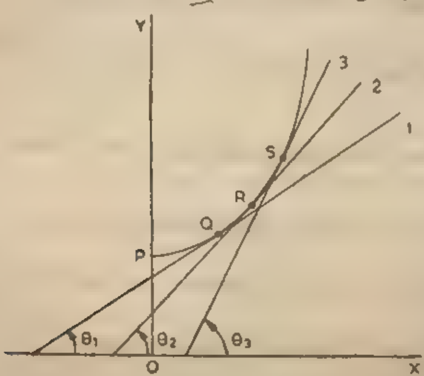


Fig. 0-2.5

if QT is indefinitely small, the straight line cuts the curve at two *coincident* points i.e. touches the curve practically at one point and hence becomes a tangent with evidently the same slope as QT; so that $\frac{dy}{dx}$ *represents the slope of the tangent* to the curve at the point Q for which the value $\frac{dy}{dx}$ is found.

"Slope of a curve" carries no precise meaning, for at different points (Q, R, S), tangents drawn have different inclinations (θ_1, θ_2, θ_3), as shown in fig. 0-2.5 but a slope of a curve at a point (say R), has a perfectly valid meaning, for a definite inclination (θ_2) to the OX axis is associated with the tangent at the point.

Note that in fig. 0-2.4, dx is a *small* displacement to the right and dy the same, upwards. They have to be small, in fact very very small—though it is not possible to show them in the diagrams. So we conclude what we have stated above, that the ratio $\left(\frac{dy}{dx}\right)$ is *geometrically* given by the tangent of the slope angle (θ) made with the x-axis by the tangent drawn to a point on the curve in question.

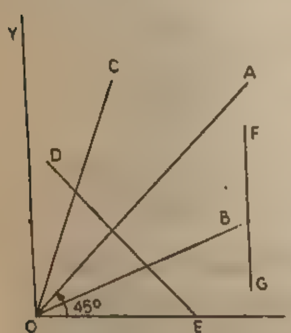


Fig. 0-2 6

In fig. 0-2.6, slope of the line OA to the x-axis is 45°; so $\frac{dy}{dx} = \tan 45° = 1$; slope of the line OB $<45°$, so $\frac{dy}{dx} < 1$, that of OC$>45°$ with $\frac{dy}{dx} > 1$. The line FG is perpendicular to OX and hence its slope is 90° and $\frac{dy}{dx} = \infty$. Finally, the slope of the line DE is *greater than 90°* with the +*ve* direction of the x-axis and hence $\frac{dy}{dx}$ *is negative*. For a st. line, $\frac{dy}{dx}$ is constant everywhere.

But it is not so for a curve as shown in fig 0-2.7. There the

slope slowly diminishes as you move from A to B and so does $\frac{dy}{dx}$. But it is everywhere +ve. In the *very small* regions CB and DF, tangents are parallel to x-axis, so the slope and hence $\frac{dy}{dx}$ are zero. Hence *differentiate a quantity and equate it to zero to get either a maximum or a minimum value* of the expression. In the CD portion the slope is

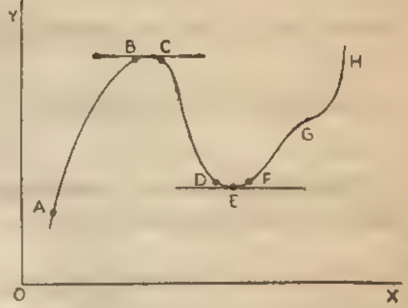


Fig. 0-2.7

greater than $\frac{\pi}{2}$ and hence $\left(\frac{dy}{dx}\right)$ negative. It is again positive in the part FG but beyond G is ∞ as the slope becomes $\pi/2$.

Questions : (1) Plot the curve $y=\frac{2}{3}x^2-5$ on a mm graph paper. Take $x=1, 2, 3, 4$ and measure the slope angle for each value.

Find the slope by differentiating the expression. Verify whether the angles agree, from a table of Natural tangents.

(2) If $y=(x-a)(x-b)$, show that at the point $x=\frac{1}{2}(a+b)$, the ratio $\frac{dy}{dx}$ vanishes.

(3) Find the differential co-efficient of the equation $y=x^3+3x$. Plot on a graph values of $\left(\frac{dy}{dx}\right)$ when $x=0, \frac{1}{2}, 1, 2$. [The result would be a parabola similar to Fig 0-2.5]

[$3(x^2+1)$; 3, $3\frac{3}{4}$, 6, 15].

(4) Find the slope at any point of the curve of equation $(x^2/9)+(y^2/4)=1$; also find the numerical values of the slope where $x=0$ and $x=1$

$\left[-\frac{4x}{9y}, 0, \mp\frac{1}{3\sqrt{2}}\right]$

0-2.9 Integration :

By this term we mean summing up of a very large number of small quantities of same nature. The direction to sum up a large number of small but finite sized terms (x) is provided by the Greek

letter Σ (sigma) which means that $\Sigma\delta x = x$. When we have to sum up a very large number of very very small quantities (dx) the direction is communicated by the symbol $\int$ which is nothing but an elongated S, i.e. $\int dx = x$

As an example of summing up small quantities, let us take a series of terms in G. P.

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \ldots$ upto infinity.

Fig. 0-2.8

You know that if we take an infinite number of such terms then they all add up to 2. Take a line segment 1″ long; add to it $\frac{1}{2}$″, then ($\frac{1}{4}$″), then again ($\frac{1}{8}$), and then again ($\frac{1}{16}$). Whenever you stop the sum will fall short of 2 by the amount you have added last; e.g if the last term added is $\frac{1}{8}$″ then you fall short by $\frac{1}{8}$″, if that is $\frac{1}{64}$″, then you are short by $\frac{1}{64}$ and so on. Only if you add *innumerable* terms then only you will reach 2. But you cannot draw beyond the tenth term ($\frac{1}{1024}$)″, you will not be able to see the 18th term under a microscope, although integration refers to summing up of innumerable terms. Integration is an alternative, fast yet simple but accurate method of summation instead of the laborious time-consuming method outlined above

We look upon integration from two different view points, namely

(1) Integration as the reverse process of differentiation, and

(2) Integration as a summation of a very large number of very small quantities.

A. The Reverse process to Differentiation : As for all other mathematical operations, a process reverse to differention is possible. If by differentiating x^4 we have obtained $4x^3$, then it must be possible

to go back from $\frac{dy}{dx}=4x^3$ to $y=x^4$; a point must however need be remembered that on differentiating a quantity $y=x^4 \pm C$ ($C=$ const) we also obtain $\frac{dy}{dx}=4x^3$. Hence as we reverse that process to integrate, the provision of an added constant must be there ; the constant is originally undetermined but can be determined from the so-called **boundary conditions**. This unknown constant is said to be the **Integration constant**, and this integrated result is said to be an **Indefinite integral**. So it may be said that if

$$\frac{dy}{dx}=nx^{n-1}.dx \text{ then } \int dy=nx^{n-1} \text{ and } y=x^n+C \qquad \text{(0-2.9.1)}$$

will be the result of integration.

Hence the working rule in this case will be : *raise the index by 1 and divide the quantity by that increased index* ; in other words, if

$$\frac{dy}{dx}=x^n \text{ then } y=\frac{x^{n+1}}{n+1}+C \qquad \text{(0-2.9.2)}$$

$$\text{Similarly if } \frac{dy}{dx}=ax^n \text{ then } y=\frac{ax^{n+1}}{n+1}+C \qquad \text{(0-2.9.3)}$$

Examples : (1) $\frac{dy}{dx}=x^3$. Then $dy=x^3.dx$

$$\therefore y=\int dy=\int x^3.dx+C=\frac{x^4}{4}+C$$

(2) $\frac{dy}{dx}=x^4 \pm x^7+5$. Then $dy=x^4.dx \pm x^7.dx+5.dx$

$$\therefore y=\int x^4.dx \pm \int x^7.dx+\int 5.dx+C$$

$$=\frac{x^5}{5} \pm \frac{x^8}{8}+5x+C$$

This is how we may arrive at indefinite integrals from differential co-efficients. A table is provided below setting out the results of differentiating and integrating the different functions we have so far studied.

TABLE FOR DIFFERENTIATION AND INTEGRATION

$\frac{dy}{dx}$	y	$\int y.dx$
1	x	$\frac{1}{2}x^2+C$
0	a	$ax+C$
1	$x \pm a$	$\frac{1}{2}x^2 \pm ax+C$
a	ax	$\frac{1}{2}ax^2+C$
$2x$	x^2	$\frac{1}{3}x^3+C$
nx^{n-1}	x^n	$\frac{x^{n+1}}{n+1}+C$
$-\frac{1}{x^2}$	$\frac{1}{x}$	$\log_e x+C$
$\cos x$	$\sin x$	$-\cos x+C$
$-\sin x$	$\cos x$	$\sin x+C$
$\sec^2 x$	$\tan x$	$-\log_e \cos x+C$
e^x	e^x	e^x+C
$\frac{1}{x}$	$\log_e x$	$x(\log_e x-1)+C$
$a^x \log_e a$	a^x	$\frac{a^x}{\log_e x}+C$
$\frac{1}{\sqrt{1+x^2}}$	$\sin^{-1}x$	
$-\frac{1}{\sqrt{1+x^2}}$	$\cos^{-1}x$	

B. The Process of Summation : The Definite Integral : The subject-matter of integral calculus arose from the attempt of finding the area of a surface bounded by curved lines.

Let us for example have an area bounded by a curved line PQ (fig. 0-2.9) two perpendicular lines PM($=y_1$) and NQ($=y_2$) and a horizontal line MN. Let PQ be a part of a curve BPQD which has an equation $y=f(x)$, and let $OM=a$ and $ON=b$. Then the required area will be

$$PMNQ=\int_a^b f(x).dx$$

To find the area we (*i*) draw a large number of perpendiculars on the x-axis so as to (*ii*) cut up the required area into a large no. of strips, then (*iii*) compute the area of any one of such strips and finaly (*iv*) add up the areas of all of these elementary strips.

Let us first compute the area of the strip ACC'A'; we have two perpendiculars y and $y+\delta y$, a horizontal base δx and at the top the slightly curved line AA'; let us take y as the average of y and $y+\delta y$ so that the area of the strip would be $y.\delta x$. Now, as we can make δx as small as we please, we make it so small that the strip becomes very narrow and its average height becomes equal to that at its mid-point. Then the area of this very narrow strip becomes $ds = y.dx$. So

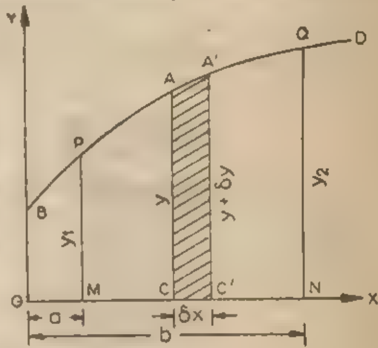


Fig. 0-2.9

$$\text{area PQNM} = \int ds = \int y.dx$$

Note that the area here is definite but limited by PM to the left and QN to the right—you are *summing up* (or *integrating*) *between two definite limits*. Here the value of x is not less than a, nor more than b—the *lower* (or *inferior*) and the *upper* (or *superior*) limits respectively. Such limited integrals are **definite integrals**. Hence

$$\text{area PMNQ} = \int_{x=a}^{x=b} ds = \int_{x=a}^{x=b} f(x).dx = \int_{x=a}^{x=b} y.dx$$

Again we may look upon the area PMNQ as the difference between two areas OBQN and OMPB. In fact any definite integral is the difference between the two integrands, one at the upper limit and the other at the lower. That is why the integration constants that appear for both integrations cancel out by subtraction. *Integration constants do not occur in definite integrals.*

Examples :

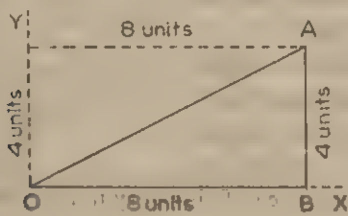
(*1*) *Find the area of a right-angled tringle of base 8 units and height 4 units by integration.*

In the triangle AOB we have the hypotenuse OA as an inclined straight line of equation $y=\frac{1}{2}x$(for $y/x=\frac{1}{2}$). Then the area would be

$$\int_{x=0}^{x=8} y.dx=\int_{x=0}^{x=8} \tfrac{1}{2}x\,dx=\tfrac{1}{2}\int_0^8 x.dx=\tfrac{1}{2}\left[\frac{x^2}{2}+C\right]_0^8$$

$$=\left[\frac{x^2}{4}+\frac{C}{2}\right]_0^8=\left[\frac{8^2}{4}+\frac{C}{2}\right]-\left[\frac{0^2}{4}+\frac{C}{2}\right]$$

$=16$ square units. [Integration const cancels out]



From elementary geometry we know that the required area is $\frac{1}{2}ba=16$ units.

(*2*) *Find the area of a circle of radius R.*

In the fig. let O be the centre of a circle of radius R ; draw an annulus of inner radius r and breadth δr. We may consider the circle to be the sum of areas of a very large number of such annuli from the centre to the circumference ($r=0$ to $r=R$).

In finding the area of one strip ($=l\times b$) we note that its length is $2\pi r$ and breadth δr, i.e. the area (δA) becomes $2\pi r.\delta r$; now making the breadth vanishingly small we shall have

$$\text{area of the circle, } A=\int dA=\int_{r=0}^{r=R} 2\pi r.dr=2\pi\int_0^R r.dr$$

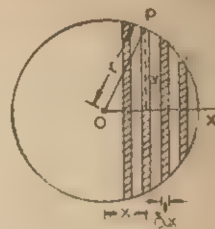
$$=2\pi\left[\frac{1}{2}r^2+C\right]_0^R=\pi\left[\left(R^2+2C\right)-\left(0^2+2C\right)\right]$$

$=\pi R^2$ [Integration const cancels out]

This is a result familiar to you from mensuration.

(3) *Find the volume of a sphere.*

On the sphere shown, put side by side a large number of paper ribbons parallel to each other. Consider one such strip PQ of radius y and width δx. The strip being circular its area is πy^2 and volume $\pi y^2.\delta x$. Note from the figure that $y^2 = r^2 - x^2$ where the value of y may change from 0 to r depending on the position of the ribbon PQ on the sphere. Hence, reducing the width to a very very small value we have



$$\text{Volume of the sphere } V = 2\int_{x=0}^{x=r} \pi y^2.dx$$

$$= 2\int_0^r \pi (r^2 - x^2)\, dx = 2\pi \left[\int_0^r r^2.dx - \int_0^r x^2.dx\right]$$

$$= \pi\left[r^2.\int_0^r dx - \left(\tfrac{1}{3}x^3\right)_0^r\right]$$

$$2\pi(r^3 - \tfrac{1}{3}r^3) = \tfrac{4}{3}\pi r^3$$

once again a result known from mensuration.

PART I

Mechanics

PART I : MECHANICS

I-1

KINEMATICS
REST & MOTION
OF A PARTICLE

I-1.1. Mechanics :

The science of mechanics deals with such ideas as motion, force, energy and their relations with one another. *Mechanics is said to be the grammar of physics*, for it enters into practically all other branches of the subject. It is fundamental since a knowledge of mechanics is a must, for properly understanding physical phenomena. Though the name implies a connection with machinery, they and other practical applications properly belong to '*applied mechanics*'. Gravitation, elasticity and the mechanics of fluids which we shall be studying later under Part II, actually form branches of applied mechanics.

Mechanics proper is broadly divided into **statics** and **dynamics.** The former treats of bodies *at rest* under the action of two or more forces. The latter deals with bodies *in motion*, again under the action of two or more forces but simplified into a resultant unbalanced force or torque. Dynamics again, is divided into two branches, *kinematics* and *kinetics*.. The former studies the nature of motion without bothering about their causes ; the latter studies the causes and laws of motion.

Mechanics itself is an important branch of Applied Mathematics. Our line of study will not be along that traditional discipline for, you shall be doing that in your mathematics classes. Our purpose is only an intelligent application of the results of mechanics. We shall confine ourselves to understanding the basic ideas and results and seeking to apply them to solving various simple problems.

Some Definitions : Body : Any piece of matter is a body. It should have mass and volume. If on applying equal and opposite forces, however large, its shape or size does not change, the body is said to be *rigid*. If they do change, the body is *elastic.* No body is

perfectly rigid; it is an impractical, idealised concept only, but helpful in easily grasping relevant laws. Whenever we talk of a body in Part I, it would be a rigid one. In Part II we shall consider elastic bodies which undergo changes in shape or size under *balanced forces.*

A rigid body is taken as a conglomeration of particles (bodies of negligible size), their separation remaining unchanged under all conditions.

Particle: It is a body of negligible dimensions and corresponds to a point in geometry. Like the latter, it has *position* but unlike it, has also *mass.* These two quantities, position and mass completely specify a particle.

A body may be considered (*i*) a particle with its mass concentrated at its centre of gravity, more appropriately, at its centre of mass* and (*ii*) also when its separation from another is very large compared to its dimensions.

I-1.2. Rest and Motion:

A body is said to be at rest when it does not change its position with respect to its surroundings. Your book-shelf fixed to the wall always remains there. You go to school in the morning return home in the afternoon day after day and you find them always there, never elsewhere. They are at rest apparently.

When a body changes position with respect to its surroundings it is said to be in motion. In a football game you are seeing, the ball appears to be every successive moment at a different position: the football is in motion. You board a bus or a train from a stoppage and you recognise it to be in motion when you find that the stoppage appears to be falling back with time.

However your home is not at rest. You know that the earth is spinning about its axis once in 24 hours. You are crossing a thousand miles in space each hour. The astronauts on the moon have clearly seen the earth to be rotating. Again, the spinning earth is moving along its orbit round the sun once a year. The sun along with the entire solar system is moving round the centre of our galaxy at about 200 miles a second. So nothing on earth is at rest.

* Both of these concepts will be treated in Statics, chapter I-7.

to an observer anywhere outside it. The passengers seated inside a moving bus or a train appear to be at rest when looking at each other but appear to a man on the road or station outside, to be in motion. Hence *rest and motion are relative*, i.e. at rest or in motion with respect to a reference or standard landmark which we consider to be at rest. So is your home, referred to some other building, at rest. In a football game the ball is in motion with respect to you, standing at a fixed point. So, for any motion on earth we take the earth to be the reference and we judge anything to be at rest or in motion with respect to it. Look out on a clear night to the north and locate the *Pole Star* : observe for a few hours : you will find other stars close to it, appearing to move round it.

Remember, *motion is itself imperceptible* : we perceive it due to the change of position of a body with respect to others : we perceive it because of sudden changes in motion due to bumps, jerks, jars, vibrations. If you are in a closed vehicle moving smoothly, such as modern high flying jet-planes or the famous Japanese bullet-trains, you will not be aware of your motion. Because the earth spins and moves quite smoothly, man till the days of Copernicus (1469-1543) took the earth to be at rest. Galeleo (1564-1642) was hauled up before the dreaded Inquisition in Rome for teaching that the earth moves and Bruno was burnt at stake for believing so ! Long before, Aryabhatta of India in a book in 499 A.D. had however declared (and was believed) that the earth is round, it spins about an axis producing day and night and also goes round the sun in company with five other planets.

I-1.3. Reference Frames :

We therefore need a reference at rest with respect to which motion is to be described. You are already familiar with such a system in drawing graphs, while solving simultaneous algebraical equations ; you draw two fixed axes XOX′ and YOY′ at right angles to each other and plot the points (x, y) with respect to these axes, of different points you select.

A **frame of reference** *is a set of lines or surfaces with respect to which we describe the motion of a particle*, by plotting its positions at different instants of time.

Reference frames are chosen according to convenience. No reference frame is intrinsically better than others : only under a set of

circumstances one may be more convenient than others. We discuss below, some of the more widely used ones.

A. One-dimensional Reference frame : If a particle moves along a straight line then its position w.r.t. a fixed point called *origin* can be expressed by the separation between them. In fig. I-1.1, O is such an origin : P the position of the moving particle at a given distance OP ($=r$) at a given instant, OR representing the one dimensional reference frame and the motion of the particle is said to be one dimensional.

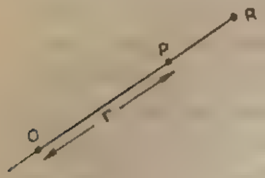


Fig. I-1.1

B. Two dimensional Reference Frame : When a particle moves in a single plane (as it will do in most of our discussions) the motion is said to be planar. Then we require two quantities to specify its position with respect to an origin. The more important two of such reference frames, are (*a*) the Cartesian co-ordinates and (*b*) the Polar co-ordinates.

(a) The Cartesian Co-ordinate system : You are already familiar with this system in drawing graphs and would study in detail when you take up co-ordinate geometry.

You draw a pair of straight lines OX and OY *at right angles* to each other (Fig. I-1.2) and choose O, their intersection point, as the origin. Any point (P) in the X-Y plane represents a position of the moving particle. From P, drop a pair of perpendiculars PQ and PR on the X and Y axes. The distances OQ and OR, from O are taken as x and y and are called *the co-ordinates* of the point P. As x, y both give us distances, their combination (x, y) i.e. the co-ordinates of the point P tells us where the point is on the plane of reference XOY. Co-ordinates indicating the position of a point are called its *space-co-ordinates.*

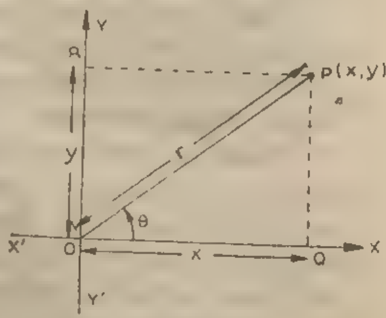


Fig. I-1.2

However, intersecting reference lines may be at any angle other than at right angles. The axes are then said to be *oblique axes* as different from the *rectangular axes* described above. We take no further notice of oblique axes.

(b) Polar Co-ordinates: In this system we express the position of a point P in terms of a distance from a fixed point ($=r$), the so-called radius-vector OP and the angle the radius-vector makes with a standard reference axis, generally OX. The polar co-ordinates of P in fig. I-1.2 is shown to be (r, θ).

The two systems can be related. If in the Cartesian system the co-ordinates of P is x and y and in the polar system r and θ, note from simple geometry that

$$r^2 = x^2 + y^2,\ \tan\theta = y/x,\ x = r\cos\theta \text{ and } y = r\sin\theta \qquad \text{(I-1.3.1)}$$

These relations would be very handy, when later we learn to resolve and compound vectors.

C. Three-Dimensional Reference Frames: When a particle moves about in space, for example a fly, we require three quantities to indicate its instantaneous position. There are three frames in general use, the Cartesian, the Spherical and the Cylindrical. The first two are of wider use. But so far we are concerned the uses of the latter two will be very few.

(a) Three-dimensional Cartesian Co-ordinates: Here we build up a system of three mutually perpendicular lines OX, OY, OZ intersecting at O, the origin. In an ordinary rectangular room they can be taken as the lines along which the floor meets two adjacent walls and the line along which these walls meet, all three lines meeting at a corner point of the room, and at right angles to each other.

In this system, to specify the position of a *point in space* (say, the tip of a hanging bulb in a room) three perpendicular distances of the point from the three planes (X-Y, Y-Z, Z-X) need be known. To do so, a normal PQ is dropped on the plane X-Y and two perpendiculars from Q, QD and QS are drawn respectively to OX and OY axes. Now OD represents x, QS represents y while PQ

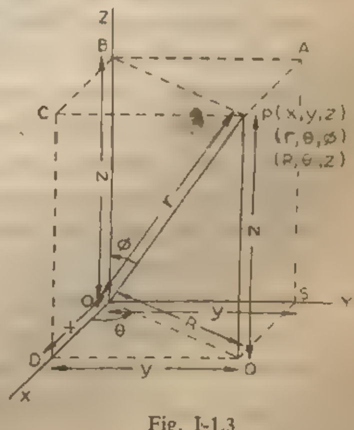


Fig. I-1.3

does so for z. Then the distance OP is given by

$$OP^2 = r^2 = x^2 + y^2 + z^2 \quad \text{(I-1.3.2)}$$

In fig I-1.3, ODQS represents the floor or X-Y plane, ABOS and BCDO the two adjacent walls, respectively the Y-Z and Z-X planes, the point P (x, y, z) the tip of the bulb hanging from somewhere.

(b) Spherical Polar Co-ordinates : In this system one distance OP $(=r)$ is taken and two angles ϕ, with the *vertical* z-axis that OP makes and θ which OQ the projection, forms with the horizontal x-axis. So P has co-ordinates r, ϕ, θ where $x = r \sin\phi \cos\theta$, $y = r \sin\theta . \sin\phi$ and $z = r \cos\phi$

(c) Cylindrical Co-ordinates : Here we use two distances $z\ (=PQ)$ and R $(=OQ)$ and as before $\theta = \angle DOQ$, in the same figure. Note that

$$x = R\cos\theta \quad y = R\cos\phi, \quad z = z$$

Any given frame of reference is generally called a *co-ordinate system.* In a broader sense, any system with reference to which we describe an event, is a reference system. An *event* is anything that happens, such as the motion of a particle, a rotation, a collision between two particles, etc. It involves time t, another dimension.

I-1.4. *Inertial and Non-inertial Frames :

From the above discussions we find that a co-ordinate frame made of three mutually perpendicular axes may give us an idea of the state of rest or of motion of a body with respect to it. Let such a frame be fixed on the surface of the earth. If a body does not change position with time, it is said to be at rest *relative* to the frame. If it changes its position with time, then the body is in motion relative to the frame. We have said so before. But let the frame be fixed inside a railway carriage that is moving uniformly. A person sitting inside, is at rest relative to this frame but he is in motion relative to a frame fixed on earth. So a body need not move itself to be in motion, but it moves if the reference frame does. Again, if two cars travel with same speed parallel to each other, passengers in them find each other at relative rest yet both their frames are moving.

The *reference frame is said to be inertial when motion of particles in them arise from their mutual interactions. This means* that there must be at least two bodies exerting forces on each other :

* The ideas presented are difficult and may be left out at a first readign.

for example, the movement of a cricket ball thrown in air or of an orbiting satellite, is due to the attraction of the earth ; an electron will move towards a positively charged body because of electrostatic attraction ; a magnetic needle rotates because the earth behaves as a giant magnet ; a moving marble comes to rest because of frictional forces exerted on it by the surface on which it moves. These interacting forces are said to be *real*, for both the agent applying the force and the body subject to it, can be identified. Later we shall come across forces which cannot be traced to any applying agent ; they are *pseudo*-forces. Newton's first law of motion tells us that in absence of nearby bodies i.e. agents responsible for exerting forces, the state of motion of a body or of rest does not change. This behaviour is said to be due to the property of **inertia**. The first law is hence said to be that of inertia and the frames of reference to which it applies are said to be inertial frames. Since forces due to mutual interactions are there, Newton's second law must also be valid in inertial frames, for the law gives us a measure of force. We shall see later that the third law is contained in the first. So we conclude that *inertial frames are those where Newton's laws of motion hold.*

Inertial frames are those (*i*) that are at rest with respect to each other or (*ii*) if moving, are doing so with unchanging velocity ; they may have different origins or their axes may intersect at any angle. A single frame in constant motion along a straight line is thus an inertial one. Though in nature, strictly speaking no inertial frame exists, its concept (like those of a material particle or frictionless plane) is idealised but very helpful. Even the earth is not such a frame as we shall presently see, but not much error arises if we take it to be so.

When a frame is accelerated with respect to an inertial frame i.e. it is either speeding up or slowing down or rotating, it is said to be *non-inertial*. The acceleration may be linear or rotational or both. In such cases forces arise due to the acceleration (a_0) of the frame which cannot be traced to any real agent. To apply Newton's second law we must include the acceleration of the frame a_0, and state that the total force acting on the body is

$$F' = F + F_0 = ma + ma_0 = m(a + a_0) \qquad \text{(I-1.4.1)}$$

or the effective force

$$F = F' - F_0 = ma \qquad \text{(I-1.4.2)}$$

where m is the mass of the body and a the acceleration generated. This force F_0 is the *fictitious* or *pseudo*-force and always opposes the applied force (F). We shall find it useful in explaining the experiences of a person in an accelerating or retarding car and also in a rotating frame. Since the earth is rotating, it is a non-inertial frame but for it a_0 is only 0.336 cm/s and hence can be neglected, to make the earth an *acceptable* inertial frame.

I-1.5. Kinds of Motion :

Two types of motion are recognised, namely (*i*) translational and (*ii*) rotational. A third type is very important, namely the periodic, which may be either of these two types.

A. Translation : In this motion all the particles of a body move along parallel paths and describe the same distance in the

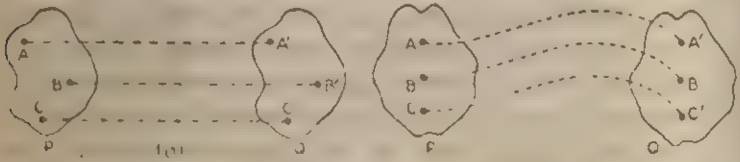


Fig. I-1.4

same time. Translation may be along, either a straight line when the motion is said to be *rectilinear*, or a curved line when the motion is *curvilinear* (Fig. I-1.4*a* and *b*). Throughout either type of motion the body however remains oriented in the same way. In the first figure representing rectilinear motion, as the body goes from P to Q, the particles A, B, C move to A′, B′, C′ where by definition AA′=BB′=CC′. *Any line AB in the body remains parallel to itself for all positions of the body.* The second figure shows translation in a curved line i.e. a curvilinear motion.

B. Rotation : In this type of motion, all the particles of the body describe concentric circles in parallel planes around some common axis. The angle, each of the particles turn through, in a given time-interval is the same. *The axis of rotation* may pass through the body or be outside it. Fig. I-1.5 shows a rotation, say of a large piece

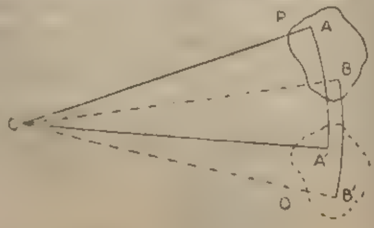


Fig. I-1.5

of stone tied to your finger by a stout piece of string, about an axis through O. The particles A and B of the body when it is at P move to A′ and B′ when the body goes to Q. Here $\angle AOA' = \angle BOB'$ while $OA = OA'$ and $OB = OB'$.

Any motion however complicated, can be shown to be a combination of a translation and a rotation. A lift moving along its guides shows translation, the tips of moving blades of a fan shows rotation. A small piece of stone sticking to the rim of a moving

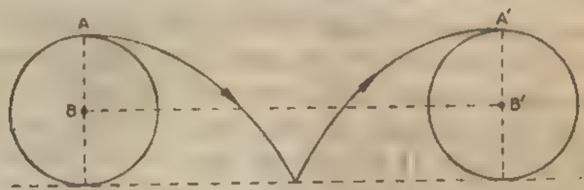


Fig. I-1.6

motorcar tyre shows a combination of both. A in fig. I-1.6 moves round B as B rolls forward.

I-1.6. Relevant terms for Linear Motion:

The simplest of motions is that of a particle along a straight line. When we shall talk of motion of a body, that of its *centre of mass* (I-7.10) should be understood—a point where the entire mass of the body can be taken to be concentrated, thus effectively reducing it to a particle. The relevant terms for such a motion are displacement, speed, velocity, acceleration, retardation and momentum.

A. Displacement: A body or a particle is said to be displaced when its position changes relative to its surroundings. Let a particle initially at O (Fig. I-1.7) move to A. Whatever be the path taken (1, 2, 3, 4 etc.) by the particle, its *displacement is measured by the shortest distance between initial and final positions*, namely OA *in the direction* O to A. At some later instant the particle may move to B; then again its further displacement is $\overrightarrow{AB}$

Fig. I-1.7

but the total or final displacement referred to O is OB. If however, the particle returns to O, the net displacement is zero.

Displacement always carries with it a sense of *direction* in addition to that of a *magnitude* and hence is a *vector*.

B. Speed and Velocity: Speed of a body or particle is the distance it covers in unit time. If it covers a distance s in a time-interval t its speed is given by s/t.

Speed is expressed in *units of length per unit time* e.g. 1 cm/s or 1 cm s^{-1} or 1 m/s (1 m s^{-1}) or again 1 ft s^{-1}; they are respectively absolute units of speed in cgs, mks and fps systems. Of course speed may be expressed in other units of length and time e.g. 1 km/hr or 1 m.p.h. (1 mile/hr) but they are not absolute units.

The *knot* is a *nautical* unit of speed equal to 1 *sea-mile* (=6020 ft) per hour. A 'speed of 20 knots' is a correct expression (per hour is needless).

If a body while moving, covers equal distances in equal time intervals however small, its speed is *uniform*. If not, the speed is non-uniform or *variable*. With variable speed, s/t represents *average* speed over the distance s or in the time-interval t. If t is very, very short, s/t is the *instantaneous* speed. Note that its dimension is therefore LT^{-1}.

Let the particle be at a distance s_1 from an origin O (Fig. I-1.8) at an instant t_1 and at a distance s_2 at an instant t_2; then it has moved over a distance $(s_2—s_1)$ over a time-interval $(t_2—t_1)$. The average speed then is $(s_2—s_1/(t_2—t_1)$ in the interval $(t_2—t_1)$. If both the differences are small then the *average* speed is

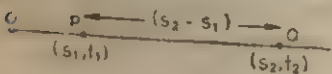


Fig. I-1.8

$$\bar{v} = \frac{\Delta s}{\Delta t} \qquad \text{(I-1.6.1)}$$

and when Δt is very very (vanishingly) small then the ratio

$$v = \mathrm{Lt}_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad \text{(I-1.6.2)}$$

represents that most important quantity, *instantaneous speed*, in the notation of calculus.

Speed is a scalar not being associated with any direction.

Example : I-1.1. *A car covers first half of the distance between two places at a speed of 40 km ph and the second half at 60 km ph. Find its average speed.*

[I. I. T. '74]

Solution : Let the separation between two stations be S km. Then

Time taken for the first half of the journey $= \frac{S/2}{40}$ hr

and Time taken for the second half of the journey $= \frac{S/2}{60}$ hr

$\therefore$ Total time required $= S\left(\frac{1}{80} + \frac{1}{120}\right)$hr $= S\frac{3+2}{40 \times 2 \times 3}$ hr

$\therefore$ Average speed $= \frac{\text{Distance}}{\text{time}} = \frac{S}{S.5/40}\ \frac{\text{km}}{\text{hr}} = 48$ km/hr.

Problem : A car completes two successive quarters of its journey at 30 km/hr and 40 km/hr respectively and the rest at 60 km/hr. What is the average speed ?

(Ans. 43.64 km/hr)

Instantaneous speed as a limiting process : To illustrate the instantaneous speed as of finding the ratio of $\Delta s/\Delta t$ for very small values, we refer to fig. I-1.8 and consider different values for s_2 and t_2 keeping s_1 and t_1 constant at values of 50 cm and 1 s respectively. We tabulate the results as below :

x_1 (cm)	x_2 (cm)	t_1 (s)	t_2 (s)	$\Delta x = x_2 - x_1$ (cm)	$\Delta t = t_2 - t_1$ (s)	$\Delta x/\Delta t$ (cm/s)
50.0	150.0	1.00	11.00	100.0	10.00	10.0
,,	130.0	,,	9.60	80.0	8.60	9.3
,,	110.0	,,	7.90	60.0	6.90	8.7
,,	90.0	,,	5.90	40.0	4.90	8.2
,,	70.0	,,	3.56	20.0	2.56	7.8
,,	60.0	,,	2.33	10.0	1.33	7.5
,,	55.0	,,	1.69	5.0	0.69	7.3
,,	53.0	,,	1.42	3.0	0.42	7.[illegible]
,,	51.14	,,	1.14	1.0	0.14	7.1

Note that as the time interval diminishes so does the speed value ($= \Delta s/\Delta t$), but ever more slowly and finally settles to a constant value as a limit—the instantaneous speed.

Velocity: This is the distance covered in unit time in a given direction i.e. a speed with direction. Thus *velocity is a vector.* Both are *time rates of change of position.* Change of position or displacement means the *shortest distance between two points travelled* and time rate refers to the *time elapsed.* Remember, velocity *always* implies speed in a given direction, e.g. a velocity of 50 m/s due north.

Difference. Speed is a scalar with magnitude only, while velocity, a vector with both magnitude and direction. Velocity is uniform only when it covers equal distances in equal time-intervals *without* changing direction. It is variable when either magnitude or direction or both, change. A body along a curved path may have uniform speed but variable velocity (Fig. I-1.9) as its direction continuously changes. We shall learn that for uniform circular motion (Chapter I-5).

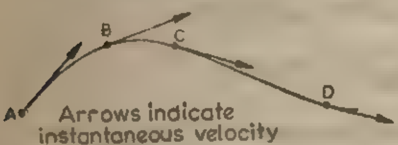


Fig. I-1.9

Instantaneous velocity: If s represents distance from a fixed point along a path then, in the notation of differential calculus, Δs represents a *short* distance along the path. If t denotes time, Δt means a *short* interval of time. In fig. I-1.10 suppose a point moves along the curved path from A to B. Let the moving point be at C at an instant t and at D at time $t+\Delta t$. The distances of C and D from A along the curved path are s and $s+\Delta s$ respectively.

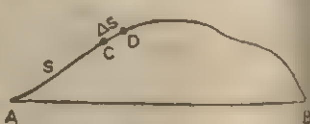


Fig. I-1.10

Then $\frac{\Delta s}{\Delta t}$ means the average speed between C and D as we have already noted. If D is so close to C as to be almost coincident with it, then $\frac{\Delta s}{\Delta t}$ is written as $\frac{ds}{dt}$. The latter symbol represents the instantaneous *value* of the *velocity* of the moving point at C. It has the *direction of the tangent of the curve* at C since, when D is very very close to C, the line joining them becomes the tangent at C.

$\frac{ds}{dt}$ = *Instantaneous velocity at a point along the tangent at the point.*

We shall, however, be concerned mostly with motion in a straight line. So the question of tangents will not arise initially.

When A is the initial position and B the final position in the motion, the straight line AB is the *displacement*. This we have already learnt.

C. **Acceleration and Retardation :** *Time-rate of change of velocity is acceleration,* the change may be either in speed (magnitude) or direction. When change is that of increase, as for a car picking up speed, it is acceleration, when that of decrease, as for a train slowing down near a station, it is deceleration or retardation. Mostly in our discussions, acceleration would mean increase in *linear* velocity (i.e. motion along a st. line).

An acceleration is *constant* or *uniform* when the velocity changes by equal amounts in equal intervals of time however small the intervals may be. Like velocity, acceleration may be *average* or *instantaneous*. It may as well be variable but we shall not consider them. Deceleration or **retardation** is negative acceleration, velocity diminishing with time.

Remember since (*i*) speed is a part of velocity (magnitude) it may be constant with variable (when direction changes) velocity but never the other way about. Again as (*ii*) acceleration is time rate of change of velocity *either* in magnitude *or* in direction, a particle may have zero velocity with acceleration (when a moving particle just reverses direction) and zero acceleration with velocity (for uniform linear velocity). Further, deceleration being negative acceleration (*iii*) a particle may have oppositely directed velocity and acceleration, as in a swinging pendulum or vertical ascent of a particle, (*iv*) acceleration is a vector as it includes both velocity and displacement.

Unit : As acceleration is time rate of change of velocity and velocity again, time rate of change of position, the unit of time will occur twice in its unit as m/s^2, cm/s^2, ft/s^2 etc.

Symbol : For a very very long time, the symbol had been *f*. But we shall be using the symbol *a* instead, for that is the recommendation of International Commission for Symbols, Units and Nomenclature. Foreign authors and those in other states of

India have taken to using a; let us also do so. So the instantaneous acceleration is

$$a \text{ (or the older } f) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2} \qquad \text{(I-1.6.2)}$$

Its dimension is thus LT^{-2}.

D. Momentum is a useful concept in the relation between force and motion. *It is measured by the product of the mass of a body and its velocity.* (To distinguish it from angular momentum (I-6.2) it is proper to call it the *linear momentum*). Consider a cricket ball thrown softly; it will be easy to stop it. But when hit hard, it is much more difficult to stop. In the latter case it has a larger momentum. A stone softly thrown has not much destructive effect. But when thrown very hard it may break open your forehead i.e. is much more damaging. In the first case the momentum is small while in the latter, momentum is much larger.

Think of a heavy iron roller lying on the ground or a four-wheeled carriage standing on a road. Because of its large mass you find it difficult to move. A continued push applied for some time gives it a certain velocity. A harder push over a shorter period will give rise to the same velocity. So, to give the body a momentum you had to *apply a force on it for a certain period.* (See 'Impulse of a force', I-3.4B). To bring the moving roller or the carriage to rest, i.e., to remove the momentum the body possesses, you must oppose the motion with a force and continue to exert the force for some time till the body stops. Production or removal of a larger momentum requires application of a larger force for a shorter time or of a smaller force for a longer time.

The concept of momentum is of particular importance in connection with problems on blows, and action and reaction between moving bodies (see I-3.4B and I-3.12).

Unit : Momentum is *mass × velocity*. So it is measured as 1 kg m/s in SI or mks system, as g cm/s in cgs system and lb-ft/s in fps system

$$1 \text{ kg m/s} = 10^3 \text{ g} \times 10^2 \text{ cm/s} = 10^5 \text{ g cm/s}$$

Its dimension is thus MLT^{-1}.

I-1.7. Kinematical Equations in One dimension or Laws of Uniformly Accelerated Motion :

We consider the motion of a particle along a straight line OAB (Fig. I-1.11) starting with a velocity u from the point A accelerating uniformly with a and reaching a velocity v at B after time t at a distance s and continuing beyond. Note that we are here concerned with *five* quantities and we shall establish four algebraical relations, each containing *four* of them. They are

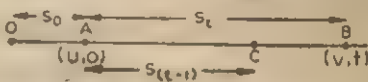


Fig. I-1.11

(*i*) $v=u+at$ (*ii*) $S=S_0+ut+at^2$ (*iii*) $v^2=u^2+2as$

(*iv*) $S_t-S_{t-1}=u+\frac{1}{2}a(2t-1)$

(i) v = u + at : By definition, acceleration is given by

$$a=\frac{\text{Change of Velocity}}{\text{Time taken}}=\frac{v-u}{t}$$

$$\therefore \quad v=u+at \qquad \text{(I-1.7.1)}$$

(ii) $S=S_0+ut+\frac{1}{2}at^2$: Refer to fig. I-1.11 and note that the particle at A is at a distance *So* from the origin at O. It starts from A with a velocity u and attains a velocity v at B, distance s away after time t. Then the average velocity between A and B is $\frac{1}{2}(u+v)$ and the distance S_t it covers, is

$$S_t=\frac{1}{2}(u+v)t=\frac{1}{2}(u+u+at)t=ut+\frac{1}{2}at^2 \qquad \text{(1-1.7.2)}$$

$$\therefore \quad S_{OB}=s_{OA}+s_{AB}=S_0+ut+\frac{1}{2}at^2 \qquad \text{(I-1.7.2a)}$$

[Samsad specimen question '79]

(iii) $v^2=u^2+2as$: This relation is obtained by eliminatng t between the other two relations. Thus

$$v^2=(u+at)^2=u^2+2uat+a^2t^2=u^2+2a(ut+\frac{1}{2}at^2)$$
$$=u^2+2as. \qquad \text{(I-1.7.3)}$$

(iv) Distance covered in a particular (e.g. *t*-th) second. It is obtained by subtracting the distance (OC) covered in a total of $(t-1)$ seconds from that (OB) covered in all the t seconds. It is *actually distance covered in 1 second.*

Thus $S_t-S_{t-1}=[\,s_0+ut+\frac{1}{2}at^2\,]-[\,s_0+u(t-1)+\frac{1}{2}a(t-1)^2\,]$

$$=u+\frac{1}{2}a(2t-1) \qquad \text{(I-1.7.4)}$$

Ex. 1-1.2. *A particle starting from rest travels first with uniform acceleration a and then with uniform retardation b. If it stops t secs after it starts and covers a distance s then show that $t^2=2s\,(1/a+1/b)$.* [H. S. '70]

Solution : Let the particle attain a velocity v after $t_1 s$ and then stop after t_2 s. Then

$$v=0+at_1 \quad \text{and} \quad 0=v-bt_2$$

$$\therefore \quad t=t_1+t_2=v\left(\frac{1}{a}+\frac{1}{b}\right) \text{ or } t^2=v^2\left(\frac{1}{a}+\frac{1}{b}\right)^2$$

If again it covers distances s_1 and s_2 during the same intervals, $v^2=2as_1$ and $v^2=2bs_2$ as it starts from rest and finally stops.

$$\therefore \quad S=\frac{v^2}{2}\left(\frac{1}{a}+\frac{1}{b}\right) \quad \text{or} \quad t^2=2s\left(\frac{1}{a}+\frac{1}{b}\right)$$

Ex. 1-1.3. *A car accelerates from rest at a constant rate of α for some time and then decelerates at a constant rate β to come to a stop. If the toal time taken be t find the maximum velocity reached and the total distance covered.* [I. I. T. '78]

Solution : Let the car accelerate for time t_1 from rest to attain a velocity of v. If then retards for time $(t-t_1)$ before stopping. Then

$$v=\alpha t_1 \text{ and } v=\beta(t-t_1) \text{ or } \alpha t_1=\beta(t-t_1)$$

$$\therefore \quad t_1=\beta t/(\alpha+\beta)$$

So the maximum velocity attained is $v=\alpha t_1=\dfrac{\alpha\beta t}{\alpha+\beta}$

Again the total distance covered is

$$s=s_1+s_2=\tfrac{1}{2}\alpha t_1^2+\tfrac{1}{2}\frac{v^2}{\beta}=\tfrac{1}{2}\alpha t_1^2+\tfrac{1}{2}\cdot\frac{1}{\beta}\cdot\beta^2(t-t_1)^2$$

$$=\tfrac{1}{2}\alpha\left(\frac{\beta t}{\alpha+\beta}\right)^2+\tfrac{1}{2}\beta\left(t-\frac{\beta t}{\alpha+\beta}\right)^2=\tfrac{1}{2}\frac{\alpha\beta}{(\alpha+\beta)^2}t^2$$

Ex. 1-1.4. *Two trains are approaching each other with velocities v_1 and v_2 along the same line. The drivers apply brakes when they are x apart producing decelerations f_1 and f_2. The trains just avoid colliding if*

$$v_1^2 f_2+v_2^2 f_1=2f_1f_2x$$

Solution : Let the trains cover d_1 and d_2 respectively. In this case $x=d_1+d_2$. Now clearly $v_1^2=2f_1d_1$, and $v_2^2=2f_2d_2$

$$\therefore \quad x=d_1+d_2=\tfrac{1}{2}\left(\frac{v_1^2}{f_1}+\frac{v_2^2}{f_2}\right)=\tfrac{1}{2}\,\frac{v_1^2 f_2+v_2^2 f_1}{f_1 f_2}$$

$\therefore$ Hence the result follows.

Prob. (1) An express train moving with velocity u_1 is overtaking a goods train moving with u_2 along the same lines. If the former applies maximum deceleration f_1 and the latter maximum acceleration f_2 when at a separation of x show that $(u_1-u_2)^2=2(f_1+f_2)\,x$ is the condition for just avoiding collision.

(2) A car A is travelling along a straight level road at 60 km/hr. It is followed by another, B at 70 km/hr. At a separation of 2.5 km, B is decelerated at 20 km/hr². After what time and at what distance will B catch up with A? (*Ans.* ½ hr., 32.5 km). [I.I.T. '66]

Ex. 1-1.5. *A particle moving with uniform acceleration f covers a distance s in time t and s' in t'. Show that*

$$f=\frac{2(s'/t'-s/t)}{t+t'}\qquad\text{[H.S. '69]}$$

Solution : If the initial velocity of the particle be u then

$s=ut+\frac{1}{2}ft^2$ and $s+s'=u(t+t')+\frac{1}{2}f(t+t')^2$

$\therefore\ s'=ut'+\frac{1}{2}f(t'^2+2tt')$

$\therefore\ s'/t'-s/t=\frac{1}{2}f(t+t')$. **Hence the result.**

Ex. 1-1.6. *A body travels 200 cm in the first two sec and 220 cm in the next four secs. What will be the velocity at the end of the seventh sec from the start?*

[I. I. T. '64]

Solution : From the given data we have

$200=u.2+\frac{1}{2}f.4$ or $u+f=100$

and $420=u.6+\frac{1}{2}f.36$ or $u+3f=70$

or $f=-15$ cm/s² and $u=115$ cm/s

$v_7=u+ft=115-15\times7=10$ cm/s

Ex. 1-1.7. *A particle moving with uniform acceleration covers 9/25th part of the whole distance covered, in the last second of its journey. If it had started from rest and covered 6 cm in the first second how long and how far has it gone?*

[C. U. '68]

Solution : From the given data

$6=\frac{1}{2}f(1)^2$ or $f=12$ cm/s² $S_t-S_{t-1}=\frac{9}{25}S_t$

$\therefore\ 2f(t-1)=\frac{9}{25}S_t=\frac{1}{2}ft^2$ or $12(t-1)=\frac{9}{25}6t^2$

or $\frac{9}{25}t^2=2(t-1)$ or $9t^2-50t+25=0$ or $(t-5)(9t-5)=0$

$\therefore\ t=5$ s the other result being less than 1, is unacceptable.

$$S_t=\frac{1}{2}ft^2=1.5\text{m}$$

Ex. 1-1.8. *A bullet loses half its velocity after penetrating 3″ of a plank. Considering the resistance offered to be uniform how much more will it penetrate?*

[Tripura H. S. '79]

Solution : Let the initial velocity be u. Then from the given datum

$\left(\frac{u}{2}\right)^2=u^2-2f.3$ or $f=\frac{1}{6}\left[u^2-\left(\frac{u}{2}\right)^2\right]=\frac{3u^2}{6\times4}$

Then $0=\left(\frac{u}{2}\right)^2-2fs$ or $S=\frac{u^2}{8f}=\frac{u^2.24}{8\times3u^2}=1''$.

Problem : A bullet moving at 200 m/s can just pierce a plank 4 cm thick. Find the velocity it must have to go through a plank 10″ thick. (*Ans.* 348 m/s).

[J. E. E. '67]

Application of Calculus: Although not explicitly stated, u and v in above equations are instantaneous velocities and t is actually built up of or summation of a very large number of very small intervals of time dt. Hence the use of calculus in deriving the above equations.

A. By definition $a=\dfrac{dv}{dt}$ or $dv=a.dt$.

Integrating we get $v=at+c_1$; c_1 const of integration

Now when $t=0$ i.e. at the initial moment v (i.e. u) $=c_1$

$\therefore \quad v=at+u$

Alternatively, $dv=a.dt$ or $\int_u^v dv=a\int_0^t dt$

or $\quad v-u=at \quad \therefore \quad v=u+at$

B. By definition $v=\dfrac{ds}{dt}$ or $ds=v\,dt=(u+at)dt$

Integrating $s=\int u.dt+\frac{1}{2}a\int t.dt+c_2$

$\qquad =ut+\frac{1}{2}at^2+c_2$

At $t=0$, we get $s=s_0=c_2$

$\therefore \quad s=ut+\frac{1}{2}at^2+s_0$

Alternatively,

$$\int_{s_0}^{s} ds=u\int_0^t dt+\tfrac{1}{2}a\int_0^t t\,dt$$

or $s-s_0=ut+\frac{1}{2}at^2$

C. By definition $a=\dfrac{dv}{dt}=\dfrac{dv}{ds}\cdot\dfrac{ds}{dt}=\dfrac{dv}{ds}\cdot v$

or $v.dv=a.ds$

Integrating $\frac{1}{2}v^2=as+c_3$

At $s=0$, $v=u$ or $c_3=\frac{1}{2}u^2$

$\therefore \quad v^2=u^2+2as$

Alternatively, $\int_u^v v\,dv = \int_0^s a\,ds$

or $\frac{1}{2}(v^2 - u^2) = ads$ or $v^2 = u^2 + 2as$

Note from the relation $a = v\,(dv/ds)$ that *acceleration is not only* time-rate of change of velocity but *also proportional to space-rate of variation of velocity.* We shall utilise this aspect in relevant cases.

The four equations I-1.7.1 to I-1.7.4 describe fully the motion of a particle in a straight line with constant acceleration. These are the basic equations of kinematics. *The first and the second equations are fundamental.* The other two can be derived from them.

Ex. I-1.8. *A particle starts from rest and acquires a velocity of 42 cm/s in 3.5s. Find the acceleration and the distance it travels in 3 seconds.*

Solution : Since the particle starts from rest, its initial velocity $u=0$.

$$a = \frac{v}{t} = \frac{42\text{cm/s}}{3.5\text{s}} = 12 \text{ cm/s}^2$$

The distance traversed in 3 seconds

$$S = 0 + \tfrac{1}{2} \times 12 \text{ cm/s}^2 \times (3\text{s})^2 = 54 \text{ cm}.$$

Ex. I-1.9. *The acceleration of a particle is 4 m/s². What will be its velocity after it has travelled 20 000 cm from rest ?*

(*Note*– In solving problems in which the units belong to different systems, they must all be reduced to the same system. Let us use the mks systems, here.)

Solution : Here $a = 4\text{m/s}^2$, $u=0$, $s = 20\,000$ cm $= 200$ m. To find v. From Eq. I-1.7.3, that is $v^2 - u^2 = 2as$, we get $v^2 = u^2 + 2as$.

Or $v^2 = 2 \times 4\text{m/s}^2 \times 200\text{m} = 1600\ (\text{m/s})^2$. $\therefore v = 50$m/s. (Try working it out using the cgs system.)

Ex. I-1.10. *A particle moves 25 cm in the 3rd second and 55 cm in the 6th second. Find how far it will travel in 8 seconds.*

Solution : (The quantities are all in cgs units. So no change of unit is necessary). Let u=initial velocity a=the acceleration of the particle in cgs units. The 3rd second is the interval between the completion of 2 seconds and 3 seconds. If s_2 and s_3 represent respectively the distance traversed in 2 and 3 seconds then $s_3 - s_2$ is the distance traversed in the third second.

$$\therefore\ 25 = (3u + \tfrac{1}{2}a.3^2) - (2u + \tfrac{1}{2}a.2^2) = u + \tfrac{5}{2}a.$$

Similarly, $55 = s_6 - s_5 = (6u + \frac{1}{2}a.6^2) - (5u + \frac{1}{2}a.5^2) = u + \frac{11}{2}a.$

Solving for u and a from these two relations, we get $u=0$ and $a=10$ in cgs units, that is, $a = 10$ cm/s².

Distance traversed in 8 seconds is

$$= 8u + \tfrac{1}{2}a.8^2 = 0 + \tfrac{1}{2}\,.\,10 \text{ cm/s}^2.(8\text{s}) = 320 \text{ cm}.$$

Problem : An accelerated particle passes 51 ft in the 4th and 75 ft in the 8th sec. How fare will it go in the 10th sec ? (Ans. 600 ft.) [H. S. '80]

I-1.8. Graphical Representations and deductions of Kinematical Laws :

Motion of a particle, varying quantities connected with it and their inter-relations when represented graphically, give us vivid pictures of what happen.

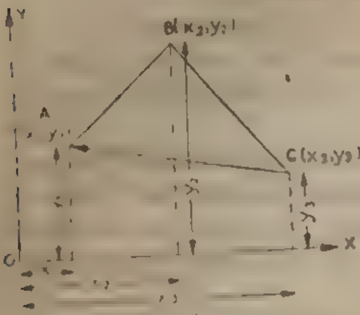


Fig. 1-1.12

A. Displacement. Referred to a reference frame in our case, plane cartesian system, displacement of a particle from A to B and from B to C and back to A is shown in fig. I-1.12. Obviously from co-ordinate geometry

$$AB = \sqrt{(x_2{}^2 - x_1{}^2) + (y_2 - y_1{}^2)}, \quad BC = \sqrt{(x_3{}^2 - x_2{}^2) + (y_3{}^2 - y_2{}^2)}$$

$$CA = \sqrt{(x_3{}^2 - x_1{}^2) + (y_3{}^2 - y_1{}^2)}$$

B. Variable Velocity : 1. Distance-velocity Graph. Let a policeman chase a run-away thief who constantly changes speed and direction inside a market place but ultimately is caught up, say after 5 minutes and 100 m away. His path of flight is shown in fig. I-1.13 We cannot decide his speed at different points and so settle for an average which is 100 m/s minutes, or 20 metres a minute. The points a, b, c, ... etc. give the

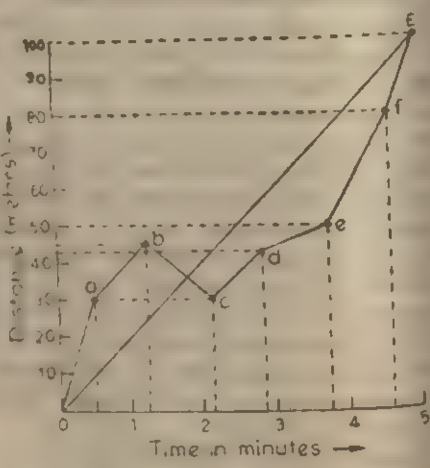


Fig. 1-1.13

points of sudden changes, horizontal broken lines measure the inter-

vals of straight flight and the vertical ones his distances from the base line. This is an example of variable velocity, where v and s are the variables.

2. Time-velocity graph. The instantaneous velocities of a moving body at successive moments may be plotted graphically also against the time at least in principle and the points connected together by a *smooth curve*. Such a line is called a *time-velocity graph*. Let the instantaneous velocities at different times be as follows :

Time in seconds	0	1	2	3	4	5
Velocity in m/s	2	2.4	3	3.5	3.8	4

Plot the time along the x-axis and the velocity along the y-axis. Join the points so plotted by a smooth line (Fig. I-1.14). This line is the time-velocity graph.

If the velocity is uniform, the graph will be a straight line AB parallel to the x-axis. *The distance transversed between any two moments will be given by the area bounded by the ordinates for the initial and final moments, the graph and the x-axis.* This statement holds *even when the velocity changes with time*. Area OABC gives the distance covered for *uniform* velocity (v) and area PQCO that for the plotted *variable* velocity (v)

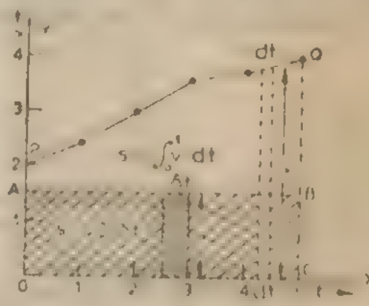


Fig. I-1.14

While studying integration as a summation process (0-2.9) we had seen that, taking $dy=f(x).dx$ and integrating the result

$$y=\int_a^b f(x)\,dx$$

represents an area bounded by the curve, the x-axis and perpendiculars to the x-axis from the end points of the curve. In the curve above, that process has been illustrated. With constant velocity (v) the distance covered, is the area obtained by summing up the elementary areas $v.\delta t$. For the variable velocity we take v' as the ordinate and dt a very small time interval, and sum up the strips $v'.dt$. Thus

Distance covered with constant velocity = Area = OABC (S) = $\Sigma v.\delta t$ and

Distance covered with variable velocity (v') = Area OPQC $(S') = \int_0^t v'.dt$

This approach will be utilised below to establish the second kinematical equation.

C. Constant Acceleration: The following table illustrates a case of *uniform acceleration.*

Time in seconds	0	1	2	3	4	5	6
Velocity in m/s	2	2.5	3	3.5	4	4.5	5

The velocity changes by 0.5 m/s every second. This is expressed by saying that the acceleration is 0.5 *metre per second per second* or 0.5 m/s^2, or m s^{-2}. In all expressions of acceleration the term 'per second' occurs *twice*, because the unit of time is involved twice in such expressions, —once in the velocity, and then again in the rate of change of velocity. [See I-1.6.3C. units]

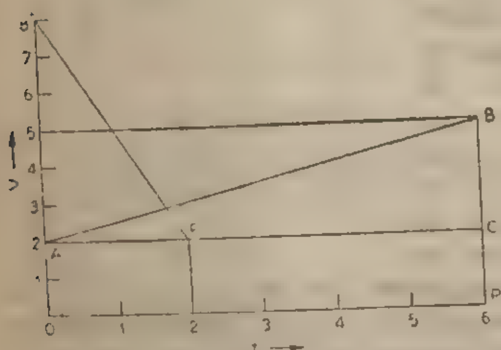


Fig. I-1.15

The time-velocity graph of the values in the above table is shown in fig. I-1.15. It is the straight line AB. When the acceleration is constant, the time-velocity graph will be a straight line, sloping upwards when positive.

The distance gone over between 0 and 6 seconds is area OABPO

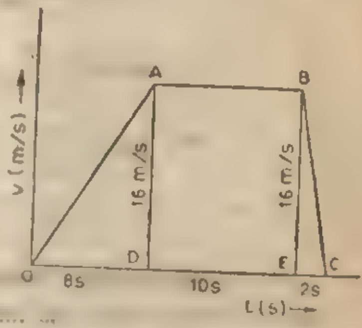
= rectangle OACP + ΔABC = OA $\times$ OP + $\frac{1}{2}$AC $\times$ BC. These areas are $2(m/s) \times 6s + \frac{1}{2}(6s \times 3m/s) = 12m + 9m = 21m$

If again the velocity diminishes uniformly from 8 m/s to 2 m/s after 2 seconds the deceleration would be $(8m/s - 2m/s)/2s = 3m/s^{-2}$. The acceleration would be $-3m/s^2$ and the time-velocity graph is a straight line (BE) sloping downwards.

Ex. I-1.11. *A car starts from rest on a straight track with an acceleration of 2 m/s². After it has reached a speed of 16 m/s, it moves for 10 seconds with the velocity it has acquired. It then decelerates at 8 m/s² until it stops. Find how long it has been on the move. Draw the time-velocity graph for different parts of its motion. From the area of the curve calculate the distance it has moved.* (ICSE)

Solution. The car accelerates from rest to a velocity of 16 m/s at the rate of 2 m/s². Hence the time taken is $t_1 = 16\ \text{m/s} \div 2\ \text{m/s}^2 = 8$ s. In the next stage of uniform motion the time $t_2 = 10$ s (given). In deceleration, the time $t_3 = 16\ \text{m/s} \div 8\ \text{m/s}^2 = 2$ s. The total time the car has been moving, is therefore, $t_1+t_2+t_3 = (8+10+2)$ s $= 20$ s.

To find the distance moved, draw the time-velocity graph as shown. Note that for the first 8 seconds the velocity v increases uniformly 0 to 16 m/s. For the next 10 seconds v = constant = 16 m/s. For the last 2 seconds, v diminishes uniformly by 8 m/s per second.



The distance traversed is the area under the line OABC. We can divide it into three parts, namely, the triangles OAD and BEC and the rectangle ABED. With the values of v and t as known for the different parts, the total area $= \frac{1}{2}\times 8\times 16+16\times 10+\frac{1}{2}\times 2\times 16 = 64+160+16$ in (m/s×s) unit that is in meters. *Answer* : 240 m.

Complete the same result from the trapezium OABCO. [See (*b*) below]

D. Derivation of Kinematic Equations :

(a) Proof of v = u + at. Referring to fig I-1.15 we find that at $t = 0$, $v_0 = 2$m/s, at $t = 1$s $v_1 = 2+0.5 = u+a$, at $t = 2$s, $v_2 = 2+0.5\times 2 = u+2a$, at $t = 3$s, $v_3 = 2+0.5\times 3 = u+3a$ etc. So we may by extrapolation say that at $t = t$

$$v = u+at \quad \text{for} \quad PB = PC+CB$$

Note that for a straight line the relation $y = c+mx$ is similar in form where $a = m$, the slope of the v vs t straight line and $u = c$, the constant intercept on the v i.e. y-axis.

(b) Proof of $S = ut + at^2$: In discussing figs. I-1.14 and I-1.15 we have noticed how the distance covered by an accelerating body may be found by graphically finding an area. Now refering to fig. I-1.15 we observe that

S = Area of the trapezium OABPO

$= \frac{1}{2}$ (sum of parallel sides) × perp. distance between them

$$= \frac{1}{2}\,(AO+BP)\times OP$$
$$= \frac{1}{2}\,[u+(u+at)]\times t = ut+\tfrac{1}{2}at^2$$

(c) **Proof of $v^2=u^2+2as$.** From fig. I-1.15 note that the trapezium OABD has an area

$$S = \tfrac{1}{2}(AO+BP)\times OP = \tfrac{1}{2}(AO+BP)\times AC$$

$$= \tfrac{1}{2}(AO+BP)\frac{AC}{BC}\times BC$$

$$= \tfrac{1}{2}(AO+BP)\frac{AC}{BC}\times (BP-PC)$$

$$= \tfrac{1}{2}(u+v)\frac{t}{(v-u)}\times (v-u)$$

$$= \tfrac{1}{2}(u+v)\frac{1}{a}(v-u)$$

$$\therefore\quad v^2-u^2=2as$$

E. Instantaneous Velocity : It has been stated already that Newton invented the infinitesimal or differential calculus when investigating the question of finding the limiting value of the ratio $\Delta s/\Delta t$ i.e. instantaneous velocity. In fig. I-1.16 is plotted a time-displacement graph (A) for a car moving along a busy thoroughfare but in a constant direction and forced to change speed at random. The distances covered in the same time-interval is different at different instants. The ratio $\Delta s/\Delta t$ which gives the average velocity over a small distance Δs is clearly tan θ i.e. the gradient of the tangent to the curve, say at a point B. Clearly this gradient at different points is different and so the *velocity is variable* and become zero at C.

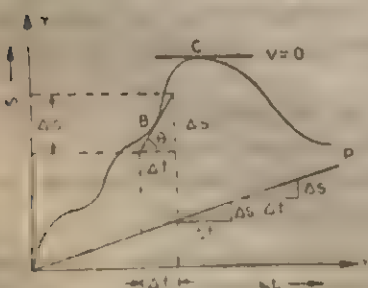


Fig. I-1.16

For the curve OP, the gradient is the same, the tangent coinciding with the curve itself and so it represents a *constant velocity*.

F. Instantaneous Acceleration : Let us again consider the same motion of the car where instead of displacement we plot velocity (fig. I-1.17) clearly variable, and the car actually stops momentarily at C. We consider two points A and B when the car is accelerating and another pair E and F when it is decelerating non-uniformly.

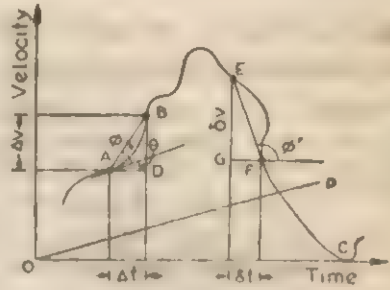


Fig. I-1.17

Clearly the average acceleration between A and B will be DB/AD = tan ϕ = $\Delta v/\Delta t$. Similarly the average deceleration between E and F will be $-$EG/GE = $-\delta v/\delta t$ = tan ϕ'. Now let B approach A. As the interval Δt diminishes the chord AB turns and finally in the limit becomes tangent at A at an angle θ to AD i.e. $\Delta v/\Delta t \to dv/dt$, the instantaneous acceleration at A. We may similarly develope the case of decleration at F becoming $-dv/dt$ from $-\delta v/\delta t$ for ϕ' being greater than $\frac{1}{2}\pi$, tan ϕ' is negative.

As before OP represents constant or uniform acceleration, as does AB in fig. I-1.15.

Ex. I-1.12. *Two particles start from a point simultaneously and move parallel to each other in the same direction. The first moves all along with a uniform velocity of 40 m/s ; the other starts with 16 m/s and accelerates with 6 m/s². Find when they meet again.* [C. U.]

Solution : Let t be the required time and s the distance civered by either. Then we have

$$S = vt \text{ and } s = ut + \tfrac{1}{2}at^2 \quad \text{or} \quad 40t = 16t + 3t^2 \quad \text{or} \quad (3t - 24)t = 0$$

$$\because \; t \neq 0 \text{ and hence we have } t = 8s.$$

Ex. I-1.13. *A train starts accelerating uniformly at 2 m/s² from rest. A person 9 m behind immediately starts running at full speed and just catches the train. Find his speed.* [J. E. E. '72]

Solution : Refer to fig. I-1.11. Let the man be at O and the rear of the starting train at A and again at B when the man catches up with it. Given OA = 9 m. Let AB be s m. Then from the given data.

$$S = \tfrac{1}{2} \times 2 \times t^2 \quad \text{and} \quad S + 9 = vt$$

or $S=t^2$ and $t^2-vt+9=0$

$$\therefore \quad t=\frac{v\pm\sqrt{v^2-4.9.1}}{2}$$

Now for t to be real, $v^2-36\geqslant 0 \quad \therefore \quad v=6$ m/s.

As it has been stated that he *just* manages to get in with $v=6$ m/s.

Problems: (1) A policeman running at u m/s sees a thief x m ahead and gives chase with a constant acceleration of α m/s. The thief starts off with an acceleration of β m/s². Show that the thief will be caught if

$$\alpha\geqslant\beta \text{ or } \alpha<\beta<(\alpha+\tfrac{1}{2}u^2/x)$$

[*Hint:* $S_P=ut+\frac{1}{2}\alpha t^2$; $S_T=S_P-x=\frac{1}{2}\beta t^2$. Subtracting $x=ut+\frac{1}{2}(\alpha-\beta)t^2$ or $(\alpha-\beta)t^2+2ut-2x=0$. Solve for t]

(2) A bus starts off with an acceleration of 1 ft/s². Show that if a man can run at 9 ft/s he can not catch it if he is more than $40\frac{1}{2}$ ft behind.

Ex. I-1.14. *The displacement of a particle x m, is related to the time taken by the equation* t s$=\sqrt{x}+3$. *Find the displacement when* $v=0$, *and displacement and* acceleration after 6 *s*. [I. I. T. '79]

Solution: From the relation given, we have

$$dt=\frac{dx}{2\sqrt{x}} \quad \text{or} \quad v=\frac{dx}{dt}=2\sqrt{x}$$

So when $v=0$, displacement x must be zero.

Again displacement after 6 s must be given from

$$\sqrt{x}=t-3=6-3. \quad \therefore \quad x=9\text{m}$$

$$\text{Acceleration } a=\frac{dv}{dt}=\frac{d}{dt}(2\sqrt{x})=2.\frac{dx}{2\sqrt{x}}.\frac{1}{dt}$$

$$=\frac{1}{\sqrt{x}}.2\sqrt{x}=2\text{m/s}^2$$

This is a const acceleration.

I-1.9. Acceleration due to gravity: By *gravity* we mean the pull of the earth on a body. A body is said to fall 'freely' when its fall under gravity is not opposed in any appreciable way. We now know that *for a body falling freely under gravity, the motion is one of* **constant acceleration.** The motion is considered however, close to the surface of earth.

The acceleration with which a freely falling body moves towards the earth is called the acceleration due to gravity. It is denoted by the symbol g. [*Do not* confuse it with the symbol 'g' for gram.]

g is 9.0 m/s^2, 980 cm/s^2, or 32 ft/s^2 in numerical problems unless some other value is given.

Equations of vertical motion under gravity: When a body moves under the action of gravity, its motion is determined by the equations

$$v = u \pm gt \tag{I-1.9.1}$$

$$h = ut \pm \tfrac{1}{2} gt^2 \tag{I-1.9.2}$$

$$v^2 - u^2 = \pm 2gh \tag{I-1.9.3}$$

$$h_n - h_{n-1} = u \pm \tfrac{1}{2}g(2n-1) \tag{I-1.9.4}$$

h is the vertical distance travelled from the starting point in the direction of u. We have put h for s and g for a in Eq.s I-1.7.1 to I-1.7.4.

The *plus sign* is used for *falling bodies*; the *minus sign* for bodies *thrown vertically upwards*. This is so because during upward motion, the velocity and the acceleration are opposite in directions.

A. Free fall under gravity. When bodies fall from rest, $u = 0$. The equation of free fall *from rest* under gravity then reduces to

$$h = \tfrac{1}{2}gt^2 \tag{I-1.9.5}$$

h is positive downward and proportional to t^2

If projected vertically downward with an initial velocity u, the equation will be

$$h = ut + \tfrac{1}{2}gt^2. \tag{I-1.9.6}$$

B. Maximum height of ascent. A body projected vertically upward gradually loses speed as it rises. This is so because it is decelerated by gravity, which is directed downward. For upward projection, h and u are positive while g is negative. The speed gradually falls to zero; it is then at the topmost point of is rise. v, the *velocity at the topmost point, is zero.* Not here, velocity is zero but acceleration exists. Thereafter it starts moving downward.

If u is the initial velocity of projection, and H the maximum height of ascent, then from the relation $v^2 - u^2 = -2gh$, we get

$$0 - u^2 = -2gH \quad \text{or} \quad H = u^2/2g. \tag{I-1.9.7}$$

C. Time for ascent. How long does the body take to rise to its maximum height? If T is this time, then from the relation $v=u-gt$, we get

$$0=u-gT \text{ or } T=u/g. \tag{I-1.9.8}$$

It is easy to show that the *time for ascent=time for descent.* The time T_1 for descent is the time required by the body to fall from rest through a height $H=u^2/2g$. Then $u^2/2g=\frac{1}{2}gT_1^2$ or $T_1=u/g$. This is also the time for ascent.

D. Two values of time to reach the same height. When a body is projected vertically upward, it will be at a given point of its path at two moments—once during rise and again during fall. Since for upward projection $h=ut-\frac{1}{2}gt^2$ we have $\frac{1}{2}gt^2-ut+h=0$. This is a quadratic equation in t. Solving for t, we get

$$t=\frac{u\pm\sqrt{u^2-2gh}}{g} \tag{I-1.9.9}$$

h must be less than the maximum height of ascent $H=u^2/2g$.

E. Same speed at the same height during ascent and descent. A body projected vertically upward will have the same speed at the same point of its path while going up and coming down. This follows from the relation $v^2-u^2=-2gh$.

$$\text{or} \quad v=\pm\sqrt{u^2-2gh} \tag{I-1.9.10}$$

The plus sign relates to the upward velocity and the minus sign, to the downward velocity.

Ex. I-1.15. *A stone falls freely from rest from the top of a tower. When it has fallen though x ft, another is let fall from a point y ft below the same top. If they reach the ground together show that the tower is $(x+y)^2/4x$ ft high.*

[H. S. '71]

Solution : The first stone in falling through x aquires a velocity of $\sqrt{2gx}$ and covers the rest of the distance $(h-x)$ during the time the second stone takes to cover $(h-y)$. Let this time interval be t. Then we have

$$h-x=\sqrt{2gx}.t+\tfrac{1}{2}gt^2 \text{ and } h-y=\tfrac{1}{2}gt^2$$

Subtracting the second from the first we get

$$y-x=\sqrt{2gx}.t \text{ or } t=\frac{y-x}{\sqrt{2gx}}$$

$$\text{Now } h-y=\tfrac{1}{2}gt^2=\tfrac{1}{2}g\frac{(y-x)^2}{2gx}=\frac{(y-x)^2}{4x}$$

$$\therefore \quad h=\frac{(y-x)^2}{4x}+y=\frac{(y-x)^2+4xy}{4x}=\frac{(x+y)^2}{4x}$$

Ex. I-1.16. *A heavy particle is projected vertically upwards from a point Q so as just to reach the point P and at the same instant another heavy particle is dropped from rest from P. Show that the spaces passed over would be 3:1 but velocities equal and opposite.* **[C. U.]**

Solution : Let them meet at a point N in between P and Q. Let PN $\downarrow = x$ and QN $\uparrow = y$. Then if t be the time for the second particle to fall to N from P and the first to rise to N from Q, we shall have

$$x-y=\tfrac{1}{2}gt^2 \quad \text{and} \quad y=ut-\tfrac{1}{2}gt^2$$

u being the *upward* velocity of projection of the first particle. Then

$$(x-y)+y=x=ut \quad \text{or} \quad t=x/u$$

Now the *downward* velocity of the particle from P at N is

$$v_1=gt=gx/u$$

Again the second particle just rises to P so that QP is the maximum height

$$\text{or} \quad 0=u^2-2gx \quad \text{or} \quad u^2=2gx$$

Then at N the *upward* velocity would be

$$v_2=u-gt=u-(gx/u)=(u^2-gx)/u$$

$$=\frac{2gx-gx}{u}=\frac{gx}{u}$$

We find then at N the velocities upward and downward are equal. Again the height descended by the particle from P to N is

$$x-y=\tfrac{1}{2}gt^2=\tfrac{1}{2}gx^2/u^2$$

and the height ascended by the particle to N from Q is

$$y=ut-\tfrac{1}{2}gt^2=x-\tfrac{1}{2}g\left(\frac{x^2}{u^2}\right)=\frac{2u^2x-gx^2}{u^2}=\frac{2x.2gx-gx}{u^2}=\frac{3gx^2}{2u^2}$$

$$\therefore \quad \frac{y}{x-y}=\frac{3gx^2}{2u^2}\div\frac{gx^2}{2u^2}=3:1.$$

Ex. I-1.17. *A particle takes t s to rise vertically through a height h. It returns t_2s later to the ground. Show that $h=\frac{1}{2}gt_1t_2$.* **[H. S. '65, '69]**

Solution: If it be projected with an upward velocity of u, then

$$h=ut_1-\tfrac{1}{2}gt_1^2$$

Since it returns to the ground (t_1+t_2) s later we write

$$0=u(t_1+t_2)-\tfrac{1}{2}g(t_1+t_2)^2$$

If we take u as +ve we are to regard g as −ve. From the two relations

$$(h/t_1)=u-\tfrac{1}{2}gt_1 \text{ and } u=\tfrac{1}{2}g(t_1+t_2)$$

$$\therefore \quad \frac{h}{t_1}=\tfrac{1}{2}g(t_1+t_2)-\tfrac{1}{2}gt_1=\tfrac{1}{2}gt_2$$

$$\therefore \quad h=\tfrac{1}{2}gt_1t_2.$$

Ex. I-1.18. *From a balloon descending with 20 ft/s a body is released and it reaches the ground in 10 s. Find the velocity with which it strikes the ground and from what height was it released? ($g=32$ ft/s^2)* **[B. H. U.]**

Solution: If v be the required velocity then we have

$$v=u+gt=20+32\times 10=340 \text{ ft/s}$$

If h be the required height then

$$h=ut+\tfrac{1}{2}gt^2=20\times10+16\times100=1800 \text{ ft.}$$

Ex. 1-1.19. *Two stones are thrown upwards together. A rises 112 ft more and returns to the ground 2s later than B does. Find their velocities of projection. ($g=32$ ft/s²)*

Solution: Let them be projected with vertical velocities u_A and u_B

Then $h_A-h_B=\dfrac{u_A^2}{2g}-\dfrac{u_B^2}{2g}=\dfrac{u_A^2-u_B^2}{64}=112$ ft (given)

and $T_A-T=\dfrac{u_A}{g}-\dfrac{u_B}{g}=\dfrac{u_A-u_B}{32}=1\text{s}$ (given)

Thus $u_A-u_B=32$ ft/s and $(u_A^2-u_B^2)=112\times64$

Dividing the second by the first $u_A-u_B=224$ ft/s

$\therefore$ $u_A=128$ ft/s and $u_B=96$ ft/s.

Problems: (1) How far will a body fall from rest in 5 seconds? Take $g=9.8$ m/s². [Ans: 122.5 m]

(2) If the same body were projected downward with a velocity of 10 m/s, how far would it fall in 5 seconds? [Ans: 172.5 m]

(3) A body is projected vertically upward with a velocity of 20 m/s. Find (a) how far it will rise, (b) how long it will take to reach to topmost point. Find also when it will be at a height which is half of the maximum. What will be its velocity then?

[*Hints*: (a) $v=0$ at the top. Put $v=0$ in $v^2-u^2=-2gh$ and find h. $h=(20\times20)/(2\times9.8)$m. (b) At the top $u-gt=v=0$. This will give t in seconds. Half the max. height is about 10.2 m. Find t from the relation $10.2=20t-\tfrac{1}{2}\times9.8\times t^2$. You will get two values of t. The shorter one is for ascent, and the longer one, for descent.]

(4) Two stones are projected from the top of a tower 100m high, each with a velocity of 20 m/s, one vertically upwards and the other vertically downwards. Calculate the time each takes to reach the ground and the velocity with which each strikes the ground.

(I. C. S. C.)

[Ans: Upward velo. = 48.59 m/s downward; time ≃ 7 s Downward: Velocity same; time = 2.92 s nearly.]

VECTORS

I-2.1. Scalars and Vectors

In course of your studies, you have come across two classes of quantities, namely scalars and vectors. Mass, speed, time, volume etc. are physical quantities that require *magnitudes only* to specify them; they are said to be *scalars*. Again another class of physical quantities, like displacement, force, velocity, *require directions to be specified in addition to their magnitudes*; they are *vectors**.

A scalar can be uniquely represented by a quantity with a unit. That a *mass* is 10 kg provides a complete description. A car is moving at 50 km a hour completely describes its *speed*, nothing else is needed. In them, 10 is the magnitude in kg units, 50 is the magnitude in km/hour. But a vector needs a direction to be specified as well. Take a simple example; your school must be in a certain direction, say north-east from your home. You require half an hour to go there, two miles away across a large field. Your velocity is 4 miles an hour in the N-E direction.

The *mathematical processes* like addition, subtraction, multiplication, division, involution, evolution etc. *are not always identical* for scalars and vectors as we shall see; e.g. scalars are added algebraically, vectors geometrically; multiplication of two scalars gives just one product, whereas multiplication of two vectors gives you two, a *dot* product (involving cosine of their included angle) and a *cross* product (involving the sine of their included angle).

We shall later come across scalars like power, work, energy, temperature, potential etc. and vectors like torque, momentum, magnetic moment, e.m.f., electric current-density, field intensity, flux etc.

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* The word comes from the Latin word *Veho* (*I carry*) which clearly includes an idea of movement in a certain direction. In olden days, vector or radius vector in astronomy, meant the imaginary line joining the moving planet to the sun. The orbit being an ellipse according to Kepler's 1st law, this radius vector continuously changed its magnitude and direction. Because of this twofold change in magnitude and direction, they came to be associated with a vector.

Use of vectors provides a particular mathematical operation only. It enjoys *two particular advantages* over others, namely

(1) Vectorial expression of physical relations (*i.e. laws of physics*) *is ideal, for it is independent of any co-ordinate system.*

(2) Vector notation is short. Laws of physics thus expressed assumes a clear, transparent easy-to-follow form; expressed in some definite co-ordinate system the same law loses clarity, as it becomes cumbersome.

* I-2.2. Geometrical Arithmetic :

In discussing vectors it pays to develop the habit of thinking of numbers graphically or in terms of co-ordinates. So we try to give you first a geometrical picture of natural numbers.

In the Cartesian system XOX′ is a horizontal straight line and said to be the *x*- or the *real* axis. The line YOY′ is a vertical line, the

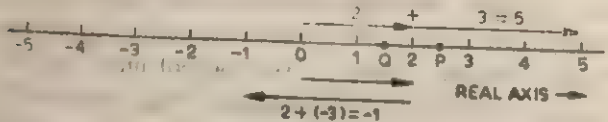


Fig. I-2.1.

y- or the *imaginary* axis. These names will be clarified when you study *Complex Numbers*. We consider now only the former on which any point O may be taken as the origin. On this line mark points 1, 2, 3......etc. to the right from O in any unit, say cm ; to the left of O let there be similar markings −1, −2, −3......etc. (Fig. I-2.1). Consider the line extending both ways to infinity.

The connection between real numbers and geometry lies in the fact that—on this straight line there is a definite point *corresponding* to any and every real number ; *each point on the line represents a real number.* Mathematicians term this fact as one-to-one correspondence. For example, the number 2.45 would lie 45% of a cm to the right of the point (P) marked 2 ; again we shall indicate by Q, the number $\sqrt{2}$ (=1.41) placed to the right of the point marked 1 by 41% of a cm and to the left of 2. Thus every geometrical point

* For the more inquisitive student. May be safely omitted by otthers.

on the line would represent a number. Their values would equal the distance of their representative points from the origin; the +ve numbers would lie to the right of O, and $-$ ve numbers to its left. We may consider the real numbers (integers, say) to be a swarm of birds perching side by side on a telegraph wire.

$2+3$ makes 5. To add geometrically we move 2 divisions to the right and then 3 more divisions—which is 5 divisions to the right of O. To add a +ve number to a $-$ve one [say $2+(-3)=-1$], we move first two divisions to the right and then *from there*, 3 divisions to the left, ending at 1 division to the left of O. To add two negative numbers, -3 to -5 we move 3 divisions to the left and 5 divisions more ending up at 8 divisions to the left, indicating -8. Decimals and fractions may be similarly added and subtracted. These results reveal a formal property of algebraical addition—*the commutative law.* This law states that the sum of numbers remain unchanged if the sequence of numbers are reversed (e.g. $3+2=2+3=5$ or $a+b=b+a$).

Multiplication is repeated addition e.g. $4\times3=4+4+4=12$—you move thrice successively, 4 divisions each time and end at 12 divisions to the right of O. Again for a division which is subtraction repeated, say $8\div4=8-(2+2+2)=2$, we move 4 times successively to the right, 2 divisions at a time and then return by 2 divisions thrice, to end up at 2 divisions to the right of O.

Another property concerning algebraic addition and multiplication is the *distributive law* stating that $c(a+b)=ca+cb$; i.e. reversing the order of multiplication does not affect the final result. It can be cleared up from this standpoint also; e.g. we move 2 divisions and 3 divisions successively and again 2 and 3 divisions finally reaching 10 divisions to the right. If we again move 2 divisions twice successively and then again 3 divisions twice we move over 10 divisions as before.

Thus we translate the abstract idea of numbers to a concrete picture through geometry. A physicist conceives a number in three steps, *first* the abstract magnitude, *second* its geometric representation by a point on a line and its distance from a chosen origin, providing a concrete picture and *thirdly* a measurable physical quantity. The numerical value of a physical quantity, gives not only its

magnitude but also states that it obeys the commutative and distributive laws.

1-2.3. Vectorial Arithmetic or Algebra :

We see then, numbers can be added or subtracted by moving to the right or left along the x- or real axis. Suppose now you move 3 m eastward and then 4 m to the north. How do we add ? As before we choose an origin O from which we move 3 divisions along the x-axis as before and then 4 divisions upwards parallel to the y-axis to P when our displacement the shortest distance, is OP measuring 5 units and at an angle of 52.1° north of east. Hence in this new process of addition, *distance and direction both are relevant* and hence for the straight line giving the sum, both magnitude and direction are necessary. This *directed number system* belongs to vectorial arithmetic or better, vector algebra. The resultant displacement is indicated by arrow-tipped line segment OP as shown in fig. 1-2.2.

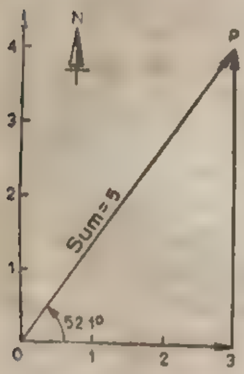


Fig. 1-2.2.

Hence a vector is represented by a *directed line segment of proportional length drawn parallel to it, with an initial point* (*O*) *and a terminal or end or final point* (P), *arrow-tipped near the end point.* Just as a number may be represented on a straight line along the x-axis by its distance from a chosen origin, a vector may be represented by an arrow-tipped line segment joining the initial to the final point. Just as one to one correspondence exists between a number and a point on the line, that also exists between a vector and the directed line segment. This latter is the pictorial representation of a vector.

Like a number, a vector is a mathematical entity, a way of representation. To the physicist, the vector is a very convenient tool to describe the physical world. Many physical entities like displacement, velocity, force, momentum etc. cannot be fully described by numbers alone—they behave as vectors do *viz.* their resultants are similar to those of vectors. Scalars are those that *behave like*

numbers ; e.g. 2 g mass added to 2 g of mass gives 4 g but adding 2 dynes of force to 2 dynes may result in *any* value between 0 and 4 dynes, depending on their directions. We say force behaves as a vector does.

Vectors like scalars, embody three ideas- an abstract mathematical entity, a concrete or real geometrical representation and a physical quantity. For scalars, geometrical representation is attractive but not essential ; for vectors it is a must. Vectors are those that obey vector algebra i.e. vector laws of addition. But all quantities with magnitude and direction do not always follow this law. For example (1) Velocity behaves like a vector so long as *it is much smaller than velocity of light*, but not when close to that value (2) a rotation has both magnitude and direction but addition of two finite rotations do not obey law of vectorial addition (3) moment of inertia (1-6.8) is a very important entity in rotational dynamics which is not a vector though possessing both magnitude and direction ; it is a *tensor*. All physical quantities with magnitude and direction then do not behave as vectors do. An infinitesimally small area is a vector, not so a finite area.

1-2.4. Representation of vectors. This may be done in three ways, namely, (i) by *line segments*, (ii) by *components* and (iii) by *coordinates*.

A. In representing a vector by a **line segment,** a straight line is drawn parallel to the direction of the vector. A segment of the line is chosen to represent the vector. The length of the segment is made proportional to the magnitude of the vector to some convenient scale. The length of the line segment which represents unit magnitude of the vector is called the *unit vector* in the given direction. An arrowhead is put on the line segment to indicate the *sense* in which (i.e. the side towards which) the vector acts along the line. We have already said these above and yet *re-emphasise.*

To represent a force by a line segment, we have to consider the point at which it is applied, and also the magnitude and direction of the force. The point of application of a force is represented by the point from which the line segment is drawn. The line segment is parallel to the direction of the force, and its length is proportional to its magnitude. For vectors like velocity, acceleration, momentum, etc., we represent them by line segments of appropriate lengths drawn parallel to the directions of the actual quantities.

When a line segment AB represents a vector and the vector acts from A towards B, we write the vector as $\overrightarrow{AB}$, putting an arrowhead above AB. $\overrightarrow{BA}$ then represents a vector equal and opposite to

$\overrightarrow{AB}$. In $\overrightarrow{AB}$, A is the *initial point* of the vector and B the *end point*. In BA, B is the initial point and A the end point.

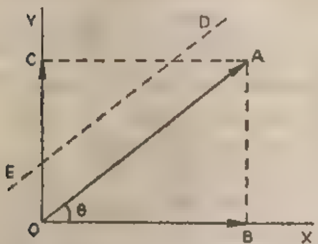


Fig. I-2.3.

B. Representation of a vector by components can be understood from fig. I-2.3. Suppose a vector **A*** acts along the line DE in the sense D to E. the line segment OA represent this vector. With O as origin draw a line OX as axis at any angle to OA. Draw another line OY as the axis perpendicular to OX in the same plane as OA and OX.

Let OB be the *projection* of OA on the OX axis and written as A_x. Then $A_x = \text{OB} = \text{OA} \cos\theta$. Similarly the projection A_y on the OY axis is $A_y = \text{OC} = \text{OA} \sin\theta$. We thus get the magnitude A of the vector **A** as

$$A = \sqrt{A_x^2 + A_y^2} \qquad \text{(I-2.4.1)}$$

Besides, the cosine of the angle between OA and the OX axis is $\cos\theta = A_x/A$; A_y/A is the value of the cosine of the angle between OA and the OY axis. So, if we know the values of A_x and A_y, we get both magnitude and the direction of the vector **A** in the reference frame (or co-ordinate system) which we have taken. A_x and A_y are called **components** of **A** in the reference frame chosen.

Above we have so chosen the plane of the axes, that it contains the line OA. But this is not essential. We may draw through O any three mutually perpendicular axes OX, OY and OZ (Fig. I-2.4). Let the projections of OA on these three axes be respectively A_x, A_y and

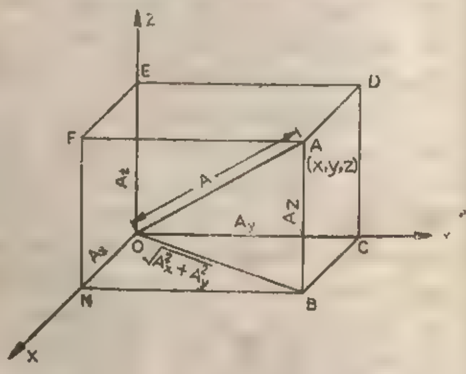


Fig. I-2.4.

* Vectors are printed in thick, capital letters, such as **A, B, P, Q** etc. To represent its *magnitude only* we write *A, B, P, Q, etc.* [that is, we use the *italic* (slant) type]. Another way to write the magnitude is |**A**|, |**B**| etc.

A_z. If the angles which OA makes with these axes are α, β and γ respectively, then $\cos\alpha = A_x/A$, $\cos\beta = A_y/A$ and $\cos\gamma = A_z/A$. Also $A^2 = A_x^2 + A_y^2 + A_z^2$.

A_x, A_y and A_z are called the components of A in the reference chosen. We thus find that when we know the values of the components of a vector (in a tri-rectangular frame of reference)., we know both the magnitude and the direction of the vector. The *triplet of numbers* A_x, A_y, A_z completely defines the vector A. (So, while a scalar requires only *one* number to represent it, a vector requires *three*). *OA* is a said to be the *position vector* for the point A.

Ex. I-2.4 : *A balloon when released rises 25 m while moving 8 m to the east and 15 m to the north. How far is it from the point of release?*

Ans: From the figure observe that

Ans. **From the fig. I-2.4a observe that**

$$OC^2 = CB^2 + OB^2 = CB^2 + (OA^2 + AB^2) = 25^2 + 8^2 + 15^2$$
$$= 914 \text{ m}^2 \qquad \therefore \; OC = 32.2 \text{ m}$$

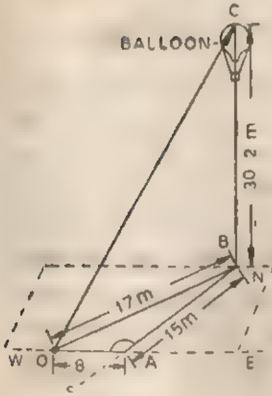


Fig. I-2.4a

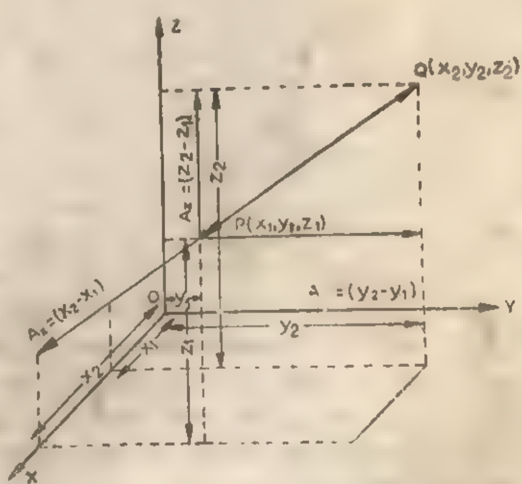


Fig. I-2.5.

C. Representation of a vector by coordinates follows from what we have already said. For this we require the coordinates of the initial and end points of the line segment which represents the vector. Let the coordinates of the initial point (in a tri-rectangular coordinate system fig. I-2.5) be x_1, y_1, z_1, and those of the final point x_2, y_2, z_2. Then the components of the vector A will be as follows:

$$A_x = x_2 - x_1 \; ; \; A_y = y_2 - y_1 \; ; \; A_z = z_2 - z_1$$

The magnitude of the vector will be

$$A=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

If α, β, γ be the angles which the vector makes with the x-axis, y-axis and z-axis respectively, then

$$\cos\alpha=A_x/A=(x_2-x_1)/\{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}^{\frac{1}{2}} \text{ etc.}$$

If the x-y plane contains the vector, the z-coordinate will be zero.

I-2.5. More Facts about Vectors:

(i) **Collinear Vectors** are those that lie along the same line or parallel lines irrespective of end or terminal points, magnitude or direction ; for example in fig. I-2.6 (a) **p, q, r** and **AC, BD** and **CB** are two collinear sets of vectors. Remember then **p, q, r** may not be in the plane of the paper and neither in the same direction.

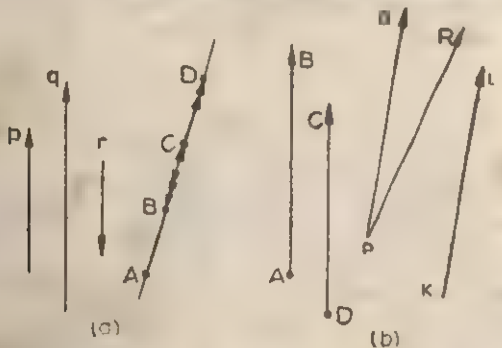


Fig. I-2.6.

Vectors will be considered **equal** only when they are of same magnitude and same direction. **AB** and **DC** are equal vectors (fig. I-2.6 (b) but not so **PQ, PR** though they are equal in magnitude. Again PR $\neq$ KL for though of same length they are not parallel. But **PQ=KL.**

Vectors equal in magnitude, parallel in direction but opposite in sign are said to be *opposite* e.g. **AB** and **CD** (i.e. **DC** reversed) in the last figure.

Null vector is one of which the initial and final points A and B or C and D coincide. Symbolised by 0, it is taken to be *collinear to any vector.*

(ii) **Unit vector** of any given vector is a *line segment of unit length but parallel to it.* In Fig. I-2.7(a) we note a the vector has a magni-

tude a and unit vector $\mathbf{r}$ of unit magnitude. A vector, in common with all other physical quantities, must possess a unit and magnitude—both scalars. In addition it has a direction. Hence if a vector be divided by its magnitude we are left with a direction and unit length—that gives the *unit vector.* As shown in fig. I-1.4 a *space vector* OA may be a directed line segment with three components A_x, A_y, A_z. As its unit vector is parallel to OA it must also have three components. In Cartesian system they are designated as **i, j, k** [fig. I-1.7(b)]. They are of great help in resolving vectors (I-2.7). It also helps in multiplying a vector by a scalar e.g. vector $\mathbf{a}=a\mathbf{r}$ where $|\mathbf{a}|$ or just plain a, is its magnitude and $\mathbf{r}_a$ its unit vector. Whatever the value of a or its unit, the vector $a\mathbf{r}_a$ will always be acting parallel to $\mathbf{r}$, hence a unit vector is said to be an *operator.*

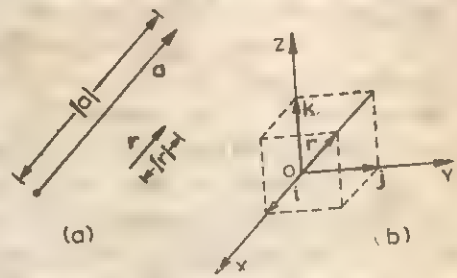


Fig. I-1.7.

(iii) Localised vector is one with a definite point of application. If on a body moving in a straight line, the point of application of the vector moves in the same line, the motion does not change. But it does change if the point of application of the vector shifts. Such localised vectors are important for rotation parameters like moment of forces and couples.

(iv) Vectors and Co-ordinate system. Though we have used co-ordinate systems to represent vectors, they are in fact independent of the co-ordinate system chosen. In fig. I-2.8 let OP ($=\mathbf{r}$) represent the position vector of P with reference to the OX-OY co-ordinate system or reference frame. Let the frame *rotate* about O anticlockwise to take up

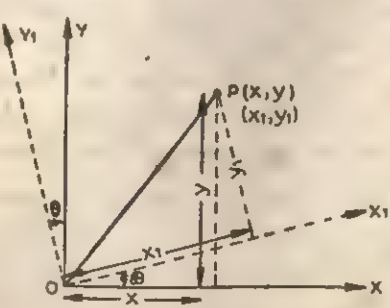


Fig. I-2.8.

the OX_1-OY_1 position. Then P has co-ordinates (x, y) as well as (x_1, y_1), though OP has not shifted. Clearly $r^2=x^2+y^2=x_1^2+y_1^2$

This means that the vector **r** represents a related pair of numbers (x, y) or (x_1, y_1), co-ordinates respectively according to two co-ordinate frames, one obtained by rotating the other. Since there can be any number of such rotations, as many pair of numbers are possible, each represented by the same vector **r**. So we find that a vector representation is not tied down to any co-ordinate or reference frame and so quite general in nature. Since a vector remains unchanged, if it is shifted parallel to itself in any direction, neither translation nor rotation of frames affect it. So the *physical law represented by the vector appears same to the observers in different frames.* Thus the physical law becomes general, compact and invariant in vector notation.

I-2.7. Composition and resolution of vectors: By the term '*composition*' of vectors we mean the *addition* of two or more vectors. We shall confine ourselves to *coplanar* vectors, that is those in the same plane. Scalar quantities are added *algebraically*. Vectors are added *geometrically* as indicated below.

'*Resolution*' of vectors means splitting (=dividing) a vector into two components, the components and the vector lying in the same plane. Resolution of a vector can be easily understood when we have learnt how to add vectors.

A. **The geometrical law of addition of vectors.** In fig. I-2.9 let the vectors **P** and **Q** to be added, be represented by the line segments OA and OB drawn from the same point. (The vectors may be a pair of velocities which a particle has at the same moment. Or they may be a pair of forces acting on the same particle at the same time.) To get their sum, called the *resultant*, complete the parallelogram OACB.

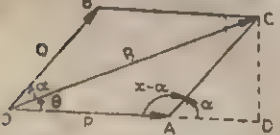


Fig. I-2.9

Then OC, the diagonal, will represent in magnitude and direction the vector sum **R** of $\vec{OA}$ and $\vec{OB}$, that is, of **P** and **Q**. In symbols.

$\mathbf{P}+\mathbf{Q}=\mathbf{R}$ or $\vec{OA}+\vec{OB}=\vec{OC}$. (I-2.7.1)

Also $\mathbf{Q}+\mathbf{P}=\mathbf{R}$ or $\vec{OB}+\vec{OA}=\vec{OC}$. (I-2.7.2)

Thus vector addition obeys *commutative law*.

This law of addition, which applies to any quantity that behaves as a vector, is known as the **parallelogram law of addition.** The law may be stated as follows: If *two vectors are represented in magnitude and direction by the two sides of a parallelogram drawn from a point, they are equivalent to a single vector represented in magnitude and direction by the diagonal of the parallelogram passing through that point.*

The single vector to which the other vectors are equivalent is called the *resultant* of the vectors.

B. **Vector subtraction.** If the sum of two vectors **P** and **Q** is **R**, that is, if **P**+**Q**=**R**, then **P**−**Q**=**P**+(−**Q**). This means that to subtract a vector **Q** from a vector **P**, we should reverse **Q** and add it to **P**. If we do so in fig. I-2.10, we shall find that **P**−**Q**=**R′** (I-2.7.3)

R′ in Fig. I-2.10 will be represented by the other diagonal of the parallelogram. **P**−**Q** will be represented by BA (=OD) and **Q**−**P** by AB (=DC). They are equal and opposite.

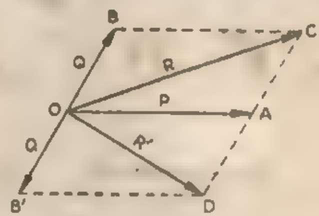


Fig. I-2.10.

$$\mathbf{Q}-\mathbf{P}=-(\mathbf{P}-\mathbf{Q}). \qquad \text{(I-2.7.4)}$$

C. **Magnitude and direction of the resultant.** Fig. I-2.9 shows the resultant **R** of two vectors **P** and **Q** inclined to each other at an angle α. Then the magnitude of the resultant is

$$R^2=OC^2=OD^2+DC^2=(OA+AD)^2+DC^2$$
$$=OA^2+(AD^2+DC^2)+2OA.AD=OA^2+AC^2+2OA.AC\cos\alpha$$
$$=P^2+Q^2+2PQ\cos\alpha. \qquad \text{(I-2.7.5)}$$

R is inclined at an angle θ to **P** where

$$\tan\theta=\frac{CD}{OD}=\frac{AC\sin\alpha}{OA+AC\cos\alpha}=\frac{Q\sin\alpha}{P+Q\cos\alpha} \qquad \text{(I-2.7.6)}$$

In *vector subtraction* α changes to $(180°-\alpha)$.

$$\therefore\quad R'^2=P^2+Q^2+2PQ\cos(180°-\alpha)$$
$$=P^2+Q^2-2PQ\cos\alpha \qquad \text{(I-2.7.7)}$$

[Remember P is the magnitude of vector **P**. So for all vectors.]

Eqquations (I-2.7.5) and (I-2.7.6) give respectively the magnitude and direction of the resultant of two vectors. (say, forces or velocities).

A theorem is trigonometry states that in a triangle the ratio of the sines of their opposite angles equal the ratio of the sines of thtir opposite angles. Applying this result to the triangle OAC (fig. I-2.9) we have

$$\frac{R}{\sin(\pi-\alpha)}=\frac{Q}{\sin\theta}=\frac{P}{\sin(\alpha-\theta)} \qquad \text{(I-2.7.8)}$$

These relations may be used for solving problems involving the resultant of two vectors.

D. Alternative construction for addition of vectors: The Triangle method. An alternative construction for adding two or more vectors is as follows: It is a graphical method.

(a) (b)

Fig. I-2.11.

Let **P** and **Q** [fig. I-2.11(a)], represented respectively by lines KL and MN, be the vectors to be added. From the end point L (fig. I-2.11b) of the straight line KL (representing the vector **P**) draw LS *equal and parallel* to MN (representing **Q**). We have stated already, that the vector does not change by such shifts.

Then the straight lint KS represents in magnitude and direction the sum of **P** and **Q**. According to vector notation.

$$\text{KL}+\text{LS}=\text{KS} \quad \text{or} \quad \mathbf{P}+\mathbf{Q}=\mathbf{R}$$

Thus, *if two sides* KL *and* LS *of a triangle taken in order, represent two vectors, then the third side* KS *taken in reverse order gives their resultant vector.* This is the *law of triangles.*

Magnitude and Direction of the Resultant. As for the Parallelogram Law, here also the magnitude of the resultant is not the algebraic sum of the magnitudes of the components. To obtain that, drop SM perpendicular, to KL extended to M. Then

$$\begin{aligned} R^2 = KS^2 &= KM^2+MS^2=(KL+LM)^2+MS^2 \\ &= KL^2+LM^2+2KL.LM+MS^2 \\ &= KL^2+(LM^2+MS^2)+2KL.LS.LM\,/\,LS \\ &= KL^2+LS^2-2KL.LS.\cos\theta \\ &= P^2+Q^2+2PQ\cos\theta \end{aligned}$$

—the same result as before in eqn. I-2.7.5. Again we find the resultant vector KS making an angle ϕ with the vector KL as

$$\tan\phi = \frac{SM}{KL+LM} = \frac{Q\sin\theta}{P+Q\cos\theta}$$

The only difference between the parallelogram and triangle method is that in the former, vectors start from the same point while in the latter they are put successively.

E. The Polygon method. This is only an extension of the triangle law. Any number of vectors may be added in this manner by *placing them head to tail* (tail of the second to the head of the first and so on) *in a continuous sequence* (fig. I-2.12). *The order in which they are taken is immaterial.* The line drawn from the origin of the first

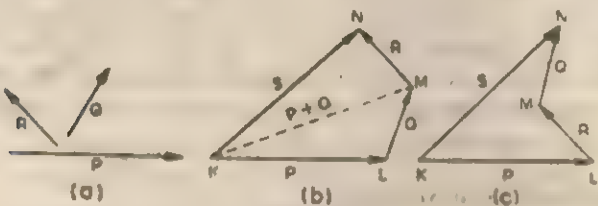


Fig. I-2.12

to the terminus of the last represents resultant in magnitude and direction. Fig. I-2.12 shows the addition of three vectors, **P**, **Q** and **R**, in this way. KN=**S** is their resultant. This is the *polygon method* of addition. For two vectors the polygon reduces to a triangle.

If we add vectorially **P**, **R**, **Q** we get the same resultant **S**. (fig. I-2.12c). Thus vector addition obeys associative law i.e. order of addition if changed leaves the resultant unchanged.

Q. *Can two vectors of different magnitudes be combined to give zero resultant? Can three vectors? Explain in brief.* [S. S. Q.]

Ans. In addition of two or more vectors by the geometrical method we lay down the vectors successively with the tail of one to the head of the previous one. The resultant is obtained by joining the initial point (tail) of the first vector to the end point (head) of the last vector.

(*i*) Two vectors of unequal magnitude will always leave a resultant, whatever the angle between them. There cannot be a zero resultant.

Proof: $R^2=P^2+Q^2+2PQ\cos\theta$. If $\theta=180°$ we have
$R^2=P^2+Q^2-2PQ=(P-Q)^2$

This is the minimum value for R. It cannot vanish unless $P=Q$. Hence the above statement.

(*ii*) But three vectors can give a zero resultant if the third vector is equal and opposite to the resultant of the other two ; or three vectors added in order head to tail form a closed triangle according to the Triangle law of vectors.

Ex. I-2.2 : *When will the magnitude of the resultant of two equal vectors added vectorially be equal to (i) $\sqrt{2}$ times the magnitude of each (ii) $\sqrt{3}$ times?* [H. S. '80]

Ans. $R^2=P^2+P^2+2P.P.\cos\theta=2P^2(1+\cos\theta)=4P^2\cos^2\theta/2$

$\therefore\ R=2P\cos\theta/2$

(i) $\sqrt{2}P=2P\cos\theta/2$ $\quad\therefore\ \cos\theta/2=1/\sqrt{2}$ or $\theta=90°$

(ii) $\sqrt{3}P=2P\cos\theta/2$ $\quad\therefore\ \cos\theta/2=\sqrt{3}/2$ or $\theta=60°$

Problem : (1) The resultant of two vectors P and Q equals in magnitude that of P. Show that the resultant of $2P$ and Q is perpendicular to Q.

(2) Show that the magnitude of the resultant vector of two vectors acting at a point can neither be greater than the sum of their magnitudes nor smaller than the difference of their magnitudes.

(3) The resultant of two vectors P and Q acting at an angle θ has a resultant of magnitude $(2K+1)\sqrt{P^2+Q^2}$ and $(2K-1)\sqrt{P^2+Q^2}$ when the inclination is $(90°-\theta)$. Show that $\tan\theta=(K-1)/(K+1)$.

I-2.8. Composition of velocities :

To compound i.e. add two velocities or forces or any other directed quantities we proceed exactly as stated for vectors, merely substituting for vectors, the word 'velocities' or 'forces' as required. Fig .I-2.9 and equations I-2.7.5 and I-2.7.6 apply with the modification that **P** and **Q** are the two velocities, or forces and **R**, their resultant.

Let us illustrate the general result by taking 'velocity' as the vector. The *law of parallelogram of velocities*, which states how two velocities are to be added together, may be stated as follows :

If a particle possesses simultaneously two velocities represented in magnitude and direction by the two sides of a parallelogram drawn from a point, they are equivalent to a single resultant velocity represented in magnitude and direction by the diagonal of the parallelogram passing through the point.

A third velocity may be added to the resultant, then a fourth and so on. Before we take up illustrations of such compositions let us learn the basic principles behind.

A. Principle of Independence of Vectors. The law of addition of vectors is based on a principle known as the principle of independence of vectors. It states that *each vector produces its own effect as if the others do not exist.* When we add two or more forces, velocities, displacements, etc., this principle operates. To emphasize this independence for velocities, we write the following statement—

B. Principle of independence of velocities. When a particle has two (or more) velocities at the same time, the velocity in one direction is not affected by that in the other direction. Each velocity produces its own motion as if the oher does not exist.

Illustration : To cross a stream in the shortest time. In solving problems in which a particle has more than one velocity at the same time, it is well to remember the *principle of independence of velocities.* When a man wants to cross a river the distance he needs to cover is the shortest if it is perpendicular to the bank. If he swims in this direction he will cross the river in the shortest time, though in this attempt he will be carried downstream by the current. According to the principle of independence of velocities, his velocity perpendicular to the stream will not be affected by the velocity of the stream. So, in his attempt to cross in the shortest time, he swims *as if there were no current* in the stream.

Ex. I-2.3 : *A river flows with a velocity of one mile per hour. A man rows a boat at an angle of 30° to the river with a velocity of 3 miles per hour. If the river is half-a-mile wide, how long will he take to cross it? How far downstream will he be carried in the meantime?*

Solution. Let us resolve the velocity of the boat downstream and perpendicular to it. The downstream component $=3\cos 30° = \frac{1}{2}3\sqrt{3}$ mi/hr.

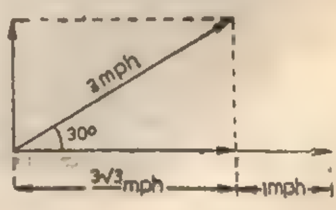
The perpendicular component $= 3 \sin 30° = \frac{3}{2}$ mi/hr.

The total downstream velocity of the man

$= \frac{1}{2}3\sqrt{3}$ mi/hr due to the boat

$+1$ mi/hr due to the stream

$= \frac{1}{2}3 \times 1{\cdot}732 + 1 = 3{\cdot}598$ mi/hr.



According to the *principle of independence of velocities* each of these compoents acts as if the other does not exist. The downstream component does not help him to cross the river.

The component perpendicular to the stream $= \frac{3}{2}$ mi/hr. It is this component which carries him across the river.

$$\text{So the time required to cross} = \frac{\frac{1}{2}\text{ mi}}{\frac{3}{2}\text{ mi/hr}}$$

$$= \tfrac{1}{3}\text{ hr} = 20\text{ minutes.}$$

The distance he is carried downstream in crossing the river $= \frac{1}{3}$ hr $\times 3{\cdot}598$ mi/hr $= 1{\cdot}199$ miles.

Ex. 1-2.4 : *A river $\frac{1}{4}$ mile wide flows at the rate of 4 miles per hour. A man who can swim at 3 mi/hr has to cross it. Find (a) the minimum time in which he can cross, (b) the actual distance he travels in crossing, (c) the direction with respect to the bank in which he moves and (d) his resultant velocity with respect to the bank.*

(The reader is advised to draw his own diagram).

Solution. From the principle of independence of velocities we understand that he must swim perpendicular to the river to cross it in the shortest time. His speed in this direction carries him across the river, and produces no displacement parallel to the bank. So the minimum time to cross

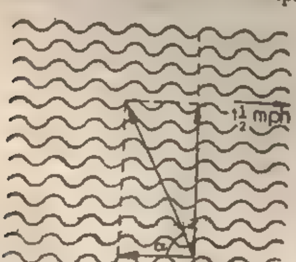
$$= \frac{\text{breadth of river}}{\text{speed across the river}} = \frac{\frac{1}{4}\text{ mi}}{3\text{ mi/hr}} = \tfrac{1}{12}\text{ hr} = 5\text{ minutes.}$$

In this time he is carried 4 mi/hr $\times \frac{1}{12}$ hr $= \frac{1}{3}$ mile downstream by the current. Since these two displacements that he undergoes are at right angles, the actual distance he moves is $\sqrt{(\frac{1}{4})^2 + (\frac{1}{3})^2}$ mile.

If the direction of his motion makes angle θ with the bank, $\tan\theta = \frac{3}{4}$.

His resultant velocity $= \sqrt{3^2 + 4^2}$ mi/hr $= 5$ mi/hr.

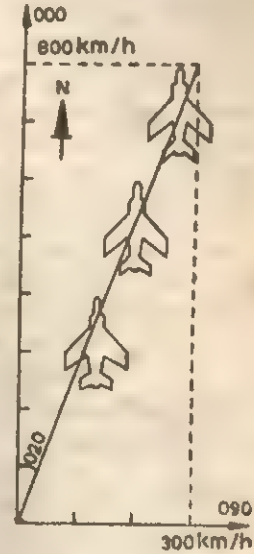
Problem. In the above example, if the velocity of the stream is $1\frac{1}{2}$ miles per hour find the direction in which the man should swim so that he may cross the river perpendicularly.



Hint : Let the direction in which he should swim make an angle α with the bank on the upstream side.

Resolving his velocity parallel and perpendicular to the bank we have parallel component $= 3$ mi/hr $\times \cos\alpha$ and perpendicular component $=$ 3 mi/hr $\sin\alpha$. Now find α.

Ex. I-2.5 : *During the closing days of the Second World War the American pilots found at a height of about 7 to 13 km a a very strong continuous air current from west to east (named the jet stream) encircling the earth reaching upto a velocity of 300 kmph. and beyond. If a jet bomber flying at 40,000 ft. straight north (000) at 800 kmph. gets into it. what will happen to it?*



Solution : The adjoining figure tells you what happens. It will be found to be flying at 854 kmph. along 020 direction i.e. 20° east of north.

Problem : An aeroplane flies at a speed of u kmph and has a range (out and back) of R km in calm weather. Prove that in a north wind v kmph its range becomes

$$R' = \frac{R(u^2 - v^2)}{u(u^2 - v^2 \sin^2\theta)^{\frac{1}{2}}}$$

Special cases of composition. Equations I-2.7.5 and I-2.7.6 are perfectly general and hold for all values of α. We take up here a few special cases :

(1) The forces or velocities are in the *same direction, i.e.,* $\alpha = 0$. Since cos $0=1$, equation I-2.7.5 gives $R^2 = u^2 + v^2 + 2uv$ or $R = u + v$.

(2) The forces or velocitits are in *opposite directions, i.e.* $\alpha = 180°$. Since cos $180° = -1$, $R^2 = u^2 + v^2 - 2uv$ or $R = u - v$. Here the resultant is the difference of the two and acts in the direction of the larger.

(3) The forces or velocities are *at right angles, i.e.,* $\alpha = 90°$ (fig. I-2.13). Since cos $90° = 0$,

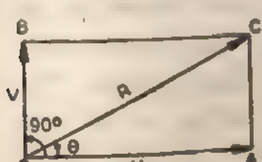


Fig. I-2.13

$$R^2 = u^2 + v^2$$

$$\text{or} \quad R = \sqrt{u^2 + v^2} \qquad \text{(I-2.8.1)}$$

$$\text{and} \quad \tan\theta = v/u \qquad \text{(I-2.8.2)}$$

Note that in this case

$$\left.\begin{aligned} u &= R\cos\theta \\ \text{and} \quad v &= R\sin\theta \end{aligned}\right\} \qquad \text{(I-2.8.3)}$$

Ex. I-2.6. *Suppose a steamer moves downstream with a velocity of 16 km per hour, while a man walks across its deck with a velocity of 6 km per hour. What velocity will this man appear to have to a man standing on the bank of the river?*

Solution : The man on the steamer has two simultaneous velocities at right angles to each other. One is that of the steamer at 16 km/hr and the other his own, at 6 km/hr,

From Eq. I-2.8.1 the resultant velocity $R=\sqrt{16^2+6^2}=17{\cdot}1$ km/hr.

If θ is the angle between the direction of motion of the steamer and the direction of the resultant velocity, then, from equation I-2.8.2, $\tan\theta=3/8$.

Thus to the man at rest on the bank of the river the other man will appear to move with a speed of about 17.1 km/hr in a direction inclined at an angle $\tan^{-1}$ 3/8 with the downstream direction.

Ex I-2.7. *A crane pulls a body vertically upward with a velocity of 4.8 m/min, while it itself moves along horizontal rails at 2 m/min. Find the velocity of the body (relative to an observer at rest on the surface of the earth).*

Solution : Here also the two velocities are at right angles.

$\therefore$ The resultant $R=\sqrt{4{\cdot}8^2+2^2}=5.2$ m/min

The angle between the vertical and the direction of the resultant R is $\tan^{-1}$ 5/12 backward.

I-2.9. Resolution of vectors: We have seen above how two vectors may be compounded into one. By following the reverse process (*i.e.*, constructing a parallelogram on a given diagonal, the directions of the sides of the parallelogram being given) it is possible to split a vector into two parts in two given directions. The case in which these two directions are at right angles, is of special importance. The parts into which a given vector is split are called the components of the vector, and the process is known as *resolution* of the vector. If the components are at *right angles*, the parallelogram reduces to a rectangle. Each component in such a case is called the *resolved part of the vector* in the direction concerned.

Resolved part of a vector. The resolved part of a vector in a direction is its effective part in that direction. The resolved part of a vector **R** (fig. I-2.14) in the direction OX making an angle θ with **R**, is given by $P=R\cos\theta$. This together with **Q** given by $Q=R\sin\theta$, equals **R**, as may be seen by the vector addition **P** and **Q**.

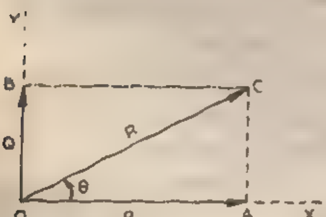


Fig. I-2.14.

The effect of **R** in the direction OX is the same as that of **P** where $P=R\cos\theta$. **P** and **Q** are two *components* of **R**.

Definition. The *resolved part* of any vector in a given direction is that vector which together with another vector in a perpendicular direcion, produces the same effect as the given vector.

If the vector is **A** and it makes an angle θ with some given direction OX the *resolved part* (*or component*) of **A** in the given direction is given by $A_x = A \cos \theta$ (I-2.9.1)

The resolution of a velocity in perpendicular directions have already been illustrated above in solving sums on crossing a river by a swimmer.

The method of resolving detailed above, is one (the rectangular type) of the innumerable types available. We discuss below the general way of resolution when the components **P** and **Q** make angles θ and ϕ with the vector **R** to be resolved (fig. I-2.15). Let OC represent **R** OA and OB **P** and **Q** respectively. In calculating these, we utilise the law of sines (see eqn. I-2.7.8) and remembering that BC=OA=P and AC=OB=Q to write

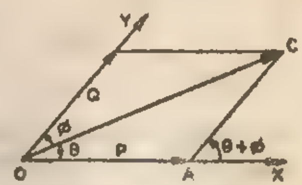


Fig I-2.15

$$\frac{OA}{\sin \phi} = \frac{CA}{\sin \theta} = \frac{OC}{\sin [\pi - (\theta + \phi)]}$$

or $$\frac{P}{\sin \phi} = \frac{Q}{\sin \theta} = \frac{R}{\sin (\theta + \phi)}$$

$$\therefore \quad P = R \frac{\sin \phi}{\sin (\theta + \phi)} \text{ and } Q = R \frac{\sin \theta}{\sin (\theta + \phi)} \qquad \text{(I-2.9.2)}$$

This resolution follows directly from the parallelogram law of vectors.

I-2.10. Addition of Vectors by Geometrical and Analytical methods. We have seen how two or more vectors may be added together by the *geometrical method*. They can be added in another way, called the *analytical method*. For convenience, we shall consider all the vectors to lie in the same plane.

First, resolve all the vectors into rectangular components along any convenient pair of axes, and then combine them into a single resultant.

This is illustrated in fig. I-2.16. V_1, V_2, V_3 are vectors to be added. Let us take the x-axis along any suitable direction and the y-axis perpendicular thereto. If θ_1, θ_2 and θ_3 be the angles the vectors make with the x-axis, then the x-components of the vectors are $V_1 \cos \theta_1$, $V_2 \cos \theta_2$ and $V_3 \cos \theta_3$. Adding these up we get the x-component R_x of the resultant R. The y-component R_y of the resulant is similarly $V_1 \sin\theta_1 + V_2 \sin \theta_2 + V_3 \sin \theta_3$. Then

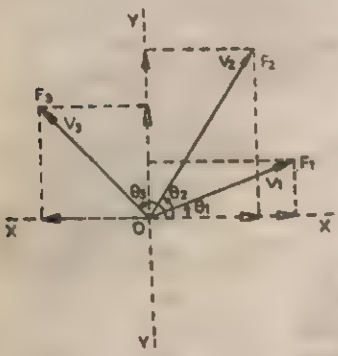


Fig I-2.16

$$R = \sqrt{R_x^2 + R_y^2}$$
$$= \sqrt{(\Sigma V_x)^2 + (\Sigma V_y)^2}, \qquad \text{(I-2.10.1)}$$

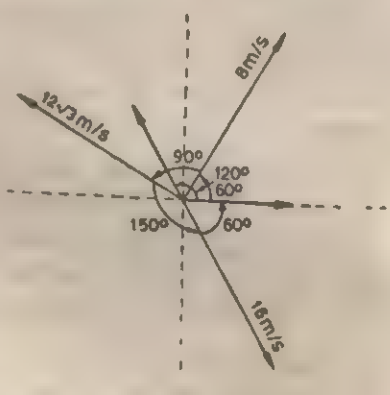
where ΣV_x is written for the *sum* of x-components of the vectors and ΣV_y for the sum of the y-components.

If α is the angle which R makes with the x-axis, then

$$\tan \alpha = \frac{R_y}{R_x} = \frac{\Sigma V_y}{\Sigma V_x}. \qquad \text{(I-2.10.2)}$$

Ex. I-2.8. *Four velocities of magnitudes 4m/s, 8m/s, 12 √3 m/s and 16m/s are simultaneously imparted to a particle. Angles between* V_1 *and* V_2 *is 60°, between* V_2 *and* V_3 *is 90°, between* V_3 *and* V_4 *is 150°. Find the resultant velocity.*

Solution : Let axes be taken along the direction of V_1 and the perpendicular to it. Resolving each velocity along the two axes we get respectively,



Along the x-axis $= 4 \cos 0° + 8 \cos 60° + 12\sqrt{3} \cos 150° + 16 \cos 300°$
$= 4 + 8.\frac{1}{2} + 12 \times \sqrt{3}(-\sqrt{3}/2) + 16.\frac{1}{2}$
$= -2$m/s

Along the y-axis $= 4 \sin 0° + 8 \sin 60 + 12\sqrt{3} \sin 150° + 16 \sin 300°$
$= 0 + 8(\sqrt{3}/2) + 12.(\sqrt{3}/2) + 16(-\sqrt{3}/2) = 2\sqrt{3}$m/s

Let the resultant velocity V make an angle θ with OX. Then $V \cos \theta = -2$ and $V \sin \theta = 2\sqrt{3}$ m/s. So squaring and adding,

$V^2 = 16$ or $v = 4$m/s and $\tan \theta = -\sqrt{3}$ or $\theta = 120°$

Hence the resultant velocity is 4 m/s at an angle of $2\pi/3$ with the first velocity.

Problem : Add displacements—(1) 8 m north of east (2) 12 m due south and (3) 20 m at 30° west of south by *vector resolution* method.

(*Ans.* : 24.06, $\tan^{-1}$ 5.45 w of s)

I-2.11. Position Vector :

This is the shortest directed line segment (OP) joining any point P with the origin O of a chosen tri-axial co-ordinate system (fig. I-2.17) with axes OX, OY, OZ at right-angles to each other —a righthanded cartesian co-ordinate system (so-called because rotation of a screw from OX to OY drives its tip along the OZ direction).

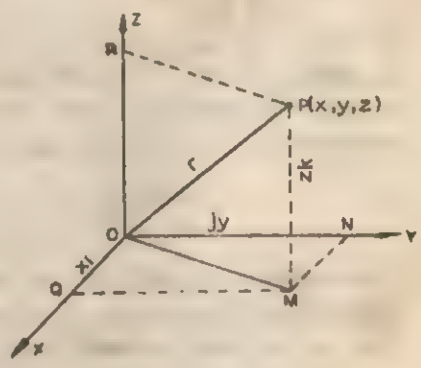


Fig. I-2.17.

From P a normal PM is dropped on the X-Y plane and again perpendiculars MQ and MN dropped on OX and OY. The line OM represents the projection of the line OP on the X-Y plane. Then $OQ=x$, $ON=y$ and $OR=z$, the co-ordinates of OP with respect to the co-ordinate system. If **i**, **j**, **k** represent the respective unit vectors along OX, OY and OZ, then by triangle law of addition of vectors

$$\mathbf{OP}=\mathbf{OQ}+\mathbf{ON}+\mathbf{OR}$$

$$\therefore \quad \mathbf{r}=x\mathbf{i}+y.\mathbf{j}+z.\mathbf{k} \qquad \text{(I-2.11.1)}$$

$$\text{where} \quad r=(OM^2+PM^2)^{\frac{1}{2}}=[(ON^2+NM^2)+PM^2]^{\frac{1}{2}}$$

$$=(x^2+y^2+z^2)^{\frac{1}{2}} \qquad \text{(I-2.11.2)}$$

The above analysis is an elaboration of representation of vectors by co-ordinates in foregoing I-2.4(c). Below we show you an application of the idea.

An idea of Instantaneous Velocity. In the last chapter we have

given you that idea—here is its pictorial or physical presentation. Let a particle be moving along a curved path OBDE of which the points B and D have position vectors **OB** ($=\mathbf{r}_1$) and **OD** ($=\mathbf{r}_2$) at instants t_1 and t_2 where $\Delta t=t_2-t_1$, and the displacement from B to D is $\Delta\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$ (fig. I-2.18). Then the average velocity (v) of the particle during the interval Δt as it moves from B to D will be given by PQ, a directed line segment ($\Delta\mathbf{r}$) parallel to BD. We go on making BD smaller and smaller till it reduces to a vanishingly small segment about the mid-point. Then the instantaneous velocity becomes

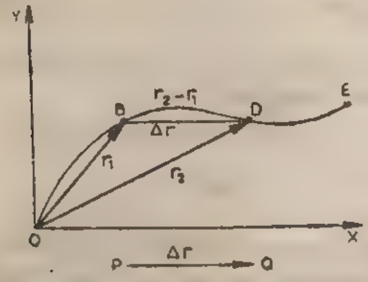
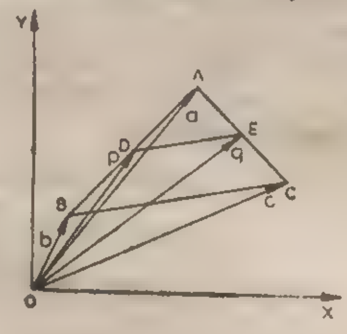


Fig. I-2.18.

$$v=\underset{\Delta t\to 0}{\mathrm{Lt}}\ \frac{\Delta r}{\Delta t}$$

Instantaneous acceleration can be similarly illustrated and has been so done for centripetal acceleration (fig. I-5.4). Both the figures show you the utility of *law of triangles of vector addition.*

Ex. I-2.9. *Show vectorially that the line joining middle points of two sides is half of and parallel to the third side.* **[H. S. '83]**



Ans.: Let **a**, **b**, **c** be the position vectors of the vertices of a tringle w.r.t. to the same origin. Let D, E be the mid points of the sides AB and AC, their position vectors being **p** and **q**. Then $\mathbf{p}=\frac{1}{2}(\mathbf{a}+\mathbf{b})$ and $\mathbf{q}=\frac{1}{2}(\mathbf{c}+\mathbf{a})$

$$\therefore\ \mathbf{DE}=\tfrac{1}{2}(\mathbf{c}+\mathbf{a})-\tfrac{1}{2}(\mathbf{a}+\mathbf{b})$$
$$=\tfrac{1}{2}(\mathbf{c}-\mathbf{b})=\tfrac{1}{2}\,\mathbf{BC}$$

Thus is established the proposition.

Ex. I-2.10. *Three vertices of a triangle ABC have position vectors* $\mathbf{i}+2\mathbf{j}$, $4\mathbf{i}+5\mathbf{j}$ *and* $7\mathbf{i}+\mathbf{j}$. *respectively. Find the position vector of D the mid-point of BC and show that* $AB^2+AC^2=2(AD^2+BD^2)$*—the well-know Appolonius theorem. Also find the lengths of the sides of the triangle.*

Solution : $\mathbf{AB}=\mathbf{AO}+\mathbf{OB}=(-\mathbf{i}-2\mathbf{j})+(4\mathbf{i}+5\mathbf{j})=3\mathbf{i}+3\mathbf{j}$

$\therefore\ AB=\sqrt{3^2+3^2}==3\sqrt{2}$

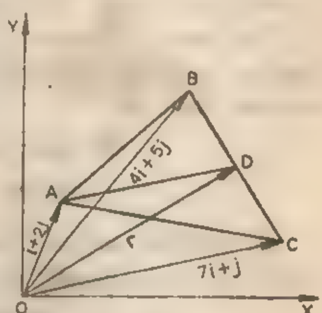
$\mathbf{BC}=\mathbf{BO}+\mathbf{OC}=(-4\mathbf{i}-5\mathbf{j})+(7\mathbf{i}+\mathbf{j})=3\mathbf{i}-4\mathbf{j}$

$\therefore\ BC=\sqrt{3^2+(-4)^2}=5$

and $\mathbf{CA}=\mathbf{CO}+\mathbf{OA}=(7\mathbf{i}-\mathbf{j})+(\mathbf{i}+2\mathbf{j})=-6\mathbf{i}+\mathbf{j}$

$\therefore\ CA=\sqrt{(-6)^2+1^2}=\sqrt{37}$

Let the required position vector of D be $\mathbf{r}$. Then



$\mathbf{r}=\mathbf{OD}=\frac{1}{2}(\mathbf{OB}+\mathbf{OC})$

$=\frac{1}{2}(4\mathbf{i}+5\mathbf{j}+7\mathbf{i}+\mathbf{j})=\frac{11}{2}\mathbf{i}+3\mathbf{j}$

$\therefore\ \mathbf{AD}=\mathbf{AO}+\mathbf{OD}=(-\mathbf{i}-2\mathbf{j})+(\frac{11}{2}\mathbf{i}+3\mathbf{j})=\frac{9}{2}\mathbf{i}+\mathbf{j}$

$\therefore\ AD^2=\frac{81}{4}+1=\frac{85}{4}$

$\therefore\ 2AD^2+2BD^2=2(\frac{85}{4}+\frac{25}{4})=55$

for $BC=5$ $\therefore\ BD=\frac{5}{2}$

Again $AB^2+AC^2=(3\sqrt{2})^2+(\sqrt{37})^2$

$=18+37=55$

So $AB^2+AC^2=2(AD^2+BD^2)$.

Ex. I-2.11. *Show that if $\mathbf{AB}=\mathbf{DC}$ in a figure ABCD, it is a parallelogram and its diagonals bisect each other.*

Solution : Draw your own diagram and let the position vectors of points A, B, C, D be $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$. Then we have

$$\mathbf{b}-\mathbf{a}=\mathbf{a}-\mathbf{d} \quad \text{and hence} \quad \mathbf{d}-\mathbf{a}=\mathbf{c}-\mathbf{b}$$

$$\text{or}\quad \mathbf{AD}=\mathbf{BC} \quad \text{or ABCD is a parallelogram}$$

Again from the above relations

$$\mathbf{a}+\mathbf{c}=\mathbf{b}+\mathbf{d} \text{ i.e. } \tfrac{1}{2}(\mathbf{a}+\mathbf{c})=\tfrac{1}{2}(\mathbf{b}+\mathbf{d})$$

Thus mid-points of the diagonals AC and BD are identical.

Problem : D, E, F are the mid-points of the sides AB, BC, CA of a triangle. Show that the vector sum of the medians is zero and they have a common point of trisection.

I-2.12. Product of Vectors :

A vector may be multiplied by a scalar (i.e. pure number) or by another vector. When two vectors are multiplied the result may be either a scalar or a vector, the former is said to be a *scalar* or *dot* product, the latter a *vector* or *cross* product.

A. The product of a vector **r** by a real number $\pm n$ denoted by n**r** or **r**n is just another vector of length $|n|$ times that of **r** having a direction same as that of **r** or in the opposite direction according as n is+ve or −ve. Momentum vector m**v**, the multiplication of velocity vector **v** by a scalar m, is an example.

Division of a vector by a real number m gives a vector of length r/m. The resultant vector may be+ve or −ve as before, parallel to the original vector. It is thus that we get a *unit vector.*

The laws of association and distribution for scalar multipliers hold as in ordinary algebra. For scalar multiplication and division we may write

$$\mathbf{r} = \pm n\mathbf{a} = \pm \mathbf{a}.n$$
$$\mathbf{r}' = \pm \mathbf{a}/n \qquad \text{(I-2.12.1)}$$

B. **Scalar Product** of two vectors **a** and **b** inclined at an angle of θ with each other is the real number $ab \cos \theta$ and is written as

$$\mathbf{a}.\mathbf{b} = ab \cos \theta = \mathbf{b}.\mathbf{a} \qquad \text{(I-2 12.2)}$$

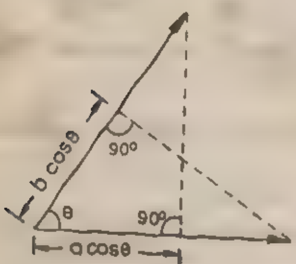


Fig. I-2.19.

The order of the vectors may be reversed without altering the value of the product. Further $b \cos \theta$ is the *resolute* (i.e. the resolved part) of **b** in the direction of **a** and $a \cos \theta$ the resolute of **a** in the direction of **b**, +ve or −ve according as θ is acute or obtuse. Hence we may state that

The scalar product of two vectors is the product of modulus (i.e. the magnitude) *of either vector and the resolute of the other in its direction* as indicated in fig. I-2.19.

From eqn. I-2.12.2 we conclude that the *scalar product* of two vectors (*i*) is a scalar itself in the same plane for a, b are scalars and $\cos \theta$ a pure number (*ii*) obeys *commutative* law of multiplication for $a.b = b.a$ and (*iii*) is indicated by putting a dot (a distinguishing feature) between them and hence called also a *dot product.*

Examples of scalar product of vectors are many, including mechanical work (**F**. **d**), power (**F**. **v**). gravitational potential energy (**W**. **h**), electric power (**E**. **i**) and electromagnetic energy density (**E**. **H**)

From definition we find that if $\theta=\pi/2$ than $\mathbf{a.b}=0$ which means that the vectors are perpendicular to each other and conversely. If $\theta=0$ $\mathbf{a.b}=ab$, the dot product becoming equal to the algebraic product for collinear or parallel vectors.

C. **Vector Product** of two vectors **a** and **b** inclined at an angle θ is a *vector* of modulus (i.e. magnitude) $ab \sin\theta$ and perpendicular to the plane containing **a** and **b** being positive for a rotation from **a** to **b** and is written as

$$\mathbf{c}=\mathbf{a}\times\mathbf{b}=ab \sin\theta \qquad \text{(I-2.12.3)}$$

To specify the direction of **c** imagine rotating a right-handed screw with axis perpendicular to the plane formed by **a** and **b** so as to turn it from **a** to **b** through an angle between them (fig. I-2.20).

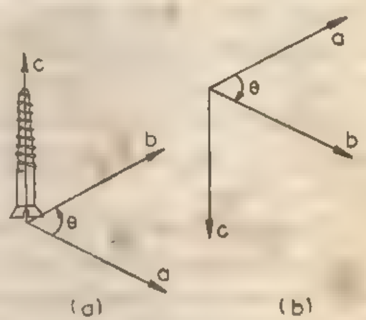


Fig. I-2.20.

From eqn. I-2.12.3 we find that the vector product of two vectors is (*i*) a vector itself, perpendicular in direction to both, (*ii*) does not obey the commutative law ; for $-\mathbf{a}\times\mathbf{b}$ means rotation from **b** to **a** reversing **c** in direction fig. I-2.20b, (*iii*) and is specified by putting a cross ($\times$) sign between them and hence called the *cross product*.

Examples are amongst others, torque ($\mathbf{F}\times\mathbf{d}$), angular momentum ($\mathbf{I}\omega$) magnetic moment ($\mathbf{M}\times\mathbf{H}$), force on a moving charge in a magnetic field (**B** e **v**). For $\theta=0$, cross-product of two vectors vanish, giving us another condition for parallelism or collinearity of two vectors.

I-2.13. Relative velocity :

Ordinarily, we consider the motion of a body with respect to an observer at rest on the surface of the earth. When the observer himself is in motion it seems to him that the velocity of the other body has changed. To a man at rest on the surface of the earth raindrops appear to fall vertically when there is no wind. But when he is moving in a train the drops seem to move slantingly even in the absence of a wind. When a person looks at a moving body, the

motion he sees is always a *relative motion*, i.e., the motion of the body relative to the observer, whether the latter is at rest or in motion.

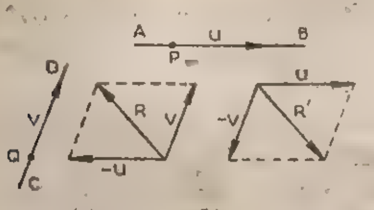


Fig. I-2.21.

The relative velocity between two bodies, both of which are in motion w.r.t. the surface of earth, may be obtained easily by the method of composition of velocities. Let two bodies P and Q move with velocities u and v along the lines AB and CD respectively (fig. I-2.21), u and v being values relative to an observer at rest. What will be the magnitude and the direction of the velocity which Q will appear to have to an observer situated on P? To picture it clearly, P and Q may be looked upon as two ships proceeding in different directions, whose velocities are u and v with respect to a person standing on the sea beach.

If we superpose on both P and Q a velocity equal to $-u$, their relative velocities will not alter. It is the same as imagining that the sea has a current whose velocity is equal and opposite to that of the ship P. P will then appear to be at rest to the observer on the beach. Q will seem to him to have two simultaneous velocities, v along CD and $-u$ along AB. The velocity of Q to both the stationary observers is the same, and is the resultant of v and $-u$. This resultant R is obtained graphically as shown in fig. I-2.21(a).

When we want the relative velocity of P with respect to Q (i.e., if the observer instead of being in P were in Q), we bring Q to rest by superposing a velocity of $-v$ on both. Then the relative velocity will be the resultant of u and $-v$. Fig. I-2.21(b) gives its value R', which is easily seen to be equal and opposite to R.

Generalising, we may say that to find the relative velocity of a body Q with respect to another body P when both are in motion, (*i*) reverse the velocity of P and (*ii*) combine it with that of Q.

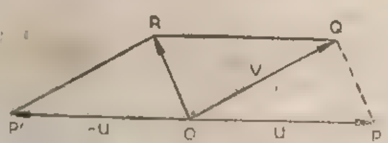


Fig. I-2.22.

The *resultant* gives the relative velocity. P may be called

the *observer* and Q, the *observed.* The working rule then reduces to—**'Reverse the velocity of the observer and combine it with that of the observed'.** The magnitude and direction of the resultant are best obtained by geometrical construction.

Note. Reversing a vector u and adding it to a similar vector v means *subtracting* u from v, that is, getting $v-u$. So relative velocity is obtained by appropriate *vector subtraction.* If OP and OQ (Fig. I-2.22) represent the two vectors u and v, then $v-u=$the vector represented by OR=PQ. $u-v$ is the vector represented by QP. So, we may say that the velocity of Q relative to P is equal to the velocity of Q *minus* the velocity of P. (Note that while OC in Fig. I-2.22 represents $P+Q$, the other diagonal represents the difference of P and Q ; ($P-Q=$ BA and $Q-P=$ AB).

The magnitude of the relative velocity is given by eqn I-2.8.1

To *determine the relative velocity* of a body do the following:—

(*i*) Identify the frame of reference or simply the object w.r.t. which you are to find the relative velocity.

(*ii*) Superpose on that body or frame a velocity equal and opposite to its own. It will apparently come to rest.

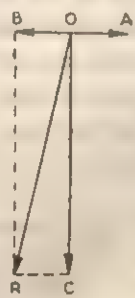
(*iii*) Compound this superposed velocity with the actual velocity of the body in question. The resultant gives you the required resultant velocity.

This has been indicated above. This is to re-emphasise.

Ex. 1-2.13. *A ship steams due east at 18 km/hr and another due south at 24 km/hr. Find the velocity of the first ship with respect to the second.*

Solution : The student is advised to draw the necessary diagram.

Here the observer is in the second ship. So the relative velocity is obtained by combining the velocity of 18 km/hr due east with 24 km/hr due north. This gives a velocity of 30 km/hr at an angle $\tan^{-1} \frac{3}{4}$ east of north.



Ex. 1-2.14. *A cyclist rides horizontally at 10 km per hour. There is no wind and rain-drops appear to him to fall at an angle of 10° to the vertical. Find the speed of the rain-drops.*

Solution: In the fig OA is the velocity of the cyclist and OC that of the rain-drops. The relative velocity OR between them is obtained by reversing OA and combining it with OC. $\angle COR=10°$, $CR=10$ km/hr. To find OC.

Now $CR/OC=\tan 10°=0{\cdot}176$ (from table of tangents).

$$\therefore\ OC=\frac{10}{0.176}\ \text{km/hr}=56{\cdot}8\ \text{km/hr}.$$

Problems : (1) The pilot of an aircraft flying on a horizontal course due south-west at 200 kmph sees a train apparently moving due north, although he knows that the railway line runs due west. Find the speed of the train.

[*Ans.* : 141.4 kmph].

(2) A steady wind blows from the west at 100 km/hr during the flight of an aircraft from airfield A to airfield B due north of A. The airspeed indicator reads 260 km/hr during the flight. It takes 1 hr 15 min for the flight from A to B (neglecting take off and landing times). Find the direction in which the pilot steers the aircraft and also the distance AB. [J. E. E. '83]

[*Ans.* : At an angle θ give by tan $\theta = 5/12$ west of the line AB. Distance AB=100 km.]

I-2.14. Relative motion in the same straight line with acceleration. Suppose two bodies, P and Q, are moving in the same direction with initial velocities u_1 and u_2, and accelerations f_1 and f_2, and that at the initial moment they are at a distance s apart (fig. I-2.23). Suppose P overtakes Q after travelling a distance x_1. Let the distance which Q travels in the same time t be x_3. Then

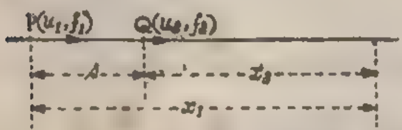


Fig. I-2.23

$$x_1 = u_1 t + \tfrac{1}{2} f_1 t^2 \quad \text{and} \quad x_2 = u_2 t + \tfrac{1}{2} f_2 t^2.$$

$$\therefore \quad s = x_1 - x_2 = (u_1 - u_2)t + \tfrac{1}{2}(f_1 - f_2)t^2. \qquad \text{(I-2.14.1)}$$

This is the same as if Q were stationary and P approached Q with a velocity $u_1 - u_2$ and acceleration $f_1 - f_2$. (Velocity and acceleration of Q are reversed and added to those of P). The time required by the faster body to overtake the slower one is that which it will take in travelling the distance s with a velocity $u_1 - u_2$ and acceleration $f_1 - f_2$.

Ex. I-2.15. *A stone is dropped from a high altitude and 3.00s later. Another is projected downward with a spread of 150 ft/s. When and where will the second overtake the first ?* (g=32 ft/s^2).

Solution : At the moment the second stone is projected the first one, *i.e.*, the slower one has fallen $\tfrac{1}{2} \times 32 \times 3^2 = 144$ ft, and its speed $u_1 = 32 \times 3 = 96$ ft/s. The second stone has $u_2 = 150$ ft/s. The acceleration is $g = 32$ ft/s^2 for both.

We, therefore, have $s=(150-96)t+\frac{1}{2}(g-g)t^2=54t$, i.e., $t=2\frac{2}{3}$ s.

The distance x, from the point of projection $=150\times 2\frac{2}{3}+16\times(2\frac{2}{3})^2=514$ ft

Problem :. A bullet is shot vertically upward with a speed of 320 ft/s, and 4 s later a second bullet is shot upward with a speed of 190 ft/s. Will they ever meet ? If so, where-?

(*Hint* : Calculate the speed of the first bullet after 4 sec. Note whether the faster bullet is approaching the slower one or receding from it. They will not meet.)

A. Accelerating Lift : Let a lift accelerate *upwards* with f when referred to a frame at rest on the surface of the earth. The acceleration of the lift referred to another frame accelerating downwards with g under gravity is obviously $[f-(-g)]$ i.e. $(g+f)$ upwards as shown in fig. I-2.24(c)

In the fig., in (a) we show separately f the upward acceleration of the lift and also the downward accelerating frame falling freely with g. In (b) side by side are shown f and reversed g. They are compounded in (c).

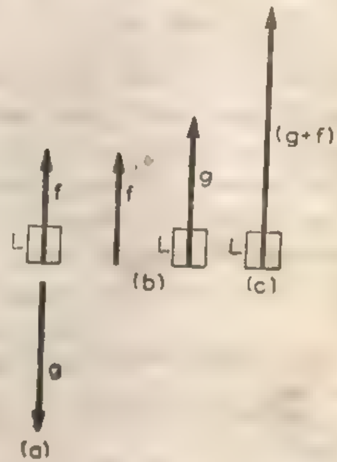


Fig. I-2.24

If the same lift accelerates *downwards* with same $f\,(<g)$ then its relative acceleration becomes $[f-(-g)]$ i.e. $(g-f)$ *upwards*.. To and observer in the rescending frame the lift would appear to move upwards with this acceleration. We shall return to this problem later (I-3.11B).

Thus for ascending and descending lifts the relative accelerations would be

$$\left.\begin{aligned} a\uparrow &= g+f \\ a\downarrow &= g-f \end{aligned}\right\}\downarrow \qquad \text{(I-2.14.2)}$$

Problem : Ane levator is ascending with an acceleration of 4.0 ft/s². At the instant when its upward speed is 8.0 ft/s, a bolt drops from the top of the cage, which is 9.0 ft from the floor. Find the time until the bolt strikes the floor and the distance it has fallen. [ANS : 0.71 s ; 2.3 ft.]

[*Hint* : For the elevator $u_1=8$ ft/s, $a_1=4$ ft/s². For the bolt $u_2=8$ ft/s, $a_2=-32$ ft/s², $s=9$ ft.)

B. Relative Acceleration along Perpendicular lines. Let there be a large drum on its side on an open lorry at rest. Suddenly the lorry is started forward. Unless somebody holds on to the drum it will roll backwards with an opposite acceleration and tumble out onto the ground. In addition to the horizontal acceleration it also has a vertical one due to gravity.

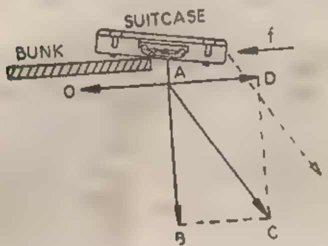


Fig. I-2.25

A similar case is shown in fig I-2.25. A running train is suddently decelerated. A suitcase on the bunk slides forward and falls on the floor. The acceleration of the train (here −ve) is represented by AO which equals −AD. The suitcase accelerates downwards represented by g=AB. Clearly the relative acceleration of the suitcase with respect to the decelerating train will be

$$f_R = AB - OA = AB + AD = AC$$

and $$\tan\theta = \frac{BC}{AB} = \frac{AD}{AB} = \frac{f}{g} \qquad \text{(I-2.14.3)}$$

Hence the suitcase will appear to fall forward at an angle of $\tan^{-1}(f/g)$.

Problem : If a moving train suddenly retards at 1.2m/s², a suitcase slips forward and falls on the floor. Find its relative acceleration and the direction of fall.

(*Ans.* $f_R = \sqrt{1.2^2 + 9.8^2}$; $\tan^{-1} 7°$ (nearly) $= \tan^{-1}\frac{1.2}{9.8}$)

KINETICS
NEWTON'S LAWS OF MOTION

I-3.1. Kinetics.

So long we have learnt *Kinematics*—motion has been discussed in its abstract form without considering what moves a body and how does it do so. Now we take up *Kinetics*, which tries to answer these two questions and throws up two new concepts central to the answer, mass (*inertia*) and *force.*. Here as before, when we say a body you should take it to be a particle.

A body moves under an *unbalanced* force (one that is not opposed by an equal and opposite force) and it moves with an acceleration. Our idea of force is associated with that of *push* or *pull*—the so-called *contact forces.* But forces may be exerted without contact; namely gravitational, electric or magnetic forces of attraction and repulsion, which need neither contact nor medium to act. They are *field forces.*

The nature of force which can affect bodies without contact, which can move a body after it has parted company with the agent e.g. a stone hurled forward by your hand) had engaged the attention of wise thinkers since the days of the academic Aristotle (384-322 B.C.) tutor to Alexander the Great till the experimenter Galeleo. He for the first time introduced experimentation, to study the behaviour of moving bodies. He carried out extensive and systematic observations on how a body moves, how does it come to a stop, how does gravity affect the motion of a body falling freely or rolling down an incline, what happens when it collides with another and others. He followed up these observations with a chain of logical arguments and arrived at some general pattern of behaviour of material objects when at rest or in motion. He concluded that (1) a body at rest tends to continue to do so, (2) likewise (but contrary to teachings of Aristotle) a moving body continues to move without change either of direction or of speed and (3) it requires a force (unbalanced and exernally applied) to change any of these states. You must realise that the idea of inertia is introduced thereby and also Newton's First

law.* Intertial frames of reference (I-1.4) are alternatively called Galelean frame to honour his pioneering efforts.

Newton (1642-1728) built upon and expanded these findings to his famous rules or laws of force. They cannot be formally proved or deduced but crystallizes human experience through the ages and form the basis of *Classical Mechanics.* They have stood the test of rigorous investigations with complete accuracy for (*i*) *gross bodies* and (*ii*) at *speeds small* compared to that of light.

Apart from (1) the idea of unbalanced forces we should get used to treating (2) a body as a particle and (3) the idea of a *closed system*, when discussing Newton's laws of motion. Interacting particles are imagined to be inside a closed surface quite *isolated* from the effects of any other body or particle.

I-3.2. Newton's Laws of Motion.**

The exact relation between force and motion was stated first in the form of three laws by Newton in his monumental work (1686-87) Principia Mathematica Philosophiae Naturalis, written in Latin. The laws are

Law I. *Every body* (i.e. a particle) *continues in its state of rest or of uniform motion in a straight line unless impelled by an unbalanced external force to change that state.*

Law II. *The time rate of change of momentum is proportional to the externally impressed unbalanced force and is in the direction of that force.*

Law III. *In an isolated or closed system, action and reaction are equal and opposite.*

I-3.3. Discussion on the First Law ; Intertia and Force.

The law introduces to us the concepts of *inertia*, those of (*a*) inertia of rest (*b*) inertia of motion and (*c*) that of *force*. The law is really a statement about reference frames and as it is called the law

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*Newton, a truly great savant, had declared "If I have been able to see more than others, that is because I stood on shoulders of giants." Galeleo was the foremost amongst these giants.

**You are surely aware of these laws from your secondary school. Yet go through them carefully. There may be significant differences in the statements.

of inertia, the frames in which tht laws are valid are called *Inertial* or Galelean frames.

I-3.3A. Inertia of Rest : The tendency of a body to continue in its state of rest is an inherent property that is called *inertia of rest.*

(I) You have already learnt that, it is this that is responsible, for falling back or leaning back of a standing or sitting passenger in a car that starts suddenly.

(2) You must have noticed while playing carrom that if you pile up carrom dices in a neat vertical column and hit the lowermost dice sharply with the striker it darts forward and the pile just settles down without toppling over.

(3) Take an empty cup. On it place a postcard and on that a steel ball. Flip the card forward sharply, it will move off but the ball will land inside the cup. Note that the carrom-board surface is generally smooth and hence friction is small ; same is the case of spherical ball on the card.

(4) A *seismograph* utilises inertia of rest to record movements due to earthquakes. It consists (fig. I-3.1) of a horizontal pendulum rod R fixed to a nearly vertical axis A. R carries a heavy block (B) at the end with a sharp stylus St, touching the surface of a smoked drum D. D rotates slowly with a constant speed, moving axially at the same time. The base of the drum D is rigidly attached to the ground. On D, St traces a uniform spiral. In an earthquake, the ground the support of D, moves before B does and hence St does and so records the quake in irregular jerks (*j*). [I-3.1(b)].

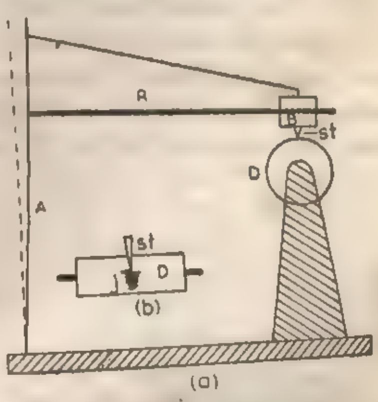


Fig. I-3.1

(5) Put a cupful of water on a smooth table and give it a sharp forward push. Water will splash back on your hand. This happens because of inertia of rest exactly as happens to the passengers in a bus which just starts.

(6) Hang a kg-wt by a thin thread, say A, from a rigid support. Attach another thin string, say B, from its lower side. Now pull B slowly and steadily and you will find A to snap, not B ; give B a sharp tug and B will break, not A. The large inertia of rest of the weight permits a momentary but very large force to develope on B and prevents it from reaching A and so B snaps, not A.

I-3.3B. Intertia of Motion : The inherent property of a moving body to tend to continue to move uniformly in a straight line was first recognised by Galeleo experimentally and is called *inertia of motion.* The *velocity vector* of a moving body tends to remain unchanged. We present below a number of illustrations.

(1) Galeleo '*diluted*' gravity by allowing a ball to roll down inside a large bowl ; he found the ball rolling up on the other side to approximately the same height irrespective of the incline.

(2) Again, the ball, if rolls out on a level plane after descending the incline, would go further and further as the level plane is made smoother and smoother. It is only the force of friction, omnipresent, that opposes the motion, reduces it and finally halts the ball.

(3) You sprinkle french chalk or powder on carrom boards and billiard tables to reduce friction. Hard-pressed plain ice surfaces are *almost* frictionless. Both provide instances of inertia of motion when the carrom striker or the billiard ball or the ice skater moves on. An ice-hockey ball is a block not a sphere (Why ?)

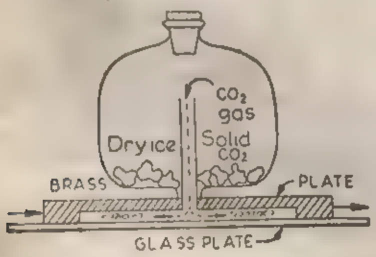


Fig. I-3.2

(4) A 'dry ice' puck is a brass plate which literary floats on a thin film of CO_2 gas and rides over a glass plate. A very slight push makes the disc move (fig. I-3.2) on and on and on in a straight line with constant speed as if by magic. Friction here is almost non-existent.

(5) An object say a spaceship, far out in interplanetary space where no air resistance exists and no appreciable gravitational attraction, follow a linear path with constant speed for ever, like the

Voyager and Pioneer space-probes sent out to learn about our Solar System by the U.S.A.

The *tendency of moving bodies to persist along a straight line* is an everyday experience. Mudguards over car or bicycle wheels are provided to protect you from the mud and slime which the whirring wheels throw up tangentially. Sparks fly off tangentially from a fast revolving grind-wheel when a knife-blade is pressed on it. A ball thrown up in a uniformly moving open railway carriage returns to hand, a vaulting performer on a slowly cantering circus-horse re-lands squarely in her seat, due to this inertia of motion both in speed and direction.

You know why—standing passengers fall forward when their bus suddenly stops; commuters alighting from moving buses must lean backwards, or fall on their faces if they do not; trunks slide and jump forward off bunks in railway compartments if a fast train suddenly brakes—all due to inertia of motion. Such experiences can be multiplied; think up some yourself.

I-3.3C. Inertia and Frames of Reference: *Inertia of rest and motion are not however, two different properties of a body. but appears so because of the frame of the observer.* We return for clarification to the case of a ball thrown up vertically inside a uniformly moving open carriage.

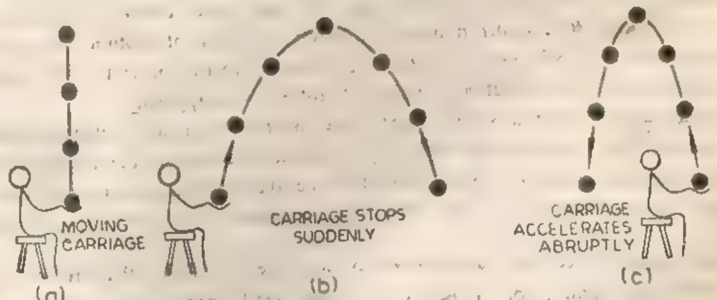


Fig I-3.3

(1) The path of the ball would appear as in fig I-3.3(a) to another passenger seated in the some car; but to a stationary observer on the platform the ball appears to describe a curved path as in (b). To the first observer the up and down path described, is due to inertia of rest, while to the second one, the forward motion would bear out inertia of motion. To the first observer the *uniformly moving* carriage is the reference frame; to the second, the unmoving platform is the reference frame—both are inertial and so to both of them Newton's first law is valid.

(2) Exactly similar observations are made by a pilot in a steadily flying bomber when he drops a bomb and a man on the ground who sees the bomb falling ; to the pilot in a uniformly moving frame the bomb descends vertically ; to the man on the ground it appears to describe half a parabola (fig. I-3.4).

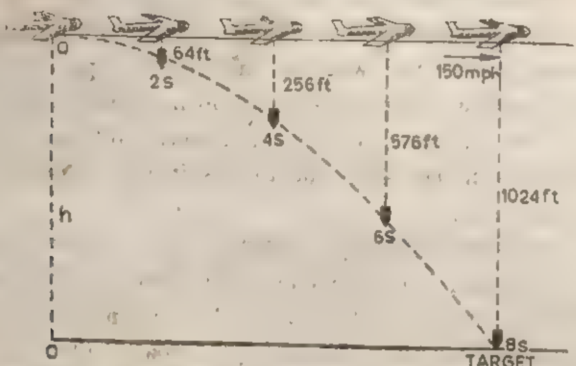


Fig. I-3.4

(3) A boy leaning out of a fast but steadily running train drops a coin and a man standing on a field beside, observes the fall ; to the boy it is a vertical fall, to the man parabolic. Later (I-3.6), we shall look at these phenomena from another angle—the Principle of Independence of Motion.

In fig. I-3.3(c) is shown what happens if the carriage uniformly accelerates immediately after the ball is thrown up. The ball lands behind. The observer on the platform will consider it normal, a result of inertia of motion. But to the seated passenger, it will appear to be a violation of inertia of rest ; for he sees the ball going back without any horizontal force being applied.

If the carriage takes a sudden bend while the ball is in air, it will not return to the boy. He will see it moving away from the direction of the bend. To the observer outside, the ball moves along the line the carriage was moving before. To him it is natural but to the boy, it is inexplicable from Newton's first law.

These happen because the passenger is no longer in an inertial frame but in an accelerated frame ; for in the former case speed changes, in the latter, the direction. We shall return presently (I-3.6) to this apparent anomaly.

Thus the first law does not differentiate bbetween bodies at rest and in uniform rectilinear motion. In absence of forces both tendencies are *"natural."* The apparent differences is all due to reference frames. Suppose on parallel tracks are a train at rest and another moving uniformly past. To a passenger in the first train, others in his compartment are at rest ; to an observer in the second, the same passengers in the first train appear to recede uniformly. To none of the two sets of observrs, the compartment at rest appear to have an acceleration and hence (from first laws) under any force.

I-3.3D. Inertia and Mass: Mass is defined as the quantity of matter in a body. The definition is not satisfactory for it is not operational—it does not provide a method of measuring mass.

Mass is a measure of inertia for we find larger the mass of a body larger is its inertia; inertia is that fundamental property of matter because of which a piece of matter resists any attempt (*i*) to start or accelerate it when at rest, (*ii*) to stop or decelerate it or (*iii*) change the direction, when in motion. Larger the mass proportionately greater, appears to be this difficulty. Compare the efforts to start or stop or divert a child's pram and a motor car and you will realise why mass is taken as the measure inertia. They are both measured in kg (MKS) g (CGS) and slug (FPS).

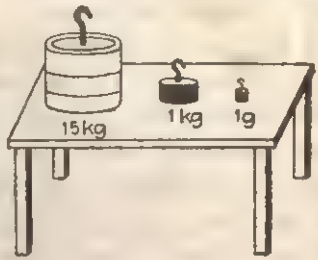


Fig. I-3.5

A *demonstration experiment* is as follows. On a toy-car moving smoothly on rails there is a vertical pivot (fig. I-3.6) which

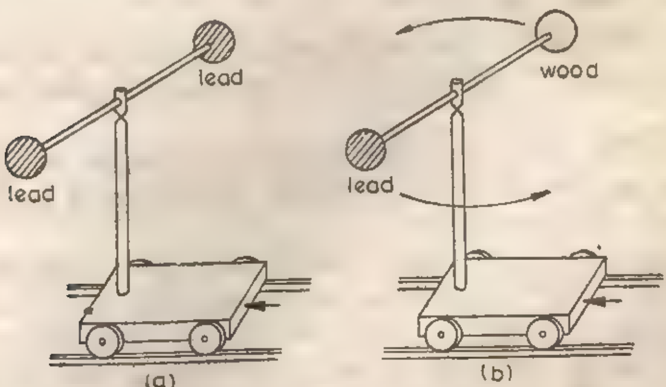


Fig I-3.6

supports a light uniform horizontal bar that carries at its two ends identical lead balls. The car may be pushed along the rails. When it starts suddenly or stops suddenly the rod remains unmoved; one of the balls is next replaced by a wooden one of same size. Now

if the car is suddenly started the rod starts rotating, the lead ball backwards, the wooden one forwards. Stop the car suddenly ; the sense of rotation reverses, the lead ball advancing, the wooden one receding. The lead ball has a larger mass and hence larger inertia ; hence its above behaviour.

A familiar occurrence illustrates the fact that *inertia is proportional to mass.* You might have observed that sometimes a railway engine cannot start a long heavy train. It then first pushes the bogies backwards and then starts the forward motion. What happens is this—when the links between the bogies are taut, to start the train acceleration has to be imparted to the entire train. This has definitely a very large mass and the engine has not power enough to do it. It gives a backward push to loosen the couplings between the bogies. Then when it exerts a pull the bogey just behind the engine, having a much smaller mass and hence less inertia, gets a forward motion. This is picked up by the successive bogies one by one and the whole train gets under way forwards.

Not for nothing has Newton's first law of motion been called the law of intertia. *Inertia is that fundamental property of matter by which it resists any attempt to change its state of motion* including that of no motion. This assertion follows from the first part of the first law.

1-3.3E. Force : The second part of the first law provides us with a qualitative* definition of force, the concept of greater interest to us.

Force is defined to be *that agency which changes or tends to change the state of uniform rectilinear motion and of no motion (i.e. state of rest) of a body.*

Suppose you push a heavy truck or pull a heavy roller and you can move neither ; this is because an opposing force, that of friction, neutralises your effort. A ceiling fan is pulled downwards by the force of gravity but it does not fall, for the opposing force of tension acting upwards through the supporting rod, neutralises the weight. The forces tend to move the truck or roller or the fan but cannot do so because they are *balanced.* If then several ropes pull a heavy body in various directions but fail to move it, that is because they

* Not the quantitative or operational one, which comes from the second time which you know, viz. $F=ma$.

add up to a zero resultant. Thus the first law does not distinguish between absence of all forces on a body and action of any number of forces on it, that add up to zero ; so the law can be alternatively phrased as *if no net force acts on a body its acceleration is zero.* Hence the importance of the adjective *unbalanced* to the word force, in the statement of the first law given above. Newton's third law tells us that in nature there can never be a single force ; there can be either an unbalanced (resultant) force or a system of balanced ones.

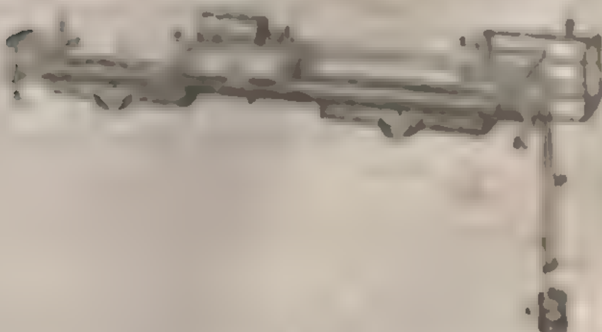
Note carefully, that an *unbalanced force must act on a body from outside to make it move.* You can never move a car by pushing it from inside, however hard you may try. When your car stalls, you must come outside and push it.

This happens because when you are inside the car you are inside a *closed system.* There are just two bodies, you and the inside wall of the car ; you push the wall and the wall pushes you back equally because of Newton's third law ; and the forces are balanced. When you push from outside you are utilising a third body, the ground. See the cases of a rickshaw puller pulling his vehicle and a horse pulling a cart [I-3.9B(3)].

Force as a Vector : As it involves acceleration, a vector, the force must also be a vector. It has both magnitude and direction and as we shall next, that it obeys the laws of vector addition and resolution. Further, the *point of application* of a force has to be specified just as for a vector. This last requirement brings out the importance of treating a body as a particle. The latter being of no size, the question of point of application does not arise ; but it does, for a body. Push a heavy book or a brick with a single finger ; you will not move it in a straight line if you push at any and every point ; it would move and rotate together. Only when, the line of push passes along a particular line (passing through the *centre of mass* of the body) it will move in a straight line. Later (I-7.10) we shall see that this point is the one where the whole mass of the body can be *taken* to be concentrated, in fact the body reduced to a particle, at least in imagination. That point is its *center of mass.*

R **Effects of Forces :** In mechanics, force is the most important entity. It can do a number of tasks, as for example

(*a*) Two or more forces may keep a body or particle at rest. Examples are many and obvious.



with a pair of parallel rails *rr* fixed upon it. The bench can be accurately levelled. A *trolley* T with frictionless wheels runs along the rails. Two pair of springs (SS) called *frictionless brakes* fixed at the terminii, arrest the motion of the trolley slowly. A frictionless *pulley* P fixed at one end helps to pull the trolley if necessary by a cord. A metal *reed* R, capable of transverse vibration carries an inked *stylus* St. It touches lightly a long paper *tape* TP fixed to the pulley at one end and passing over a smooth roller carries a *load* L at the other end.

The base plate is made horizontal by levelling screws. The stylus tip is inked and the trolley slowly pulled towards B_1. A straight line referred to as the *zero-line* is thus drawn along the medial line of the tape. The stylus is then set vibrating at right angles to the zero line and the trolley released at B_1. The load pulls the trolley forwards with a constant aceleration. The tiny inked stylus traces out a wavy curve on the tape sliding beneath it. Time for each vibration bebing constant, this device gives small time-intervals. The points of insection of also a convenient method of accurately measuring the wave and the zero-line, *a, b, c, d*...are marked and the distances *ab, bc, cd*...accurately measured.

Fig. I-3.7(b)

If the period of vibration of the reed be T then the *average* velocity in succeding intervals of time would be ab/T, bc/T, so that the change in velocity in the first interval is $(bc—ab)/T$ and so the acceleration $(bc—ab)/T^2$. The load L and the trolley mass (M) remaining constant, acceleration comes out to be the same for each successive segments like $(cd-bc)$. Thus we find that acceleration (a) to be proportional to the force—the weight of the load L.

I-3.4A. Units of force. (a). Absolute units: A unit of force defined according to the equation $F=ma$ is called an absolute unit.

In cgs units the absolute unit of force is the *dyne*. The *dyne** is that force which, acting on a mass of 1 g, gives it an acceleration of 1 cm/s^2.

$$1 \text{ dyne} = 1 \text{ g} \times 1 \frac{\text{cm}}{\text{s}^2} = 1 \frac{\text{g cm}}{\text{s}^2} = 1 \text{ g cm s}^{-2}.$$

The dyne (symbol dyn) can be replaced by its equivalent g cm/s^2

In *mks units*, the unit of force is called the *newton***. It is the force required to produce an acceleration of 1 m/s^2 in a mass of 1 kg. Thus

1 newton (symbol N) $= 1 \text{ kg} \times 1 \text{ m/s}^2 = 1 \text{ kg m s}^{-2}$

The newton (N) can be replaced by its equivalent kg m s^{-2}

In the British system the absolute unit of force is called *poundal*. The *poundal*† is that force which, acting on a mass of 1 slug, (or, lb) gives it an acceleration of 1 ft/s^2.

Relation between newton and dyne: Let us write 1 newton $=$ 1 kg $\times$ 1 $m/s^2 = x$ dynes $= x$ g cm s^{-2}

$$\therefore \quad x = 1 \frac{\text{kg}}{\text{g}} \cdot \frac{\text{m}}{\text{cm}} = 1000 \times 100 = 10^5$$

$$\text{or } 1 \text{ N} = 10^5 \text{ dyn.} \qquad \text{(I-3.4.3a)}$$

Relation between the poundal and the dyne:

$$\begin{aligned} 1 \text{ Poundal} &= 1 \text{ lb} \times 1 \text{ ft/s}^2 \\ &= 453.6 \text{ g} \times 30.48 \text{ cm/s}^2 \\ &= 453.6 \times 30.48 \text{ g cm/s}^2 \\ &= 1.382 \times 10^4 \text{ dyn} = 0.1382 \text{ N.} \end{aligned} \qquad \text{(I-3.4.3b)}$$

(b) Gravitational units of force. We use such terms as 'a kilogram of force or a kilogram-weight' and 'a gram of force or a gram-weight', etc. These are *gravitational units of force* and represent the attraction of the earth on a unit mass, i.e., the weight of a kilogram or a gram or a pound. As the weight of a body varies slightly at different places it is necessary for the *sake of accuracy* to specify where this weight is measured.

A *kilogram of force* (written kg-wt or kgf) is the weight of a kilogram of mass at 45° latitude and mean sea-level.

A gram of force (or a gram-weight. written g-wt or gf) is the weight of a gram of mass at 45° latitude and mean sea-level.

* The dyne is a very small force, being about the weight of a mosquito.

** A newton is about equal to the weight of 100 g.

† The poundal is about equal to the weight of a 14-gram mass.

A *pound of force* (or a pound-weight, written lb-wt or lb) is the weight of a pound of mass at 45° lititude and mean sea-level. Nowadays in FPS system **Slug** is the unit of mass and **pound** that of force.

For most practical purposes the slight variations in weight due to differences in location may be neglected.

Newton, dyne, poundal are *absolute units* for they do not change. But kgf, gf or lb are not, as they depend on the value of g which is not constant everywhere.

Conversion from gravitational to absolute unit. A kilogram of force makes a kilogram of mass move with an acceleration g (=9.8 m/s²). Hence, to convert a force in gravitational unit into one in absolute unit, multiply by the appropriate value of g. Thus

1 kg-wt or kgf=9.8 N (because $g=9.8$ m/s² in mks units)
1 g-wt or gf=980 dyn (because $g=980$ cm/s² in cgs units)
1 lb-wt or lb=32 poundals (because $g=32$ ft/s² in fps units)

Where extremely high precision is needed we use the *standard value of g*, which is 980.665 cm/s² or 32.1741 ft/s². This value closely represents *the value of g at 45° latitude and mean sea-level.*

Note from the above that if in the relation $F=kma$, we take $k=g$ in the same system of units as m and a and put m and a equal to unity, then F becomes the gravitational unit of force, *i.e. k the proportionality const, is not unity*, as for absolute units.

I-3.4B. Alternative derivation of F=ma: Statement of the law includes the expression the time rate of change of momentum ($d\mathbf{p}/dt$) and you know that momentum (**p**) of a body is measured by the product of its mass (m) and velocity (**v**). Note that momentum is a vector—*mass* (a scalar) *times the velocity vector* of a moving particle. The second law may then be expressed as

$$\mathbf{F} \propto \frac{d\mathbf{p}}{dt} \quad \text{or} \quad \mathbf{F} = k\frac{d\mathbf{p}}{dt} = k\frac{d}{dt}(m\mathbf{v})$$

$$= k\left(m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}\right) \qquad \text{(I-3.4.4)}$$

When m is constant (it is not always so, as has been pointed out before) the second term inside the bracket vanishes and we are left with

$$\mathbf{F} = k\,m\,(d\mathbf{v}/dt) = k\,m\mathbf{a} \qquad \text{(I-3.4.5)}$$

With suitable choice of units k becomes 1 and the magnitudes are related by

$$F = ma \qquad \text{(I-3.4.6)}$$

I-3.4C. Derivation of the First law from the Second : If we put $F=0$ in equation (I-3.4.5) we have $(m\, d\mathbf{v}/dt)$ vanishing. Since $m \neq 0$ we must have $d\mathbf{v}/dt=0$ *or* $\mathbf{v}$ const. Thus without an external force acting, the body must be moving with velocity unchanging both in magnitude and direction. The condition must also include the case of $\mathbf{v}=0$ i.e. the body at rest continues to do so.

The continuation of a moving body without changing either speed or direction may be more elegantly established as follows : the velocity vector may be represented as

$$\mathbf{v} = |v|\,\mathbf{r}$$

where $|v|$ represents the magnitude of velocity (the speed) and $\mathbf{r}$ the associated unit vector i.e. the direction. Then

$$\frac{d\mathbf{v}}{dt} = \frac{d}{dt}\Big[\,|v|\,\mathbf{r}\Big] = \Big[\frac{dv}{dt}\mathbf{r} + v\,\frac{d\mathbf{r}}{dt}\Big]$$

For $d\mathbf{v}/dt$ to vanish, each of the terms in the bracket must separately vanish. Since again for a moving body, neither $\mathbf{v}$ nor $\mathbf{r}$ can vanish we must have separately the result that

$$\frac{dv}{dt}=0 \quad \text{and} \quad \frac{d\mathbf{r}}{dt}=0$$

which means that neither speed nor direction of a moving body changes if no resultant force acts upon it.

We shall later obtain the third law from the first. So it seems that we could have done with the second law alone, for the first and third laws can be derived as corrolaries from it.

I-3.4D. Inertial mass : From the second law of motion, mass comes out to be a ratio of externally applied unbalanced force **(F)** to the resultant acceleration **(a)** i.e. the *ratio of cause to effect*, a ratio of two vectors which is itself a scalar. It is so called, as mass is measured by inertia.

Intertial mass shows the following three properties :—

(*i*) The inertial mass of two bodies is the (algebraic) sum of the masses of the two.

(*ii*) Inertial mass of a body does not depend on the shape or composition of the body.

(*iii*) Chemical reaction does not change the total mass of the reactants.

To compare the masses m_1 and m_2 of two bodies, we may apply the same force F to them and find the accelerations they acquire. We shall then have $F=m_1a_1=m_2a_2$ or $m_1/m_2 = a_2/a_1$. The accelerations produced will be inversely proportional to the masses. If

$m_1=1$ kg, $m_2=a_1/a_2$ kg. In I-3.4F we describe a method of so doing.

Solution of problems based on the equation F=ma.

In applying the equation $F=ma$, remember that

(*i*) When an unbalanced force F acts on body of mass m, it moves with an acceleration $a=F/m$.

(*ii*) This acceleration is in the direction of the applied force.

(*iii*) The acceleration continues so long as the force acts.

(*iv*) When the force ceases to act, the acceleration also ceases. The body then continues to move with the velocity, it had at the moment when the force stopped.

(v) 1 dyn=1 g. cm/s². 1 N (newton)=1 kg. m/s²=10^5 dyn.

(*vi*) In equations, unit symbols cancel like algebraic quantities.

Examples I-3.1. *Sand drops vertically from a nozzle, 100 g each second on a conveyor belt moving horizontally at 10 cm/s. Find the force on the belt.*

Solution: The velocity of falling sand particles is zero horizontally. But the time rate of change of mass is 100 g/s. From equation I-3.4.4 we deduce that the force in this case is

$$v\frac{dm}{dt}=10\frac{\text{cm}}{\text{s}}\times 100\frac{\text{g}}{\text{s}}=1000 \text{ g-cm/s}^2=1000 \text{ dyn.}$$

Examples I-3.2. *A string AB of length L lies on a frictionless table. It is pulled at A with a force F. Find the tension at a point C distant l from A.*

[I.I.T. '78]

Solution: The acceleration generated on the string as a whole is F/m. Then the mass of the portion AC$=m\times l/L$.

$\therefore$ **Tensional force at C** = mass of length AC (l) $\times$ acceleration

$$=\frac{ml}{L}\times\frac{F}{m}=\frac{Fl}{L}$$

So tension is independent of the mass of the string.

Ex. I-3.3. *A force acts on a mass of 120 lbs for 3 s and then stops. In the next 3 s the body describes 108 ft. Find the force in lbs-wt.*

[H.S. (Tripura) '80]

Solution: So long as the force acts the body accelerates. After the force stops the body moves on uniformly with the velocity acquired. So

$$v\times 3 \text{ s}=108 \text{ ft} \quad \text{or} \quad v=36 \text{ ft/s}$$

Again $v=at$ or $a=\frac{v}{t}=\frac{36 \text{ ft/s}}{3\text{s}}=\frac{12\text{ft}}{\text{s}^2}$

[since the body starts from rest]

$\therefore$ Acting force $F=ma=120$ lb $\times$ 12ft/s² = 1440 poundals

= 1440/32 lbs-wt = 45 lbs-wt

Note again, mass is nowadays measured in slugs and force in lb. not lbs-wt.

Ex. I-3.4. *How will the equation F=ma be modified if Newton is the unit of force, metric ton that of mass and km/hr. min. the unit of acceleration.*

Solution : 1 metric ton $=10^3$ kg. So to produce on it an acceleration of 1 km/hr. min. the necessary force would be

$$10^3\,\text{kg}\times\frac{10^3\ \text{m}}{3600\times 60\ \text{s}^2}=\frac{10^4}{36\times 6}\frac{\text{kg-m}}{\text{s}^2}\simeq 4.63\ \text{N}$$

$\therefore$ In $F=kma$ we have to put $F=4.63$, $m=1$ and $k=1$ or $k=4.63$

so that F (newtons) $=4.63\times m$ (tons) $\times a$ (km/hr. min).

Problems : (1) A force acts on a body of mass 16 g for 3 s and then stops. In the next 3 s the body describes 81 cm. What is the force on the body ?

(*Ans.* 144 dyn) **[H.S. (Tripura) '82]**

(2) If the unit of mass be 1 kg, unit of length 1 decimetre and that of time 1 hectosecond (=100 s) in a certain system show that the unit of force is 1 dyn.

I-3.4E. Impulse of a force : The product of a force (F) and the time (t) for which it acts on a body is known as the *impulse of the force.* An impulse (J) produces a change of momentum. For

$$F\times t=m\times a\times t=m(v-u)=mv-mu \tag{I-3.4.7}$$

or *Impulse of a force=Change of linear momentum of the body.*

A large force acting for a short time is called an *impulsive force.* Kicking a football, hitting a cricket or tennis ball, are examples of impulsive forces. The effect of an impulsive force is to change the momentum of the body on which it acts. If the body was at rest, it starts with a velocity because of it. The velocity will be greater the longer the force acts on the body. Hence a kick to a football will be more effective if the foot *follows through*, i.e., maintains a longer contact with the ball. So also for a bat hitting a ball, the 'follow-through'.

Unit. Unit of impulse is that of momentum i.e., g cm/s or kg-m/s or again dyn-s or N-s which is 10^5 times the former.

Now, an impulse is denoted by **J** and linear momentum by **p**. So the relation between them is given by $\mathbf{J}=\mathbf{p}t$. From the statement of the 2nd law we have seen already that $\mathbf{F}=d\mathbf{p}/dt$.

Graphical representation of Impulse. Since force and the time of its application are involved in impulse we may plot relations among them. In fig. I-3.8 the straight line (a) AB parallel to t-axis

shows their relation when the force is const and the curved line (b) when the force is variable. As impulse is the product of F and t, the area between the curve and the two axes gives the measure of impulse. We have already graphically found the distance (s) covered for a moving body in fig. I-2.14, in the same way.

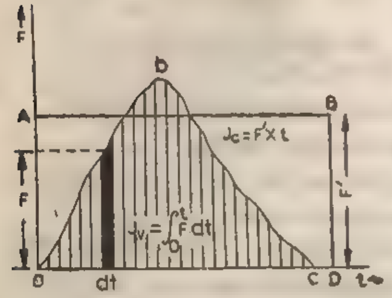


Fig. I-3.8

For a variable force, we break up the time-interval in very small bits dt and find the area under the curve, shown shaded and add up all of them [the method is that of finding a definite integral : see fig 0-2.8]. During each small bit of time however the force is taken to remain constant. Thus we find that the magnitude of the variable impulse is

$$J_v = \int_0^t \mathbf{F}.dt^* = \int_0^t \frac{d\mathbf{p}}{dt}.\,dt = \int_0^t dp = \mathbf{mv} - \mathbf{mu} \qquad \text{(I-3.4.8)}$$

where **mv** is the final and **mu** the initial momentum.

Vector Representation of momentum and its change. The situation is illustrated in fig. I-3.9 where **pq** represents initial momentum both in magnitude and direction. A force **F** now acts on the body for a time interval t bringing about a change in momentum along its own direction, during that interval represented in magnitude and direction by **qr.** Vector **qr** then gives the impulse of the force and **pr** the final momentum.

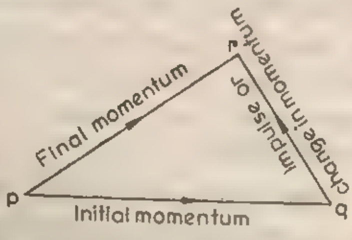


Fig. I-3.9

Ballistics. This is the branch of dynamics dealing with the

* Impulse J_v is said to be the *time-integral* of force for you are summing up time-intervals. Later (I-8.1) we shall see work is *space-integral* of force.

motion of a body under an impulsive—a very large—force acting for a very short time e.g. a kick on a football, a powerful smash in tennis or badminton, a firm hit on a cricket ball, firing a rifle, canon or an ICBM. A force changes the (*i*) position and (*ii*) momentum of a body and that, *only so long as it acts*. For an impulsive force, it is *assumed* that the body changes only momentum, not position, the time-interval being so very small. The effect of an impulsive force is thus a sudden change in a very short time. The subsequent motion of the body belongs to ballistics.

The expression I-3.4.8 is said to be *Impulse-Momentum theorem.*

Ex. I-3.5. *A force of 20 dynes acts on a body of mass 5 g initially at rest What is the acceleration produced? If the force ceases to act after 5 seconds how will the body move?*

Solution: From the relation that acceleration = force ÷ mass, we have $a = 20$ dyn/5 g = 20 g cm/s^2 ÷ 5 g = 4 cm/s^2.

As the force acts for 5 seconds, the increase in velocity in this interval is 4 cm/s^2 × 5 s = 20 cm/s. Since the body was at rest initially, this is the velocity when the force ceases to act. By Newton's first law of motion, the body will continue to move with this constant velocity after the force ceases. The motion is in the direction in which the force acted.

Ex. I-3.6. *Under action of a force a body of mass 10 g moved 40 cm from rest in 4 seconds. What is the magnitude of the force? If the force acts altogether for 5 seconds, how far will the body move in 3 seconds after the force has ceased to act?*

Solution. Here $u=0$, $s=40$ cm, $t=4$s. To find the force we must find the acceleration. From the relation $s=ut+\frac{1}{2}at^2$, we get

$$40 \text{ cm} = \tfrac{1}{2}\, a \times (4\text{s})^2 \text{ or } a = 5 \text{ cm/s}^2.$$

Since the force produces this acceleration on a mass 10 g,

$$\text{the force} = 10 \text{ g} \times 5 \text{ cm/s}^2 = 50 \text{ g cm/s}^2 = 50 \text{ dyn}.$$

The velocity acquired comes from the relation $v=u+at$. Since $u=0$, we have $v=5$ cm/s^2 × 5s = 25 cm/s. This is the velocity of the body when the force stops. The body travels on with this constant velocity thereafter, and in the next 3 seconds covers 25 cm/s × 3s = 75 cm.

Ex. I-3.7. *A football player takes a spot kick as a result of which the ball moves with a speed of 10 m/s. If the ball weighs 350 g and its time of contact with the foot during the kick is 0.1 second, what is the force applied?*

Solution. In the relation $Ft=m(v-u)$ we have $u=0$, $m=350$ g. $v=1000$ cm/s.

$$\therefore \quad F = \frac{350\text{g} \times 1000 \text{ cm/s}}{1/10\text{s}} = 3{\cdot}5 \times 10^6 \text{ dyn} = 35\text{N}.$$

Ex. 1-3.8. *You throw a stone weighing 60 g vertically upward. It reaches the top of a three-storeyed building which is 10 m high. If in throwing the stone you swung it for 0.2 second, what force did you apply on it?* ($g=9.8$ m/s^2).

(Note that the values are in mixed units. Convert all of them into the same system, cgs or mks.)

Solution. It is an impulsive force. If it started upward with a velocity u, then from the relation $H=u^2/2g$, we get $u^2=2gH=2\times9.8$ m/s$^2\times10$ m or $u=14$ m/s. Hence the momentum imparted was 0.06 kg$\times$14 m/s$=$0.84 kg m/s. This momentum was imparted in 0.2 s. Hence force (or rate of change of momentum)$=$0.84 kg m/s$\div$0.2 s$=$4.2 N. Try working out in cgs units.

Ex. 1-3.9. *A pile-driver (called a 'monkey') weighing 400 lb falls from a height of 16 ft and comes to rest in 0.2 sec after striking the pile. What is the force exerted on the pile?*

Solution. The velocity acquired by the fall is obtainable from the relation $v^2=2gh$. With $g=32$ ft/s^2 and $h=16$ ft, we get $v=32$ *ft/s*. The momentum of the pile-driver is 400×32 lb ft/s. The force$=$momentum$\div$time$=(400\times32\div0.2)$ poundals$=$64,000 poundals$=$2000 lb-wt.

(This is the force of the blow. The total force on the pile wil be this plus the weight of the pile driver, i.e. 2400 lb-wt.)

[In the above problem read 400 kg for 400lb. 5 m for 16 ft. and work out the force, $g=9.8$ m/s^2.]

Ex. 1-3.10. A *cricket ball of mass 150 g moving at 12 m/s reverses direction after hitting a bat, with a velocity of 20 m/s, the contact lasting for 0.1s. Find the average force:* **[I.I.T. '74]**

Solution. Change in momentum $p_2-p_1=150\text{g}\,[12-(-20)]$ m/s

$$=480\times10^3 \text{ g.cm/s}$$

$$\therefore \text{ Average force}=(p_2-p_1)/t=480\times10^3 \text{ gm-cm/s}\div\tfrac{1}{100}\text{ s}$$

$$=480 \text{ N}\simeq5 \text{ kgf.}$$

Ex. 1-3.11. *A horizontal jet of water 5 cm in diameter with a speed of 10 m/s hits a vertical wall and then spills down the wall. Find the average force exerted if density of water is 1000 kg/m^3*

Solution. The mass of water falling per sec on the wall *horizontally* is

$$V\rho=v\times\pi r^2\times\rho=10 \text{ m/s}\times\pi\times\left(\frac{0.05}{2}\right)^2 \text{m}^2\times1000 \text{ kg/m}^3$$

$$=10\times(25/4)\times10^{-4}\times3.14\times1000 \text{ kg/s}$$

Its forward momentum per sec normal to the wall$=mv=\frac{25}{4}\times3.14\times10$ kg-m/s^2 and backward momentum is zero as water descends vertically

$\therefore$ Average force$=$196.25 N.

Problem. A brickbat weighing 5 kg moving with a velocity of 6 km/hr hits your head. The time of contact is 5 milliseconds. Find approximately in kgf the force exerted.

[*Ans*: About 170 kgf. (It makts a painful bump)]

I-3.4F. Inertial, Impulsive or Dynamical method of Comparing Masses: This gives us a method of measuring mass of a body if a standard mass is available.

We measure the mass of a body by (*i*) finding its *weight* with a spring balance and then by dividing that by g or by (*ii*) comparing it with the weight of another standard mass in a common balance. The mass we find thus, is said to be *gravitational*; for the force of gravity pulls it downwards and the pull is balanced in both the balances; the determination is thus a **static** method. It is widely used, for it is quite is simple to carry out; but it cannot be universal, for in an orbiting satellite, in a freely falling system, far out in space, bodies become weightless. Still these bodies have mass. That can be measured only by inertial i.e. dynamical method.*

In fig. I-3.10, is shown a Fletcher's trolly. On the heavy horizontal metal base plate run a pair of smooth rails along which may roll a pair of identical toy carriages or trolleys; they are stopped at the ends by a pair of buffers. Trolleys carry a pair of equally strong magnets (NS) as shown and held bound close to each

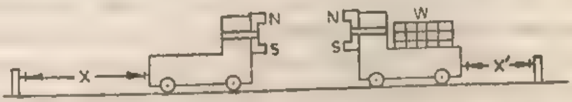


Fig. I-3.10

other by a piece of cord, about the middle of the bed. One of the trolleys is made heavier by placing a heavy mass (W) on it. If the connecting thread is suddenly cut or burnt, the magnetic repulsion sets the cars moving simultaneously in opposite directions till stopped by the buffers. The *forces being impulsive* (i.e. very short and sudden) we may take the cars to be moving with constant velocities over the small distances involved. By trial and error the position of the cars are adjusted till they hit the buffers simultaneously after release.

Let M be the mass of each trolley so that the heavier one has

* It is a very fortunate coincidence that *inertial* and *gravitational* mass of a body is totally equivalent. Newton experimentally proved it to be so (Refer to II-1.4). There appears no logical explanation for the coincidence to hold; serious difficulties would have arisen had it not been so.

a mass of $M+W$. Since the same force acting for the same time interval pushes them apart their momentum must be the same i.e. $Mv_1=(M+W)\ v_2$. They travel over different distances X_1 and X_2 for the same time. So

$$\frac{M+W}{M}=\frac{v_1}{v_2}=\frac{X_1/t}{X_2/t}=\frac{X_1}{X_2}$$

If we know M and obtain X_1 and X_2 from the experiment, W comes out.

We need not know M if we want to compare two masses m and m'. We then carry out two experiments by putting them separately on one trolley and carrying out the above experiments.

1-3.5. Reference Frames and Newton's Laws.

Newton's first two laws, remember, are valid only in inertial frames. Let us take a simple example : a ball lies on the floor of a stationary carriage ; let it suddenly move ; the ball rolls back. When the moving car halts suddenly the ball rolls forward. In both cases, Newton's first law appears to be violated, for the ball moves without any force acting upon it. We explain these facts by invoking the ideas of inertia of rest and motion. We have an alternative explanation.

When the car suddenly starts or as suddenly stops, it is no longer an inertial (i.e. uniformly moving) frame but an accelerated one. The behaviour of the ball shows that the first law is not valid in such a frame. As the first law is but a special case of the second, the latter is also not valid in an accelerated frame. But a slight modification validates this law in such a frame. Instead of $F=ma$ we use $F-F'=ma$ and the behaviour becomes explicable. A *fictitious* or a *pseudo force* F' comes in operation, opposite in direction to the acceleration of the frame. Since we cannot find any agent applying this force we call it a fictious or **pseudo-force.**

Now recall that, when a bus suddenly starts or stops or takes a bend, you seem to feel a force that is opposite in direction to that of motion. This is the pseudo-force. This is what pins you to the back of your seat when a plane taxis on the runway and becomes air-borne ; this is what you are advised to guard against, by fastening your seat belt when a plane lands and decelerates to a halt ; if you don't do as advised you may fall forward. Hence whatever you have explained by inertia of rest or motion may be also interpreted as an apparent failure of Newton's laws of motion in a non-inertial frame.

1-3.6. Principle of Independence of Forces :

An unbalanced force produces accelerated motion. Several such, may act on a particle simultaneously. The motion is determined by the resultant. As we have noticed in I-2.8 this is governed by the universal principle named above. It states that

If a number of forces act simultaneously on a particle at rest or in motion, their combined effect on the system is found by considering the effect of each force as if the other did not exist and then adding up these effects by parallelogram, triangle or polygon law of addition of vectors.. Basically all these laws are the same. This is only a special case of Principle of Independence of vectors.

Similarly, a motion whether uniform or accelerated is a vector and hence follows this principle of independence. We have already learnt two examples (1) A horizontally flying plane dropping a bomb, a mailbag or food packets and (2) A circus performer jumping up from a slow-cantering horse and landing back on it.

The following examples also illustrate the physical independence of motion, one not being affected by the other.

Refer to fig. 1-3.4. The plane has dropped a load at a height of 1024 ft when vertically above the point O. The load has simultaneously two motions (1) a uniform speed of 220 ft/s horizontally as the plane itself and (2) an accelerated vertical fall under gravity. Observe that when the load reaches the target the plane is directly overhead i.e. the load has covered the same horizontal distance as the plane, all the time falling vertically. Hence its horizontal motion a uniform velocity, has not been affected by its accelerated vertical fall. Had a stationary balloon directly overhead of O dropped a load from the same height and simultaneously with that from the plane both of the loads would reach the ground, at the same time.

The apparatus in fig 1-3.11 illustrates the above conclusions. On a table is a spring plunger. Its one end goes smoothly through a ball A and its other end is a little away from the ball B at the end of the table. If you suddenly release the spring, H suddenly moves to the right, releasing A vertically downwards and knocking B horizontally forward. Both the balls reach the ground together.

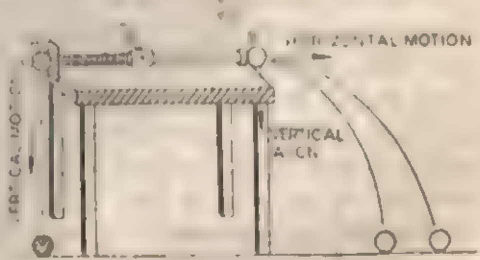


Fig. 1-3.11

The Monkey and Hunter Experiment (fig. 1-3.12). In a favourite lecture demonstration a *toy gun* (A) is sighted at an elevated target, a toy *monkey* (M) on a toy tree. The monkey is

released by a trip mechanism, the instant of the gun is fired. No matter what the initial speed of the bullet is, it will always hit the falling monkey.

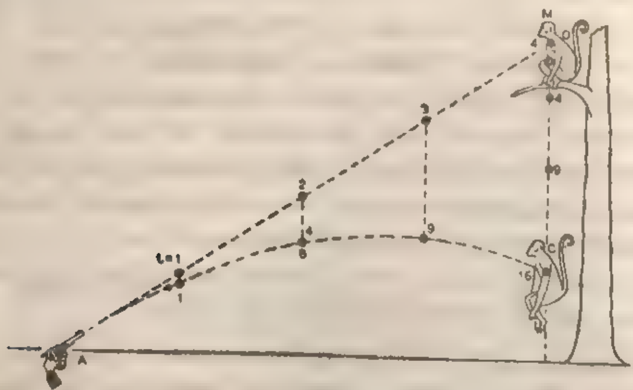


Fig. I-3.12

The explanation is simple. Had there been no gravity, the bullet will travel straight along AM to hit the monkey for then it will not fall. Gravity causes each to accelerate downwards at the same rate from the position it would otherwise have. In time t the monkey falls through $\frac{1}{2}gt^2$ i.e. MC, the bullet does the same, it describes the path ABC and hits the monkey. During each fraction of a second indicated by $t=1, 2, 3, 4$ both fall the same distance from their gravity-free positions. Had the bullet moved faster it would have reached the line of fall higher but quicker ; but during this shorter time the monkey would have fallen less and so would get hit. Had the bullet moved slower it would hit the monkey lower down than C by the same arguments.

More examples: (1) In problems about crossing a river by a boat, you have already worked with two motions, that of the boat and that of the river at right angles to each other ; they occur independently of each other. The same happens to the plane flying across a gale.

(2) Marksmen shooting flying birds point their guns ahead of the bird to allow for its horizontal displacement during the flight of the bullet. If the marksman be galloping on a horse with the bird sitting overhead, he must aim behind the bird to allow for the forward motion of the bullet, sharing his own motion.

(3) The case of the boy throwing up a ball vertically when running forward uniformly and recovering it ahead, illustrates the independence of forward uniform motion and vertical declerated motion followed by equally accelerated motion of the same ball.

(4) The dropping of a coin from a running car and its path observed by a stationary observer is the same as the load dropped from a plane. If the height descended be h and the time of fall t, then we have $h=\frac{1}{2}gt^2$. The horizontal distance covered by the car (or the plane) during that time is $s=vt$. Eliminating t between the two we get the equation of a parabola.

$$s^2=\frac{2v^2}{g}h. \qquad \text{(I-3.6.1)}$$

[Compare $x^2=ky$. v and g are constants]

We get here half a parabola with a vertical axis. If therefore a projectile is fired from the ground at an angle θ with the ground, it will describe a full parabola before returning to the ground as shown in fig. I-3.13. All projectiles, from a kicked football to an ICBM going from one continent to another, would follow a parabola, had air-resistance been absent.

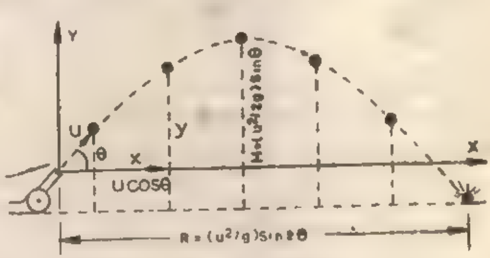


Fig. I-3.13

So you see that we can explain one and the same phenomenon, say the load dropped from a flying plane, or the motion of a ball thrown up in a uniformly moving railway carriage from three view-points namely, (1) physical independence of motion, (2) inertia of motion, (3) inertial frames of reference in motion.

I-3.7. Composition and Resolution of forces: A. Composition:

As indicated above, composition of forces follow from physical independence of more than one force acting together on a single body; and as for velocities, obey the same laws of parallelogram or triangle or polygon. The *parallelogram law* of forces can be stated as

If two forces acting simultaneously on a particle be represented

in magnitude and direction by two adjacent sides of a parallelogram, then they are equivalent to a single force represented in magnitude and direction by the diagonal of the parallelogram through the point. The magnitude and direction are given by

$$R^2 = P^2 + Q^2 + 2PQ \cos \alpha$$

$$\text{and} \quad \theta = \tan^{-1} \frac{Q \sin \alpha}{P + Q \cos \alpha}$$

Illustrations: (1) Verification of the parallelogram law of forces—Fig I-3.14 shows three cords tied together at a point and loaded at their free ends by weights W_1, W_2, W_3; the cords of first two pass smoothly over pulleys P_1 and P_2, all the loads hanging vertically against a board. When the cords come to equilibrium their traces are carefully drawn on the board.

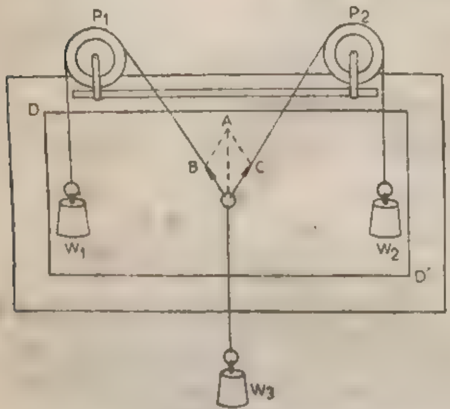


Fig. I-3.14

From them, lengths AB and AC proportional respectively to forces W_1 and W_2 are cut off, the parallelogram ABCO completed and the diagonal AO is drawn. Length AO will be found proportional to W_3 and passing through A vertically. The point A being vertically above W_3 it follows that the load W_3 balances the resultant of the loads W_1 and W_2.

(2) Flinging a stone with a catapult (Fig. I-3.15) :—The boy has put a stone in his catapult and is pulling back the elastic backwards. A sort of V results with forces **P** and **Q** acting along the two arms. When released the stone moves along the direction of the resultant **R**, but not along any of the directions.

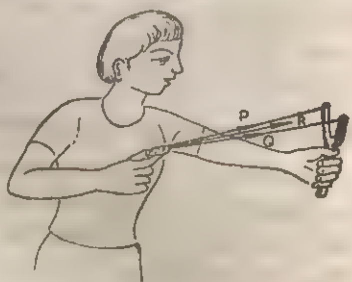


Fig. I-3.15

(3) Towing a boat with two ropes: In a canal where rowing becomes difficult a boat may be towed forward as shown in fig. I-3.16 by a pair of men pulling equally with two ropes and moving

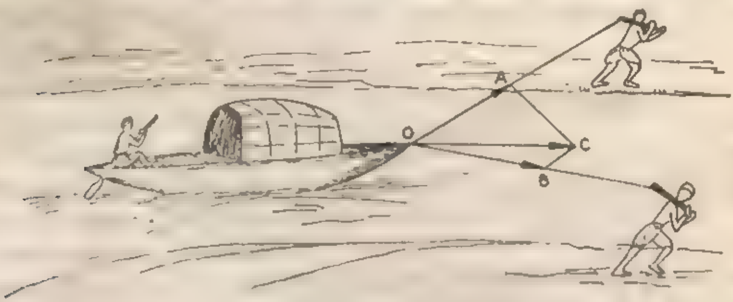


Fig. I-3.16

along the banks. If their pulls are represented by two vectors along **OA** and **OB** the boat moves forward along the force vector **OC**.

Problems. *(1) A boat is towed by two ropes, each inclined at 30° to the stream, but on opposite sides of it. If the pull along each rope is 50 kgf find the magnitude and direction of the resultant force.* [*Ans*: $R=86.6$ kgf ; $\theta=30°$.]

(2) Forces 3 gf and 4 gf act at a point. Find the angle between their lines of action if they have a resultant of (i) 7 gf, (ii) 6 gf, (iii) 5 gf, (iv) 4 gf, (v) 1 gf. [*Ans.* (i) 0°, (ii) 62°42′, (iii) 90°, (iv) 131°48′, (v) 180°.]

(4) Soaring of a bird (fig. I-3.17): A bird in flight flaps its wings against air which reacts in opposite directions by Newton's 3rd

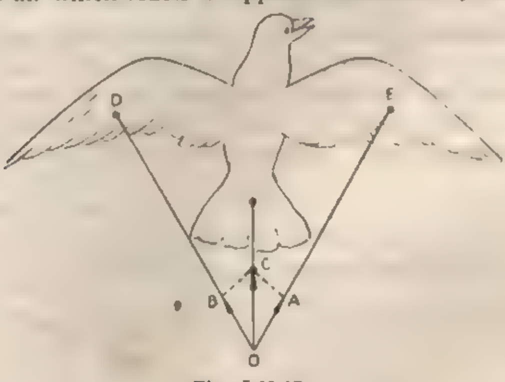


Fig. I-10.17

law. If the wings flap along DO and EO, air reactions will act along OD and OE. Proportional length OA and OB represent them in

magnitude and direction. Their resultant OC makes the bird soar up when it exceeds the weight of the bird or fly forward as and when required.

B. Resolution of Forces: The subject-matter is identical with §I-2.9. The resolved part of a force is its effective part in this direction of motion. It may be understood with the help of the following example. Suppose O represents a carriage standing on rails parallel to OX (fig. I-3.18); and is being pulled along OC with a force R. R is equivalent to P along OX and Q along OY. The action of Q cannot displace the carriage along the rails. The motion of the carriage along the rails is therfore determined entirely by the resolved part (P) of R along the rails.

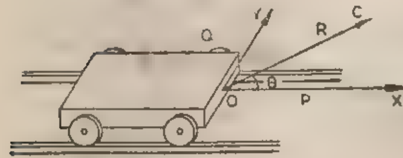


Fig. I-3.18

Problems. (1) A railway truck is pulled along level rails by a horse pulling a rope with a force of 500 kgf in a direction making an angle of 30° with the rails. What is the force pulling the truck along the rails ? What is the force pulling it sideways against the rails ? [*Ans.* 433 kgf ; 250 kgf.]

Note that a boat may be similarly towed by a single rope instead of two as shown in fig. I-3.16. The resolved part of the pull will tow the boat downstream.

(2) A gale forces a mooring cable of a captive balloon to an angle of 30° with the vertical. The tension in the cable is found to be 50 kgf. Find (a) the horizontal force exerted by the wind and (b) the net up-thrust on the balloon. [*Ans.* (a) 25 kgf ; (b) $25\sqrt{3}$ kgf.]

Further Examples: To move a heavy box along the floor, generally a rope is tied to one end of it and pulled in an inclined direction (Fig. I-3.19). The upward vertical component somewhat lightenes the weight while the horizontal component $F \cos \theta$ pulls it forward. Note that greater the inclination with the ground smaller the effective pulling component. Variants of this you must have seen in plenty.

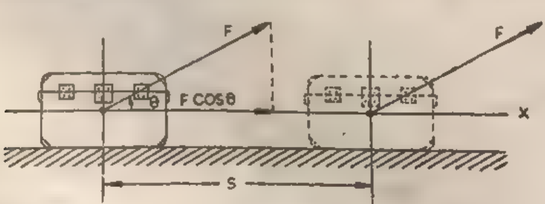


Fig. I-3.19

Instead of pulling, if you push the box in an inclined direction the vertical component acts downwards adding effectively to its weight and friction. That is

why one feels it easier to pull a heavy roller on the cricket pitch than pushing it. [Sec I-4.7., fig. I-4.6]

Flying a kite as well as a plane or navigating a sail boat, all utilise suitable components of air pressure exerted on the kite, wings of a plane or the sail of a boat. Try to think up other examples—there are lots of them around you.

When a number of forces act simultaneously on a particle we may resolve each of them separately in perpendicular directions, add up the components and finally compound the results as in fig. I-2.16.

Ex. 1-3.12. In fig. I-2.16, let $F_1 = 60$ gf, $F_2 = 100$ gf, $F_3 = 75$ gf, $\theta_2 = 60°$, $\theta_3 = 135°$. Resolving along F_1 and perpendicular thereto, we shall have $\theta_1 = 0°$, or $F_1 \cos\theta_1 = F_1$. The calculations may be tabulated as follows :

Force (F) in gf	Angle (θ)	x-component $= F\cos\theta$ (gf)	y-component $= F\sin\theta$(gf)
$F_1 = 60$ $F_2 = 100$ $F_3 = 75$	0° 60° 135°	+60 +50 −53 $\Sigma F_x = +57$	0 +86.6 −53 $\Sigma F_y = +33.3$

$$R = \sqrt{(57)^2 + (33{\cdot}3)^2}\text{ gf} = 66\text{ gf}.$$

$$\tan\alpha = \frac{33.3}{57} = 0{\cdot}588 \quad\text{or}\quad \alpha = 30{\cdot}4°.$$

Problems. (1) Two equal forces, inclined at 60°, act on a particle. Resolve the forces along the bisector of the included angle and perpendicular thereto. Hence find the magnitude and direction of the resultant. [*Ans.* $\sqrt{3}$ times the magnitude of either force : along the bisector of the included angle.]

(2) Three coplanar forces 1, 2 and 3 gf respectively act at a point. The angle between the first and the second is 60° : the angle between the second and the third is 30° : and the angle between the first and the third is 90°. Find the magnitude of the resultant and the angle it makes with the first force.

[*Ans.* 5.14gf, making an angle of 67°5′ with the first force.]

I-3.8. Projectiles : Whenever a body is projected i.e. thrown at an angle to the horizontal it is said to be a projectile and its motion provides a very important case of resolution of motion. When a footballer takes a spot kick, a canon fires a shell (fig. I-3.13) or a superpower releases an ICBM, the moving bodies are projectiles. They are activated by *impulsive forces* (large forces of very short duration) which are *supposed to have stopped before the affected bodies have appreciably moved.* The body then moves on with constant speed and in absence of air resistance, along a parabolic path, as already established.

In fig. I-3.13 let the shell be fired with an initial velocity u at an angle θ to the horizontal or x-axis. u is therefore equivalent to $u \cos\theta$ along the x-axis and $u \sin\theta$ along the vertical or y-axis. Replacing u by $u \sin\theta$ in the equations for the vertical upward motion we find that the maximum height H $(=Y_{max})$ attained by the shell and the time taken for so doing would be

$$H = \tfrac{1}{2}u^2 \sin^2\theta/g \quad \text{and} \quad T = u \sin\theta/g \qquad \text{(I-3.8.1 and 2)}$$

Now then the time for the shell to attain the maximum height and then to return to the earth must be $2T$. During this interval the shell would be moving with a constant forward velocity $u \cos\theta$ so covering a distance,

$$x = u \cos\theta . 2T = u \cos\theta . 2u \sin\theta/g$$

This distance is called the range (R) of the canon. So

$$R = u^2 \sin 2\theta/g$$

This has a maximum value for the given canon when $\sin 2\theta = \pi/2$ or $\theta = 45°$.

Ex. I-3.13. *An obstacle 19.6 m from an observer is 14.7 m high. He projects a body with an initial velocity of 19.6 m/s so that it just passes over the obstacle. Find the angle of projection. Neglect air friction and take g= 9.8 m/s².* [J. E. E. '80]

Ans. In absence of air resistance the projectile follows a parabolic path of which 14.7 m represents the highest point ; so from the relation

$$Y_{max} = \frac{u^2 \sin^2\theta}{2g} \quad \text{we get}$$

$$14.7 = \frac{(19.6)^2 \sin^2\theta}{2 \times 9.8} \quad \text{or} \quad \sin\theta = \left(\frac{14.7 \times 2 \times 9.8}{19.6 \times 19.6}\right)^{\frac{1}{2}} = \frac{\sqrt{3}}{2} \quad \text{or} \quad \theta = 60°$$

Problem : A shot after leaving a gun just passes over a wall 64 ft high and 192 ft away, horizontally. Find the velocity and direction of the shot. [U.P.B.] $(g = 32)$ ft/s².

(*Ans.* $u = 32\sqrt{13}$ ft/s ; $\tan\theta = 2/3$)

I-3.9. Discussion. Third law : Action and reaction. The first and the second laws apply to the body on which the force acts. *The third law introduces the body (or agency) which exerts the force.* The terms *action* and *reaction* are simply other names for forces. If a body A applies a force on another body B, which we call the *action*, then, accodring to the third law, the body B will apply an equal and opposite force on A, which is called the *reaction*. The law thus states that *in nature forces always occur in pairs*, and that

each force of the pair acts on a different body. Whether the bodies are at rest or in motion, touch each other or are separated by a distance (as for two magnets), the law is of universal application.

(1) Consider a book lying on a table. The weight of the book exerts a downward force on the table. According to the third law, the table will exert an equal and opposite force on the book. If the former force is called the action, the latter force is the reaction. Note that *reaction lasts only so long as the action is there.* Besides, *action and reaction act on two different bodies, and cannot, therefore, produce equilibrium.* For equilibrium the forces on one and the *same* body must be balanced.

(2) When a ladder leans against a wall it presses upon the wall. This pressure, here called the action, tends to overturn the wall. The counter-thrust exerted by the wall on the ladder is the reaction. It keeps the ladder in position.

Observe that in both the examples we consider the two interacting bodies (the book and the table, the ladder and the wall) only, as if nothing else exists and no other force by any agent acts on any of them ; e.g. air pressure exists on each but we disergard it. A moving carrom striker hits a dice and both move off in different directions ; in considering their interaction we likewise disregard friction and air pressure. Hence the pair of interacting bodies form **an isolated system** and no external force acts upon it.

Remember then, when two bodies *A* and *B interact*

(1) They may do so either by contact or from a distance.
(2) The pair of forces are said to be action and reaction.
(3) The forces are perfectly reciprocal i. e. equal and opposite,

$$\mathbf{F}_{AB} = -\mathbf{F}_{BA}$$

(4) Either of them may be the action, the other one then the reaction.
(5) Action and reaction act *always* on two different bodies, *never* on the same. No question of equilibrium therefore arises.
(6) Equality of action and reaction holds whether the bodies are at rest or in motion, in an isolated system.
(7) They arise simultaneously and hence bear no cause and effect relation to each other.
(8) Reaction lasts so long as action does.

Isolating a body : Let a block hang by a cord as shown in fig. (I-3.20). The block pulls the cord down with a force

F_g (say) while the cord pulls the block upwards with equal reaction force F. The block then remains at rest. To consider the problem, the body is isolated by drawing a dotted line around it. Only

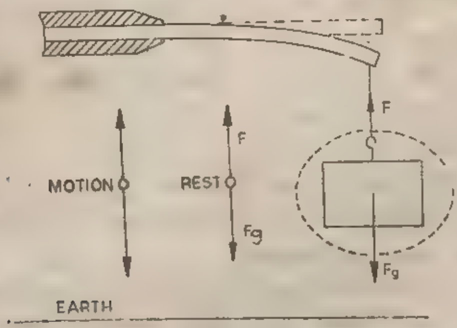


Fig. I-3.20

those forces acting on it from outside this boundary, determine its state of rest or motion. F_g is the force with which the earth attracts the block while the cord is pulling it up with an equal but opposite force F (Note that F_g and F are *not* an action and reaction pair though equal and opposite). Vectors add up to a null result and the block is at rest. When $F_g > F$, the string will snap under a downward resultant vector $(F_g - F)$.

I-3.9A. Demonstration Experiments : (1) The hook of a stout spring balance is attached to another and the pair hung from a rigid support as shown in fig. I-3.21. Now pull the lower one and notice that both the balances showing the same reading. This means that the force with which the lower spring pulls the upper one downwards is the same with which the upper one pulls the lower one upwards.

The same reading will be given by both the spring balances if you take the above pair in your two hands and pull them apart. Now this result elucidates the following case—that of a **tension** *transmitted through a rope.*

Consider a rope tied to a tree and a man pulling at its other end. The force exerted by the man on the rope is the action. The reaction is the equal and opposite force exerted by the rope on the man.

Fig. I-3.21

The force exerted by the man on the rope is transmitted through the rope to the tree. The rope thus exerts an action on the tree. The tree exerts an equal and opposite reaction on the rope. The rope is in equilibrium under the action of the pulls exerted on it by the man and the tree.

[In this example the rope transmits two forces through it—one exerted by the man which acts through the rope on the tree, and the other, the reaction to this force which is exerted by the tree on the man. The rope is stretched and is said to be in 'a state of tension'. *Either force* transmitted through the rope is called a *tension.* This would show that when two men pull at two ends of a string, each with a force of 20 kg, the tension in the string is still 20 kgf and not 40 kgf]

Problems. (1) One end of a string is tied to a fixed post. Two boys pull the string at two places, each with a force of 20 kg-wt. What is the tension in the string (*a*) between the post and the first boy, (*b*) between the two boys ?

[*Ans* : (*a*) 40 kgf, (*b*) 20 kgf]

(2) In a tug-of-war each team pulls with a force of 100 kgf. If a spring balance is inserted in the string between the teams what will it read ? (100 kgf)

(2) A small electric toy railway engine is mounted as shown in fig. I-3.22 on a track provided by a horizontally mounted bicycle wheel ; the wheel is free to move about its vertical axle. You notice that as the engine moves, the wheel starts rotating backwards. The spinning wheels of the engine pushes on the track backwards and the track pushes them forwards equally, the two forming an action-reaction pair. If friction be very small, the engine and the track will be moving uniformly but oppositely for quite some time after the electricity is disconnected, as no resultant force acts on the engine-track system.

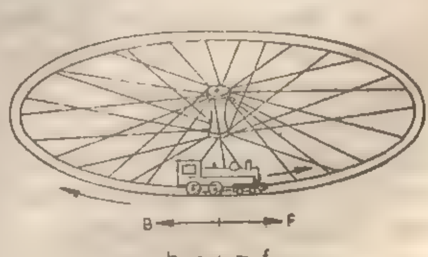


Fig. I-3.22

In a circus you may have observed a baby-elephant on a very large sphere performing as above.

(3) Barker's mill (chap. II-4) is a tall cylinder with a few L-shaped

horizontal nozzles fixed horizontally near its bottom ; the cylinder can rotate freely about its axis. It is filled with water keeping the nozzles closed by fingers. When they are opened water rushes out through the nozzles and the cylinder begins rotating by reaction to the water pressure in the opposite direction. The *rotary lawn sprinkler* is only an adaptation for practical application ; water escaping from the nozzles sets up backward rotation of the container thus sprinkling water uniformly over a large circle.

I-3.9B. A few Apparent Anomalies and their clarifications : (1) A falling apple—

It is said that at 23 years Newton started thinking about Gravitation by observing a ripe apple falling from a tree. Now, the third law poses the apparent anomaly—why the apple falls and not the earth moves up if the forces between them are equal but opposite ?

The explanation lies in the vast disparity between the masses of the earth and the apple. From the second law of Newton

$$F_{EA}=m_A g \quad \text{and} \quad F_{AE}=M_E a$$

Since $F_{EA}=F_{AE}$ we have $m_A g=M_E a \quad \therefore \; a/g=m_A/M_E$

As the earth-mass M_E is so much greater than the mass of the apple m_A, the acceleration of the earth a is so much smaller towards the apple than g, than that of the apple towards the earth.

When a person jumps ashore from a country boat the latter is visibly pushed back but not so when he jumps out from an ocean liner. The mass of the boat is comparable to that of the man but that of the liner is so much greater. Hence the difference.

(2) Moon attracted by the Earth stays up : Both the apple and the moon are subject to gravity ; but it is the apple that falls to the earth but not the moon : why ?

Because the moon moves uniformly round the earth. You will learn in Chap I-5 that a centrifugal force acts on the circling moon away from the earth which just balances the pull of the earth. The same happens to all our artificial moons. Stop them in their tracks and all of them would fall to the earth just as the apple does.

Remember that the point to be strongly emphasized in dealing with the third law is that *action and reaction are exerted on different*

bodies. So the question of their producing equilibrium does not arise. Equilibrium of a body is determined by the forces acting on the body. Any force the body itself exerts on any other body or bodies does not come in the picture.

(3) **Horse pulling a cart and a Rickshaw-puller pulling his vehicle :** The horse pulls the cart *forward* through the traces that binds him to the latter. By Newton's 3rd law the cart pulls him *backward equally* through the same traces. How then does the cart advance ? How does the rickshaw move when pulled ?

Because, the pull and the equal counter-pull between the respective units are not the only active forces ; there arise other unequal forces which start and accelerate the system. They are shown in fig. I-3.23(a) and (b).

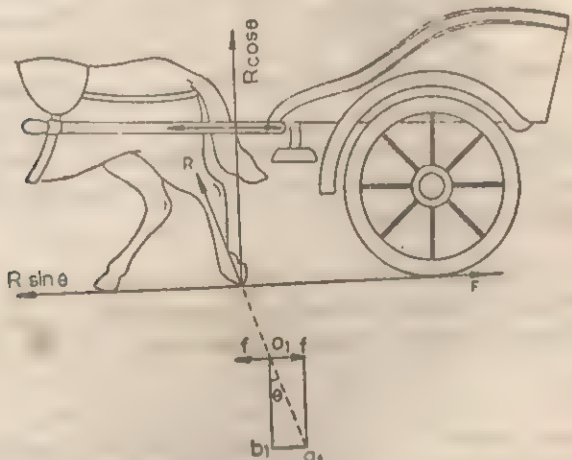


Fig. I-3.23(a)

We must isolate and specify the forces acting on the pulled (the cart or the vehicle) and the puller (the horse or the man).

(*a*) The forces acting on the cart (fig. I-3.23a) or the rickshaw (fig. I-3.23b) are two in number.

(*i*) the tension applied by the puller through the traces or the handle acting in the forward direction and

(*ii*) the frictional forces acting backwards at the wheels opposing the motion.

(*b*) The forces exerted on the horse or the puller are also two—

(*i*) the reaction force of tension P acting through the traces applied by the cart or the vehicle backwards and

(*ii*) horizontal forward component H ($=$R $\sin\theta$) in both cases. H is the component of the reaction R of the ground to the pressure applied by the horse or the man. Either of the pullers press the ground with his foot in an inclined direction which brings out an equal and opposite reaction R. Its horizontal component in the forward direction is H. Only when $H > f$ or F, (the friction) would the cart or the vehicle roll forward.

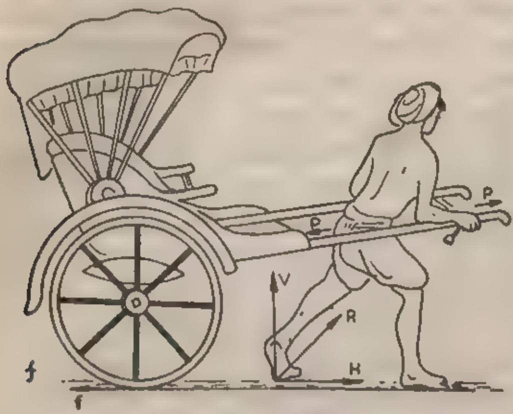


Fig. I-3.23(b)

The reaction exerted by the cart or the vehicle merely retards the motion of the horse or the man. The action exerted by the latter on the earth helps to accelerate the system forwards provided $H > f$, or F.

The force diagram is in the lower part of the first diagram. The horse presses the ground *obliquely* along o_1a_1 and its reaction acts along a_1o_1. The muscular action of the horse in its legs while pressing the ground along o_1a_1 provides the *action* bringing about the *reaction* R along a_1o_1 ; its horizontal backward component at the start, must be balanced by the frictional force represented by o_1f offered by the ground so that the horse does not slip. When the cart starts moving forward, the frictional force f switches over backwards as the horizontal forward component exceeds it.

[Remember friction always opposes any tendency to motion whatever be the direction].

Problem : Both statements are true—(*i*) In a tug-of-war both parties must pull equally on the rope ; (*ii*) The party that pushes the ground harder wins. Justify. [J. E. E.]

(4) Walking. Before a man starts to walk he stands erect and lifts one leg. His weight is supported by the vertical reaction of the ground. As his leg muscles bend his other leg, he leans forward and at the same time pushes forward his other leg. A large number of leg and body muscles take part in this action (which is too complicated to discuss). But in this position the point of application of the reaction of the ground on him moves forward (and the heel may be raised). The reaction increases in magnitude than before and is directed up the inclined leg in touch with the ground. The vertical component of this reaction supports the man's weight. The horizontal component prevents the man from slipping.

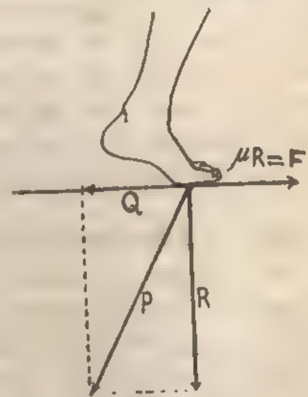


Fig. 1-3.24

If the frictional force between the foot and the ground in the *forward* direction does not balance the *backward* component (Q) of his pressure P on the ground, he will slip and fall forward on his face. We shall return to this discussion in the next chapter.

(5) Braking an Automobile to Rest ; We cannot start or stop a car by pressing from inside it. For then the force applied is internal and not external. Newton's 1st law prohibits any change in motion unless a force is applied from outside. How then a bus or a car is stopped by the driver who is inside the car ?

When you have a clear idea of friction (Chap I-4) and moment of a force (Chap I-6) later, you would appreciate the solution provided. All auto-cars like buses, trams, railway engines or motor cars, roll forward on wheels which revolve or rotate. Rotation is produced by a *couple*, i.e. a pair of equal unlike forces acting at two different points of a finite body (not a particle ; why not ?). The moment of

the couple (by which its ability to generate rotation is measured) is a *vector product* of any of the forces and the perpendicular separation of their points of application.

The wheels in rotation are under a couple. When the driver brakes, an opposing couple tending to decelerate the wheels, act on them. But then if the linear motion is not decelerated at the same time, the wheels continue skidding i.e. slipping on the road surface ; this often, happens when the roads are snow-bound, wet or slushy. Ordinarily friction of the road-surface opposes skidding or forward motion. So finally the friction between the road surface and the tyres is the external force that stops the braked automobiles.

I-3.9C. Further Illustrations of the Thrid Law : Examples from daily life is legion. Try to kick open a tightly shut door. As it flies open your feet decelerates and if your foot is bare you will realise the magnitude of the reaction. Like a karate-master try to split open a brick with your palm, or just box a wall—you will be convinced of the accuracy of the third law. Try to hit over the head of a fast bowler in cricket ; if you don't have your timing proper, you may find your bat knocked out of your hand. Fire a rifle with the butt loosely held to your body ; you may find yourself knocked flat on your back. Jump ashore from a country-boat, the boat moves back ; but during jumping make sure that the agent supplying the reaction is appropriate. Jumping down from a moving swing or run on and jump from sand or thick mud (for high jump or broad jump) is not advisable for you don't get the appropriate reaction.

When a gun is fired the explosion of the gun-powder or the strike of the hammer propels the bullet forward and the gun recoils by reaction. Canons used to be mounted on wheeled platforms so that after firing the shell it could safely recoil. You might see pom-pom guns firing upwards, telescoping back into slightly wider barrels. When a rocket is fired either in amusement or as a missile or space-shot, the burning gases come out with great momentum downwards and by reaction the rocket soars upwards.

We have learnt above how reactions of the ground enable the horse to pull the cart or the rickshaw-puller pull his vehicle or the

groundsman the heavy roller on the cricket ground, or you to walk or a party to win a tug-of war match. In all these cases we find also illustrations of resolution of forces. A very similar case is that of starting a country-boat. The boatman pushes obliquely (fig. I-3.25) through a pole on the river-bed, the forward component of the reaction puts the boat moving.

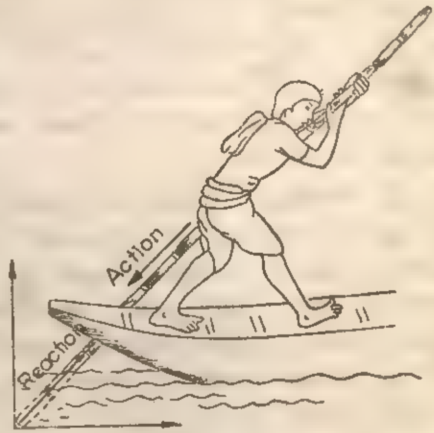


Fig. I-3.25

Put a strong magnet near an iron piece. If you hold down the magnet the iron piece moves towards it ; hold down the iron piece, the magnet would move towards it. The mutual attraction is an action and re-action pair.

The list of examples cannot be exhaustive.

I-3.10. Different kinds of Action and Reaction :

(1) Tension or Pull : It has already been discussed above. The role it plays in the horse and cart problem or in supporting a hanging load we have seen. Presently we shall enquire into its role in the motion of a connected system.

In a tug-of-war match tension generated by either of the teams through the rope must be equal and opposite. But the members of the team that presses the ground more, wins the macth. (Why ?)

This type of action-reaction pair tends either to bring the two interacting bodies nearer or to prevent them from moving away from each other. Such forces act through either a flexible medium like springs or strings (the traces in the cart and the horse problem) or through solids (the long handle of the rickshaw).

(2) Push and thrust : This action-reaction pair is opposite to pull or tension, and tends either to separate the inter-acting bodies or prevent them from coming together but acts through a solid.

Walking or starting a country boat (fig. I-3.25) are examples we have studied. You kick open a door, hit a cricket ball by a bat, try to throw down a wall, you apply an action and feel a reaction. A push distributed over a finite area is a *thrust* e.g. the force exerted on a table by a book and that it experiences, so that the two are at rest.

(3) Friction: Whenever a body tends to slide over another, such as a sledge on ice or through a fluid such as a submarine through water or a plane through air, it faces a force of reaction from the latter. This is the well-known force of friction which always tends to oppose relative motion and stops moving bodies.

For obvious reasons all these action-reaction forces are termed *contact forces.*

(4) Attraction and Repulsion: Gravitational, electric and magnetic forces tend to pull bodies under suitable conditions together or push them away but without any visible contact. They are said to be *field forces.*

The earth pulls the moon and keeps her in orbit, the moon pulls the earth equally and raises tides on her oceans. We have seen a magnet attracts a piece of iron just as the latter pulls the former. These are attractive action and reaction pairs. Hang a pair of magnets or a pair of charged balls with similar poles or charges closely side by side. Both in each pair would move away from each other.

I-3.11. Action and Reaction—Two particular Examples:

A. Motion of Connected Systems: Fig. I-3.26 shows a bock of mass m_1, on a smooth horizontal table pulled by an unstretchable string of negligible mass attached to a load m_2 hanging over a massless smooth pulley. The pulley is so considered to simplify the problem so that it merely changes the direction of the tension T of the string at that point. The weights are so chosen that they move.

To study the motion of the system we mentally isolate the block m_1 and reduce it to a point at O: the acting forces on it are (1) T the horizontal tension on the string, (2) m_1g the downward pull of the earth on it and (3) its reaction N, the vertical force exerted by the table on the block upwards. The block will be *constrained* to

accelerate only horizontally and so no acceleration along the vertical direction. Hence we write, considering magnitudes only

$$N-m_1g=m_1(a_1)_y=0 \quad \text{or} \quad N=m_1g \text{ and } T=m_1(a_1)_x \qquad \text{(I-3.11.1)}$$

To find T we consider the motion of m_2, once again considered a particle and under two forces (1) T upwards (2) m_2g downwards. As the system is accelerating downwards under the resultant of these two, we have

$$m_2g-T=m_2\,(a_2)_y \qquad \text{(I-3.11.2)}$$

Since the string is unstretchable and the direction of tension changes at the pulley we must have $(a_1)_x=(a_2)_y$ or the common acceleration of the system as a. Thus from the above two relations

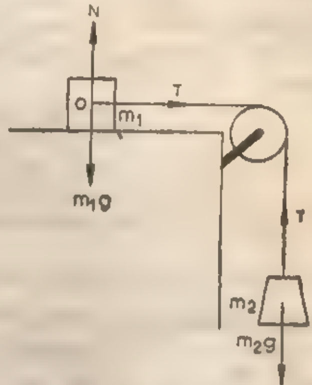


Fig. 1-3.26

$$T=m_1a \text{ and } m_2g-T=m_2a$$

Adding the two we get $m_2g=(m_1+m_2)\,a$

$$\text{or} \qquad a=gm_2/(m_1+m_2) \qquad \text{(I-3.11.3)}$$

$$T=m_1m_2g/(m_1+m_2)$$

From these relations we find that for accelerated motion to continue always $T<W\ (=m_2g)$ and $a<g$.

Problem : A load of 1 lb is pulling a mass of 15 lb on a smooth table by a light inextensible string. Find the acceleration of the system and the tension of the string. [Ans. $T=32$ poundals ; $a=2$ ft/s²] **(S. S. Q. 1979)**

Ex. I-3.14 : *Two cubes of masses m_1 and m_2 lie on two smooth slopes of a block A on a horizontal plane. They are connected by a string passing over a frictionless pulley. What horizontal acceleration is to be imparted to A such that the blocks do not slide down the planes ?* **(I.I.T. '78)**

Solution : Recall that in explaining the behaviour of bodies (e.g. men) in a horizontally accelerated frame (e.g. buses starting) we had introduced oppositely directed pseudo-forces. We do so here.

When A accelerates towards right with an acceleration a then the two masses experience pseudo-forces m_1a and m_2a to the left. Isolate m_2—the forces on it are (i) pseudo-force m_1a directed to the left (ii) vertically downward weight m_2g

and (*iii*) the tension T parallel to the plane inclined at β to the horizontal. As it is at rest we must have (Draw your own figure)

$$T+m_2g\sin\beta=m_2a\cos\beta$$

Similarly isolating m_1 and considering its equilibrium we have

$$m_1g\sin\alpha-T=m_1a\cos\alpha$$

Adding we get $g\,(m_1\sin\alpha+m_2\sin\beta)=a\,(m_1\cos\alpha+m_2\cos\beta)$

$\therefore$ The acceleration required for the block A and the tension of the string are

$$a=\frac{m_1\sin\alpha+m_2\sin\beta}{m_1\cos\alpha+m_2\cos\beta}.g$$

$$\text{and}\quad T=\frac{m_1m_2\sin(\alpha-\beta)\,g}{m_1\cos\alpha+m_2\cos\beta}$$

[Do you notice similarities with eqn. I-3. 11. 3 ?]

In fig. I-3.27 we have another arrangement where both the loads move vertically, one up the other down, connected by a light unstretchable string and passing as before over a smooth light pulley. The load m_1 being lighter than m_2, will move up. From the diagram we may write

Fig. I-3.27

$$T-m_1g=m_1a \quad\text{and}\quad m_2g-T=m_2a$$

for in both cases T acts upwards as one of an action-reaction pair. Adding them we get

$$g(m_2-m_1)=(m_1+m_2)a \text{ or } a=(m_2-m_1)g/(m_1+m_2) \qquad \text{(I-3.11.4a)}$$

Substituting this value of a we get

$$T=2\ m_1m_2g/(m_1+m_2) \qquad \text{(I-3.11.4b)}$$

Ex. I-3.16 : *A light inextensible string passing over a smooth pulley carries at its two ends loads of 2 and 3 kg respectively. Find its common acceleration and the tension on the string.*

Solution : The net downward force acting on the system is (3−2 or) 1 kgf, for the lighter one is coming up, the heavier going down. So

acting force = 9.8N and the total mass = 3+2 = 5 kg

$\therefore$ The net acceleration of the system $=\dfrac{9.8\text{ N}}{5\text{ kg}}=1.96\text{ m/s}^2$

[You get the same result by directly applying the above equation]

We know that the 3 kg load is descending with this acceleration. Then

$$1.96\text{ m/s}^2=\frac{3\times9.8\text{N}-T}{3\text{ kg}} \quad\text{or}\quad T=3\times9.8\text{N}-3\text{ kg}\times1.96\text{ m/s}^2=2.4\text{ kgf.}$$

[Same result follows from directly putting the formula]

Problem : A mass A (=500 g) is placed on a smooth table with a string attached to it which passes over a smooth pulley and supports another mass B (=200 g) hanging vertically. At $t=0$, A is 200 cm from the pulley and moving away at 50 cm/s from it. What will be its position and speed at $t=1$ s. ?

[I. I. T. '75]

Solution : B is moving up under a force m_1g-T and A is moving say to the left with a force $ma(=T)$ where a is the common acceleration. Neither the pulley nor the string has weight

$$\therefore\ 200\times g-T=200\,a \text{ and } 500\times a=T$$

$$\therefore\ a=\frac{200\times 980}{700}=280 \text{ cm/s}^2.$$

Now $u=-50$ cm/s $\therefore\ v=u+at=-50+280\times 1=230$ cm/s.

and $s=ut+at^2=-50\times 1+\frac{1}{2}\times 240\times 1=70$ cm.

B. Reactions in a Moving Lift : Most of us are familiar with a peculiar feeling ('butterflies in the stomach') when (1) the lift we are in, *suddenly* starts *downward* (2) or when the large vertical merry-go-round in festivals, starts descending fast (3) the aeroplane gets into a rarer air pocket and bumps downwards or (4) during sea-bathing the retreating waves pull out sand layers from beneath our feet. Again, when the lift suddenly starts ascending, the seat in the merry-go-round start climbing or the plane gets into a denser air pocket bumping upwards, we seem to feel an increased pressure at our feet. When again in a great hurry we try to pull up fast a heavy weight with a string, it snaps ; but had we pulled it up slowly the string would have supported the weight. When the rising lift decelerates to rest we feel lighter but when it decelerates while descending we feel havier.

In all these cases an extra force is applied to the moving system and an added reaction results. As generally we are unprepared for the reactions, we have such unaccustomed feelings. Let us consider the various cases of the moving lift, with a person of mass m kg standing in it.

(*a*) *Lift rising with a constant velocity* :—The weight of the man mg acts vertically downwards and presses on the floor of the lift. Clearly the floor reacts on the man with R, both acting along the same line and through the C.G. of the man. This R *gives us our sensation of weight*. When the lift *rises or descends uniformly*, according to Newton's first law there is no resultant force on the man, his weight and its reaction remaining equal and opposite.

He feels nothing unusual. Note that (*i*) the third law holds equally for bodies in motion and (*ii*) whether at rest or in uniform motion, the lift being an inertial frame.

(*b*) *Lift rising with a constant acceleration a* : The floor of the accelerating lift would also accelerate i.e. apply an upward force on the man and he will exert an equal reaction *ma* downwards in addition to his weight to that the total reaction on the floor becomes

$$R\downarrow = mg + ma = m(g+a) \qquad \text{(I-3.11.5a)}$$

Hence the man feels himself heavier.

(*c*) *Lift descending with an acceleration a* ($a<g$) :. Here with the lift the man also descends with same acceleration. So the force on him will be less than his weight and hence his reaction on the lift floor will also be less, so that

$$mg - R = ma \qquad \text{or} \qquad R\uparrow = m(g-a) \qquad \text{(I-3.11.5b)}$$

The man feels lighter.

Note that in these two cases the lift is an accelerated frame and hence a pseudo-force arises in both cases opposite to the direction of motion.

In I-2.14 we had arrived at these very expressions for resultant accelerations from the standpoint of *relative acceleration* along the same line.

(*d*) *Lift falling freely* ($a=g$). **Weightlessness :** Suppose the cable supporting the lift snaps and it comes down with acceleration g. From above formula we find, R vanishes. So the man feels weightless as both he and the floor accelerate down at the same rate and contact between his feet and the floor no longer exists. Then, a brief-case in his hand need not be supported.

(*e*) *Lift accelerating down with* $a'>g$: **Superweightlessness :** This unusual case can be simulated if a powerful engine drives the lift downwards. The man will accelerate down only with g but the lift moves down faster ; so the man will get detached from the floor and to an observer strapped to the lift, will appear to *rise inside* the lift (though actually falling) until his head reaches the ceiling and press on it upward with a force $m\,(a'-g)$. This phenomenon is *superweightlessness* and may prove fatal to a pilot thrown out from a nose-diving plane.

Ex. I-3.16 : *(I) A man weighing 60 kg is inside a lift. What will be the reaction of the floor on the man when the lift is (i) at rest (ii) accelerates up at 490 cm/s^2 (iii) ascends with a decleration of 490 cm/s^2 (iv) rises with uniform velocity.* $g=980$ cm/s^2 **[J.E.E. '82]**

Solution : (*i*) With the lift at rest $R=mg=60$ kgf

(*ii*) $R=m(g+a)=m\times\frac{3}{2}g=90$ kgf $\because$ $a=490$ $cm/s^2=\frac{1}{2}g$

(*iii*) $R=m$ $[g+(-a)]=\frac{1}{2}$ $mg=30$ kgf

(*iv*) $R=60$ kgf.

Ex. I-3.17 : *A weightless rope can support a maximum load of Mg. Find the greatest load that can be raised through h from rest with uniform acceleration for t s after accelerating up uniformly.*

Solution : If the acceleration is a then the tension *on* the string pulling up a mass m is $T=m(g+a)$

Obviously $T_{max}=Mg$ and hence with the above value of T,

$$m_{max}=Mg/(g+a)$$

As the mass rises through a height from rest h in time t with an acceleration a, we have

$$h=\tfrac{1}{2}ft^2 \quad \text{or} \quad a=2h/t^2.$$

Substituting this value of a in the above relation we have

$$m_{max}=M/(1+a/g)=M/(1+2h/gt^2)$$

Problems : (1) A lamp hangs vertically from a cord in a descending elevator which decelerates at 8 ft/s^2 before coming to a stop. If the tension of the cord is 20 lb-wt. find the mass of the lamp. If the elevator ascends with same acceleration what is the tension ? $g=32$ ft/s^2 (*Ans* : 0.5 lb, 20 lb-wt)

(2) A sand-glass is weighed twice in a sensitive balance—once when the sand is falling slowly from the upper compartment to the lower, and again when all the sand has collected in the lower. Will you get the same weighings ?

(*Ans.* No. Larger in the second.)

(3) A body A rests on an aeroplane B. State and explain the conditions in which the action and reaction between A and B will be

(*i*) Equal to the weight A ; (*ii*) Greater than the weight of A ; (*iii*) Less than the weight of A ; (*iv*) Zero.

I-3.12. Procedure in problem work involving forces. In solving problems involving the action of more than one force on a body or a system of bodies, much difficulty may be avoided by following the procedure suggested below :

(*i*) Select one of the bodies for consideration, usually the one of which rest or motion is to be discussed. Consider this body to be isolated from the rest of the system by an imaginary closed surface, as in fig I-3.20. Mark all forces which act on this body from *outside this surface.*

(*ii*) Construct a 'force diagram' in which the selected body is represented by a point, and represent by suitable vectors all of the forces acting *on* this body from outside the imaginary surface. Be careful not to omit any force, but *never include any force which this body exerts on other bodies.* If any force is unknown, represent it by a vector, but mark it as unknown.

(*iii*) From the force diagram find the resultant of the forces. If it is zero, the body is at rest. If, otherwise, this is the unbalanced force which gives the body an acceleration in its own line of action.

A beginner may avoid much confusion if he remembers that

(*a*) A force like a push or a pull is exerted by contact.

(*b*) An electric, magnetic or gravitational force acts without contact.

(*c*) When two bodies are in contact, the reaction of one on the

other, is exerted at the point of contact. Action and reaction act in the same line.

(*d*) In a flexible cord stretched between two points, the tension is the same throughout. The tension acts on the body at the end of the cord, and exerts a pull on the body along the cord.

Worked Examples : Let us take a few examples to illustrate the procedure sketched above.

Ex. 1-3.18 : *Find the tension in a vertical cable when it pulls a 2000 lb elevator upwards with an acceleration of 4 ft/s².*

Solution : *Step (i)*—Since there is only one body to consider, application of step (1) is obvious.

Step (ii)—The force diagram is represented in the attached fig. *O* represents the body. It is acted on by two vertical forces, (*a*) the downward pull *W* due to the earth and (*b*) the upward pull *T*, here unknown, due to the cable.

Step (iii)—Since the body moves upwards $T > W$ and the balance $T - W$ gives the body an acceleration of 4 ft/s².

Now since the weight of a body = its mass × acceleration due to gravity, we have

$W = 2000 \times 32$ poundals (taking $g = 32$ ft/s²). $\therefore$ $T - 32 \times 2000 = 4 \times 200$

Or, $T = 72000$ poundals.

Ex. 1-3.19 : *In the same problem, what will be the tension if the elevator moves downwards with the same acceleration?*

Solution : The force diagram is similar, but now $W > T$ and the unbalanced force becomes.

$W - T$ = mass of the elevator × its acceleration.

$$32 \times 2000 - T = 2000 \times 4$$

Or, $T = 56000$ poundals.

Ex. 1-3.20 : *A mass of 5 lb is pulled along a horizontal smooth table by a light inextensible string passing over a smooth pulley and carrying a mass 3 lb. Find the tension in the string and the acceleration of the system.*

Solution: *Step* (*i*)—Select the 3 lb mass for consideration and imagine it isolated from the rest of the system by an imaginary closed surface. (Refer to fig I-3.27).

Step (*ii*) Construct the force diagram of the selected body. The forces on it are (*a*) the weight W_1 of the body$=3\times 32$ poundals acting downwards and (*b*) the tension T of the string acting upwards.

Step (*iii*)—Since the mass moves downwards with an acceleration a (say), we have Total downward unbalanced force$=$mass$\times$acceleration of the moving body.

$$\text{or,}\quad 96-T=3a. \tag{A}$$

Consider now the 5 lb mass similarly. The forces on it are (*a*) the weight W_2 $=5\times 32$ poundals acting vertically downwards, (*b*) the reaction R of the table acting vertically upwards and (*c*) the horizontal pull T on it due to the string. As there is no vertical motion W_2 and R must balance each other. Hence the unbalanced horizontal tension T provides the acceleration of this mass. Since the two masses are connected by an inextensible string, their accelerations are the same. Hence

$$T=5a \tag{B}$$

Solving equations (A) and (B), we have

$$T=60 \text{ poundals and } a=12 \text{ ft/s}^2.$$

Ex. I-3.21: *Two masses 500g and 400g are connected together by a light inextensible string passing over a smooth fixed pully. Discuss the motion and find the tension in the string as also acceleration of the system. (Draw the necessary diagram yourself.)*

Solution: **Obviously** the heavier mass moves downwards, and, as a result, pulls up the lighter one.

Let T be the tension in the string and a the common acceleration of the system consisting of the two masses.

Select the 500 g mass for consideration. In constructing its force diagram remember that the forces on it are (*a*) its weight 500×980 dynes acting downwards, and (*b*) the tension T acting upwards. The unbalanced force $500\times 980-T$ gives it the acceleration a.

$\therefore$ $490\,000-T=500\,a$ (A)

Similarly for the 400 g mass, we get (B)

$$T-400\times 980=400\,a$$

Solving (A) and (B), we have

$T=435\,600$ dynes and $a=108.9$ cm/s^2.

Ex. I-3.22. Inclined Plane : *A body of mass m starts from rest and slides down along a smooth plane inclined at an angle θ to the horizontal. Find the acceleration.*

Solution : *Force diagram*—The body is acted on by two forces, (*a*) its weight mg acting vertically downwards, and (*b*) the reaction R of the plane, here unknown and perpendicular to the plane.

Resultant : As the motion occurs along the plane the resultant of these two forces must be directed parallel to the plane. The magnitude of the resultant may be obtained in the ordinary way by combining mg and R and remembering that the angle between them is $180°-\theta$.

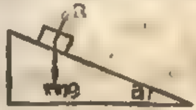


Fig. 3.13

It is however easier to obtain the magnitude of the resultant by resolving mg along and perpendicular to the plane. These resolved parts are $mg \sin\theta$ and $mg \cos\theta$ respectively. As there is no motion perpendicular to the plane, the forces R and $mg \cos\theta$ must balance, *i.e.*, $R-mg\cos\theta$.

Acceleration : The remaining unbalanced force is $mg \sin\theta$ parallel to the plane. This produces motion in the body and gives it an acceleration

$$\frac{mg\sin\theta}{m}=g\sin\theta.$$

Ex. I-3.22 : *A man weighing 120 lb stands in a lift which starts moving downwards with an acceleration of 2 ft/s^2. Calculate the thrust he exerts on the floor. If the lift moved upwards what is the thrust the man will experience ? ($g=32$ ft/s^2).*

Solution : Consider the man isolated and draw the force diagram of the forces acting on him. The vertical forces are (*a*) his weight$=120\times 32$ poundals acting

downwards and (*b*) the reaction R of the floor acting upwards. As a result of these forces he gets a downward acceleration of 2 ft/s^2.

$$120 \times 32 - R = 120 \times 2$$

Or, $R = 3600$ poundals.

The thrust he excerts on the floor is equal and opposite to R.

If the lift moved upwards R is also upwards and we should have

$$R - 120 \times 32 = 120 \times 2 \text{ or } R = 4080 \text{ poundals.}$$

His weight is however $120 \times 32 = 3840$ poundals.

Note. It will be seen from the above that when a lift moves downwards with acceleration, a man in the lift feels lighter because the thrust between him and the floor is smaller than his weight. When the lift moves upwards with an acceleration, this thrust is larger than his weight. Hence he feels heavier.

I.3.13. Principle of Conservation of Linear momentum :

This very important principle follows from Newton's third law and the **momentum-impulse theorem.**

A. Statement : *The total linear momentum of an* **isolated system** *of particles is not altered by the actions and reactions between the particles comprising the system, if no external force acts upon it.* For 'particles' we can read 'bodies', the bodies obeying Newton's third law. The principle applies only to closed systems.

The term 'system of bodies' as used in physics should be understood in the following sense. We often select for our discussion two or more bodies which exert forces on one another. For convenience of analysis we imagine the bodies under discussion to be isolated from all other bodies, as if all that existed were the bodies in question and nothing else in the rest of the universe. Such bodies are referred to as a **system of bodies, an isolated system** or a **closed system.**

B. Derivation from the 3rd Law : Let a particle A act on another particle B with a force F for a time t. The impulse Ft of the force gives the change in linear momentum of B. Since B exerts an equal and opposite force on A, this reaction acts for the same time t on A, and produces a change of momentum Ft in A which is opposite in direction to that produced in B. The changes in momenta due to action and reaction are thus equal and opposite.

If A has a mass m_1 and while moving with a velocity u_1, it interacts with B of mass m_2 moving with velocity u_2, in the same direction so that their velocities decrease to v_2 respectively then

$$Ft = m_1u_1 - m_1v_1$$

$$\text{and } Ft = m_2v_2 - m_2u_2$$

$$\text{or } m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{(I-3.13.1)}$$

Hence the total momentum of the *system* consisting of the particles A and B is not changed by the action and reaction between them. This is true whatever be the number of particles in the system.

Remember, bodies inside a closed system are not supposed to react with whatever may exist outside an imaginary surface (see fig. I-3.20) which encloses only the bodies under consideration. We have said it before (§ I-3.9 ; Isolating a body) and emphasise it again.

Alternatively, if $\mathbf{P}_1$ and $\mathbf{P}_2$ be the momenta of masses m_1 and m_2 respectively then from second law

$$\mathbf{F}_A = \frac{d\mathbf{p}_1}{dt} \quad \text{and} \quad \mathbf{F}_B = \frac{d\mathbf{p}_2}{dt}$$

$$\text{and from 3rd law} \quad \mathbf{F}_A = -\mathbf{F}_B$$

$$\therefore \frac{d\mathbf{p}_1}{dt} = -\frac{d\mathbf{p}_2}{dt} \quad \text{or} \quad \frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0$$

Integrating $\quad \mathbf{p}_1 + \mathbf{p}_2 = \text{Const.}$ (same as I-3.13.2)

Note that (1) the principle holds regardless of the character of reactions as no assumptions are being made regarding the nature of forces, F_A and F_B (2) the principle requires that the total momentum should remain constant both in magnitude and direction for momentum is a vector. (3) the principle implies that momentum of a system can be changed only by unbalanced external forces. (4) the principle is more fundamental than Newton's laws for the latter do not hold in the micro-world i.e. dimensions less than atomic radii, but principle of conservation of linear and angular momenta do.

C. **Derivation of the 3rd Law** from the principle of conservation of linear momentum is quite easy—just reverse the order of the above deduction. From the principle of conservation of momentum we have

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{const} \quad \text{or} \quad \frac{d\mathbf{p}_1}{dt} + \frac{d\mathbf{p}_2}{dt} = 0 \qquad \text{(I-3.13.3)}$$

$$\text{or} \quad \mathbf{F}_A + \mathbf{F}_B = 0 \quad \text{or} \quad \mathbf{F}_A = -\mathbf{F}_B$$

The 3rd law is contained in the first. We have deduced the 3rd law from conservation of linear momentum principle. 3rd law emphasises the fact that internal forces cannot change the momentum of a system as a whole. To do so, some force must be acting from outside. That is what the first law states. Hence we may say that the 3rd law is included in the first. Thus we might have done with the Second Law alone. The 1st law and Principle of momentum conservation state the same fact in different languages.

D. To change the linear momentum of a system of bodies, or particles, a force must be applied from outside the system. (1) Thus a cyclist cannot slow down his cycle by merely pulling at the handle. The change in momentum due to this pull on the cycle is balanced by that due to the forward reaction on the cyclist. A person sitting in a chair cannot lift it. You cannot move a car pushing from inside. (2) A shell bursting in mid-air has a certain momentum before it explodes. When the fragments fly off in all directions after the explosion, each one of the fragments has a momentum of its own. According to the principle of conservation of momentum the sum total of the momenta of the fragments must be equal to the momentum of the entire shell. (3) When a meteorite burns up on entering the earth's atmosphete it transfers its momentum to the air molecules with which it collides and scatter them.

Ex. I-3.24 : *A projectile of mass 50 kg is shot vertically upwards with an initial velocity of 100 m/s. After 5 s it explodes into two fragments. One of 20 kg mass shoots vertically upwards at 150 m/s. Find the velocity of the other at that instant. Take g=9.8 m/s.* [I. I. T. '73]

Solution. Velocity of the projectile 5 s after projection is

$$v=u-gt=500-9.8\times5$$

$$=51 \text{ m/s}$$

The explosion is due to internal forces only, no external force acts. Then

Total momentum before explosion = Total momentum after explosion

$$50\times51=20\times150+30\times v'$$

$$\therefore \quad v'=-15 \text{ m/s}.$$

i.e. the bigger fragment will be hurled downwards with an initial velocity of 15 m/s

Problems : (1) A shell falling vertically at 100 m/s explodes into two fragments of ratio 2 : 1 in masses. If the heavier one descends at 200 m/s just after the explosion what was the velocity of the lighter one at that instant ?

[J. E. T. '76] (*Ans.* 100 m/s down)

(2) A shell, moving northward with velocity v, explodes in midair into two equal pieces. The force of explosion projects one piece in the backward direction with velocity $v/2$. Will there be any change in the velocity of the other piece ? If so, what? [*Ans* : Yes. The velocity will increase to $3v/2$.]

(4) Squid, an ancient sea animal, ejects a jet of water from a flexible funnel, for swimming forward or backward as the need may be. This is possible to conservation of linear momentum.

(5) **Gun firing a shell.** When a gun fires a shell, the shell acquires a momentum in its direction of motion. Before firing, the system consisting of the gun and the shell was at rest, and their combined momentum was zero. Since the explosion exerts equal and opposite forces on the gun and the shell, their combined momentum will not change due to the explosion. The gun therefore acquires a momentum equal and opposite to that of the shell. The backward velocity which the gun acquires on firing is the *velocity of recoil.*

Let M and m be the masses of the gun and the shell respectively and $-V$ and v their velocities.

Then $-MV+mv=0$ or $MV=mv$. (I-3.13.4)

Ex. I-3.25 : *A bullet weighing 8 g is fired from a 5 kg gun. If the speed of the bullet is 400 m/s, find the velocity of recoil of the gun.*

[H. S. '74]

Solution : From the relation $MV=mv$,

we have $V=mv/M=8\text{ g}\times 400\text{ m/s}\div 5\text{ kg}=64\text{ cm/s}$

Ex. I-3.25 : *A parachutist with a machine gun has snapped the cord of his parachute and falling freely. With the parachute he weighs 100 kg. How many bullets of mass 20 g each per sec he must fire downwards at 1 km/s to check his fall?*

Solution. Let n per sec be the required number. Then the reaction of the firing would nullify the downward fall. So we have

$$n/s\times 20\text{ g}\times 1000\text{ m/s}=100\text{ kg}\times 9.8\text{ m/s}^2$$

$$\text{or} \qquad n=49.$$

Ex. I-3.27 : *Determine whether the kinetic energy of recoil of a gun is more, equal to or less than the forward kinetic energy of the bullet.* [I. I. T. '78]

Solution. Let the mass of the gun be M, *much larger than that of the bullet* (m). Let their recoil velocity and the forward one be V and v respectively. Now we know that their kinetic energiees will be $\frac{1}{2}MV^2$ and $\frac{1}{2}mv^2$ and $MV=mv$

$$\therefore E_g=\tfrac{1}{2}MV^2=MV.\tfrac{1}{2}V=mv.\tfrac{1}{2}\frac{mv}{M}$$

$$=\tfrac{1}{2}mv^2\frac{m}{M}=E_b.\frac{m}{M} \qquad (\text{I-3.13.4})$$

$\because$ $M>>m$, we must have $E_g<<E_b$ i.e. recoil energy of the gun is far less than that of the bullet.

This is why we can stand up to the recoil of a gun but are felled by a bullet fired from it. Bodies with identical momenta may not

have same kinetic energy when their masses differ. Because of tremendous recoil energies of big canon firing big shells they used to be either mounted on open railway carriages or themselves provided with wheels which you may have seen in old guns on exhibit.

Problems : (1) Find the force required to hold in position a machine gun firing 10 bullets per sec., each with a mass of 0.005 kg projected with 2.5 m/s.

(*Ans.* 125×10^{-4}N)

(2) A canon is mounted on a railway track standing on a straight track. The total mass of the cannon, the carrier, the operators and shells is 2000 kg while that of a single shell is 25 kg. The shell is fired horizontally with a velocity of 1 km/s. Find the speed of canon just after firing.

(*Ans.* $12\frac{2}{3}$ m/s)

(6) The flight of **rockets** is another application of this principle. The rocket has solid fuel. One of the constituents of the fuel provides the oxygen necessary for the combustion of the other constituents. The products of combustion within the rocket are gases of small quantities that come out with a very high velocity from the tail end of the rocket (fig. I-3.29). *The main body of the rocket gets an equal and opposite momentum and moves forward.* To change the direction of a rocket-ship in space, small jets are fired from the sides of the rocket. These may be oriented as desired.

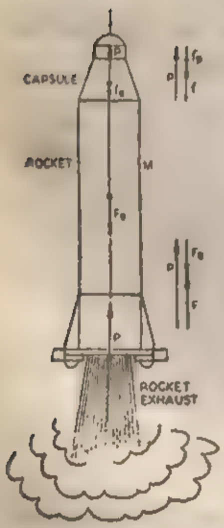


Fig. I-3.29

Let a rocket of mass M be moving up with an instantaneous velocity v (fig. I-3.29). For the present we ignore g. Let the combustion products of mass δm be continuously ejected

per sec from it backward with a *constant speed* u relative to the rocket. Thus as the mass of the rocket diminishes to $(M-\delta m)$ its velocity increases to $v+\delta v$, during a time-interval δt. During that interval the average velocity of the rocket will be $(v+\frac{1}{2}\delta v)$ and that of the ejected gas $(v+\frac{1}{2}\delta v-u)$ in the direction the rocket is moving.

Initial momentum of the rocket is Mv and *after* ejection of δm of the fuel, it is $(M-\delta m)(v+\frac{1}{2}\delta v)$ whereas the momentum of the ejected gas is $m(v+\frac{1}{2}\delta v-u)$. Then from linear momentum conservation principle we have

$$(M-\delta m)(v+\tfrac{1}{2}\delta v)+m(v+\tfrac{1}{2}\delta v-u)=Mv$$

$$\text{or} \quad M.\delta v-mu=0$$

$$\text{or} \quad (\delta m/M)=(\delta v/u),$$

neglecting the product of two small quantities $\delta m.\delta v$.

Now mass of fuel δm burnt out is also δM the *diminution* in mass of the rocket. (Earlier, in deriving the relation $F=ma$ we had indicated that mass of a moving body is not necessarily const and rockets provide such an example).

$$\therefore \quad \delta m=-\delta M$$

$$\text{and so} \quad -\frac{\delta M}{M}=\frac{\delta v}{u} \qquad \text{(I-3.13.5)}$$

Let the initial mass of the rocket+fuel be M_0 and its inital velocity v_0 which after time of flight respectively reduces to M and increases to v. So on integration between these limits we have

$$\int_{M_0}^{M} -\frac{\delta M}{M} = \frac{1}{u} \int_{v_0}^{v} \delta v$$

$$\text{or} \quad -\log_e \frac{M}{M_0} = \frac{v - v_0}{u} \quad \text{or} \quad M = M_0 \, e^{-(v-v_0)/u}$$

$$\text{and} \qquad v = v_0 - u \ln(M/M_0) \tag{I-3.13.6}$$

As the rocket rises the fuel burns and the mass of the rocket diminishes. If the fuel burns at a constant rate it provides a constant force (F) acting on the rocket. Hence the acceleration of the rocket goes on increasing so long as the fuel lasts. The same applies to jets. Application of simple mathematics shows that

$$\text{acceleration} = \frac{\text{downward velocity of gas relative to rocket}}{\text{mass of rocket}}$$

$\times$ rate of loss of mass *minus* the acceleration due to gravity.

$$\text{i.e.} \quad a = \frac{u}{m} \cdot \frac{\delta m}{\delta t} - g \tag{I-3.13.7}$$

With increase of height above the earth g diminishes. If the fuel burns at a constant rate the acceleration goes on increasing due to loss of mass.

Now in actual cases, u and ($\delta M/\delta t$) remains more or less constant as fuel burns out steadily but with height both g and M go on dimi-

nishing so a goes on increasing, i.e. acceleration does not remain constant. Hence the galloping acceleration of the rising rocket you may have noticed in TV and wondered.

Ex. I-3.28 : *A rocket of total mass 6000 kg, of which 5000 kg is the propellant fuel, is to be launched vertically. If the fuel is consumed at the steady rate of 60 kg/s, what is the least velocity of the exhaust gas so that the rocket will just lift off the launching pad after the firing ? What is the thrust ? Take* $g=9.8$ m/s^2.

[J. E. E. '83]

Solution. Let u=initial velocity of the exhaust gas,

dm/dt=rate of mass of the gas ejected, downward and

mg=weight of the rocket.

Then the upward thrust=rate of change of momentum=$u.\ dm/dt$. For the upthrust to lift the rocket immediately after firing we must have $-u\ dm/dt=mg$. Substituting values we get required least value of the velocity=980 m/s.

Required thrust$=u\ dm/dt$

$=980\ \text{m/s}\times 60\ \text{kg/s}=58.8\times 10^3\ \text{N}$

Problems : (1) Find the minimum rate of fuel consumption for a 3000 kg rocket ejecting burnt gases at 300 m/s at the start to take-off vertically.

(*Ans.* 98 kg/s)

(2) In 1 s after vertical take off a rocket loses $\frac{1}{50}$th of its mass. If gas is ejected at 5 km/s find its upward acceleration. [*Hint* : Use Eqn I-3.13.5]

(*Ans.* 100 m/s^2)

(7) **Jet planes** utilize the same principle as rockets for motion. In the jet engine, air blows in at the front, is compressed and led into a combustion chamber. Here it is mixed with fuel and burned. The high pressure of the resulting gases is made to work a turbine. The turbine not only works the compressor which compresses the incoming air, but also drives the gases through the tailpiece of the

engine at a temendous speed. *The plane gets a forward momentum equal to the backward momentum of the exhaust gases and moves forward.*

Thus the principle on which the rocket and jet propulsion acts are identical. But they differ in their working as much as jets utilise oxygen from the air sucked in, whereas a rocket carries inside it oxygen as liquid or sold fuel. Hence jets *depend on the atmosphere* for their flight whereas *rockets can go to outer space* ; there they have the added advantage of frictionless flight. Rockets had been invented long ago by the Chinese and had been in use in fireworks and war in the Middle Ages.

(7) When a man jumps ashore from a boat it moves back. We have explained that by Newton's 3rd law. The conservation principle applies there just as in a rifle firing a bullet. If the person walks briskly along the deck of the vessel, say towards the shore, the boat will move back.

For example, let a person of mass 50 kg move so 3 m on a boat of mass 250 kg. Let his original distance from the shore be 5 m. Now the walking man and the boat act and react on each other. So those forces, 50 a and 250 a' must be equal and opposite i.e. $a=5\ a'$. Now they have interacted for same time t and so developed velocities $v=at$ and $v'=a'\,t$ which makes $v=5v'$ and directed oppositely. Clearly the man and the boat must have moved over opposite distances x and x' where $x=\ -5x'$. But $x-x'=3$ m i.e., $x'=0.5$ m and $x=2.5$ m. So the boat moves away by 0.5 m from its position placing the man $(5-2.5$ i.e.) 2.5 m from the shore.

Problem : A boat is in perpendicular direction to the shore, its nearest end 0.6 m from the shore. A 50 kg man walks away on the boat which weighs 200 kg and is 2.5 m long. Show that the boat will not reach back to the shore. Find by how much it falls short. *(Ans.* 0.5)

I-3.14. Elastic collisions. You are familiar with collisions, such as between a man and a cyclist, two motors, two trains, etc. Mathematical analysis of any such real collision is about impossible. But in idealized simple cases, it is possible to do so.

Answers to all problems on elastic collisions are based on two conservation principles, namely,

(i) *Conservation of momentum* and (ii) *Conservation of kinetic energy.* Collisions in which the momentum principle holds, but kinetic energy is not conserved, are called **Inelastic Collisions**. In practical cases most collisions are inelastic. Part of the kinetic energy is converted into some other form, while linear momentum is conserved.

A. **General :** Suppose two particles collide with each other while moving along the same straight line. A collision is said to be **perfectly elastic** when the sum of the kinetic energies of the particles remains the same before and after a collision. The linear momentum is of courseconserved. Atoms, molecules, nuclei, electrons, etc. may suffer elastic collisions.

Let two particles of masses m_1 and m_2 move *along the same line* with velocities u_1 and u_2 respectively (Fig I-3 30). The velocities

Fig. I-3.30

will be taken as positive if the motion is from left to right. Momenta are in the direction of velocities. If any of the particles moves from right to left its velocity and momentum will be taken as negative.

Suppose after the collision, the velocity of m_1 becomes v_1, and that of m_2 becomes v_2. From the principle of conservation of momentum, we get

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \text{(I-3.14.1)}$$

Since in elastic collisions, kinetic energy is also conserved, we have

$$\tfrac{1}{2}m_1u_1^2+\tfrac{1}{2}m_2u_2^2=\tfrac{1}{2}m_1v_1^2+\tfrac{1}{2}m_2v_2^2 \qquad \text{(I-3.14.2)}$$

These two equations provide answers to all questions of elastic collisions in a straight line. Let us consider a few cases. Note that we are taking $u_1 > u_2$ and $v_2 > v_1$.

(i) Relative velocities before and after collision

From 1-3.14.1 $m_1(u_1 - v_1) = m_2(v_2 - u_2)$ (I-3.14.3)

From 1-3.14.2 $m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$ (I-3.14.4)

Dividing ·4 by ·3 we get,

$$u_1 + v_1 = v_2 + u_2$$

or $u_1 - u_2 = v_2 - v_1 = -(v_1 - v_2)$ (I-3.14.5)

This shows that for (perfectly) elastic collision *relative velocity of approach before collision is equal to the relative velocity of separation after collision.*

(ii) Dependence of final velocities on the colliding masses.

From 1-3.14.5, we have $v_2 = u_1 - u_2 + v_1$. Putting this value in 1-3.14.3 we get $m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$

or, $v_1(m_1 + m_2) = (m_1 - m_2)u_1 + 2m_2u_2$

or, $v_1 = \dfrac{m_1 - m_2}{m_1 + m_2}u_1 + \dfrac{2m_2}{m_1 + m_2}u_2$ (I-3.14.6)

Similarly, $v_2 = \dfrac{2m_1}{m_1 + m_2}u_1 + \dfrac{m_2 - m_1}{m_1 + m_2}u_2$. (I-3.14 7)

(*a*) *If both masses are equal* $(m_1 = m_2)$

$v_1 = u_2$ and $v_2 = u_1$ (1-3.14.8)

It shows that the particles *interchange* their velocities after collision.

(*b*) m_2 *initially* at rest (*that is,* $u_2 = 0$).

In this case $v_1 = \dfrac{m_1 - m_2}{m_1 + m_2}u_1$ and $v_2 = \dfrac{2m_1}{m_1 + m_2}u_1$ (I-3.14.9)

(*c*) m_2 *initially at rest and* $m_1 = m_2$.

In this case $v_2 = u_1$ and $v_1 = 0$. (1-3.14.10)

This means that the first particle comes to rest after collision and the second starts off with the velocity of the first.

(*d*) *The second particle is very heavy relative to the first* $(m_2 \gg m_1)$ *and is initially at rest* $(v_2 = 0)$. In this case, we get from Eq.s 1-3.14.6 and 1-3.14.7

$v_1 \cong -u_1$ and $u_2 \simeq 0$. (1-3.14.11)

(This sign $\cong$ means 'about equal to')

This means that the lighter particle turns back practically with the same velocity after collision while the heavy particle remains practically at rest.

It follows that on colliding with a rigid wall, a particle will turn back with the same speed. m_2 in this case is taken as infinite.

(*e*) *m_2 is very small compared with m_1 and is at rest.* We neglect m_2 compared with m_1 and take $u_2 = 0$ in Eqn. 1-3.14.6 and. 7. This gives

$$v_1 \simeq u_1 \text{ and } v_2 \cong 2u_1 \qquad \text{(I-3.14.12)}$$

This means that the velocity of the heavier particle remains practically unchanged, while the lighter particle shoots off with a speed about twice that of the heavier particle. Hence however hard you take a spot kick in football or hit a golf ball, you can never propel it faster than twice the velocity of your foot or the stick.

B. Classification of Collisions and Energy Relations :

Collisions between two bodies may be of four types (*i*) perfectly elastic or simply elastic (the last article) (*ii*) perfectly inelastic (*iii*) partly elastic (*iv*) hyper-elastic. They may be defined respectively as cases wherein (*i*) relative velocity of approach is *same* in magnitude but *opposite* in direction to relative velocity of separation (*ii*) no relative velocity after interaction (*iii*) relative velocity of approach is *greater* in magnitude but opposite in direction to the velocity of seperation (*iv*) relative velocity of approach *less* than that of relative velocity of separation. Again, in (*i*) linear momentum and kinetic energy *are* conserved so far as initial and final conditions are concerned ; here *conservative* forces are at work. In (*ii*) the masses stick together, linear momentum is conserved but *not* the kinetic energy ; dissipative forces are at work here, always generating heat sometimes light or sound. In (*iii*) also kinetic energy is *not* conserved as again there is some dissipation. In (*iv*) also, kinetic energy is *not* conserved, it is gained at the expense of vibrational energy.

The first two cases are idealsations ; mostly it is the third type that occurs and rarely the fourth. Ivory or steel or glass balls hitting others of their types provide *very nearly* perfectly elastic collisions. A lump of mud or putty falling on a hard surface is a case of inelastic collision ; a bullet fired and stuck into a target hanging or fixed, is

such an example. Most collisions that you observe are of the third type. The examples of the fourth type are rare ; you throw a marble at a large swinging body along its direction of motion and may be surprised to find it bouncing back with a much greater velocity ; this happening if the marble hits when the other body is approaching and is at its lowest position. When two vibrating (said to be *excited*) molecules collide, kinetic energy may be increased at the expense of vibrational energy as in *Raman Effect.* We have already said that collisions of atoms, protons and other fundamental particles (including photons—lumps of energy) are perfectly elastic.

I-3.15. Energy transfer ; (a) Perfectly elastic Centric collision : Let a mass m_1 moving with a velocity u_1 hit another of mass m_2 moving in the same direction with u_2, centrally i.e. head-on. Let them continue to move in the same direction with changed velocities u_2 and v_2.

The momentum conservation law gives

$$m_1u_1+m_2u_2=m_1v_1+m_2v_2 \qquad \text{(i)}$$

From the original definition of *perfectly elastic collision* since *relative velocities of approach and separation must be the same,* we must have

$$u_1-u_2=v_2-v_1 \qquad (\textit{Note}:\ e=1 \text{ here}) \qquad \text{(ii)}$$

From these two equations as we have already obtained (I-3.14.6 and 7)

$$v_1=\frac{(m_1-m_2)u_1}{m_1+m_2}+\frac{2m_2u_2}{m_1+m_2}. \qquad \text{(iii)}$$

$$\text{and}\quad v_2=\frac{2m_1u_1}{m_1+m_2}+\frac{(m_2-m_1)u_2}{m_2+m_1} \qquad \text{(iv)}$$

Hence the total kinetic energy before impact is

$$K=\tfrac{1}{2}m_1u_1^2+\tfrac{1}{2}m_2u_2^2=\tfrac{1}{2}\,\frac{1}{m_1+m_2}\,[\,(m_1+m_2)(m_1u_1^2+m_2u_2^2)\,]$$

$$=\frac{1}{2(m_1+m_2)}\,[\,m_1^2u_1^2+m_2^2u_2^2+m_1m_2(u_1^2+u_2^2)\,]$$

$$=\frac{1}{2(m_1+m_2)}\,[\,(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2\,]$$

$$=\frac{1}{2(m_1+m_2)}\,[\,(m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2\,]$$

$$= \frac{1}{2(m_1+m_2)} [(m_1+m_2)(m_1 v_1^2 + m_2 v_2)^2]$$

$$= \tfrac{1}{2} m_1 v_1^2 + \tfrac{1}{2} m_2 v_2^2 = \text{total kinetic energy after impact} \quad \text{(I-3.15.1)}$$

Hence *kiniic energy is conserved in perfectly elastic collisions.*

Condition of maximum energy tran'fer : We consider the case when m_2 is initially at rest. Then from I-3.14.1 we get

$$m_1(u_1 - v_1) = m_2 v_2$$

From I-3.14.5 we get $u_1 = v_2 - v_1$ or $v_2 = u_1 + v_1$

$\therefore \quad m_2 v_2 = m_1(u_1 - v_1) = m_1[u_1 - (v_2 - u_1)] = m_1(2u_1 - v_2)$

or $\quad v_2(m_2 + m_1) = 2m_1 u_1$

$\therefore$ Kinetic energy aquired by m_2 i.e. transferred from m_1 is

$$\tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_2 \frac{4m_1^2 u_1^2}{(m_1+m_2)^2} = \tfrac{1}{2} m_1 u_1^2 \frac{4m_1 m_1}{(m_1+m_2)^2}$$

$$= \tfrac{1}{2} m_1 u_1^2 \left\{ \frac{4m_1 m_2}{(m_1 - m_2)^2 - 4m_1 m_2} \right\} \quad \text{(I-3 15 2)}$$

Now $\frac{1}{2} m_1 u_1^2$ is the original K.E. For $\frac{1}{2} m_2 v_2^2$ to be a maximum the bracketed term must be a maximum. Obviously it is so when $m_1 = m_2$. Under this condition the entire energy is transferred ; the first body stops the other moves off with same velocity as in equation I-3.14.10. Fig. I-3 31 shows how the fraction of energy transferred (b)*, varies with the fraction m_2/m_1. It is unity when $m_1 = m_2$.

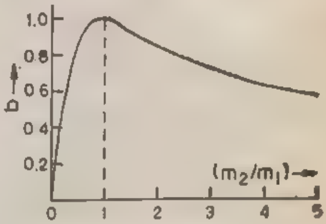


Fig. I-3.31

Problems. (1) For elastic collision of a body of mass m with another of mass km the energy lost by the former on a head-on collision is maximum when $k = 1$. Prove by applying the laws of conservation of energy. [J. EE. '79]

(2) A mass m_1 hits head-on another mass m_2 at rest with a velocity u. The two move on with velocities v_1 and v_2. If the collision is perfectly elastic show that the velocity and energy gained by the mass at rest are

$$v_1 = \frac{2um_1}{(m_1+m_2)} \quad \text{and} \quad \text{K. E.} = \frac{4m_1 m_2}{(m_1+m_2)^2}$$

$$^* b = \frac{4m_1 m_2}{(m_1+m_2)^2} = \frac{4(m_2/m_1)}{(1+m_2/m_1)^2}$$

(3) Two masses m_1 and m_2 moving along the same straight line with velocities u_1 and v suffer a perfectly elastic collision. Show that there will be a transfer of momentum from the first to the second of magnitude $2m_1m_2(u-v)/(m_1+m_2)$

(4) Two perfectly elastic flat discs **A** and B one k times as massive as the other rest on a smooth horizontal table. A is then made to move with a velocity u to hit B head-on. Apply conservation laws to find an expression for kinetic energy transferred from A to B. Also show that the fraction transferred is irrespective of whether A is the bigger or the smaller one. [J E E '78]

(5) A neutron collides head-on with a C^{12} nucleus at rest in the graphite moderators of a reactor. Show that the neutron loses 28% of its energy. The collision is elastic and the mass number of neutron is unity.

(b) **Perfectly elastic slightly Excentric collision :** Collision which is not head-on but slightly off the central line make the bodies move along diverging directions. You must have noticed that, when in playing carrom or billiard you deliberately hit the dice or the ball at rest, off th centere line. The dice and the striker or the billiard bal's after impact no longer continue in thes ame line, their paths diverge.

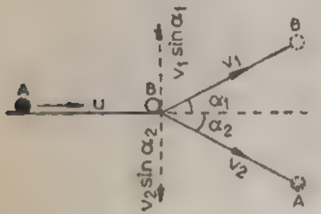


Fig. I-3.32(a)

Let a ball moving with a velocity u hit an identical ball at rest slightly off the center. They will fly away with velocities v_1 and v^4 respectively in directions making angles α_1 and α_2 respectively with the direction of u (fig. I-3 32a). These angles are said to be angles of *scattering*.

The balls being identical have the same mass and the collision is elastic so that the total energy and forward momentum are conserved. So

$$\tfrac{1}{2}mu^2=\tfrac{1}{2}mv_1^2+\tfrac{1}{2}mv_2^2$$

and $mu+0=mv_1\cos\alpha_1+mv_2\cos\alpha_2$ (I-3.15.3)

$\therefore\ u=v_1\cos\alpha_1+v_2\cos\alpha_2$ (I-3 15.4)

and $v_1\sin\alpha_1-v_2\sin\alpha_2=0$ or $v_1\sin\alpha_1=v_2\sin\alpha_2$ (I-3.15.5)

There being no initial component of velocity in the direction perpendicular to u. The velocities can be found if the angles of scatter are known and the angles, if the velocities are known.

Ex. I-3 29. *A ball moving with a velocity of 0.5 m/s hits an identical ball at rest elastically and moves after collision at 0.3 m/s. Find the velocity of the second ball and show that they move away at right angle to each other.* [I.I.T '72]

Solution **:** Refer to the fig above and the three equations ; we have

$50^2 = 30^2 + v_2^2$ (for $\frac{1}{2}$ m in the first equation cancels out).

or $v_2 = 40$ cm/s

From the 2nd equation $50 = 30 \cos \alpha_1 + 40 \cos \alpha_2$

or $3 \cos \alpha_1 + 4 \cos \alpha_2 = 5$ (i)

from the 3rd relation $3 \sin \alpha_1 = 4 \sin \alpha_2$ (ii)

Squaring (i) and (ii) and adding we have

$9 \sin^2 \alpha_1 + \cos^2 \alpha_1) = 16(\sin^2 \alpha_2 + \cos^2 \alpha_2) + 25 - 40 \cos \alpha_2$

or $40 \cos \alpha_2 = 25 + 16 - 9 = 32$

$\therefore \cos \alpha_2 = \frac{4}{5}$ and hence $\sin \alpha_2 = \frac{3}{5}$

Substituting this value of $\sin \alpha_2$ in eqn (ii) we get,

$\sin \alpha_1 = \frac{4}{3} \times \frac{3}{5} = \frac{4}{5}$ and so $\cos \alpha_1 = \frac{3}{5}$

Now $\sin(\alpha_1 + \alpha_2) = \sin \alpha_1 \cos \alpha_2 + \cos \alpha_1 \sin \alpha_2$

$= \frac{4}{5} \times \frac{4}{5} + \frac{3}{5} \times \frac{3}{5} = 1$

$\therefore \alpha_1 + \alpha_2 = 90°$.

Problem. A ball moving at 9 m/s strikes a stationary identical ball such that each moves on at 30° to the original direction. Find the speeds of the two balls after collision. (**Ans.** 3 $\sqrt{3}$ m/s). [I.I.T. '75]

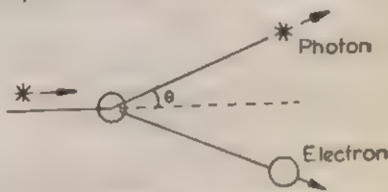


Fig. I-3.32(b)

Compton Effect relates to scattering of X-rays by electrons. The observed facts could be explained *only* by considering radiations to be finite bundles or *particles of energy*, the so-called *photons*. They collide with an electron and both get scattered because of perfectly elastic collision. Their paths are shown in fig. I-3.32(b).

Demonstration of conservation of Linear momentum : "Newton's Cradle".

The apparatus is shown in Fig. I-3.33 and consists of solid identical steel balls suspended as shown, by a pair of threads each, from a pair of horizontal supports so that the balls stay in the same horizontal line. If one ball is allowed to swing down and hit, none moves but the last one, which immediately swings out through an arc equal to that of the striker. The action depends on *i*) conservation

laws and (*ii*) the fact that collision of steel balls are very nearly perfectly elastic. If two balls swing down together to hit, the last

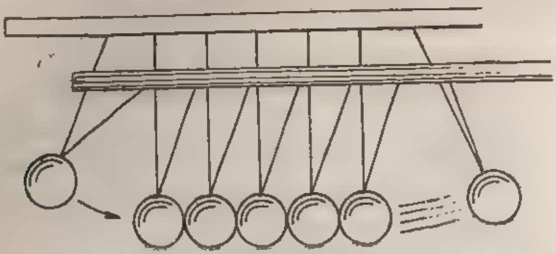


Fig. I-3.33

two balls swing out and so on for any number of balls Refer to eqn I-3.14.10.

The same will happen if the same steel balls are put in a horizontal groove in contact with one another and one two etc. of their number roll in and hit them. Even with a row of identical coins on a smooth surface the effect can be demonstrated. Compare the children's game, the bagatelle, or hitting a row of carrom dice head-on with the striker.

Ex. I-3.30. *A 1 lb steel sphere hangs by a light thread 27" long. It is pulled to a horizontal position and then let go. It strikes a 5 lb mass at its lowermost position elastically, kept on a smooth table. Find the velocities of the masses.*

Solution : (Draw the relevant diagram). The ball descends through 27" and hence developes a horizontal velocity of

$$u = \sqrt{2gh} = \sqrt{2.32\,\tfrac{27}{12}} = 12 \text{ ft/s}.$$

After collision let the ball recoil with a velocity v_1 imparting a forward velocity of v_2 on the stationary mass. Then

$$m_1 u = m_2 v_2 - m_1 v_1$$

$$\text{or } 1 \times 12 = 5v_2 - 1 \times v_1 \text{ or } 5v_2 - v_1 = 12 \qquad \text{(i)}$$

Again from Newton's law of collisions

$$u = v_2 - (-v_1) \text{ or } v_1 + v_2 = 12 \qquad \text{(ii)}$$

On adding (i) to (ii) $6v_2 = 24$ or $v_2 = 4$ ft/s and $v_1 = 8$ ft/s

(c) Perfectly inelastic Collision : To simplify matters we take the second mass m_2 to be initially at rest. *After collision the two bodies stick together and move off with a common velocity v.* Then we have $m_1 u = (m_1 + m_2)v$ or $v = m_1 u/(m_1 + m_2)$

Now the kinetic energy of the combined mass is

$$\tfrac{1}{2}(m_1 + m_2)v^2 = \tfrac{1}{2}(m_1 + m_2)\frac{m_1^2 u_1^2}{(m_1 + m_2)^2}$$

$$= \tfrac{1}{2}m_1u_1^2\frac{m_1}{m_1+m_2}=\frac{m_1}{m_1+m_2}\cdot\text{(Original K.E.)} \qquad \text{(I-3.15.6)}$$

Thus after collision a fraction (a) of the original K.E. remains in the combined mass and let that fraction be denoted by

$$a=\frac{m_1}{m_1\,1+m_2/m_1)}=\frac{1}{(1+m_2/m_1)} \qquad \text{(I-3.15.7)}$$

Obviously the rest of the kinetic energy is dissipated in the form of heat and sound. *Energy is not conserved* in this case. Amount of energy dissipated depends upon (m_2/m_1) and we consider three important cases—(*i*) $m_1 \gg m_2$ (*ii*) $m_1/m_2=1$ and (*iii*) $m_1 \ll m_2$.

In the *first* case, $(m_2/m_1 \to 0)$ and $a \cong 1$ and all of the initial energy remains and none is dissipated. This happens when a big mass hits a dust particle at rest.

In the *second*, we have $a \simeq \frac{1}{2}$ so that half the kinetic energy is dissipated.

In the *last* case $m_2/m_1 \to \infty$ and $a \simeq 0$ so no kinetic enery is retained, the whole of it dissipated. This is exemplified when a lump of mud or butter falls on the ground and does not bounce.

Ex. I-3 31. *A body of mass m moving with a velocity V in the X-direction collides with a body of mass M moving along the Y-axis with a velocity v and they coalesce. Find the momentum of the combined mass and the fraction of the energy converted into heat during the collision.* (I.I.T. '77)

Solution : Let the combined mass $(m+M)$ move off with a velocity u at an angle θ to the X-axis. Resolving its momentum along X and Y directions respectively we get, (Draw the relevant figure yourself).

$$mV=(m+M)u\cos\theta$$

and $$Mv=(m+M\,u\sin\theta$$

$$\therefore\quad m^2V^2+M^2v^2=(m+M)^2u^2$$

$$\therefore\quad \text{(Velocity)}^2 \text{ of combined mass}=\frac{m^2V^2+M^2v^2}{(m+M)^2}$$

$$\text{Its (Momentum)}^2=m^2V^2+Mv^2$$

and the direction of momentum is given by

$$Mv/mV=\tan\theta \text{ i.e. } \theta=\tan^{-1} Mv/mV$$

Initial kinetic energy $E_1=\frac{1}{2}mV^2+\frac{1}{2}Mv^2$

Final kinetic energy $E_2=\frac{1}{2}(m+M\,u^2$

$$=\tfrac{1}{2}(m+M)\frac{m^2V^2+M^2v^2}{(m+M)^2}$$

$$=\tfrac{1}{2}\frac{m^2V^2+M^2v^2}{m+M}$$

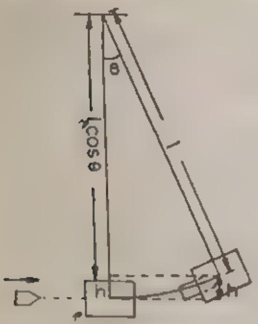
Hence the dissipated fraction of initial kinetic energy will be

$$\frac{E_1-E_2}{E_1}=\frac{mM}{m+M}\,\frac{V^2+v^2}{Mv^2+mV^2}$$

Ex. II-3. 2 *A 15 g bullet travelling at 363 m/s horizontally gets embeded in an 1.8 kg block of wood hanging at rest by a 3 m long thread. Find the angle through which the combined mass swings out ? Take $g=10\ m/s^2$.*

Solution : Refer to the adjoining fig.

Initial momentum $=15\times10^{-3}\times363+0$



Final momentum $=(1.8+\cdot015)v$

Velocity of the combined mass

$$v=\frac{363\times0.015}{1.85}$$

Its Kinetic energy $=\frac{1}{2}(m+M)v^2$ and Potential energy at the end of the swing $(m+M)gh$

$$\tfrac{1}{2}(m+M)v^2=(m+M)gh$$

$$\therefore\quad h=\frac{v^2}{2g}=\frac{363^2\times15^2\times10^{-6}}{2\times10\times(1.815)^2}=\frac{9}{20}m$$

Now, from the figure $h=l-l\cos\theta$

$$\therefore\quad 3(1-\cos\theta)=h\frac{9}{20}m\qquad\therefore\quad\cos\theta=\frac{17}{20}.$$

Problem. Two masses A and B of masses 100 and 400g approaching each other at 100 and 10 cm/s respectively suffer a head-on collision and stick together.

Find (*i*) the direction of movement of the combined mass (*ii*) distance covered in 10 s and (*iii*) rise in temperature if the sp, heat of either is 0.1. [I. I. T '71]

Ans (*i*) as of A (*ii*) 1.20 cm (*iii*) 2.3×10

(d) **Partly Inelastic collisions :** In discussing it an important quantity is necessary, the **Co-efficient of Restitution**. For two bodies moving along the same line with velocities u_1 and u_2 and colliding, the relative velocity of approach is (u_1-u_2) and that of their separation is (v_2-v_1). Newton from experiments arrived at the law, that the *ratio between these relative velocities for given materials is*

almost a constant. This ratio, the coefficient of restitution is defined as

$$e=\frac{\text{Relative velocity of separation}}{\text{Relative velocity of approrch}}=\frac{v_2-v_1}{u_1-u_2} \qquad \text{(I-3.15.8)}$$

When $e=1$, the collision is said to be perfectly elastic, (egn I-15.ii, p 124) when $e=0$, it is perfectly inelastic, when $e<1$ it is partly elastic and for hyperelastic collision $e>1$.

A term **resilience** is relavant in this connection. Perfectly elastic bodies are perfectly resilient and such bodies bounce back very quickly and they are very hard Softer the bodies, less resilient they are and more slowly do they rebound. *Resilience of a body is its ability to suffer elastic deformation without being permanently deformed. Restitution* is ability to recover from such deformation.

In *partly inelastic* collision *colliding bodies do separate after collision but the relative velocity of separation is less than that of approach.* From the equation above we have

$$e=\frac{v_2-v_1}{u_1-u_2} \text{ or } (u_1-u_2)e=v_1-v_2 \qquad \text{(i)}$$

Also, $m_1u_1+m_2u_2=m_1v_1+m_2v_2$ (ii)

Now by multiplying (i) by m_2 and adding to (ii) we obtain

$$v_1=\frac{u_1(m_1-em_2)+m_2u_2(1+e)}{m_1+m_2} \qquad \text{(iii)}$$

$$\text{and similarly } v_2=\frac{u_1m_1(1+e)+u_2(m_2-em_1)}{m_1+m_2} \qquad \text{(iv)}$$

Loss of energy is given by

$$(\tfrac{1}{2}m_1u_1^2+\ m_2u_2^2)-\tfrac{1}{2}(m_1v_1^2+\tfrac{1}{2}m_2v_2^2)$$

$$=\tfrac{1}{2}\cdot\frac{1}{m_1+m_2}\left[(m_1-m_2)(m_1u_1^2+m_2u_2^2)-(m_1+m_2)(m_1v_1^2+m_2v_2^2)\right]$$

$$=\tfrac{1}{2}\cdot\frac{1}{m_1+m_2}\left[(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2-(m_1v_1+m_2v_2)^2+m_1m_2(v_1-v_2)^2\right]$$

$$=\tfrac{1}{2}\cdot\frac{1}{m_1+m_2}\left[(m_1u_1+m_2u_2)^2+m_1m_2(u_1-u_2)^2-(m_1u_1+m_2u_2)^2-m_1m_2e^2(u_1-u_2)^2\right]$$

[applying the results (i) and (iii) above]

$$=\tfrac{1}{2}\cdot\frac{m_1m_2}{m_1+m_2}\cdot(u_1-u_2)^2(1-e^2) \qquad \text{(I-3 15.9)}$$

Since $e<1$ and $u_1>u_2$, the expression is a positive one and hence there is a loss of energy for partially inelastic collisions.

When $e=1$ i.e. collision is elastic there is no loss of energy according to the above equation, as we have already deduced in I-3.14.1. If $e=0$ i.e. the collision is inelastic and the bodies get stuck, maximum loss of energy occurs, for one of them stops.

Ex. I 3.33. *Two spheres of masses 5 and 2 kg travelling at 10 and 5 m/s respectively in the same direction, collide. Find their velocities after impact and energy wasted if $e=0.6$.*

Solution : From definition $v_2-v_1=0.6(10-5)=3$ m/s
From momentum conservation, $5\times10+2\times5=5v_1+2v_2$
Solving from the two expressions $v_1=7.7$ m/s and $v_2=10{\cdot}7$ m/s
From equation I-3.15 9 *Loss of kinetic energy* is

$$\tfrac{1}{2}\cdot\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2(1-e^2)=\tfrac{1}{2}\frac{5\times2}{5+2}(10-5)^2(1-0\ 6)^2=11.43 \text{ J}$$

(d) Partially Inelastic collision between a sphere and a plane : For simplicity we arrange to drop a sphere from a height h on a fixed horizontal plane. Its velocity just before striking the plane will be obtained from $u^2=2gh$ and that after striking $v^2=2gH$ where v is the velocity of rebound and H the maximum height it rises. The collision is direct i e head-on, for both u and v are vertical. Here the relative velocity of approach between the sphere and the plane is u and that of separation between them v. So coefficient of restitution is

$$e=\frac{v}{u}=\frac{\sqrt{2gH}}{\sqrt{2gh}} \quad \therefore \quad H=e^2h \tag{I-3.15.10}$$

If a ping-pong ball falls on a hard floor it will jump up and down a number of times before stopping, each time rising less than the previous one fig. I-3.34). We find below the *total distance covered* before it stops.

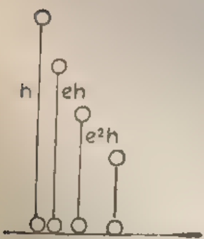


Fig I-3 34

After the first rebound the ball rises by $H=e^2h$ and falls through the same height. So just previous to the second rebound the total distance covered is $h+2H=h+e^2h$.

On bouncing a second time the sphere should rise through $e^2(e^2h)$ or e^4h and falls through the same height and thereby cover up $h+2e^2h+2e^4h$. Similarly after the 3rd

rebound the height will be $e^2(e^4h,$ and cover a total distance of $h+2e^2h+2e^4h+2e^6h$.

$\therefore$ Total distance covered $=h+2e^2h+2e^4h+2e^6h+\cdots$ to ∞

$$=(2h+2e^2h+2e^4h+\cdots)-h$$

$$=2h(1+e^2+e^4+e^6+\cdots\text{to }\infty)-h$$

$$=\frac{2h}{1-e^2}-h$$

$$=\frac{2h-h+he^2}{1-e^2}=\frac{h(1+e^2)}{1-e^2} \qquad \text{(I-3.15.11)}$$

Total Time taken can be found similarly. Time to fall to the floor for the first time is $t=\sqrt{2h/g}$ as the ball is falling freely. It jumps up with a velocity $v=eu$ and the time for it to go up and down will be $2(v/g)=2eu/g=2et$. After the second bounce it starts with an upward velocity of $v=e^2u$; so the time to go up and down will be $2e^2u/g=2e^2t$ and so on.

Hence the total time taken will be

$$t+2t_1+2t_2+\cdots\text{to }\infty=t+2et+2e^2t\cdots\text{to }\infty$$

$$=2t(1+e+e^2+e^3+\cdots)-t$$

$$=\frac{2t}{1-e}-t=\frac{t(1+e)}{(1-e)}=\sqrt{\frac{2h}{g}}\cdot\frac{1+e}{1-e}. \qquad \text{(I-3.15.12)}$$

I-4

FRICTION

I-4.1. What It Is. It is a force brought into play whenever there is a relative motion or tendency to it between two surfaces. It is one of the action-reaction pairs of the 3rd law of Newton. Newton's First law says, no body moving uniformly along a straight line should ever stop; but our every day experience shows just the opposite fact —all moving bodies stop after covering some distance *apparently* without cause. Force is required to maintain it moving uniformly in a straight line. If Newton's law is true then this fact indicates that *movement of bodies is always opposed.* That is, moving bodies always face a resisting force which comes into play; even when a body tends to move but as yet does not, it is opposed as we know. This is the **frictional force** defined as that force *which is brought into play whenever there is relative motion between two surfaces in contact* or even a tendency to such, this force acts at points of contact between the surfaces and *always* opposes that motion or tendency to it, irrespective of the direction of motion.

So we have to spend more energy for motion than we need to, because of friction, it always leads to wastage in the form of irrecoverable heat energy, it causes wear and tear of moving parts of machinery and shorten their lives; much effort and ingenuity go to just reducing friction, it cannot be eliminated. We have listed only a few of its *disadvantages*. But in our daily lives, friction is very important, for it has many *advantages* as well. Without it we could not walk or lean, could not hold a pen and it would not write, a wall or a piece of wood would not hold a nail, wheeled transport as we know, would not be possible for they could not be braked to a stop. Such advantages flowing from friction can be multiplied, without number.

The disadvantages of friction have made it the most important of **dissipative forces**. Whatever be the direction of motion, friction will be there i.e. it opposes motion and that too independently of direction. The kinetic energy that is so dissipated, always appear as heat and under favourable circumstances as light and sound as you must have noticed.

I-4.2. Types of Friction. Friction appears between two surfaces in contact. The surfaces may be both solids, one solid the other fluid or two of same or different fluids. This last is said to be **viscosity**. Solid surfaces may be two extended planes e g. a large block *sliding* over a table-top or an inclined road ; or one *rolling* over another. The surfaces may be dry or wet and of course of diverse materials. Fluid friction occurs when a solid moves through a fluid, like a boat through water or a balloon through air or when a fluid (liquid or gas) flows past stationary solids like tall buildings or bathers in a river. Again there is friction when there is relative motion (*kinetic* friction) between surfaces and when there is no motion but a tendency to move (*static* friction) Only one of these diverse and complex cases we shall study in some detail—*sliding friction of the static type between two dry, plane, solid surfaces.*

Note that *friction involves bodies, not particles.* (*Why* ?)

I 4 3. Sliding friction Consider a *heavy* body resting on a

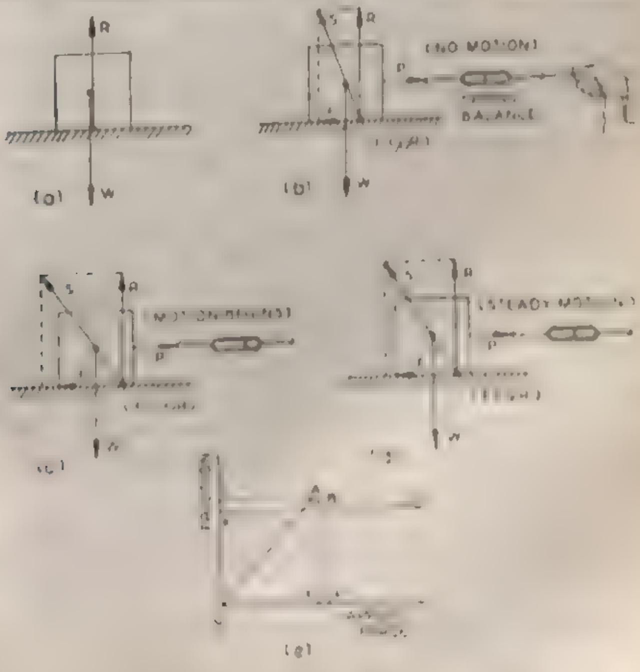


Fig. I-4.1

horizontal table [fig. 1-4.1 a)]. Its weight W acts vertically

downwards. The reaction R of the table acts on it vertically upwards. Since the body is at rest, these two *opposite* forces are *equal.* They act along the *same* line through the C.G, but are shown separated in the figure for clarity. If we now apply a small horizontal force P to the body it will not move [fig. I-4.1(b)]. This force will be balanced by the force of friction F which now comes into play at the surface separating the body form the table. P and F must be equal and opposite. Their lines of action are parallel. S is the resultant of F and R. The spring balance records the force applied.

As we increase P, F also increases and still balances P. But when P exceeds a certain value, the body starts to move [Fig. I-4.1(c)]. After the motion has begun, the value of the force of friction slightly decreases (Fig. I-4.1(d). The force of friction reaches the maximum value just before the motion begins This value is called the *limiting friction* (or the *limiting value of static friction*) The slightly smaller value when the body is moving uniformly is called the *force of kinetic friction.*

Fig. I-4.1(e) is a graphical representation of the relation between the force applied as read by the spring balance and the friction developed. OA represents the condition of rest. Applied force is increasing while enough of friction is being generated as to prevent motion ; this part represents *static friction.* When the force applied is OE, the body is on the point of starting. OL represents the limiting friction. A force greater than OE makes the body move. If the body moves with constant velocity it does so with a slightly lower but constant kinetic friction OK. Pull a heavy cricket roller from rest to uniform motion ; you will realise that the above statements are true.

Summarising, we may say that

(*i*) Friction is a *self-adjusting' force,* increasing from zero to a maximum. So long as there is no motion, just as much of it comes into play as is necessary to prevent motion.

(*ii*) It always acts in the plane of sliding and its direction is always opposite to that in which the motion occurs or tends to occur.

(*iii*) When the surfaces are just on the point of relative motion, the force of friction is the greatest. This value is known as the **limiting friction**, and the equilibrium is called *limiting equilibrium.*

(*iv*) When motion occurs, the force exerted by friction against the motion is *less* than the value of the limiting friction and is called the **force of sliding friction,** (also **kinetic** or **dynamic** friction).

Static friction relates to the friction so long as the body does not move. **Kinetic friction** is the friction during motion, uniform or not.

I·4.4. Laws of static friction. Some general results relating to *static friction between two dry solid surfaces*, known as the 'laws of friction', are as follows. They are approximately true within limits and summarise human experience. They were formulated by Coulomb, but initiated by Leonerdo da Vinci.

(*i*) For a given pair of surfaces limiting friction F is proportional to the normal force R acting at right angles to the plane of contact between the surfaces. The ratio $F/R=\mu$ is called the **coefficient of static friction** between the two surfaces.

$$\text{Coeff. of static friction } \mu=\frac{F}{R}=\frac{\text{Limiting friction}}{\text{Normal reaction}}. \qquad \text{(I-4.4 1)}$$

(*ii*) The limiting friction between two bodies in contact depends upon the following two factors :—

(*a*) the *nature* of the substances of which the bodies are made (such as wood, iron, leather etc.) ;

(*b*) the *condition* of the surfaces (moist or dry, plane or rough, etc.)

(*iii*) The limiting friction is independent of the area of contact *so long as the normal force pressing the bodies together, remains unchanged.* Thus a brick lying flat on a floor, or standing on edge, will suffer the same frictional resistance to its motion. This is surprising, but true.

(*iv*) When motion* occurs, the force F' of sliding or kinetic friction is proportional to the normal force R between the surfaces. The ratio $F'/R=\mu_K$ is then called the **coefficient of sliding** or **kinetic** or **dynamic friction.**

$$\text{Coeff. of kinetic friction } \mu_K=\frac{\text{Force of sliding friction}}{\text{Normal reaction}} \qquad \text{(I-4.4.2)}$$

* This is a *relative motion without acceleration.*

(*v*) Within wide limits the force of sliding (or kinetic) friction is *independent of the relative velocity between the surfaces,* and their areas of contact provided the normal reaction is the same. Kinetic (or sliding) friction is by far the more important in practice. When a body is at rest, but a force of friction is operating, its value is rarely the limiting value. In practical cases we are more concerned with bodies in motion with friction opposing motion. Thus *it is the (kinetic or sliding) friction which is the one to be considered.*

Table I Coefficient of Sliding friction)

Wood on wood	0.2 to 0 5	Wood on stone	0.6 to 0.7
Leather on metal (dry)	0.56	Earth on earth	0.25 to 1.0
" " (oily)	0.15	Smooth oiled surfaces	0.03 to 0.036
Metal on metal (dry)	0 15 to 0.2	Iron on stone	0.4

Experimental Verification : Horizontal plane method. The arrangement is as represented in fig. 1-4.2. A string passing over a pulley connects the movable block *A* with a scale pan on which different weights *W* can be placed. The load on the pan is slowly increased until *A* starts moving. If *F* is the total weight of the scale pan and the load on it, and *R* is the normal reaction on *A*, then $F/R = \mu_s$, the coefficient of static friction. *R* is obviously the weight of the block *A* together with that of any additional load which may be placed on it. *Alternatively*, hang a small spring balance from the string and pull downwards, gradually pulling harder. The spring balance indicates the force exerted as shown in the first figure.

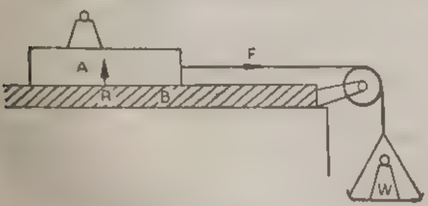


Fig. I-4.2

With a slightly smaller load than *F*, *A* will move *with a constant speed* if started by a light push. The ratio of this load F' to *R* is μ_K, the coefficient of kinetic friction ; i e. $\mu_K = F'/R$.

Start the experiment anew with an *identical* block on top of *A* and gradually go on loading the pan You would find that double the previous load is necessary to start or maintain the steady motion. The same will happen if the second block is placed behind and tied

to A for starting the motion. This shows that static friction depends on the total weight of the blocks i.e. the normal reaction.

Again, try with the block standing on its shorter side. Same load would start moving it, as when it was on the larger side, i.e. friction is independent of the area of contact.

Lastly if the table-top is very smooth, rough or wet or is of glass or wood or metal, different loads will initiate motion in each case.

I 4.5. **Rolling friction.** *Wheels change sliding friction into rolling friction.* It is far less than sliding frietion.

When one body rolls on another, the supporting surface is slightly depressed at the place of contact (fig. I-4.3), which results in the formation of a ridge (P) in front of the rolling body. The result is that the rolling body is constantly climbing a minute bill. Rolling frictlon is mainly due to this behaviour of the supporting surface; molecular forces are also believed to play an important role. The rolling body is slightly flattened at the place of contact. The deformations are temporary and both surfaces recover as soon as the contact point shifts. The effect is smaller the harder the two surfaces.

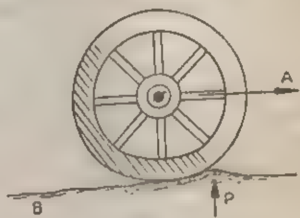


Fig. I-4.3

The laws of rolling friction are not so well known. The static value is, however, greater than the kinetic value. The coefficients of rolling friction are much smaller than those of kinetic friction. The force required to overcome rolling friction is inversely proportional to the radius of the roller.

In rotating machinery friction is reduced by using ball bearings; sliding friction is changed by this means to rolling friction. We shall elaborate it later.

The appended table gives you an idea of static or limiting friction (μ_S), dynamic or kinetic friction (μ_K) and rolling friction (μ_R) coefficients for a few pair of surfaces.

Table II (Coefficients of Different Frictions)

Materials	μ_s	μ_K	μ_R
Steel on steel	0.15	0.09	0.002
Rubber tyre on concrete	1.0 (Dry) 0.7 (Wet)	0.7 0.5	0.03
Cast iron on steel	0.15	0.10	0.004

I-4.6. Causes of Friction.

Surfaces of solids are generally uneven. Even plane polished, surfaces on suitable magnification, are found to be far from being absolutely plane. Coulomb had carried out many experiments on friction, formulated the *empirical* laws of friction given earlier and distinguished kinetic from static friction. He concluded that friction between two *solid*, *dry* surfaces arises from their roughness which produce interlocking between their projections and depressions or *hills* and *dales*. Hence polished surfaces should have less friction as is generally found. In fact *smooth surfaces are taken to be frictionless*, though that is not actually the case.

For, experiments show considerable forces of friction between a pair of *optically flat* metal planes in a vacuum e. g. a copper block on a copper surface, both very smooth ; one will not slide over the other until placed nearly verticaly. Intermolecular forces must then be holding them together and prevent sliding. These forces called weak *Van der waals forces* of attraction (§II-2.5) operate upto a separation of about 10Å ($\simeq 10^{-9}$ m) and assume importance when the surfaces are very smooth and hence close.

A clean metal plate left in air gathers a fine film of air or oxide or moisture on it. The force of friction between two such plates is less than that between a pair of them in vacuum and arise due to forces of adhesion between the molecules of two surfaces of different materials.

The main *causes* of friction are believed to be the following :

(*a*) Interlocking of surface irregularities

(*b*) Action of molecular forces that causes adhesion, cohesion and local welding

(c) Ploughing of harder projections through softer ones

(d) Action of electrostatic forces.

The principal role is believed to be played by molecular forces.

All cases of friction may be classified into three types: (1) *dry friction* where there is no separating layer of air, oxide or moisture, (2) *boundary friction* when there are films of air, oxide or moisture on the two surfaces in contact, (3) *fluid* or *floatation friction* where there is a liquid lubricant in between the two surfaces.

Minimising friction : It is imperative to lessen wastage of energy and prolong the lives of machinery by reducing wear and tear by minimising friction. Several methods are in common use.

(i) *Polishing surfaces :* Generally, polishing of contact surfaces of relatively moving parts to a high degree, lead to less friction, particularly if the materials are hard.

(ii) *Rolling friction.* Table II shows how much smaller than sliding friction is the rolling friction between same materials. Hence small steel ball bearings (fig. I-4.4) are used to separate two rotating

Fig. I-4.4

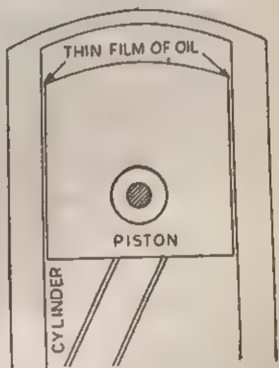


Fig. I-4.5

surfaces as in modern fans and cycle paddles. As the shaft rotates, the balls roll in the groove called a *race* preventing sliding altogether. In some cases *rollers* i.e. cylindrical bearings, replace ball bearings to better effect.

(iii) *Babbitting process.* Some alloys called *antifriction* metals, like one of Pb and Sb, diminish friction. Bearings are packed with

this alloy which gives for steel less friction than is the case with steel sliding over steel.

(*iv*) *Use of Lubricants :* **A lubricant** is a substance which, when placed in between two surfaces, minimizes the force of friction between them. It separates the two surfaces so completly that either the projections and irregularities on the two surfaces do not touch or there is a great reduction in their inter-locking. Lubrication changes sliding friction into fluid friction.

It should be remembered that the force of friction will depend on the properties of the lubricant, specially its *viscosity.*

Formerly a thin film of a soft metal like lead or iridum was spread on steel or copper to reduce the force of friction. Later, it was found that a liquid lubricant serves the purpose better. To be good as a lubricant, a liquid should have the properties (1, of viscosity (2) of adherence to solid surfaces and (3) of chemical stability These properties are possessed by mineral oils. They serve as good lubricants and maintain a multi-molecular film between two surfaces which minimizes the force of friction between the surfaces (fig. I-4.5). Vegetable oils like castor oil, have the property of forming adhering films on solid surfaces due to the presence of fatty acids in them. In one respect therefore they serve better as lubricants. Now-a-days, a mixture of mineral oils and vegetables oils is prepared as a lubricant. Sometimes a little of colloidal graphite is added to the mixture to great advantage.

In *Hovercrafts* and specialised railroad transports air layers are being used as fluid lubricants.

Again, *friction is* sought to be *increased* in many cases. Belts coupling two revolving wheels are made broad, rough and of yielding materials so as to increase the number of contact points and greater interlocking. Brake-shoes of cars are made especially rough, tyres and shoe-soles of special design and rough, yielding materials so as to have a better grip on the roads ; sand and earth are sprinkled on rain-wetted steel rails to prevent slipping of locomotive wheels or car tyres on muddy roads. Writing paper is not made very smooth or glossy.

I-4 7. A few Relevant Problems : (1) *It is easier to pull a roller than to push it.* Let the push *R* be exerted along the rod *AO* (Fig. I-4.6a) where *O* is the centre of the

axis of the roller OC represents R. Resolve OC into two components OB and OD, parallel and perpendicular to the ground respectively and call them P and Q. The effect of Q is merely to press the roller to the ground. Q is opposed by the reaction of the earth on the roller, and does not produce any motion. The component P is the effective part of which moves the roller along the ground. While pushing, the component Q presses the roller to the ground and increases the frictional resistance to the motion of the roller.

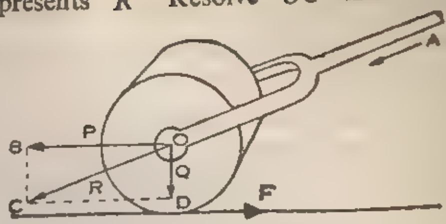


Fig. I-4.6(a)

When the force applied to the roller is a pull (fig. I-4.6b), Q is directed vertically upwards. It acts in opposition to the weight of the roller and thus reduces the total force with which the roller is pressed to the ground. This diminishes the frictional resistance to the motion. It thus becomes easier to pull a roller than to push it.

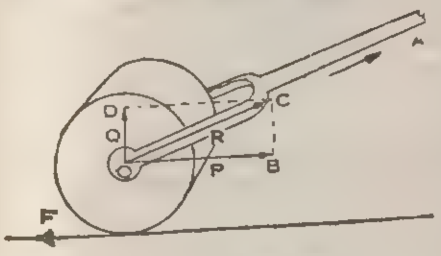


Fig. I-4.6(b)

(2) It is easier to move a barrel full of pitch by rolling it than pushing it. [*Hint* : $\mu_R < \mu_K$]. Same is the case for large heavy tyres being rolled along a road. Wheels had been devised by early man for heavy transport vehicles as he had observed that it was easier to roll logs or tree-trunks along, than either pushing or pulling it over rough ground.

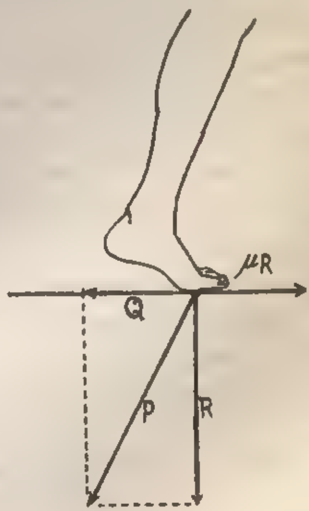


Fig. I-4.7

(3) *In walking over smooth ice short steps are necessary.* While

walking on ice (or a polished surface) we should take short steps if we are to avoid a slip and a fall. In taking a longer step we exert on the ground a force P which is more inclined to the vertical than to step were short (fig. I-4.7). Its horizontal component Q may then exceed the maximum value μR of friction. The result is that friction can no longer support the foot which slips.

(4) *Why can we not walk on a frozen lake? Explain briefly.* (S.S Q) [S. S. Q. →Samsad Specimen Questions]

Ans. In walking, the horizontal component of the reaction between the foot and the ice provides the force for forward motion Friction allows such a force to be exerted. The coefficient of friction between the foot and the ice is almost zero. Hence friction is unable to exert any horizontal force on the foot-step. In attempting to walk therefore the man will fall forward. Further, *regelation* i.e., melting of ice underfoot, by pressure into water, reduces whatever little friction there is.

(5) *How could a person completely at rest in frictionless ice cross a pond to reach the shore, rolling, jumping or kicking his feet? Explain briefly.* (S S Q)

Ans. The man is practically an isolated or closed system. No horizontal force can act on him in the absence of friction. In rolling or kicking the feet, only the internal forces in the closed system do work. According to the principle of conservation of momentum, such action cannot impart momentum to the system. So he cannot move towards the shore.

If, however, he had a sufficient number of stones at his disposal he could, by throwing them in one direction, move in the opposite direction by recoil.

(6) *Why is it difficult to walk on a slippery surface?*

Ans. In walking we push back on the ground with one foot. The ground must be capable of exerting a force equal and opposite to the horizontal component of this thrust This is ordinarly supplied by friction.

A slippery surface has a very small coefficient of friction. So it is incapable of exerting much horizontal force If we use very short steps it may be possible to walk on such a surface. But if we use longer steps, the thrust on the ground the foot exerts will be more

inclined to the vertical. Its horizontal component becomes larger. The surface may not exert an equal and opposite frictional force. The result will be a fall

Note that the last four problems are basically the same.

To increase the friction underfoot mountaineers, runners, atheletes, footballers and cricketers use spiked boots.

Example I-4.1. *A 70 kg skater on ice throws a 3 kg stone with a velocity of 8m/s in a horizontal direction. How far does he recoil if $\mu_g = 0.02$?*

Solution : We have from conservation of momentum $70 \times v = 3 \times 8$ where v is the initial velocity of recoil.

The frictional force opposing his recoil is

$$F = \mu_g R = 0.02 \times 70 \times 9.8 \text{ N}$$

$\therefore$ Retardation produced is $f = F/m = 0.02 \times 9.8$ m/s^2

If s be the distance of recoil then $0^2 = u^2 - 2fs$

or $(24/70)^2 = 2 \times 0.02 \times 9.8 \times s$ whence $s = 0.3$ m

It is particularly difficult to walk or drive over smooth ice not only because of very small $\mu_g (\simeq 2/100)$ but also because of melting of ice under pressure (see Regelation under Heat), when water produced lubricates the path, and makes it slippery.

(7) *A smaller force is necessary to keep a heavy roller moving than starting it ?* [**Hint :** Think of μ_s and μ_g.]

(8) *To move a tall block on a rough table it must be pushed at points nearer to the table-top.* If that is not done, the block will topple over instead of moving forward. It is a question of **Stability due to friction.** Consider a rectangular block ABCD resting on a rough horizontal plane (fig. I-4.8). Let a horizontal force P be applied at K. If μ is the coefficient of static friction between the surfaces and W the weight of the block, the block will not slide until P exceeds μW. Before P reaches this value the block may topple. The turning moment of P about B is $P \times$ KB. This is opposed by the moment $W \times$ EB. So long as $W \times$ EB is greater than $P \times$ KB, the body will be in stable equilibrium. But if $P \times$ KB exceeds $W \times EB$, the body topples over.

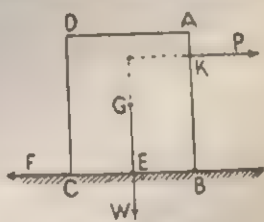


Fig. I-4.8

The limiting condition for toppling is $P \times$ KB $= W \times$ EB i.e. P/W

= EB/KB. The limiting condition for sliding is $P/W = \mu$. The body will slide if $P > \mu W$, and topple if $P > W \times$ EB/KB, but less than μW.

We return to the problem in the Chap I-7. when discussing stability of a regular body on a rough incline.

(9 *Multiplication of Pull by Friction.* The **Rope Brake.** A man pulling at a long rope wound a few times around a stout wide cylinder can hold even a ship fastened at its other end. In the chapter on Hydrostatics you will come across a device called the Hydraulic Press where a small force applied at one end can develope force enough to crush sackfuls of oil-seeds or chunks of stone.

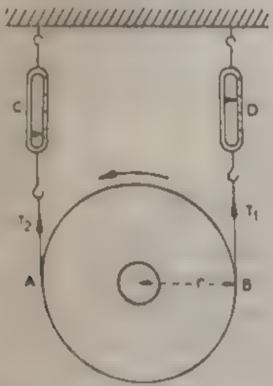


Fig. 1-49

If a pull T_1 is applied at the end of a wide band coiled a few times round a cylinder it is found to balance much larger pull T_2 at the other end, the ratio of the two tensions (T_2/T_1) being $e^{\mu\theta}$ where μ is the coefficient of friction between the surfaces of the band and the cylinder, θ the *angle of lap in radians* of the band, round the cylinder and $e = 2.7128$ the exponential function. Clearly larger the number of turns greater becomes θ, and greater the ratio T_2/T_1.

1·4 8. Rest and Motion on a Rough Horrizontal Plane :

A Rest So long as the force applied parallel to the surface of contact does not exceed the *limiting* frictional force, the body will not move. At the limiting condition the applied force $P = F_s = \mu_s N$ where μ_s is the coefficient of friction between the two surfaces and N the normal reaction force between the two surfaces.

Ex 1-4 2 *A man holds a book weighing 2 lbs between his two palms and just prevents its falling by pressing the palms each with a force of 5 lbs. Find μ between the book and the palm.* [H. S. '81]

Solution : The normal reaction on each face of the book is 5 lbs-wt and the frictional force at right angles to it must be $5\mu_s$ lbs-wt. Since this balances the weight of the book, we have

$$5\mu_s \text{ lbs-wt} = 2 \text{ lbs-wt or } \mu_s = 0{\cdot}4$$

Problem A man lifts a 16 kg can of mustard oil by pressing his palms against its vertical sides. Find the force applied by each of his palms if $\mu_s = 0{\cdot}25$. (**Ans.** 64 lbs-wt)

Ex I-4.3 *An engine driver reduces the speed of a train in 3.3s from 47.5 km/h to 30 km/h What must be the limiting coefficient of friction between a suitcase and the rack so as just to prevent slipping ?*

Solution : The retardation of the train

$$f_r = \frac{(47{\cdot}5 - 30) \times 1000/60 \times 60}{3\,3} \text{ m/s}^2$$

$\therefore$ The force on the suitcase $= mf_r$

The normal reaction between the suitcase and the rack $= \mu_s mg$.

For just preventing slipping, $mf_r = \mu_s mg$

$$\mu_s = f_r/g = \frac{175}{36 \times 3.3 \times 9.8} = 0\,15$$

(See Relative acceleration fig. I-2,25)

B. Motion : To produce motion on a rough surface the applied force P must exceed the force of *kinetic* friction F_k and the effective force parallel to the surface of contact producing motion is

$$P - F_k = ma$$

$$\text{or } P = \mu_k N + ma = \mu_k mg + ma = m(\mu_k g + a) \qquad (I\text{-}4.8.1)$$

Ex. I-4.4. *A 50 kg block rests on a horizontal table A minimum force of 44 kg wt. acting at 60° through a rope above the horizontal just starts the block which may be kept moving uniformly with 36 kg. wt of force along the same direction. Find μ_s, μ_k and the frictional forces ; also the frictional force when the pull is 12 kg wt along the rope.*

Solution : The applied forces parallel to the direction of motion are $P_s \cos 60°$ and $P_k \cos 60°$ i.e. 22 kg-wt. and 18 kg-wt. respectively

$\therefore$ $\mu_s \times 50 = 22$ kg wt. $\therefore$ $\mu_s = 0.44$

$\mu_k \times 50 = 18$ kg. wt. $\therefore$ $\mu_k = 0.36$

Frictional forces are 22 kg.-wt, 18 kg.-wt and 6 kg.-wt.

Ex I-4 5 *A chain of length l is on a table with a portion l' of it hanging from the edge of the table Find the maximum value for l' so that the chain is on the point of sliding.*

Solution : If ρ be the density of the material of the chain then the weight of its hanging portion is $\alpha l'\rho g$ where α is its cross-section. Now the limiting frictional force between the chain and the table just prevents motion The weight of the portion of the chain on the table is $(l - l')\alpha\rho g$ So for terminal equilibrium

$$\mu(l - l')\alpha\rho g = \alpha l'\rho g$$

$$\therefore \quad l' = \mu l/(\mu + 1)$$

Problem : A one-meter long iron chain lies partly on a stone table with 28.6 cm. length hanging from the end, just on the point of slipping. Find μ between iron and stone. **Ans** 0.4 (nearly)

Ex. I-4 6. *The take-off velocity of a plane of mass 10,000 kg. is 80 km/h and it requires 100 m of the runway to gain that speed. What force must the engine develope to achieve this if μ_k between the runway and the tyres is 0.2 ?* [I.I.T. '77]

Solution : Final velocity $= 80$ km/h $= (200/9)$ m/s

Acceleration developed $= \frac{1}{2}(v^2 - u^2)/s = \frac{1}{2}.\frac{(200/9)^2}{100} = \frac{200}{81}$ m/s^2

Force required to build up this acceleration

$$F_1 = ma = 10^4 \times 200/81 \text{ N}$$

Force to overcome friction $F_2 = \mu mg = 0.2 \times 10^4 \times 9.8$

$\therefore$ Total force $= F_1 + F_2 = 10^4(\frac{200}{81} + 1.96) = 10^4 \times 4.43$ N.

Ex. I 4.7. *A train of mass M is travelling on a level line ; the last carriage of mass m becoming uncoupled, the driver notices it after moving a distance l and then shuts off steam. Show that when both the parts come to rest the separation between them is $Ml/(M+m)$, if the resistance due to line and air friction be uniform and proportional to the weight, the pull of the engine being constant.* [Cambridge]

Solution : Let the train be moving with a velocity u before uncoupling. Since it is uniform, the pull of the engine = Opposing frictional force on the train By question this frictional force $\propto$ its weight i.e $F = KMg$. The pull exerted by the engine is also the same KMg.

Now this pull remains unchanged for a distance l, for the driver is unaware of uncoupling but the mass of the train has diminished to $(M-m)$. Again because of uncoupling, the frictional force reduces to $K(M-m)g$ as the mass of the train has diminished. So the unbalanced force exerted by the engine on the truncated train is

$$KMg - K(M-m)g = Kmg$$

and hence the acceleration of the train $a = Kmg/(M-m)$

Next, if v be the final velocity of the truncated train in covering the distance l then we have,

$$v^2 = u^2 + 2al = u^2 + \frac{2Kmg}{M-m}. l$$

The frictional force on the detached bogey is Kmg and its deceleration will be

$$a_1 = Kmg/m = Kg$$

Let it stop after covering s_1 ; then $u^2 = 2a_1s_1 = 2Kgs_1$

As the driver shuts off steam after covering a distance l, the deceleration of the truncated train would be

$$a_2 = \frac{K(M-m)g}{(M-m)} = Kg$$

If the train stops after covering s_2, then $v^2 = 2a_2 s_2 = 2Kgs_2$

Now $v^2 = u^2 + \frac{2Kmgl}{M-m}$ or $2Kgs_2 = 2Kgs_1 + 2K\frac{mg}{M-m}.l$

$\therefore \quad s_2 = s_1 + \frac{m.l}{(M-m)}$ or $s_2 - s_1 = \frac{ml}{M-m}$

The separation between the detached bogey and the truncated train with therefore be

$$s = l + (s_2 - s_1) = l + \frac{ml}{M-m} = \frac{Ml}{M-m}$$

Ex. 1-4 8. *A stick presses on a floor with a force equal to the wt of 2 kg at an angle of 20° to the normal to the floor. If μ between the floor and stick be 0.4, will the stick slip ? Cos 20° = 0.9397*

Solution : (*a*) The force exerted by the stick on the ground is 2×9.8 N. Its vertical component (V) will be $2\times9.8 \cos 20° = 19.6 \times 0.9397$ N. The frictional force $F = \mu_s V = 0.4 \times 19.6 \times 0.9397 = 7.37$ N will be acting along the floor.

Now the horizontal componet of the applied force is

$$H = 2\times9.8\times\sin 20° = 2\times9.8\times\sqrt{1-(0.9397)^2}$$
$$= 6.70 \text{ N}$$

Frictional force being greater, the stick will *not* slip.

(*b*) The conclusion can be confirmed from a consideration of angle of friction β. See below I-4.9B. Now from Eq. I-4.9.3

$\tan \beta = \mu = 0\ 4.$ Then $\beta = 22.8°$ (From tables)

Since the angle of application is 20°, the stick will *not* slip.

C. Measurement of the coefficient of friction

Inclined plane method. In this method a block of one material (A ; fig. I-4.10) is placed on an inclined plane of the other material. The inclination of the plane with the horizontal is gradually increased until A starts sliding down the plane. The angle of inclination α is measured. From Eq. I-4.9.1, $\tan \alpha = \mu_s$ the coefficient of static friction between D and B.

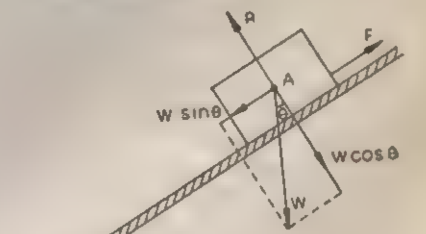


Fig. I-4.10

At a slightly smaller angle than the above, A will slide down the plane *with a uniform speed* if started from rest by a gentle push. If this angle is θ, then $\tan \theta = \mu_k$ the coefficient of sliding friction.

The component $W \sin\theta$ of the weight parallel to the plane just overcomes the frictional force during the downward motion without acceleration.

To find μ_s or μ_k several observations have to be made for each and the average value taken.

In fig. I-4.2 we have already learnt how to measure them on a horizontal plane.

I-4 9. Equlibrium on a Rough Incline. Unlike the horizontal plane a force arising from ever-present **gravity,** its component along the incline, tends to move the body and friction opposes it. We introduce two new concepts to deal with this motion.

A. **Angle of repose and angle of friction.** Let a body A of weight W (fig I-4.10) rest on an inclined plane B whose inclination θ to the horizontal can be changed at will. If the weight W be resolved along and normal to the inclined plane then, for equilibrium of the body, we must have normal reaction $R = W \cos\theta$, and $F = W \sin\theta$, where F stands for the force of friction.

Let the inclination of the plane be gradually increased so that the body is just on the point of sliding Let the inclination now be $\theta = \alpha$. In this limiting condition, we get the coefficient of static friction,

$$\mu = \frac{F}{R} = \frac{W \sin\alpha}{W \cos\alpha} = \tan\alpha. \qquad \text{(I-4.9.1)}$$

The angle α is called the **angle of repose.** It may be defined as the maximum angle of inclination that a plane may have before a body on it begins to slide under the action of its weight.

B. **Angle of friction.** For a body in limiting equilibrium, if the friction and the normal reaction be compounded into a resultant force (S; fig. I-4.11) the angle which this force makes with the normal reaction is called the angle of friction and the resultant force is called the **resultant reaction.**

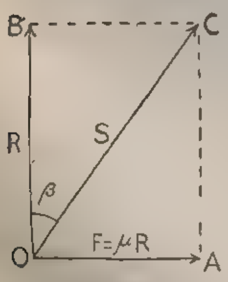


Fig I-4.11

In fig. I-4.11 let O be the point of contact of two bodies and let OA and OB represent the limiting friction $F = \mu R$ and the normal reaction R respectively. Let OC represent the resultant reaction S.

Then the angle $BOC=\beta$ is the angle of friction. From the figure we see that

$$S=\sqrt{\mu^2R^2+R^2}=R\sqrt{1+\mu^2}. \tag{I-4.9.2}$$

and $$\tan\beta=\mu R/R=\mu. \tag{I-4.9.3}$$

Hence the coefficient of static friction is equal to the tangent of the angle of friction. From Eq. I-4.9.1 and I-4.9.3 it will be seen *the angle of friction and the angle of repose are equal.*

It follows from this equation that the greatest angle which the resultant reaction can make with the normal reaction, i.e with the normal at the point of contact, is $\tan^{-1}\mu$, as we see below.

C **Cone of Friction.** Let two bodies be in contact Draw a cone (fig. I-4 12) with their common normal ON as axis, the point O of contact as the vertex, and $\beta=\tan^{-1}\mu$ as the semi-vertical angle. Then the resultant reaction will lie within or on the surface of this cone, called the *cone of friction.*

No force P whose line of action lies within the cone of friction can produce sliding motion, no matter what the magnitude of P may be. As may be seen from fig. I-4.12 the tangential component (parallel to the surface) of such a force is less than the limiting friction. Hence no sliding motion can occur. See Ex. I-4.8 above.

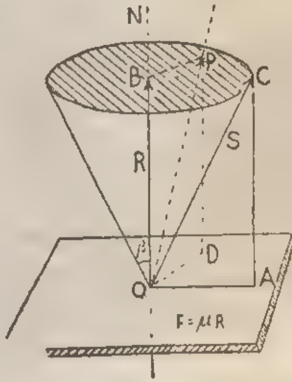


Fig. I-4.12

Ex. I-4.9. *A body just starts sliding down an incline of 15° and keeps sliding without acceleration when the tilt is lowered to 12°. Find μ_s and μ_k.*

Solution : $\mu_s=\tan\alpha=\tan 15°=0.2679\approx 0{\cdot}27$
$\mu_k=\tan\alpha'=\tan 12°=0.2126\approx 0{\cdot}21$

I-4.10. A Force down an inclined plane. If in fig. I-4.13 the angle of inclination θ of the plane, is greater than the angle of repose *the body will slide down the plane.* Let μ_s be the coefficient of static friction and μ_k the coefficient of kinetic friction. Then $F=W\cos\theta$ is the normal reaction, $\mu_s R$ = limiting static friction and $\mu_k R$ = limiting kinetic friction.

The body starts sliding when $W \sin \theta > \mu_s R$. After motion starts friction reduces to $\mu_k R$ from $\mu_s R$. The resultant force down the plane is therefore

$$W \sin \theta - \mu_k R = W(\sin \theta - \mu_k \cos \theta) \quad \text{(I-4.10.1)}$$

and acceleration $a = (W/m)(\sin \theta - \mu_k \cos \theta)$ (I-4.10.2)

B. Equilibrium with the angle of inclination exceeding the Angle of Repose.

Obviously a force up the incline must balance the resultant force downwards as calculated above. This force P parallel to and up the

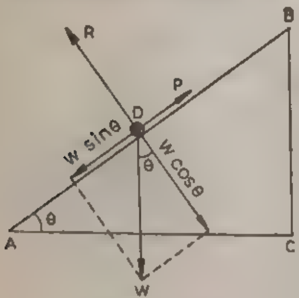


Fig- I-4.13

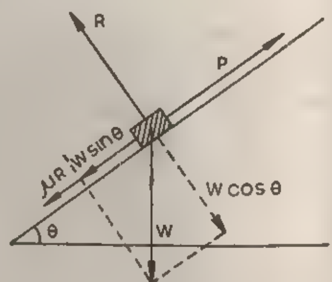


Fig. I-4.14

incline may (*i*) just prevent the body sliding down or (*ii*) just start pulling it up the incline.

(*i*) Referring to Fig I-4.13. we shall have for just preventing slipping down

$$W \sin \theta = P + \mu_s R \text{ or } P = W \sin \theta - \mu_s W \cos \theta \quad \text{(I-4.10.3)}$$

If now $\mu_s = \tan \alpha$ where α is the angle of repose then

$$P = W(\sin \theta - \tan \alpha . \cos \theta) = W\left(\sin \theta - \frac{\sin \alpha . \cos \theta}{\cos \alpha}\right)$$

$$= W\frac{\sin(\theta - \alpha)}{\cos \alpha} \quad \text{(I-4.10.4)}$$

(*ii*) Refer to fig I-4.14. When P is large enough just able to start the body *up* the incline we shall have

$$P = \mu_s R + W \sin \theta = \mu_s W \cos \theta + W \sin \theta$$
$$= W(\tan \alpha \cos \theta + \sin \theta)$$
$$= W \sin(\theta + \alpha)/\cos \alpha \quad \text{(I-4.10.5)}$$

Hence to keep the body in equlibrium on the plane the force P must lie between the values $W \sin(\theta \pm \alpha)/\cos \alpha$

C. **Minimum Force required to pull a body up a rough incline.**

Let a force P be applied up the plane at an angle λ to it. Then resolving P parallel and perpendicular to the plane we find

$$P\cos\lambda = W\sin\theta + \mu_s R \quad \text{and} \quad R = W\cos\theta - P\sin\lambda$$

Then substituting the value of R in the first expression and re-arranging we get

$$P(\cos\lambda + \mu_s \sin\lambda)$$
$$= W(\sin\theta + \mu_s\cos\theta)$$

or $$\frac{P(\cos\lambda\cos\alpha + \sin\lambda\sin\alpha)}{\cos\alpha}$$

for $\mu_s = \tan\alpha$

$$= \frac{W(\sin\theta\cos\alpha + \cos\theta\sin\alpha)}{\cos\alpha}$$

or $$P = W\frac{\sin(\theta+\alpha)}{\cos(\lambda-\alpha)} \quad (1\text{-}4.10.6)$$

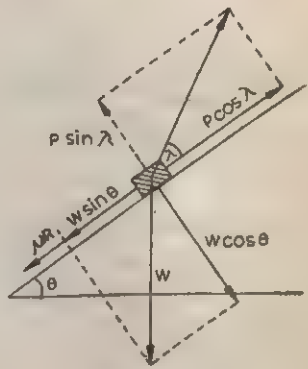


Fig. I-4.15

P will be a minimum when $\cos(\lambda-\alpha)$ is a maximum. This maximum value is unity when $\lambda=\alpha$. Hence *the force required to move the body up the plane will be least when it is applied in a direction making with the inclined plane an angle equal to the angle of friction.*

Ex. I-4.10. *A block of mass 2 kg slides on an incline of 30° where μ is $\sqrt{3}/2$ Show that the block will not slids due to its weight only. Find (i) the force needed to move the block down without acceleration and (ii) the force required to move the block up without acceleration.* [I.I.T. '76]

Solution : Let θ be the maximum angle of inclination for equilibrium. Then the normal reaction $R = 2\times 9.8\cos\theta$ and the frictional force would be $\sqrt{\frac{3}{2}}\, R = 2\times 9.8\sin\theta$

$\therefore\ \tan\theta = \sqrt{3/2} = 1.225 \qquad \therefore\ \theta = 50.8°$ (i.e. $>45°$)

But the given angle is only 30°. So the block will not slide down by itself.

(i) This is uniform motion i.e. $P + W\sin\theta = \mu R = \mu W\cos\theta$

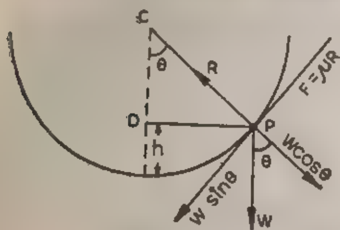
$$\text{or}\quad P = \sqrt{\frac{3}{2}}\cdot 2\times 9.8\times\frac{\sqrt{3}}{2} - 2\times 9.8\times\tfrac{1}{2} = 10.9\ \text{N}$$

(ii) This is also uniform motion where $P-\mu R = W\sin\theta$

$$\text{or } P=\sqrt{\frac{3}{2}}\cdot 2\times 9.8\times\frac{\sqrt{3}}{2}+2\times 9.8\times\tfrac{1}{2}=30.5 \text{ N}$$

Ex. I-4.11. *How high a particle may be inside a hollow hemisphere of radius r if μ_s be $1/\sqrt{3}$.*

Solution : Let the highest point reached, be at a height h at the point P inside the sphere, for the particle of weight W. The normal reaction R acts radially along PC and this equals the component $\cos\theta$ W of the weight. Its other component $W\sin\theta$ acting downwards tangentially to the particle, is neutralised by the frictional force $F=\mu_s R$.



For equilibrium $F=\mu_s R=\mu_s W\cos\theta = W\sin\theta$

$\therefore \tan\theta=\mu_s=1/\sqrt{3}$ or $\theta=30°$

From the ΔPDC we find

$$h=r-r\cos\theta=r(1-\cos\theta)=r\,\frac{2-\sqrt{3}}{2}=0.134\ r$$

I-4.11. Internal friction or Viscosity The property of a liquid by virtue of which it opposes a relative motion of its layers is called *viscosity.* When two beakers, one containing alcohol and the other oil, are tilted from side to side, much less mobility is observed in oil than in alcohol. Oil offers more resistance to the relative motion of its layers and is said to be the more viscous of the two liquids.

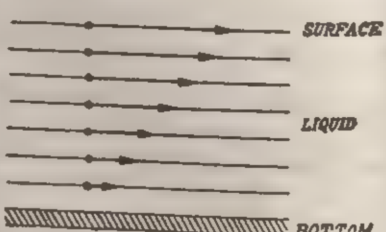


Fig. I-4.16

When a liquid flows slowly over a flat surface, the layer of liquid in contact with the surface remains stationary because of adhesion. The next upper layer moves slowly over the first (fig. I-4.16), the third layer over the second and so on. The speed

of each layer increases with its distance from the solid surface Between two adjacent layers, *cohesion** brings tangential forces into play, the faster layer tending to accelerate the slower and the slower layer tending to retard the faster. These tangential forces endow the liquid with the property called *viscosity.* They depend on the nature of the liquid and also on the area and the *velocity gradient* of the layers, i.e., the relative velocity of two layers unit distance apart.

The viscous forces do not act so long as the liquid is at rest. They come into play only when there is a relative motion between the layers. **The force is thus of the nature of friction, and viscosity is sometimes called internal friction** When water in a bowl has been churned, it is brought to rest after a time by the internal friction between the layers of water.

Gases also exhibit internal friction or viscosity. A raindrop falling through air is retarded by the viscocity of air. The viscous 'drag' increases with the velocity of the drop. After a time the drag reaches a value equal to the force of gravity on the drop. When this condition is reached the velocity of the drop increase no further. For the rest of its path it continues to fall with a constant velocity, called the *terminal velocity.*

I-4.12 Coefficient of viscosity. When adjacent layers of a liquid are in relative motion, tangential forces operate between them tending to accelerate the slower layer and retard the faster. If A is the area of such a layer, v is the relative velocity between two layers distance x apart, and F is the tangential force acting on A, then F is proportional both to A and to v/x. We may, therefore, write

$$F \propto Av/x.$$

---

* A molecule which moves relative to other molecules is successively linked with and parted from the molecules with which it comes into contact in the course of its motion.

The constant of proportionality in this relation is called the coefficient of viscosity η of the liquid. Hence

$$F=\frac{\eta A v}{x} \tag{I-4-12.1}$$

$$\text{or} \quad \eta=\frac{F}{A}\cdot\frac{1}{v/x} \tag{I-4 12.2}$$

The quantity F/A is of the nature of a shearing stress. The quantity v/x is called the *velocity gradient.*

We may, therefore, say that *the coefficient of viscosity of a liquid is the tangential force per unit area per unit velocity gradient.*

Using the notation of calculus we may write dv/dx for v/x. Then

$$\frac{F}{A}=\eta\frac{dv}{dx}. \tag{I-4.12.3}$$

The dimensions of η is $[ML^{-1}T^{-1}]$.

The cgs unit of the coefficient of viscosity is called the **poise** * A coefficient of viscosity of 1 poise means that 1 dyne of force is required to maintain a tangential velocity difference of 1 cm per sec between two surfaces each a sq cm in area and 1 cm apart.

* After poiseuille, a french physician, who investigated the bood circulation through capillary veins.

I-5

UNIFORM CIRCULAR MOTION

I-5.1. **Uniform motion in a circle :** The motion of a particle with uniform angular velocity in a circle corresponds to the motion of a particle in a straight line with uniform speed (that is rectilinear motion with uniform velocity) This is the simplest case of rotation.

Consider a particle P (fig. I-5.1) moving in a circle of radius r. As it moves, the line drawn from the centre to the particle sweeps out gradually increasing angles. (This line OP is the *radius vector.*) Let this angle be measured from some standard position of the radius vector (say, from OA on the line OAx). Angle θ which the radius vector makes with the standard line is called the **angular displacement** of the particle. It is generally measured in radians.

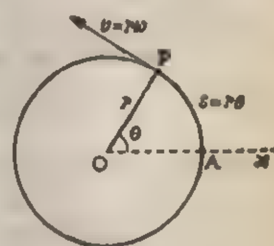


Fig. I-5.1

The **angular velocity** of the particle is the time rate of change of its angular displacement, or the angle turned through by it in unit time. If θ_1 and θ_2 are the angular displacements at instants t_1 and t_2 respectively, then the *average* angular velocity

$$\bar{\omega}=\frac{\theta_2-\theta_1}{t_2-t_1} \qquad \text{(I-5.1 1a)}$$

Angular velocity is expressed in **radians per second** It is treated as *a vector along the rotation axis.* The magnitude of angular velocity is **angular speed** So long as the direction of the axis remains fixed, it does not matter whether we call ω, the angular velocity or the angular speed. Like linear velocity or speed, ω *may be constant or variable.* When t_2-t_1 is very very small, ω is called the *instantaneous angular velocity.* $\omega = \delta t_{\delta t\to 0}\frac{\delta\theta}{\delta t}=\frac{d\theta}{dt}$ (I-5.1b)

The number of revolutions executed in one second is called the **frequency** of rotation of the particle. The time required to execute

one revolution is called the **period** of rotation. If the angular speed is ω rad/s, the period

$$T = 2\pi/\omega \text{ seconds.} \qquad \text{(I-5 2.2)}$$

The number of revolutions executed in one second, i.e., **frequency**

$$n = 1/T = \omega/2\pi \text{ per second.} \qquad \text{(I-5.3.3)}$$

Ex. I-5 1. *A particle executes 200 revolutions in 5 seconds. Find (i) its frequency (ii) angular speed (iii) period and (iv) the time required on turn through 90°.*

Solution: Frequency n = number of revolutions per second (rps in brief)

$$= \frac{200}{5s} = 40 \text{ per second or } 40\ s^{-1} \text{ or } 40/s.$$

[The unit of frequency has been named the **hertz**, symbol H_z.]

Angular speed $\omega = 2\pi n = 80\pi$ radians/second (or 80π rad/s). ω is also called **angular frequency**.

Period $T = \frac{1}{n} = \frac{1}{40 \text{ per second}} = \frac{1}{40}$ second $= 0{\cdot}024$ s

Time to turn through $90° = \dfrac{\text{angle turned through}}{\text{angular speed}}$

$$= \frac{90°}{80\pi \text{ radians/second}} = \frac{\pi/2 \text{ rad}}{80\pi \text{ rad/s}} = \frac{1}{160}\text{s}.$$

Relation between angular and linear speed. Suppose the particle (fig. I-5.1) turns through an angle θ radians in t seconds. Then its angular speed is $\omega = \theta/t$ (rad/s). In this interval the particle describes an arc AP (Fig. I-5.1) of length s.

Since angle in radians $= \dfrac{\text{length of arc}}{\text{radius}}$, we have $\theta = s/r$ or $s = r\theta$.

Dividing by t, we get $s/t = r.\theta/t$

or *linear speed = radius × angular speed* (in radians per second)

In symbols, $v = r\omega$ (I-5.1.4)

In a curved trajectory $S = r\theta$ or $ds = r.\ d\theta + \theta.dr$

$$\text{or} \quad \frac{ds}{dt} = v = r\frac{d\theta}{dt} + \theta.\frac{dr}{dt} \qquad \text{(I 5.1.5)}$$

i.e. velocity has a second part involving change in the radius vector. This does happen in planetary motion along *elliptic* orbits (§II-1.15). In circular motion r is const and hence eqn (I-5.1.4), holds.

Ex I-5 2. *A stone is whirled at the end of a string 1 metre long and describes 6 revolutions in 4 seconds. Find its angular and linear speeds.*

Solution : In 6 revolutions the stone describes $6 \times 2\pi = 12\pi$ radians.

$$\therefore \quad \text{Angular speed} = \frac{\text{angle described}}{\text{time taken in doing so}} = \frac{12\pi \text{ rad}}{4 \text{ s}} = 3\pi \text{ rad/s.}$$

$$\text{Linear speed} = \text{radius} \times \text{angular speed in radians per sec.}$$
$$= 1 \text{ metre} \times 3\pi \text{ rad/s} = 300\pi \text{cm/s} = 942 \text{ cm/s.}$$

I-5.2. Centripetal Force. According to Newton's first law of motion, a moving particle will, due to inertia, continue to move in a straight line unless acted upon by some external force. This property is the *directional inertia.*

If we want a stone tied to a string to describe a circle, such as ABCD (Fig. I-5.2.), with a constant speed, we must apply some force here Mg on it to keep it on the circle i e. to overcome directional inertia. At every point of its path the stone tends to move along the tangent to the path, *i.e.*, along AA′ at A, along BB′ at B, etc. If you cut down the string the stone will fly off tangentially as you can verify yourself. *Since the speed is to remain constant, the force applied cannot have any component in the direction of motion of the particle ;* if it has, it will then produce an acceleration in the direction of motion and thus change the speed. The force must therefore everywhere be perpendicular to the path, *i.e., it must be directed towards the centre of the circle,* along the radius ; for radii are normals at every point on the circumference.

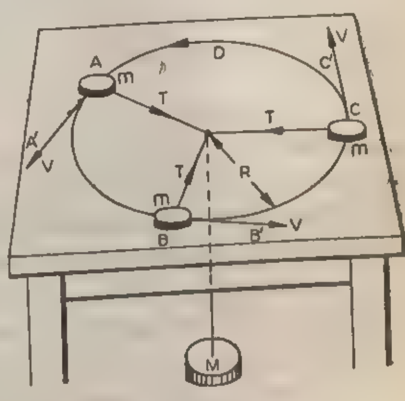


Fig I-5.2

When a body moves in a circle, *the inward force which, acting towards the centre of the circle, keeps the body moving along the circle is called the* **centripetal centri-center, petes-seeking force.** The agent here, your finger or the hanging load M which exerts the centripetal force is acted on by reactional force called *centrifugal (fugis-*flying away) *force* in engineering practice. But correctly, it should be called the **centrifugal reaction.** The centrifugal reaction

is thus *equal and opposite to the centripetal force, acts on the agent which restrains the moving body, and is directed outwards from the centre of the circle. It does not act on the body.* We shall discuss the correct meaning of *centrifugal force* in Sec. I-5.4,

Magnitude of the centripetal force. Suppose a particle of mass m moves round in a circle with a constant speed v and is at the point A (Fig I-5.3.) at a given moment. Consider a *very short* interval of time t. If there were no force acting towards the centre, the particle would move a distance AB$=vt$ tangentially in time t, which is very short. But, because of the centripetal force F there will be an acceleration $a=F/m$ normal to the track (*i.e.*, to AB). Due to this acceleration the particle would move a distance $\frac{1}{2}at^2$ perpendicular to AB in time t, and be brought to a point D on the circle where BD is parallel to AO. Drawing DE parallel to AB, we have from geometry

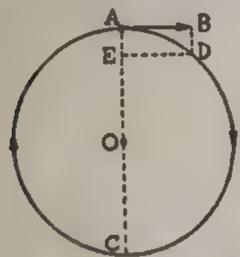


Fig. I-5.3

$DE^2=AE.EC$, where AC is the diameter

$$=BD\,EC=\tfrac{1}{2}at^2\,(2r-\tfrac{1}{2}at^2)=rat^2,$$

$[(\tfrac{1}{2})at^2]^2$ may be neglected in comparison with $2r$, as t is small.

Thus $DE^2=AB^2=v^2t^2=rat^2$ or $a=v^2/r$.

∴ **Centripetal or central) acceleration** $=\mathbf{v^2/r=r\omega^2}$ (I-5.2.1)

and **Centripetal force** $\mathbf{F=mv^2/r=m\omega^2r}$ (I-5.2.2)

Ex I-5.3. *A 70 g stone is being whirled by a 20 cm. long string, 10 times a second with uniform speed. Find the acting centripetal force.*

Solution : $F=mv^2/r=m\omega^2r=m.(2\pi n)^2r$

$$=70\times4\times(22/7)^2\times10^2\times20=55\,22\text{ N}$$

Alternative proof. The moving particle has a velocity v along the local tangent to the circle. Its speed does not change with time, but the direction of motion changes continuously at the uniform rate of 2π radians in T seconds, that is, at the rate of $2\pi/T=\omega=v/r$. Acceleration means rate of change of velocity, that is, of both speed and direction. Here speed does not change; only the direction changes at the rate v/r Therefore the acceleration is $v\,v/r=v^2/r$. We

have seen earlier that the force, hence the accleration, is directed towa·ds the centre. Thus the centripetal acceleration is v^2/r and centripetal force is $mv^2/r = m\omega r$.

As ω is the angular velocity of the particle, T its time period and n its frequency, we have

$$v = \omega r = 2\pi r/T = 2\pi nr. \qquad \text{(I-5.2.3)}$$

When necessary we may substitute any of these values in Eq.s I-5.2.1 or I-5 2 2.

The above analysis shows that if we want to make a particle of mass m and speed v move round in a circle of radius r, we must apply to it a force $F = mv^2/r$ *perpendicular to v.*

To deduce the expression for centripetal acceleration with the help of vector diagram

Let a particle be moving with uniform angular velocity ω in a circle of radius r (fig. I-5.4). The vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ represent the instantaneous velocities at P and Q. The vectors have the same length but different directions. The average acceleration $\mathbf{a}$ is the *vector change in velocity* between P and Q, divided by the time taken. The vector change in velocity means the *vector difference* $\mathbf{v}_2 - \mathbf{v}_1$. Let us write is at δv.

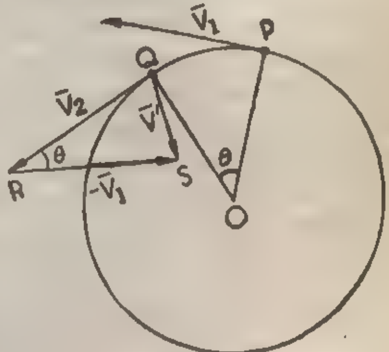


Fig. I-5.4

Now, $v_2 - v_1$ is the vector *sum* of v_1 and $-v_1$. $-v_1$ is a vector equal and opposite to v_1. To get δv, we lay down $-v_1$ from the end point of v_2, and join the initial point Q of v_2 to the end point S of $-v_1$. The line segment QS represents the vector difference δv.

If the particle took a short time δt in moving from P to Q then the average vector acceleration between P and Q is

$$\bar{a} = \delta v/\delta t.$$

It points inwards in the same direction as δv. Let v be the magnitude of either v_2 or v_1, and $\delta\theta$ the angle between OP and OQ,

that is, between v_2 and $-v_1$. Then from the isosceles triangle QRS, QS $= 2$QR $\sin \frac{1}{2}\delta\theta$ If $\delta\theta$ is very small we can write $\sin \frac{1}{2}\delta\theta = \frac{1}{2}\delta\theta$. (See eqn. 0-2.7.1) So, when P and Q are very close together, the magnitude of δv is

$$\delta v = QS = 2QR.\tfrac{1}{2}\delta\theta = v\,\delta\theta.$$

When P and Q are infinitely close together, both $\delta\theta$ and δt are infinitesimal We then get the instantaneous acceleration by dividing both sides by δt This acceleration is directed towards the centre O of the circle. Its magnitude is

$$a = \frac{\delta v}{\delta t} = v.\frac{\delta\theta}{\delta t} = v\omega$$

Or, centripetal acceleration = linear speed × angular velocity.

[Since $\omega = v/r$, we can also write $a = v^2/r$. The instantaneous acceleration points towards the centre O and has magnitude $a = v^2/r = v\omega$ The corresponding (centripetal) force $F = mv^2/r = mv\omega$, where m is the mass of the moving particle. It has the direction of the centripetal acceleration, that is, the centripetal force is directed towards the centre of the circle.]

Since the moving particle has no component of displacement along the direction of the force, the centripetal force is a **no work force**, for work done by a force is measured by $F.s \cos\theta$, F representing the force and $s \cos\theta$ the component of displacement in its direction

Ex 1 5 4. *A small body of mass 200 g revolves in a circle on a horizontal frictionless surface, being attached by a string, 20 cm long, to a pin in the surface. Find the tension in the cord if the body executes 90 revolutions per minute.*

Solution : Angular velocity $\omega = 2\pi n = 2\pi\, 90 \div 60)$ rad/s. The tension in the string supplies the centripetal force P.

$$\therefore\quad F = m\omega^2 r = 200\text{g} \times (3\pi)^2\ \text{s}^{-2} \times 20\ \text{cm}$$
$$= 9\pi^2 \times 4000\ \text{g cm s}^{-2} = 3\ 55 \times 10^5\ \text{dyn}.$$

Problem (1) A particle is on the top of a smooth sphere of radius r. What is the smallest horizontal velocity that should be given to it so that it may leave the sphere without sliding down it ?

(See Eqn. I-5.5 1). [Ans $v = \sqrt{gr}$]

(2 A body of mass m lies on a horizontal rotating table at a distance r from the rotation axis. The coefficient of friction between the body and the table is μ. What is the magnitude of the force of friction if the table is rotating at n rps ? At what angular velocity will the body begin to slide ? [Ans. μmg ; $\omega = \sqrt{\mu g/r}$]

(*Hint :* The force of friction provides the centripetal force).

[**Ans.** $\omega = \sqrt{\mu g/r}$.]

I-5.3 Centripetal acceleration : We know that acceleration is a vector and the time rate of change of another vector, velocity. A vector involves a magnitude and a direction With time, *only magnitude* of velocity may change (when we say speed is changing) or *only direction* may change ; in both cases there is acceleration. Symbolically, as we have seen before that

$$\mathbf{a} = \frac{d}{dt}(\mathbf{V}) = \frac{d}{dt}[(v)\mathbf{r}] = \mathbf{r}\frac{dv}{dt} + v\frac{d\mathbf{r}}{dt} \qquad \text{(I-5.3.1)}$$

Both the terms on the right hand side individually represent an acceleration—the first term, speed changing without changing direction as exemplified by acceleration due to gravity (g), and the second representing uniform speed with changing direction—as for centripetal acceleration v^2/r.

The most *significant point* about centripetal acceleration is that it constantly accelerates the particle towards the centre normally (*i.e.*, at right angles to its path) and yet the particle *maintains its distance from the centre unchanged* ; it does never approach the center. This happens also for planets and the satellites. This is because the the acceleration being normal to the direction of motion has no component along that direction for it to change the speed.

Note that, this is an example of *independence of two mutually perpendicular vectors*—operation of one an acceleration, on a particle does not at all affect the operation of the other, a velocity on the same particle. Another that you have learnt, is the parabolic path followed by a particle projected *horizontally* from a height with uniform speed and simultaneously subjected to a *vertical* acceleration due to gravity.

We therefore see, that the centrally directed constant force is used up completely in continuously changing the direction of a particle moving along a circle, leaving none to move the particle towards the centre. This state of affairs hold for (*i*) an artificial satellite circling the earth and (*ii*) an electron circling a nucleus in an atom. In neither cases the central force acting towards the center can draw the circling particle towards it and for the same reason.

I-5.4. Centrifugal force. The centrifugal reaction is called the centrifugal force by many modern authors, particularly in engineering practice. It is exerted by the body on any thing that makes the body follow a circular path. It is equal and opposite to the centripetal force. Application of the term 'centrifugal force' to mean the 'centrifugal reaction' is definitely a misuse.

The *true sense of the term centrifugal force* may be represented in the following way. (*i*) Consider a horizontal table rotating about a vertical axis at a constant angular speed. Imagine, further, an observer situated at the centre of the table and rotating along with it, (fig. I-5 5) but unconscious of the motion of the table. He holds in his hand one end of a spring, to the other end of which is attached a ball. When the whole system rotates uniformly, the ball appears to him to be at rest. But he is conscious of the pull he exerts on the ball. How is it that the ball on which he exerts a pull does not come nearer to him, but maintains a constant distance? It then *seems* to him that the ball is being acted on by a force equal and opposite to the pull he exerts on it. This force is called the **centrifugal force**. It must not be confused with the centrifugal reaction which the stone exerts on the observer.

Fig. I-5.5

(*ii*) Or, as shown in the same fig. a small ball hanging vertically assumes and maintains a constant inclination from the vertical. This is also due to the centrifugal force for as soon as the rotation stops the ball returns to the vertical.

To an observer standing at rest outside the table the centrifugal force has no existence. To the observer who has the same motion as that of the table, this force is as real as any other force. When the centripetal force ceases, the stationary observer finds that the stone flies off tangentially to its path; but to the rotating observer it appears to move radially outwards because of the centrifugal force.

(*iii*) When a car suddenly rounds a curve a passenger seated in the car *feels* a force pushing him radially outwards. *This subjective feeling of a force directed outwards is the centrifugal force in its original sense.* It exists relative to one who shares the motion of the car. *To a stationary observer outside the car* the passanger tends to move tangentially to the track due to inertia, while the car moves in a circle. This reduces the distance between the passenger and the outer side of his seat, which soon reaches him, and presses him inwards, thus supplying the necessary centripetal force to make him move in the required circle. Some prefer to call the radial thrust exerted by the passenger on the body of the car as the centrifugal force.

Definition. The *centrifugal force* may then be defined *as a force equal and opposite to the centripetal force, acting radially outwards on a body rotating in a circle aud existing only* **relatively** *to an observer who has the same rotational motion as that of the body.*

Centrifugal force a 'pseudo' force. It is considered as a *pseudo* (false) or *fictitious* force for above examples tell us that outside the rotating system or frame it has no existence We cannot locate an agent applying the force. For all *real* forces we can. For, all real forces arise due to interaction between two bodies either in contact or at a distance. Examples are, push or pull, tension, friction (bodies in contact) electric and magnetic repulsion and attraction, gravitational attraction (in contact or connected or at a separation), in each a pair of bodies being involved. Centripetal force is real for it involves interaction between two bodies but centrifugal force is pseudo for there is no interaction. It is sometimes called a 'g' force.

Origin of centrifugal forces or any other form of pseudo-forces are due to non-inertial or accelerated frames of reference.

Newton's second law of motion $F=ma$ holds in a frame of reference which is either at rest or moving with a uniform velocity. It does not hold in the given form when the reference frame is rotating. To write the law of motion in the Newtonian form when the reference frame is rotating, it is necessary to add other forces to the applied force. *Centrifugal force* one is such force. *Coriolis force* is another. These added forces are *pseudo-forces as they are not due to interaction between bodies.*

Any object undergoing a circular motion may be referred to as

a rotating frame of reference, which is after all an accelerated (centripetal) frame, with the same angular velocity as that of the circular motion. Centrifugal force is due to the rotation of the frame of reference where the observer is. Like all pseudo forces it acts opposite to the acceleration, in this case away from the center.

Note : In mechanical problems we have two alternatives—

(*i*) Choice of an *inertial frame* where Newton's laws of motion holds and real forces i. e. assosciable with definite bodies that can be located. (*ii*) Choice of *non-inertial* frames where to apply Newton's second law a pseudo force has to be subtracted so that $F-F'=ma$ would hold in place of $F=ma$.

Though we do with the first alternative mostly, sometimes we have to fall back upon the second one, as in the present case.

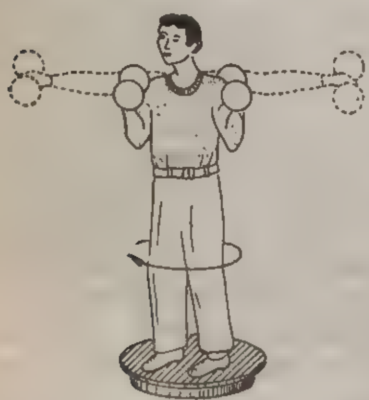


Fig I-6.5(a)

Reality of centrifugal force. A man stands on a rotating turn-table fig. I-5.6(a) holding two weights on his outstretched hands. In pulling the weights towards his body his muscles will have to do work against the centrifugal force. The man is in a rotating reference frame.

If a briefcase lies loose on the seat of motor car it will move sideways and reach the wall of the car, if it takes a quick turn.

Measuring the Centrifugal force (fig. I-5.6b). The device can be fitted to a hollow axis which can be rotated pretty fast. On the frame is a ballthat can slide along the rod AB. A thread is attached to the bottom of the ball which passes over a pulley (P) and supports a scale-pan (S). When the frame rotates, the ball tends

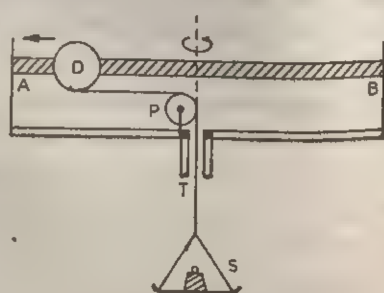


Fig. I-5.6(b)

to slide away from the axis towards the end and thus tenses the string (T). By putting weights on S, the ball is prevented from leaving its position. The weight on S equals the tension of the string and hence measures the centrifugal force. Faster the frame rotates greater is the weight required, to stop the sliding motion.

We can also regard the weight on the pan as measuring the centripetal force for the tension of the string may be regarded as supplying the centripetal force which equals the centrifugal force in magnitude.

Ex I-5 5. *A small disc of mass m rests on a friction'ess table at the end of a long string. Its other end passes through a hole at the center of the table and supports a heavier mass M. Find the condition with which m must circulate so as to keep M at rest.* **[See fig. I-5 2]**

Solution : The condition is $mv^2/r = Mg \; v^2/r = (M/m)g$

Ex I-5.6. *What would be the length of a day be if the earth spun fast enough for a body on the equator to become weightless ? Equatorial radius 6378 km, $g = 9.78 \; m/s^2$. How much faster would the earth spin then ?*

Solution : To realise the given condition we may arrive at the same relation by considering either (*a*) the centripetal force being supplied by the pull of the earth on a body, or (*b*) centrifugal force nullifying that pull.

In either case we have $m\omega^2 r = mg$ or $(4\pi^2/T^2)r = g$

or $T = 2\pi\sqrt{r/g} = 1.41$ hrs $\simeq \sqrt{2}$ hrs

This would be the length of a day. The relative angular speed would be

$$\frac{\omega}{\omega_0} = \frac{T_0}{T} = \frac{24}{1.41} \simeq 17$$

The earth then would spin *17 times faster.*

Remember as a summary—

All the three forces, Centripetal, Centrifugal reaction and centrifugal force have same magnitudes—$m\omega^2 r$, indicated in fig. I-5.7.

(*i*) *Centripetal force*—It acts *on the moving body radially inwards* and is applied by *an external agent* and hence is *real.* ($F_{C.P}$)

(*ii*) *Centrifugal reaction*—It acts *on the steady agent at the center radially outwards* and is exerted by the moving body as a *reaction* to the above force and so is *real.* ($F_{CFR.}$)

(*iii*) *Centrifugal force*—It acts *on the moving body radially out-*

wards appearing only in *rotating frames.* No agent applying the force can be recognised and so the *force is fictitious.*

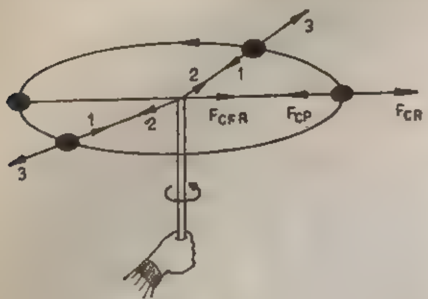


Fig. I-5.7

In fig. I-5.7 they have been marked as (1), (2) and (3) respectively.

Note again, though equal and opposite the centripetal and centrifugal forces cannot be action and reaction for they act on the same body.

I-5 5. Some Illustrations of Centrifugal forces.

(*i*) *Whirling of a bucketful of water in a vertical plane* (fig. I-5.8) can be done fast enough such that though directly overhead water will not fall off. This is because the vertically upward centrifugal force balan.es the downward directed weight of water, i.e. $mg = mv^2/r$ or $m\omega^2 r$ so that $v = \sqrt{gr}$. (I-5.5.1)

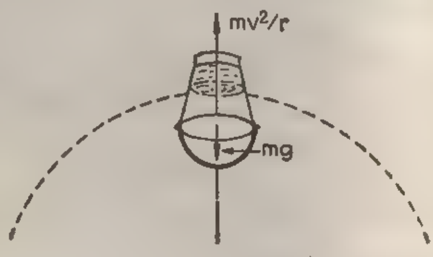


Fig. I-5.8

The same occurs when the pilot is pinned to his seat head downwards as he *loops a verical loop* fast enough so as to develop centrifugal acceleration of $5g$ to $10g$ and experience strange and physically damaging cerebral and visceral sensations—a case of *superweightlessness.*

(ii) **Flattening of the Spinning Earth. The Equatorial bulge.** This well-known fact, is explained by the fact that at its birth and till much much later, the earth was a semifluid and semi-solid sphere spinning about an axis through the poles and much faster. As one proceeds towards the equator, points on earth describes progressively larger circles about the axis of rotation. So a point on the equator

must spin fastest as it describes the largest circle. Hence there would be the largest centrifugal force there, producing the *equatorial bulge* against the gravitational pull of the earth. Newton had proposed this explanation and calculated that the difference between equatorial and polar radii as $\frac{1}{300}$ of the average radius. Data provided by artificial satellites have corrected the value to 1/297.5.

This state of affairs can be *simulated* by a simple device shown in fig. I-5.9. To a thin vertical rod attach four metal strips so as to from two circles at right angles. Their upper ends are attached to a collar which can slide easily up and down the rod. Now spin the rod very fast when the upper collar slides down producing (as shown lower) an oblate spheroid with an equatorial bulge.

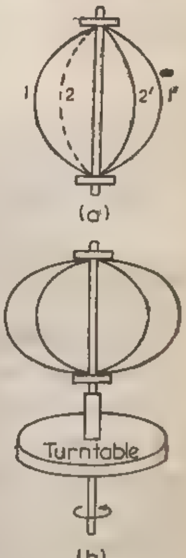


Fig. I-5.9

(iii) **Reduction of Weight due to the Spin of the Earth:** To an imaginary observer at rest out in space everything on earth would be spinning with the same angular velocity ω. This necessarily produces a centrifugal force on each, the magnitude depending on the latitude or the radius of the great circle where the body is. That lessenes the weight mg by $m\omega^2r$. A the equator, loss for 1 kg becomes

$$4\pi^2r/T^2 = \frac{4\pi^2 \times 6378 = 10^5}{(24 \times 3600)^2} = 33.7 \text{ g}.$$

This can be explained from the standpoint of centripetal force also; $m\omega^2r$ being deducted from the weight mg to provide the necessary centripetal force. We return to this point in II-1.6(3).

(iv) We have seen that if the earth spun 17 times (Ex. I-5.6) as fast as it does now, bodies on the equator would become *weightless*. If it spun still faster, so-called *superweightlessness* would result, bodies will fly off radially and the earth may burst at the seams, because of strong centrifugal forces. Here the centrifugal reaction may be said to exceed the centripetal force.

That is why a flywheel or armatures in giant electromagnetic alternators *should not be rotated very fast* lest they burst.

(v) A ball rolling down an incline may achieve enough speed at the bottom so as to describe a vertical circle i.e topping the loop before moving up an incline as shown in fig. I-5.10. The motion is similar to as in our first illustration. We shall see that the *minimum* height of the incline should be $5/2r$, where r is the radius of the radius of the vertical circle described. [Ex I-5.12]

Fig. I-5.10

(vi) **Centrifuge** It is a device in which small tubes or vials, hinged near their tops, are spun at a high speed in a horizontal circle by air pressure or electrically driven gears. As the machine spins, the lower ends of the tubes rise (as the governors of steam engine). The tubes ultimately become practically horizontal and move ina horizontal circle. (fig. I-5.11)

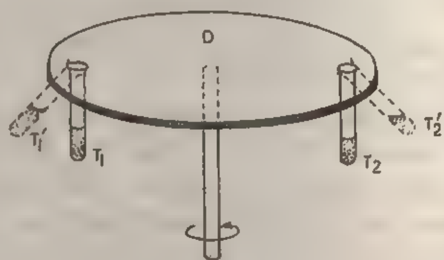


Fig. I-5.11

The tubes contain particles in suspension in a liquid of different density, such as fat particles in milk, the red corpuscles in blood, etc. The purpose of the centrifuge is to separate these particles from the liquid.

In the centrifuge, a particle of volume V, density σ and angular speed ω needs a force $V\sigma\omega^2 r$ to keep it rotating in a circle of radius r. The surrounding liquid can exert on it a centripetal force $V\rho\omega^2 r$ where ρ is the density of the liquid. If $\rho > \sigma$, the particle moves towards smaller values of r, i.e., towards the axis of rotation. If $\sigma > \rho$, the particle moves towards larger values of r, i.e., away from the axis of rotation.

The rate of sedimentation under gravity is principally determined by the difference in densities between the sediment and the liquid.

When ω is large, the force $v(\sigma-\rho)\omega^2 r$ tending to displace the particle is much greater than that due to gravity which is $v(\sigma-\rho)g$. Now $\omega^2 r$ can be made very much greater than g. With modern techniques it is possible to obtain values of $\omega^2 r$ approaching a million times that of gravity. Hence sedimentation is brought about much more quickly by a centrifuge than by gravity.

The centrifuge is of great value in various fields, particularly in medical and biological research, e g, in separating proteins, hormones, viruses etc, from sera or other liquid media. It is also used for separating cream from milk and has many other similar uses.

A cream separator filching out cream from milk, works on the same principle. You might have observed milkmen doing the same by rotating with their palms rapidly, a rough wooden stick plunged in milk. If you set dirty water in a bucket into fast rotation with your hand you can achieve quick sedimentation.

(vii) **Centrifugal drying machine.** It consists of a cylindrical vessel with perforated walls and can be spun rapidly round its axis. Damp clothes are placed in the cylinder, which is set into rapid rotation. As a result of the centrifugal force, the water is forced out through the perforations in the walls of the cylinder, and clothes are dried thereby. Alternatively, we may say that as there is not enough centripetal force, the water particles fly tangentially away through the holes.

(viii) **Centrifugal Pump :** Unlike the force or lift pumps (§II-7.2) this pump raises water *in a continuous stream* or circulates it through a system of pipes. It has three parts—Casing (C), impeller (IM) and spindle (S) as indicated in (Fig. I-5 12). I and O represent the inlet and outlet pipes. IM is a hollow revolving wheel with vanes. Coupled to a driving gear the spindle rotates the impeller. Water in C is then flung outwards by the centrifugal force and is delivered through O and fresh liquid flows in through I. The pump must be

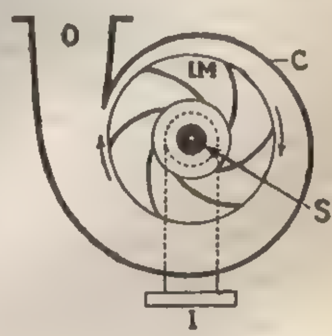


Fig. I-5.12

full of water before it starts working. If the peripheral liquid attains a velocity v, the water in the discharge tube would climb to a height h, where $v^2 = 2gh$.

They also serve as blowers and exhaust fans for blast furnaces planing mills, vacuum cleaners, ventilating systems and wind tunnels.

Ex. I-5.7 *A centrifugal pump raises water to a max height h and has a blade radius r. Find the necessary n in rps, neglecting friction.*

Solution : Equating K.E. to P.E we get

$$\tfrac{1}{2} m\omega^2 r^2 = 2mgh \text{ or } 4\pi^2 n^2 = \frac{4gh}{r^2} \text{ or } n = \sqrt{gh}/\pi r$$

(ix) **Conical Pendulum** (Fig. I-5.13) is a small sphere hanging at the end of a long thread fixed at the upper end and describing a circle. The tension T along the string has two components $T\cos\theta$ balancing the weight mg of the bob and $T\sin\theta$ balancing the centrifugal force mv^2/r. Thus

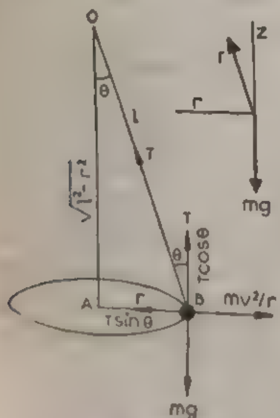


Fig I-5.13

$$\left.\begin{aligned} T\sin\theta &= mv^2/r \\ T\cos\theta &= Mg \end{aligned}\right\}$$

$$\therefore \quad \tan\theta = v^2/gr = \omega^2 r/g \quad \text{(I-5.5.1)}$$

$$\text{Again } \omega = 2\pi/T = \frac{\sqrt{g\tan\theta}}{r}$$

$$= \sqrt{\frac{g\tan\theta}{l\sin\theta}}$$

$$= \sqrt{g/l\cos\theta} \qquad \text{(I-5.5.2)}$$

This expression is very important and would recur repeatedly. Faster the bob rotates, greater is the angle of inclination and obviously higher is the plane of rotation of the bob.

Ex I-5.8 *A 1 m long thread fixed at one end carries a mass 100 g at the lower end and makes 2/π rev. per sec around the vertical axis through the fixed end. Find the angle of inclination and the linear velocity of the mass.*

Solution : From the given conditions we find

$$\omega^2 = \frac{g}{l\cos\theta} \text{ or } 4\pi^2\frac{4}{\pi^2} = \frac{9.8}{l\cos\theta} \text{ or } \cos\theta = 9.8/16$$

$$\therefore \quad \theta = 52°12'$$

Again $v = \sqrt{gr \tan\theta} = \sqrt{gl \sin\theta \tan\theta}$

$$= \sqrt{9.8 \times 1 \times 0.7902 \times 1.2892} = 3.16 \text{ m/s.}$$

(x) **Watt's Steam Governor** : This acts somewhat on the principle of conical pendulum. It governs various rotational speeds of the machines. A and B (fig. I-5.14) represent two heavy balls supported by rods pivoted at both ends. The pivots at C are fixed to a central shaft. Those at D are attached to a movable collar. As the shaft spins faster centrifugal forces push A and B further out and higher and so raise the collar D. The collar then operates a system of levers which control and check supply of steam to a steam engine. As D rises, the steam valve openes, steam rushes out, steam pressure falls, slowing down the shaft.

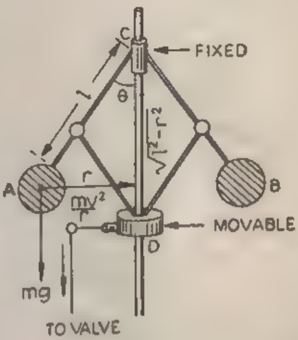


Fig I-5.14

Then A and B descend, the collar D descends and closes the steam valve where pressure and speed builds up again. The instrument has been modified to operate a brake as in a gramophone motor.

In developing its theory consider the Δ CAE where AE represents the centripetal force mv^2/r making the bob describe the circle. Then from the law of sines

$$\frac{mv^2/r}{mg} = \frac{AE}{CE} = \frac{\sin ACE}{\sin CAE} = \frac{\sin\theta}{\sin(90-\theta)} = \frac{r/l}{\sqrt{l^2-r^2}/l} = \frac{r}{\sqrt{l^2-r^2}}$$

$$\text{or} \quad v^2 = gr^2/\sqrt{l^2-r^2} \qquad \text{(I-5.5 3)}$$

(xi) **Rotor or Death-well. A** In amusement parks or circus-shows a hollow large diameter vertical cylinder is set up that can be set rotating at a very high speed. A man stands on a platform against the wall, which is rough and so is the clothing of the person When the drum rotates fast enough, the platform is moved out from below

the performer who does not fall off and remains pinned up against the wall. (fig. I-5.15.)

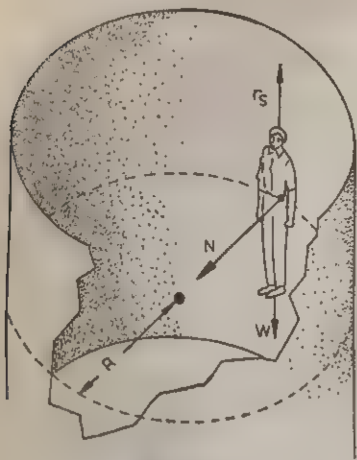


Fig. I-5.15

We find the minimum speed of rotation for this to happen. On him we have (i) his weight W acting down wards which is balanced by (ii) the force of static friction f_s acting upwards. This arises due to the normal force $m\omega^2R$ pressing the man laterally to the rotor surface; friction acts at right angles to this centrifugal force, upwards, for the tendency of the man to move is downwards. Now if the coefficient of friction between wall and clothes materials be μ, then

$$f_s = \mu N = \mu m\omega^2 R = \mu(W/g)\omega^2 R$$

This balances the weight W of the man

$$\therefore \quad \mu = \frac{W}{N} = \frac{W}{(W/g)\omega^2 R} = \frac{g}{\omega^2 R} = \frac{gR}{v^2} \qquad \text{(I-5.5.4)}$$

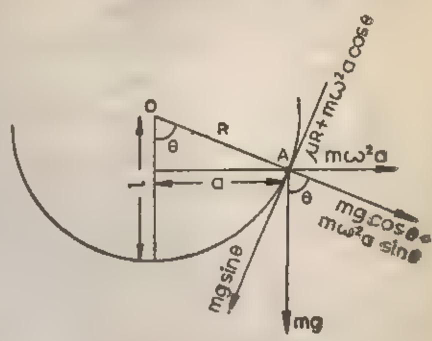
Notice that the result is independent of the weight of the man.

B. In a variant, the drum remains static and along its walls, a rider drives furiously a motor cycle. The rider and the cycle assumes a horizontal stance, yet they do not fall off. This is a case of dynamical equilibrium when the outward centrifugal force is balanced by the reaction of the wall. The latter may alternatively be thought to supply the necessary centripetal force. A motor cyclist looping the loop inside a spherical cage at a circus-show along all the possible circles utilise this principle.

Problem. A vertical steel cylinder of diameter 20 cm when rotating, carries a small steel piece pinned to it when the rotation exceeds 200 rev. per min. It falls down at lower r.p.m. Find μ between steel surfaces. (Ans. 0.22)

[Lond Univ.]

Ex I-5.9. *A small iron piece is at the bottom of a hemisphere of radius a; μ between the two is the coefficient of friction As the vesesl is rotated with an angular velocity of ω, the piece rises inside it, till the radius joining its maximum height to the center makes an angle θ with the vertical. Show that*



$$\frac{a\omega^2}{g}=\frac{\sec\theta-\mu\,\text{cosec}\,\theta}{1+\mu\tan\theta}$$

Solution: Let the particle rise to A; it does so because of centrifugal force and held there against gravity by friction which are respectively $m\omega^2 a$ and mg. Both of them are resolved and the forces are as shown in the figure. For equilibrium we have, equating tangential and normal forces

$$mg\sin\theta=\mu R+m\omega^2a\cos\theta$$
$$\text{and } R=mg\cos\theta+m\omega^2a\sin\theta$$
$$\therefore\quad mg\sin\theta=m(\mu g\cos\theta+\mu\omega^2\sin\theta)+m\omega^2a\cos\theta$$
$$\text{or}\quad g\sin\theta=\mu g\cos\theta+\mu\omega^2a\sin^2\theta+\omega^2a\sin\theta\cos\theta$$
$$\text{or}\quad 1=\mu\cot\theta+\frac{\mu\omega^2a\sin\theta}{g}+\frac{\omega^2a}{g}\cos\theta$$
$$=\mu\cot\theta+\frac{\omega^2a}{g}(\mu\sin\theta+\cos\theta)=\mu\cot\theta+\frac{\omega^2a}{g}\cos\theta\,(\mu\tan\theta+1)$$
$$\therefore\quad \frac{\omega^2a}{g}=(1-\mu\cot\theta)/(\mu\tan\theta+1)\cos\theta$$
$$=\frac{(1-\mu\cot\theta)\sec\theta}{1+\mu\tan\theta}=\frac{\sec\theta-\mu\,\text{cosec}\,\theta}{1+\mu\tan\theta}$$

I-5 6. Friction and Circular motion. Example I-5.9 above indicates the role of preventing relative motion between surfaces in circular motion, by friction. We consider more cases.

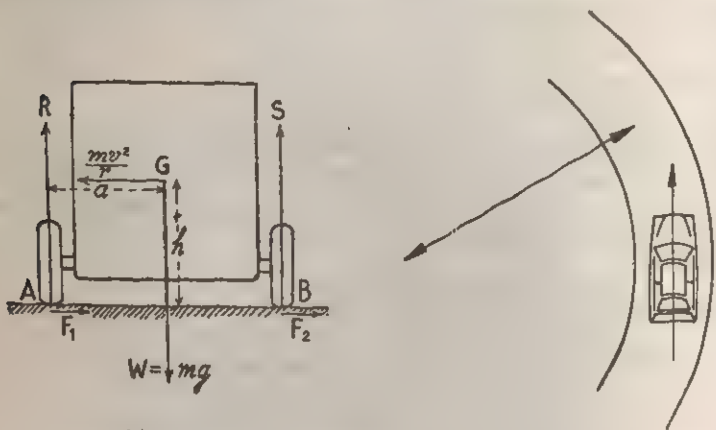
1) When a knife blade is sharpened on a spinning grinding wheel you may have observed glowing particles on the wheel rotating with it at lower speeds. Adhesion and friction prevents relative motion. But they shower out tangentially at high enough speeds when frictional forces are insufficient to provide the necessary centripetal force to hold them on the wheel.

(2) A muddy wheel holds mud patches at low speeds but throw them up tangentially when rolling fast. It is against them that *mud-guards* are provided on all carriage wheels.

When cars on a curved path takes a bend at low speeds on rough roads, sideways friction provides the necessary centripetal force. But the magnitude is small and uncertain.

*(3) **Friction determines when a car will skid or overturnn :**

Fig I-5.16(a) represents a wheeled car on a level road in contact



(a)

(b)

Fig. I-5.16

at A and B where R and S are normal reactions. Consider the vessel to be turning in a circle with the center to the left in the plane of the diagram (fig I-5.16*b*)

A motor car cannot of itself change the direction of its motion.

When it takes a bend it moves in a circle. To make it so move, it requires an external force to be applied towards the centre of the circle. On a level road this force can be supplied only through friction between the wheels and the road. The required force is mv^2/r But the friction which is to supply this force is small and of uncertain magnitude. So the car should be driven slowly ; otherwise it may skid in its tendency to continue in a straight line. Skidding occurs when v is too large and r too small.

* For the inquisitive student.

Skidding F_1 and F_2 are frictional forces which supply the necessary centripetal forces mv^2/r. If W is the weight of the vehicle and μ the coefficient of friction between the road and the wheels, the maximum frictional force being μW, the vehicle will skid i.e. side-slip if

$$mv^2/r > \text{Max. value of } F_1+F_2=\mu W=\mu mg$$

$$\text{or } mv^2/r>\mu mg \text{ or } v>\sqrt{\mu gr} \qquad \text{(I-5 6 1)}$$

Overturning : If the car overturns, it turns about the points of contact of the outer whaels. Then S and F_2 vanish. The moment of the force (§I-6.4) tending to overturn it is $h\times mv^2/r$ where h is the height of the line of action of the centrifugal force above the ground. This force acts through G the C. G. of the car. The moment of the restoring force due to the weight of the car is $Wa=mga$. The car will overturn if

$$mv^2h/r>mga \text{ or } v>\sqrt{gra/h} \qquad \text{(I-5.6.2)}$$

Note that mass of the car is quite irrelevant for either.

(i) If $\mu>a/h$, the vehicle will *overturn* rather than skid. If h is large this condition is more easily satisfied, and the vehicle overturns. In sports cars h is kept low and a is made large so that the car may not overturn.

(ii) If $\mu<a/h$, it will skid before the speed is high enough to cause overturning. If h is small the vehicle is more likely to *skid* than to overturn.

Ex I-5 9. *C. G. of a goods train is 3 ft above the rails 4 ft apart. What is the safe speed limit in taking a turning of 108 ft. radius ?*

Solution : The centrifugal force mv^2/r acts horizontally and the wt of the train acts vertically through the C. G. As shown above we take momen's about the outer rail and equate them

$$\frac{mv^2}{r}h=mga \quad \text{or} \quad v^2=gar/h=(32\times2\times108)/3$$

$$\therefore \quad v=48 \text{ ft/s}$$

Prob A car negotiates a curve of radius R, of which the whees are b apart and C. G. at height h from the ground. Find the velocity of its turning such that pressure on the inside wheel vanishes.

$$\left[\textbf{Ans.} \quad v^2=\tfrac{1}{2}\cdot\frac{Rgb}{h}\right]$$

I-5.7. Some examples of Centripetal force and its reactions

(i) **Banking of Roads :** To remove this chance of skidding or overturning the road bed is inclined to the horizontal at the bends,

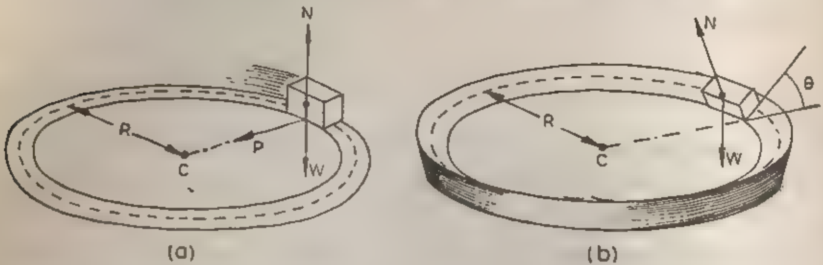


Fig. I-5.17

the slope being towards the centre of the bend as indicated in fig. I-5.17(a) and (b)

Fig I-5.17(c) shows a moving car on such an inclined road bed. The weight W of the car acts vertically downward at its centre of gravity. The normal reaction P of the road on the car is perpendicular to the bed, friction being left out of account. The vertical component of P, i.e., $P \cos\theta$, balances the weight, while the horizontal component of P, i.e., $P \sin\theta$, supplies the necessary centripetal force. Thus we have $P \sin\theta = mv^2/r$, and $P \cos\theta = W = mg$.

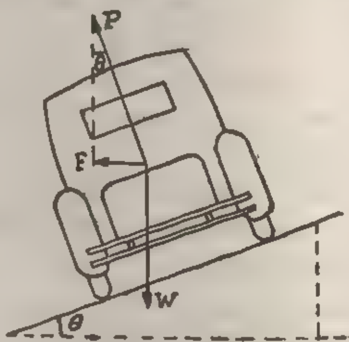


Fig. I-5 17(c)

$$\text{whence } \tan q = v^2/gr. \qquad \text{(I-5.7.1)}$$

This equation gives the proper angle for banking of a curved road of given radius r, for a pre-determined speed v of a car.

In the case of a **railway train**, the rail on the side of the circle away from the centre is raised. The forces are as for a motor car on an inclined road bed. When the track is tilted to an angle $\tan\theta = v^2/gr$, the stress between flange and rail vanishes. At other speeds either flange must press against the corresponding rail.

If the distance between the rails is d then *since θ is small* (fig I-5.17d) we may put $\tan \theta = AC/BC = AC/AB$. Thus the elevation of one rail above another will be

$$AC = (v^2/g_r)d \qquad \text{(I-5.7.2)}$$

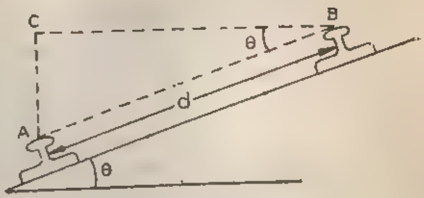


Fig. I-5.17(d)

Problem. A train takes a curve of radius half a mile at 30 mph. The separation of the rails being 5 ft how much higher the outer rail will be above the inner one so as to neutralise thrust on the wheel flangs.

(**Ans** nearly 4")

(ii) **Cyclist taking a bend.** The case of a cycle may be similarly treated. While rounding a curve the cyclist moves in a circle, so that necessary centripetal force must be supplied. The cyclist leans towards the centre of the circle (fig I-5.18) in which he wants to move and derives his centripetal force from the horizontal component of the reaction $P(=CS)$ of the ground. P is made up of a vertically upward component $R\,(=CR=AR)$ equal and opposite to W, the weight of the cycle and the cyclist and a horizontal component F ($=CD=AF$) due to friction. $P^2=W^2+F^2$. The maximum value of F is μW, where μ is the coefficient of friction between the road and the tyres. Hence the maximum available centripetal force is μW. If however mv^2/r exceeds μW the cycle skids. If θ is the inclination of

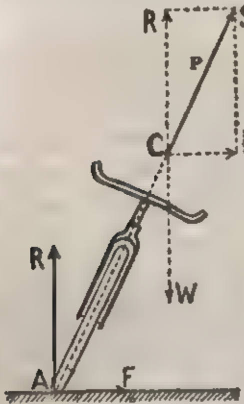


Fig 7.6
Fig. I-518

the cyclist ($\angle SCR$ or $\angle RAC$) to the *vertical*, $P\cos\theta = W$ and $P\sin\theta = F = mv^2/r$. So that again $\tan\theta = v^2/rg$. For larger speeds and smaller radii θ must increase. The maximum inclination θ which the cyclist can have without skidding is given by $\tan = \mu W/W = \mu = v^2/rg$. i.e. the condition must be $v^2 < \mu rg$, for no skidding.

Problem. (1) A cyclist takes a bend of 20 ft radius at 10 mph. At what angle must he lean, and what must be the smallest coefficient of friction between the tyre and the road? (Ans. 18°35′; 0.336)

(2) A cyclist moving at 8 mph wants to turn about. What is the radius of the smallest circle he can take if the coefficient of friction between the road and the tyres is 0.4? (Ans 10.8 ft.)

(3) A cyclist is describing a circle of 20 m radius at a speed of 18 km per hour. What is his inclination to the vertical? (Assume the rider and the cycle to be in one plane.) [H. S. '78]

(iii) **Planets** move round the sun in *approximately* circular paths and so do the satellites around their planets, the necessary centripetal forces being supplied by *gravitational* attraction of the sun and the planets as the case may be.

(iv) **Electrons** in an atom move round the nucleus similarly, the centripetal force being provided by the *coulomb force* of attraction between opposite charges.

Note: In the last two examples the forces of interaction act at a distance without an intervening medium and not through contact as in other cases.

I 5 8. **Motion in a vertical circle.** Fig. I-5.19 represents a small body of mass m whirled in a vertical circle with centre O. It is attached to the centre by a string of length r. *The motion is circular, but not uniform. The body gains speed on its way down and loses speed on its way up.* Recall the whirling of a bucket of water vertically without water spilld out.

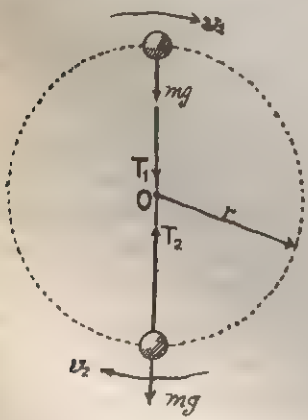


Fig. I-5.19

Let v_1 and v_2 represent the linear spieeds at the highest and lowest points, and T_1 and T_2 the corresponding tensions in the string. Then we shall have

$$T_1+mg=mv_1^2/r, \qquad \text{(I-5.8.1)}$$

and $$T_2-mg=mv_2^2/r. \qquad \text{(I-5.8.2)}$$

As the particle moves from the highest to the lowest point it loses $2mgr$ of potential energy, and gains equally in kinetic energy.

$$\therefore \quad \tfrac{1}{2}mv_2^2=\tfrac{1}{2}mv_1^2+2mgr, \text{ or } v_2^2-v_1^2=4gr \qquad \text{(I-5.8.3)}$$

There is a critical velocity below which the string becomes slack at the highest point. It may be obtained by putting $T_1 = 0$ in Eq. I-5.8 1. This gives $mv_1^2/r = mg$ or $v_1 = \sqrt{gr}$. (I-5.8.4)

Combining Eq. I-5.8.3 and .4 we find that the miniumum horizontal velocity with which a particle, hanging freely at the end of a string of length l, must be projected so as to describe completely a vertical circle is given by $v_2^2 = 5\ gl$.

Note that at any point of its path the sum of the kinetic and potential energies of the particle will be constant, and the resultant force on it will be directed towards the centre of the circle of motion.

Ex. I-5.10 *An arched bridge is to be so designed that cars may pass over the bridge safely at speeds upto 180 km/hr without jumping off the road. Find its radius of curvature.* [J. E E. '75]

Solution : For the maximum speed the weight is completely used up to supply the necessary centripetal force or just neutralised by the centrifugal force, i.e.

$$mv^2/r = mg \quad \text{or} \quad r = v^2/g = \frac{(\frac{1}{20})^2(\text{Km/s})^2}{9.8\ \text{m/s}^2} = \frac{2500}{9.8}\text{m} \simeq 255\ \text{m (nearly)}$$

Ex I-5.11 *A small body tied to an inextensible almost massless string of length 5 m is rotated in a vertical circle. Find the minimum velocity at the top, for the string not to slacken and the velocity and angular speed at the lowermost point.* [I I. T. '67]

Solution :
$$v_1 = \sqrt{gr} = \sqrt{9.8 \times 5} = 7\ \text{m/s}$$
$$v_2 = \sqrt{5gr} = 15.652\ \text{m/s}$$
$$\omega = v_2/r = 3.13\ \text{rad/s}$$

Ex. I-5.12 *A small ball rolls down a smooth incline from its top at a heigh h from the ground. It describes a vertical circle after reaching the bottom Find a relation between h and r the radius of the circle.* (See fig. I-5.11)

Solution : To move in a vertical circle the ball must earn a minimum velocity of $\sqrt{5gr}$ at the bottom where its K.E is $\frac{1}{2}mv^2$ at the cost of P.E which is mgh. $\therefore$ $mgh = \frac{1}{2}m\,5gr$ or $h = \frac{5}{2}\,gr$.

Problems. (1) A motor car of weight W travels at a uniform speed v (*a*) on a horizontal bridge, (*b*) on a convex and (*c*) on a concave bridge, both of same radius r. Find the force exerted by the car on the bridge in each case with the car at the middle of the bridge.

[*Ans* (*a*) $F = W$; (*b*) $F = W - mv^2/r$; (*c*) $F = W + mr^2/r$.]

(2) The string of a pendulum of length l is displaced 90° from the vertical and released. What i the tension in the string as it passes the position of equilibrium ? [*Ans.* $3mg$]

S.H.M

1 5.9. Projection.

From geometry perhaps you have already learnt the meaning of the term *projection of a line segment* on another straight line. You drop a pair of perpendiculars from the two ends of the given line segment on the given straight line. The distance between the feet of these two perpendiculars gives the line segment on the given line.

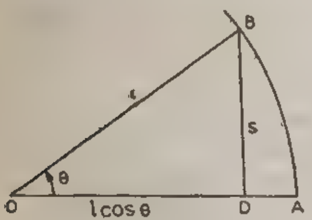


Fig. I-5.20

In fig I-5.20 we have a pair of equal lines OA and OB. We seek to find the projection of OB on OA each of length l; this is a particular case of projection relevant to discussing S.H.M. BD is the perpendicular dropped from B on OA and OD = OB cos θ gives the projection of OB on OA.

Projection of uniform circular motion on a diameter. In fig I-5.21 let us have a particle moving along the circle CADB with uniform circular velocity ω in a clockwise direction where the particle has reached P at any moment. Drop a perpendicular PK on AB. As P goes the circle round and round, the point X moves to and fro along AB. The motion of X is the projection of uniform circular motion on the diameter AB.

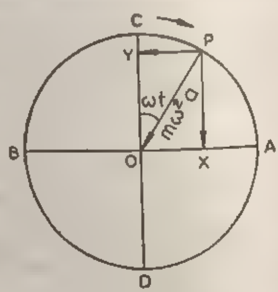


Fig. I-5.21

Similarly a perpendicular PY may be dropped on the diameter CD. As before with P moving round the circle, the point R moves up and down between C and D. Motion of R also is the projection of uniform circular motion on CD.

Models : (1) The bob of a conical pendulum you know, describes a circle in a horizontal plane. Place your eye in the same plane a little distance away and observe carefully the motion of the bob with one eye closed. You shall find it appearing to move to and

fro along a diameter (fig. I-5.22*a*) perpendicular to the line of sight.

(2) In fig. I.5.22(*b*) is shown a disc and a bar AB coupled together by a pin (P) and slot (SL) arrangement. P projects from the disc

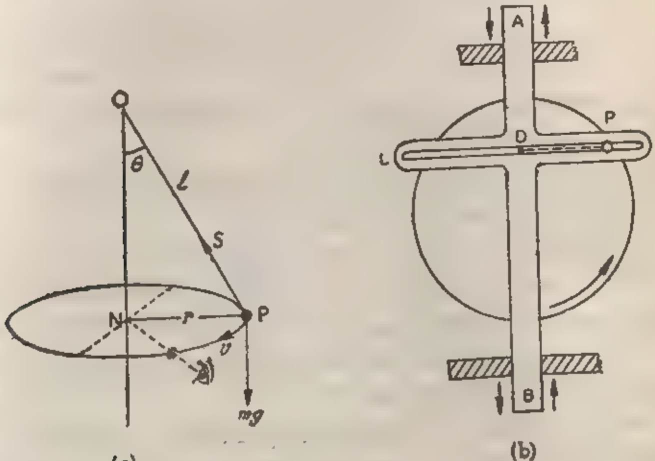


Fig. I-5.22

and engages SL. AB can move only along a diameter of the disc when it rotates. As P rotates round and round, AB moves to-and-fro. So does the point D, the foot of the perpendicular dropped from S ; as P moves round and round D moves to and fro.

(3) On a gramophone turntable beside a white wall, place a large piece of cork carrying a long vertical pin near its circumference. Let the turntable *rotate slowly* ; on one side place a strong source of parallel light. You observe the tip of the pin describing a circle while its shadow on the wall moving to and fro

In each model you observe the projection of uniform circular motion on a diameter of it.

I-5.10 Projection of uniform circular motion on a diameter is an S H. M.

Simple harmonic motion (S.H.M) and uniform circular motion are two of the simplest of that very large class of motions, the periodic motion—a motion that repeats itself along the same path over and over again and at regular intervals of time. The proposition given above connects the two. Both of them occur under *central forces* ;

i.e. forces directed towards a fixed point. In uniform circular motion, the motion is along a circle under a constant force directed towards its center; in S. H. M the motion is vibratory along a short st line or a short arc under a variable force always directed towards a 'mean or equilibrium position.

Definition : Projection of uniform circular motion on any diameter of it is simple harmonic. This is said to be a *kinematical* or geometrical definition, elucidating the peculiarities of the motion without however explaining the origin.

The kinetic or physical definition provided in III-1.2 shows S.H M to have the following **characteristics** .—

(i) The motion is to and-fro (periodic) and along a straight line

(ii) The force and hence the acceleration is proportional to the instantaneus particle-displacement from the mid-point of the motion.

(iii) The force and acceleration are opposite in direction to the instantaneous particle displacement.

(iv) At the mid-point the particle displacement is zero, so is the acceleration and force; but particle velocity is a maximum. Conversely, at the end-points velocity vanishes while the force attains a maximum.

If the acting force is P and the displacement at an instant is x measured from the mid point, then the relation between them would be given by $P = -kx$ where k is the constant of proportionality.

Projection of uniform circular motion on a diameter show all these characteristics and hence is an S. H M.

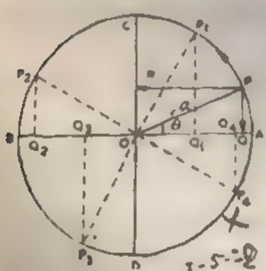


Fig. I-5.23

Proof : Refer to fig. I-5.23. As we have seen, (i) As P moves through P_1, C, P_2, B, P_3, D, P_4 back to A, the projection Q moves to-and-fro successively through Q, Q_1, O, Q_2, B, Q_3, O, Q_4 finally back to A. So the motion of Q is to-and-fro along a straight line. So would be that of R. New refer back to fig. I-5.21.

(ii) As the particle P moves along a circle it is subjected to the centripetal force $m\omega^2 a$ along PO. This is however equivalent to

componeus $m\omega^2$ PO cos ωt and $m\omega^2$.PO sin ωt along PX and PY respectively As X can move only along AO, the force acting on it is parallel to PY and hence directed towards O, the mid-point Similarly the force on Y will be found to be parallel to PX and hence directed again towards O.

The same would hold wherever on the circle, P may be. Thus the *force* on X or Y, the projections, is *always directed* towards *the mid-point.*

(iii) Again, $m\omega^2.OP \cos \omega t = m_\omega{}^2.OP\,(OY/OP) = m_\omega{}^2.OY$

and $m\omega^2\, OP \sin \omega t = m\omega^2\, OP(OX/OP) = m\omega^2.OX$

i.e. the restoring *force is proportional to the displacement* from the mid-point.

Thus all the characteristics of S.H.M. are shown by the projection of uniform circular motion on any diameter.

I-5 10.A. Equation for particle displacement in S H M. We consider the motion from the moment when the rotating particle crosses the point *C* (fig. I-5.21). *CD* is the diameter perpendicular to *AB*. When the particle is at *C*, *X* was at *O*, the centre of the circle. We measure displacement from this point. *O* is the position of rest of the particle in S.H.M. If after a time *t* from the moment considered the rotating particle reaches *P* (fig. I 5.21), *X* moves from *O* to *X*. So the particle displacement *x* in time *t* is $x = OX$. If $OP = a$ is the radius of the circle, then

$x = OP \sin COP = a \sin \omega t$ (I-5.10.1)

and $y = OP \cos COP = a \cos \omega t$

If at the initial moment the rotating particle was at *A*, and we measured particle displacement from *O*, (fig. I-5.24) will show that

$x = OX = OP \cos AOP = a \cos \omega t$ (I-5.10.2)

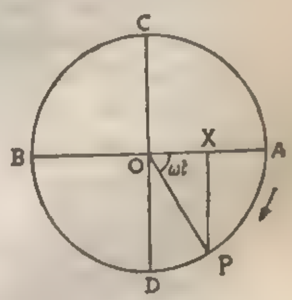


Fig. I-5.24

Eq. I-5.10 or .2 represents the same S.H.M. *The difference in form lies in the choice of the initial moment.* If the motion is considered from the moment the particle crossed the midpoint of its path, Eq. I-5.10.1 (the sine form) will apply. If we considered the motion from the moment the particle was at the end

of its path. Eq. I-5.10.2. (the cosine-form) will apply. So we may use Eq. I-5.10.1 or I-5.10.2 according to our convenience.

We might have considered the motion from any other moment, say, when the particle was at E (fig. I-5 25). If $\angle EOC = \epsilon$ (Greek letter, pronounced 'epsilon'), the figure would show that

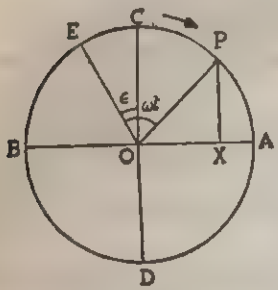


Fig. I-5.25

$$x = OX = a \sin(\omega t - \epsilon) \qquad \text{(I-5,10.3)}$$

If we took the position of E relative to the line OA then $\angle EOA = \epsilon$ and

$$x = OX = OP \cos(EOA - EOP)$$
$$= a \cos(\omega t - \epsilon) \qquad \text{(I-5.10.4)*}$$

Eq.s I-5.10.1 to I-5.10.4 represent the same S.H.M. The difference in form is due to the choice of the initial moment. ϵ may be positive or negative. We shall ordinarily use Eq. I-5-10.1 to represent an S.H.M.

a is called the *amplitude* of the S.H.M. ω is called its *angular frequency*. The periodic time T is the time in which the rotating particle goes once round the circle. This gives us

$$T = 2\pi/\omega \qquad \text{(I-5 10 5)}$$

The frequency n of the S.H.M. is $n = 1/T = \omega/2\pi$ (I-5.10 6)

I-5 10B. Particle velocity in S.H.M. Suppose the constant speed with which the circle is being described is $v_0 = a\omega$. The velocity v_0 of the particle at P is perpendicular to OP, and is represented by a line segment say, PR. The projection of PR ($= SR$) is the velocity v of the particle in S.H.M. when it is at X. From fig. I-5.26.

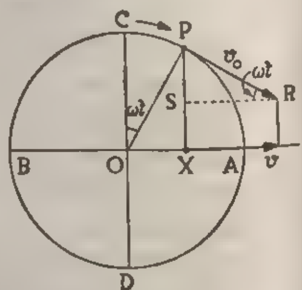


Fig. I-5.26

$$v = SR = PR \cos \omega t$$
$$= v_0 \cos \omega t = a\omega \cos \omega t \qquad \text{(I-5.10.7)}$$
$$= a\omega \sqrt{1 - x^2/a^2} = \omega \sqrt{a^2 - x^2}$$

(I-5.10.8)

Particle velocity is maximum when $x = 0$, that is, at the midpoint of the motion. It is zero at the end of the swing ($x = a$).

* $\cos(-\theta) = \cos\theta$

I-5.10C Acceleration of a particle in S.H.M. We know (See I-5.2.1) that the acceleration of a particle moving in a circle of radius a with constant angular velocity ω is $a\omega^2$, and that the acceleration is directed towards the centre of the circle (fig I-5.27). The projection of this acceleration on the diameter AB is the acceleration of the parcticle moving simple harmonically along AB. If in fig. I-5.27, the line segment PQ represents the acceleration at P, its projection XS on AB is the acceleration of the parcticle in S.H.M when it is at X. From fig. I-5.27 $XS = RQ = PQ \sin \omega t = a\omega^2 \sin \omega t$. But XS is directed towards O and is opposite to the particle displacement OX. If we take particle displacement x as +ve, particle acceleration f is −ve for their directions are opposite. So, the acceleration f at X is given by

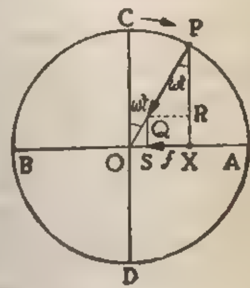


Fig. I-5.27

$$f = -a\omega^2 \sin \omega t = -\omega^2 x \tag{I-5.10.9}$$

Acceleration in S.H.M. is directed oppositely to the displacement. This means that as soon as the particle is displaced from its position of rest, an acceleration (and therefore a force) acts on the particle which tends to bring the particle back to its position of rest. The acceleration is greatest when x is largest, i.e., when $x = a$. It is zero when $x = 0$, i.e. when the particle is at the mid-point of its motion. Comparing with the case of v, we find

when $x=0$, $v = a\omega$ (maximum), $f = 0$ (minimum)

and when $x = a$, $v = 0$ (minimum), $f = -a\omega^2$ (maximum).

the fourth characteristic of S.H.M.

I-5 10D. Force on a particle in S.H.M. If m is the mass of the moving particle and $f = -\omega^2 x$ is its acceleration, the force acting on it is

$$F = mf = -m\omega^2 x \tag{I-5.10.10}$$

Clearly, the force is proportional to the displacement and is directed opposite to it. This is said to be a *linear, restoring force.* If there is no force, the particle will be at $x = 0$, the position of rest.

I-5.10E Periodic time in S.H.M. Eq. I-.5.10.5 gave us that the periodic time $T = 2\pi/\omega$. Eq. I-5.10.9 shows that the *magnitude* of th

acceleration is $f=\omega^2x$ whence $\omega^2=f/x$. Thus $\omega=\sqrt{f/x}$=square root of the acceleration at unit displacement. Therefore

$$T=2\pi/\omega=2\pi/\sqrt{f/x}=2\pi/\sqrt{\text{acclrn. at unit displacement.}} \quad \text{(I-5.10.11)}$$

This relation is very important, as we see below.

I-5.11. Example: Simple Pendulum: A simple pendulum in practice is a small metal bell hung by a long light soft string from a rigid support (fig. I-4.28(a). When oscillating with a *small* amplitude its motion is simple harmonic. Its path may be taken practically straight. The motion is repeated at equal intervals of time given by $T=2\pi\sqrt{l/g}$ (see eqn. 0-I.9.1). It starts moving towards the mid-point from its maximum displacement position increasing its velocity from zero to the maximum as it crosses the mid-point, then gradually slows down to come to rest at the other end of the swing. This is half the motion. The other half is an exact replica. As it crosses the mid-point and slows down, the force acting and hence the acceleration must be opposite in direction to the displacement. Below we prove that the force is proportional to the displacement i.e. motion of the bob of a simple pendulum is simple harmonic.

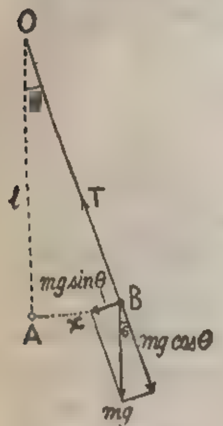


Fig. I-5.28(a)

Time-Period of a Simple Pendulum: In 0-1-9.1 we found it to be $K\sqrt{l/g}$ where K is the numerical dimensional constant. Now we find also the value of K

Consider a simple pendulum (fig. I-5 28b) of length l displaced through the angle θ. Let m be the mass of the bob and g, the acceleration due to gravity. The only force acting vertically downwards on the bob is its weight mg. Resolve it into two components, one along the string and the tober perpendicular thereto, of magnitudes $mg\cos\theta$ and $mg\sin\theta$

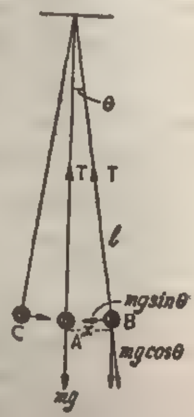


Fig. I-5.28(b)

respectively. $mg \cos\theta$ will be balanced by the tension T in the string. The component $mg \sin\theta$ is the unbalanced force which causes the bob to move towards A from its position of rest.

If θ is very small, (less than 4°) we may take $\sin\theta \simeq \theta$.* The distance of the bob from A will then be $l\theta = x$ (say). The very small arc in which the bob moves is then almost a straight line. The force acting on the bob is then $mg\theta = mgx/l$.

Clearly, it is a restoring force proportional to the displacement of the bob from its position of rest. Thus we get

$$mf = -mg.\, x/l \quad \text{or} \quad f = -(g/l)x = -\omega^2 x$$
$$\text{where } \omega = \sqrt{g/l} \qquad \text{(I-5.11.1)}$$

So the motion is executed with a periodic time T, given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}} \qquad \text{(I-5.11 2)}$$

The bob moves with simple harmonic motion about its position of rest.

If θ is not small and while measuring T, amplitude falls from θ_1 to θ_2 then it can be shown that the corrected time period is

$$T = T'\,(1 - \theta_1\theta_2/16)$$

where T' is the observed time period. Thus we find that if $\theta > 4°$ the observed time-period rises.

I-5.12 Time displacement graph in S H M The time-displacement graph (y vs. t) in S.H.M. can be easily obtained geometrically from the uniform circu'ar motion of which the S.H.M. is the projection. Refer to fig I-5.29. The circular motion is along DBECD.

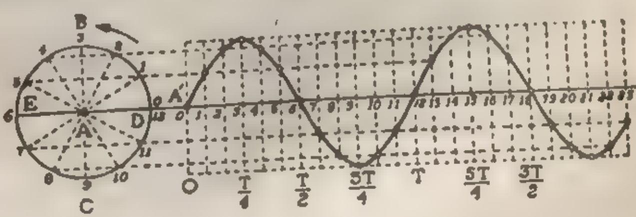


Fig. I-5.29

* Look up the logarithmic tables for values of $\sin\theta$ and θ, those of angles converted from degrees to radians. You will find values upto 4° expressed in radians, identical with those of sine values till you reach sin 4° but not beyond. Also see eq. 0—2.7.1.

The projection is taken on the vertical diameter BC. At the initial moment the particle is at D where ED is perpendicular to BC. Divide the circumference into a convenient number (here 12) equal parts. Mark the points of division successively as 0, 1, 2, 3 etc starting with D as zero. (The mark 12 falls on D or 0). Extend the line ED to the right and mark along it a number of equidistant points, taking the first of these points (A′) as zero. Number the points successively. (There are altogether 24 such points in the figure).

Suppose at $t=0$ the moving particle was at D (*i.e.* zero) and its motion was anticlockwise. Its angular velocity is $\omega=2\pi/T=2\pi n$. The foot of the perpendicular dropped on BC from the moving point describes the S.H.M. along BC.

From the marked points on the circumference drop perpendiculars on BC. As the moving particle on the circle crosses a particular mark, the foot of the perpendicular drawn from this point on BC gives the position of the point in S.H.M. Its displacement at the moment is the distance of the foot of the perpendicular from the centre A of the circle. At each marked point on ED extended draw a perpendicular line. Intersect these lines by lines drawn parallel to ED from the marked points on the circle. Mark those points of intersection which correspond to the same number on ED produced as on the circumference (such as 0, 0 ; 1, 1 ; 2, 2 ; ...... ; 5, 5 ; ...... ; 7, 7 ; etc.). Draw a smooth, continuous line through the points so marked. This is the time-displacement graph of the given S.H.M. The axis along ED produced, is the time axis, each interval on which is $T/12$. The ordinates are the corresponding displacements. The amplitude is the maximum of the curve If r is the radius of the circle, the displacement is confined between the limits $+r$ and $-r$.

ACCELERATED ROTATION

I-6.1. Angular Acceleration: We now take up angular acceleration of a circulating particle as also of a revolving rigid body of finite size. You know that a rigid body is one which retains both its shape and size unaltered under forces however large they may be. It is an ideal solid, an abstraction introduced for simplicity, but the solids we generally use are fair approximations to rigid bodies,

Such bodies may rotate about a point or an axis inside the body or outside it. Spin axis of the earth is through it while the sun taken as a point round which it moves, is outside it. A spinning top rotates about central axis through it, doors revolve about an axis through its hinges, a stone tied to a string does so about your finger as axis. By the term rotation, we shall understand that of a finite body either with uniform angular speed or acceleration that may be constant or variable. As we have done for linear acceleration, we consider here only *constant* angular acceleration.

Time rate of change of angular velocity is angular acceleration. When the time-interval considered, is very small it becomes instantaneous acceleration. In symbols

Average acceleration is $\alpha = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\delta\omega}{\delta t}$ (I-6.1.1)

and instantaneous acceleration

$$\alpha = \underset{\delta t \to 0}{\mathrm{Lt}} \frac{\delta\omega}{\delta t} = \frac{d\omega}{dt} \qquad \text{(I-6.1.2)}$$

$$= \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2} \qquad \text{(I-6.1.3)}$$

Unit of angular acceleration is rads/s^2

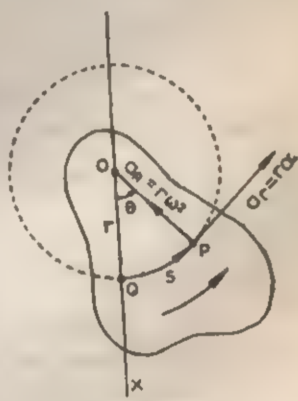


Fig. I-6.1

Relation between Linear and Angular acceleration: Let a finite body rotate about a perpendicular axis through O (fig. I-6.1), a point within the body, with *constant angular acceleration* α. A particle Q in it then moves along a circle of radius r. Its linear velocity v must be *tangential* to the arc at Q.

In moving to the point P, v must change by dv in time dt. So its linear acceleration is

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(\omega r) = r\frac{d\omega}{dt} + \omega\frac{dr}{dt}$$

Now in circular motion r is const.* So

$$a_t = r\frac{d\omega}{dt} = r\alpha \qquad \text{(I-6.1.4)}$$

∴ Linear acceleration = radius vector × angular acceleration. The particle simultaneously experiences a normal acceleration, the centripetal one, of magnitude

$$a_n = r\omega^2$$

that you already know

I-6.2. Rotation with Constant Angular Acceleration : The simplest of accelerated angular rotation is when angular acceleration is constant. From definition we have

$$\alpha = d\omega/dt = \text{const.}$$

$$\text{or } \int_{\omega_0}^{\omega} d\omega = \alpha . \int_0^t dt$$

$$\text{or } \omega - \omega_0 = \alpha t \text{ or } \omega = \omega_0 + \alpha t \qquad \text{(I-6.2.1)}$$

as the angular velocity grows from ω_0 to ω in the time-interval t.

Again, since $\omega = d\theta/dt$ we have

$$\int_0^{\theta} d\theta = \int_{\omega_0}^{\omega} \omega . dt = \int_0^t \omega_0 dt - \int_0^t \alpha t \, dt$$

$$\therefore \quad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \qquad \text{(I-6.2.2)}$$

Finally we may write

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta} . \frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

$$\text{or } \int_0^{\theta} \alpha . d\theta = \int_{\omega_0}^{\omega} \omega . d\omega$$

$$\text{or } \alpha . d\theta = \tfrac{1}{2}(\omega^2 - \omega_0^2) \text{ or } \omega^2 = \omega_0^2 + 2\alpha\theta \qquad \text{(I-6.2.3)}$$

These equations are identical replicas of those for linear accelerated motion (equations I-1.7.1 to I-1.7.3)

* When motion occurs along an ellipse as for a planet round the sun, a satellite about a planet or an electron round the atomic nucleus, r is no longer constant but a variable (See fig. II-1.12)

Example I-6.1 *A wheel is rotating about its axis with an angular velocily of 5 revolutions per sec. At the end of 40 secs the velocity drops to 4 revolutions per sec. If in the next 20 secs the angular velocity increases to 8 revolutions per sec find th number of revolutions the wheel makes during the minute, assuming that that the changes have been uniform*

Solution : During the first 40 secs the average angular velocity is $(5+4)\div 2 = 4.5$ rps (revolutions per second). During the next 20 secs it is $(4+8)\div 2=6$ rps. The total number of revolutions executed in the minute is therefore, $40\times 4{\cdot}5+20\times 6=300$

Ex I-6.2 A wheel has a constant angular acceleration of 2 rad/sec². It turns through an angle of 500 radians in 5 sec. If it started from rest, what was its angular velocity at the beginning of the 5 sec interval ?

Solution : Let ts be the time which passed between the start of the motion and the beginning of the interval. At the end of the interval the time is $(t+5)$ sec. In ts the wheel has turned through an angle $\theta=\frac{1}{2}\alpha t^2$; in $(t+5)$ sec it turned through $\theta'=\frac{1}{2}\alpha(t+5)^2$. Hence in the last 5 sec it turned through $\theta'-\theta=\frac{1}{2}\alpha(10t+25)$. Since $\theta'-\theta=200$ and $\alpha=2$, we get $t=47.5$ sec. The required angular velocity is therefore $\omega=47{\cdot}5\times 2=95$ rad/sec.

Problem.. A body revolving with a const angular acceleration of 5 rad/s² starts with an angular uelocity of 300 r.p.m. Find the time before it attains 940 r.p.m. of angular velocity and also the time before which 120 π revolutions are completed. (*Ans.* 4π s, 8π s)

I-6.3. Angular moment on Moment of momentum : Let a particle of mas m move with a linear velocity v ; it will have a *linear momentum of mv* in the direction of v. Let the perpendicular distance from the line of action of v be d (fig. I-6,2a). Then the product **d × mv** is called the moment of momentum or angular momentum of the particle about the point O which is

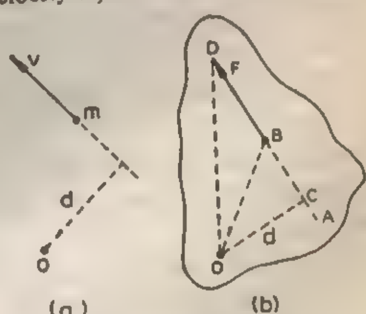


Fig. I-6.2

$$\mathbf{L} = \mathbf{mv} \times \mathbf{d}$$

$$\text{or} \quad L=mvd \qquad \text{(I-6.3.1)}$$

Angular momentum **L** is a vector quantity like angular velocity and angular acceleration and a very important quantity in rotational motion. Like its linear counterpart **angular momentum is a conserved quantity** and hence its importance. In fig. I-6,1 if the

particle at Q is of mass m, its angular momentum about O the center of the circle, will be mvr when v is the linear velocity at Q. Since $v=\omega r$ we have $L=mvr=mr^2\omega$ (I-6.3.2)

In both the above equations L is the magnitude of angular momentum. The dimension of angular momentum is therefore ML^2T^{-1} and its unit gcm^2s^{-1} or kgm^2s^{-1}.

Ex. I 6.3. *An electron in a hydrogen atom revolves round the proton once in 15×10^{-15} s along an orbit of radius $0.6\mathring{A}$. If its mass is 9.1×10^{-28} g find its angular momeutum. $1\ \mathring{A}=10^{-8}$ cm*

Solution : $L=mr^2\omega=mr^2.2\pi/T$

$$=9.1\times10^{-28}\ g\times(0.6\times10^{-8})\ cm^2\times2\pi/15\times10^{-15}\ s$$
$$=1.36\times10^{-27}\ gcm^2\ s^{-1}$$

Note : (1) In explaining the origin of line spectra of the hydrogen spectra, Bohr postulated the possible electron orbits to be such that there, the angular momentum of the electron is *quantised* i.e. must be *an integral multiple of $h/2\pi$* where h, the Planck's const which like c the velocity of light in vacuum and G, the gravitational constant is another universal constant. In this case we have $L=nh/2\pi$ where $n=1, 2, 3,$ ......etc.

(2) Kepler's second law of planetary motions (§II-1.15, is found to be a consequence of conservation of angular momentum.

Marvel at the scales involved ; mass of an electron of the order of 10^{-31} kg, that of our earth of about 10^{+24} kg, the electron orbit of radius the order of 10^{-10} m, that of earth of 10^{11} m yet both are treated as point masses.

I-6 4. Moment of a Force about a point This is defined as follows :—

Moment of a force about a point is the product of the force and the perpendicular distance between the point and the line of action of the force.

In fig I-6 2b AD represents the line of action of the force **F** (=BD) at a perpendicular distance of OC from a point O. Then the moment of **F** about O is

$$\mathbf{M}=\mathbf{F}\times\mathbf{d} \qquad \text{(I-6.4.1)}$$

M here measures the turning effect of **F** about O and is a vector. If O lies anywhere on AD, the line of action of **F**, the moment vanishes and **F** can not produce any rotation about O.

The product $\mathbf{F} \times \mathbf{d}$ i.e. $DB \times OC$ represents twice the area of the $\triangle OBD$ obtained by joining the ends of the force vector to the point of rotation and this is the moment of **F** about O.

If a number of (coplanar) forces act on the particle together, none of the lines of action passing through O, then the resultant moment about O is the algebraic sum of moments individually developed i.e.

$$\mathbf{M} = \mathbf{m}_1 + \mathbf{m}_2 + \mathbf{m}_3 + \ldots\ldots \text{ or } \mathbf{R} \times \mathbf{d} = \mathbf{F}_1 \times \mathbf{d}_1 + \mathbf{F}_2 \times \mathbf{d}_2 + \mathbf{F}_3 \times \mathbf{d}_3 + \ldots\ldots$$

when **R** is the resultant force and **d** its moment arm.

I-6 5 A. **Moment of a Force about an Axis :** In fig I-6.3 we consider a plane horizonital lamina rotating about a vertical axis AOB. Let a force P be acting *along the lamina* at an angle θ to the

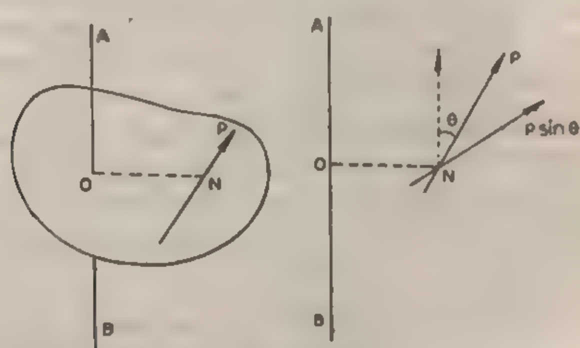


Fig. I-6.3

line perpendicular to the horizontal radius vector or moment arm ON. Then $P \sin \theta$ is the effective force component along the lamina producing the rotation. Then the moment of P about O is given by

$$M = P \sin \theta \, ON = P \sin \theta . d = \vec{P} \times \vec{d} \qquad (1\text{-}6.5.1)$$

Note that moment of a force is a vector product of two vector those of force and displacement for their magnitudes are multiplied by the sine of angle between them (See I-2.12.3) The same vectors multiplied by the cosine of the included angle gives work, a scalar product (chap I-8).

Next we consider a heavy door hinged about a vertical axis AB (fig. I-6.4). Experience tells us that a smaller force can swing the door open when it is applied *perpendicularly* to a point closer to the ring as shown than when force is applied closer to the axis AB. If we measure the two forces by a spring balance we find that *to produce a slow uniform rotation of the door about the axis*

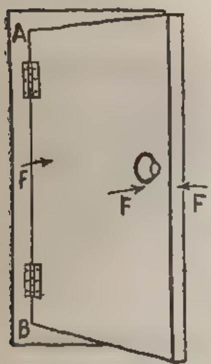


Fig. I-6.4

the force × distance of its point of application = const

The moment arm **d** here also called the *lever arm*, is as above, the perpendicular distance of the point of application of a *perpendicular* force F from the rotation axis AB. As above, the product $\mathbf{F} \times \mathbf{d}$ is the **moment** of the force or **torque** *above the given axis. Torque* however also means the moment of force about a point.

So the *rotational effect of a force is measured* not only by its own magnitude but also *by that of its distance about the given axis.* A force produces the same turning effect when applied on a body at a certain distance from the rotation axis as half that force applied at double the distance.

If *a force is applied along the door* as shown by **F** at the right extreme, *its line of a action passes through the axis and no rotation* can result for **d** the moment arm is zero then.

Inclined force : If F is applied not perpendicular to the door but at an angle then the effective force becomes smaller (for $F \sin \theta$ is always less than F; $\sin \theta < 1$) and so does the torque. Then we resolve it in components $F_x (= F \cos \theta)$ along and $F_y (= F \sin \theta)$ perdendicular to the plane of the door. F_x produces no turning, all of which comes from F_y and the effective moment or torque becomes

$$\mathbf{F}_y \times \mathbf{d} = F \sin \theta \times d \qquad \text{(I-6.5.2)}$$

B. Moment as a Vector. Being a *cross product* of two vectors, moment is itself a vector, acting along the axis of rotation. Observe that an *anticlockwise rotation* drives the axis along the $+ve$

direction of x-axis and a clock-wise rotation drives the same along the $-ve$ direction of x-axis. Hence conventionally, moments that tend to produce anticlockwise rotation are taken to be positive and those tending to anticlockwise motion, are taken to be negative (fig. I.6.5).

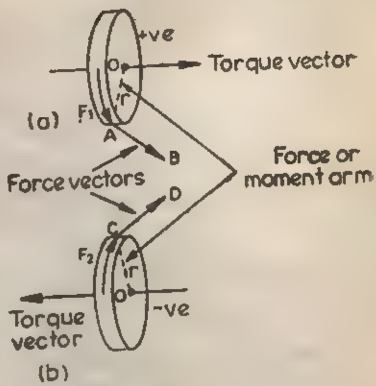


Fig. I-6.5

Let a plane lamina hang from a horizontal nail at O in a wall (fig I-6.6) and two small nails are on it at A and B with two pieces of string hanging from it. If you pull that from A by a tension F_1, the lamina *moves anticlockwise* providing a **positive** moment while tension F_2 on the string form B produces a *clockwise* motion and a **negative** moment.

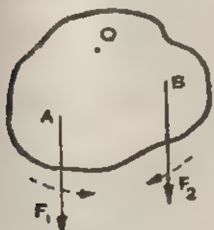


Fig. I-6.6

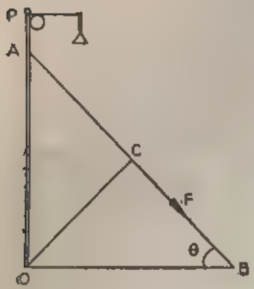
Ex. I-6.4. *A 10 kg load hangs from a string wrapped round a vertical wheel 10 cm in diameter. Find the torque exerted about the axle of the wheel.*

Solution : Refer to fig I-6.5a and imagine F_1 to be provided by the load. Then the torque $L = F_1 \times r = 10 \times 9.8\ \text{N} \times 0.05\ \text{m} = 4.9$ N-m, (consider r in the fig. to be horizontal and F_1, vertical.

C. **Equilibrium of Moments.** Two moments or torques balance each other when they are equal and opposite. Forces need not be equal. Consider the forces F_1 and F_2 in fig I-6.6. The axis of rotation at O is perpendicular to the plane of the lamina and F_1, F_2 forces in its plane. If the moment arms be respectively a and b about O, then there will be no rotation when $F_1 \times a = F_2 \times b$ i.e. the moments be equal and we have seen above that they are opposite.

Ex. I-6.5. *Where would you fasten a rope on a vertical lamp-post so that you can overturn it most easily by pulling at the other end of the rope from the ground level ?*

Solution : Let OP be the pole AB($=l$) the rope, A the point of fastening of the rope and B the position of your hand pulling with a force F. Let $\angle ABO$ be θ and OC the normal on AB.



Then $OC = OB \sin\theta$ and $OB = AB \cos\theta$. Hence

$$OC = AB\cos\theta . \sin\theta = \tfrac{1}{2}l \sin 2\theta$$

So the moment of F about $O = F\times \frac{1}{2}l \sin 2\theta$

For the maximum moment we have

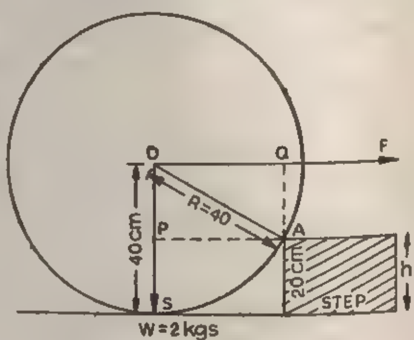
$$\sin 2\theta = 1 \text{ i.e. } \theta = 45^\circ$$

$$\text{and } OA = AB \sin\theta = l \sin 45^\circ = l/\sqrt{2}$$

So the fastening point **A** must be at a height equal to $l/\sqrt{2}$.

Ex I 6 6. *A 2 kg wheel of radius 40 cm rests against a step half as high. Find the minimum horizontal force that must be applied as shown will force the wheel climb up the step.* [I.I.T. '76]

Solution : F acts through O the center of the wheel, of which the weight acts vertically downwards through the same point. Perpendiculars AP and AQ are dropped on the vertical radius and the line of action of F.



When the moment of F about A exceeds that of W about the same point will the wheel mount the step So the condition for minimum F is

$$F\times AQ = W\times AP$$

$$\text{or } F\times \sqrt{40^2-2^2} \text{ cm}$$

$$= 2 \text{ kgf}\times \sqrt{AO^2-OP^2} \text{ cm} = 2 \text{ kgf}\times \sqrt{40^2-20^2}$$

$$= 2 \text{ kgf}\times 20\sqrt{3} \text{ cm}$$

$$\therefore\ F = 2\sqrt{3} \text{ kgf}$$

Prob. Show that the condition to be fulfilled to push a heavy wheel of radius R and weight W above an obstacle of height h by a horizontal force F though its center is

$$F > W\sqrt{\frac{2hR-R^2}{R-h}}$$

I-6.6A. Couples *A pair of equal unlike parallel forces whose lines of action on a finite body are not the same, is said to form a couple.* Its effect is to produce rotation. When we turn a door knob, wind a watch spring or an old-fashioned wall clock key, use a screw-

driver or a cork borer, spin a top or operate a tap (fig I-8. a) we apply a couple in each case with our fingers. A pair of bullocks or men turning a vertical spindle in a village oil-press apply a couple by a long horizontal pole.

In fig. I-6.7 a couple is formed by a pair of forces F and $-F$ as shown acting at the two points A and B of an extended body. The $-$ve sign merely implies that one of the forces are opposite to the other in sense.

Moment or **torque** of a couple is the algebraic sum of the moments of the component forces (forming the couple) about any point in the plane of the couple. If O be such a point and OC and OD their *lever arms* $F \times OC$ and $F \times OD$ are the moments of the components about O. From the figure we find that both tend to produce rotation in the same sense. Hence the algebraic sum of moments is $F(OC+OD)=F \times l$. So the **moment or torque of a couple is the product of either force and the perpendicular separation between their lines of action.** Torque of a couple like the moment of force measures its ability to produce rotation.

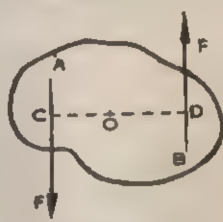


Fig. I-6.7

Axis of the couple is a line perpendicular to the plane in which the forces lie. When a couple acts on a freely movable body its axis of rotation passes though its *center of gravity*. The torque is +ve when looking to the body along its axis of rotation, it appears to be anticlockwise (fig I-6.5). It is $-$ve when rotation appears to be clockwise. We have indicated before, that torque is a vector along the direction of its axis.

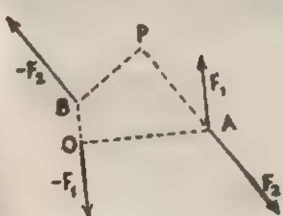


Fig. I-6.8(a)

B. Equilibrium of Couples : A force acting singly cannot balance a rotation due to a couple. To do so another couple is necessary. Two couples would be in equilibrium (i.e. would not rotate a body) if their couples are equal and opposite. In fig. I-6.8(a) $(F_1, -F_1)$ and $(F_2, -F_2)$ represent a pair of *coplanar couples* acting on the same

rigid body. Let the +ve forces meet at A and the −ve forces at B ; let the lever arm of the first couple be AO and BP that of the second. If now $AO \times F_1 = BP \times F_2$ the torques will be equal. If further, their senses be opposite no net rotation of the body would occur.

A common balance while *weighing equal masses* (I-6.8b) provides a simple example of balancing couples. Let the balance-arm AB be pivoted on a knife edge K at its mid-point The standard mass W and the experimental one M hung from the two ends of this *equal-arm* balance are attracted by the earth with forces *Wg* and *Mg* respectively. Each of them produce equal reactions at K thus producing a pair of opposite couples. The lever arms being equal the couples are equal when AB remains horizontal. So for that position the forces and hence the masses would be equal.

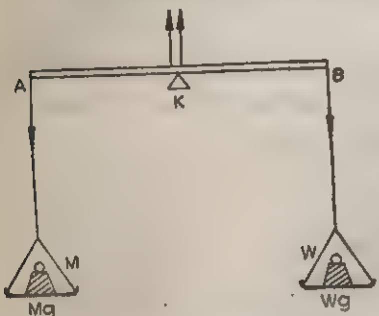


Fig. I-6.8(b

I-6.7. **Relation between angular momentum and torque**

For simplicity, let us suppose that a particle of mass m is moving with velocity v The force on the particle is F (fig I-6.2) both the same direction. O is a point at a perpendicular distance d from the line of action of F or v.

The magnitude of the angular momentum of the particle about O is $F = mvd$ The moment of the force F about O is $\mathbf{M} = F \times d$. Now $F = ma$ mv/t or $F.t = mv$.

$$\therefore \quad L = mvd = Ft.r = Mt \text{ or } L/t = M \qquad \text{(I-6 7.1)}$$

In words, we may say

The time rate of change of angular momentum of a particle about a point is equal to the moment of the force (the torque) about the point.

In rotational motion, we get an important result from eq. I-6.7.1

$$M = \frac{L}{t} = \frac{mvr}{t} = \frac{mr^2\omega}{t} = mr^2 = \left(\frac{\omega}{t}\right) = mr^2\alpha \qquad \text{(I-6.7.2)}$$

where $\alpha = \omega/t$ = angular acceleration of m. So we find that **a torque (or the moment of a force) causes angular acceleration.**

In linear motion, a force gives rise to linear momentum and linear acceleration. In rotational motion, a moment or torque gives rise to angular momentum and angular acceleration. *Both linear momentum and angular momentum are conserved quantities.*

I-6.8. **Moment of Inertia :** The quantity mr^2 is said to be the *moment of inertia.* The term is used specifically for a finite rigid body rotating about a perpendicular axis through any point through it. Then m is the mass of a particle in it at a distance r from the axis of rotation ; since the body is made up of innumerable particles of masses m_1, m_2, m_3......etc at distances r_1, r_2, r_3......etc from the axis of rotation, its moment of inertia about that particular axis is

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots m_nr_n^2 = \Sigma mr^2 = MK^2 \quad \text{(I-6.8 2)}$$

where $M(=\Sigma m)$ is the mass of the whole body and $K(=\sqrt{\Sigma r^2})$, the radius is gyration of the body. K represents the distance from the axis of rotation such that with the whole body reduced to a particle of mass M and moving in a circle of radius K has the same $K.E$ as the body itself revolving as a whole around the given axis.

The equation I-6.7.2 then can be alternatively expressed as

$$\text{Torque} \quad M = I\alpha \quad \text{(I-6.8.2)}$$

with which you compare $F=ma$ in linear motion. So that I, the moment of inertia in rotational motion, plays the same role as m the mass, plays in linear motion.

Thus the **moment of inertia** of a body can be *defined as the ratio between torque and the angular acceleration* $(I=M/\alpha)$ or *the torque required to produce unit angular acceleration.* Note that it is the ratio of cause (torque) and effect (angular acceleration) just as mass $(m=\mathbf{F}/\mathbf{a})$ is. Between these two is a difference that mass is constant for a body but its M.I. varies with the position as well as the direction of rotation

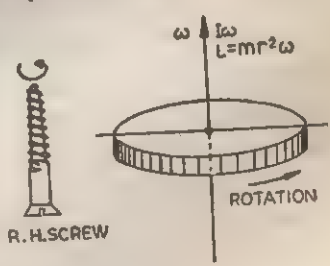


Fig I-6.9

axis. Further mass is a scalar, while M. I. is not, neither it is a vector ; it is a **tensor.**

But as we have repeatedly stressed that angular velocity, angular momentum and torque are all vectors. They have the direction of the motion of the tip of a right handed screw when it is driven forward. Fig. I-6.9 shows the relation between the direction of rotation and all these vectors. Note that by substituting I-6 8.1 in I-6.7.2 we get

$$L = I\omega. \qquad \text{(I-6.8.3)}$$

I-6 9. Principle of Conservation of Angular momentsum

In eqn I-6.8.2. we found that torque = Moment of inertia × angular acceleration i.e. $M = I\alpha$. This relation can be put as

$$M = I\frac{d\omega}{dt} = \frac{d}{dt}(I\omega) \qquad \text{(I-6.9.1)}$$

Again, from I-6.8.3 we get angular momentum $L = I\omega$. Hence

$$M = \frac{d}{dt}(I\omega) = \frac{dL}{dt} \text{ or } M.dt = dL \qquad \text{(I-6.9.2)}$$

Note that $M.dt$ is analogous to $F.dt$, the impulse of force in linear motion. Hence $M.dt$ is said to be the *angular impulse* or the *impulse moment*, which equals the change in angular momentum of the rotating body.

From the relation $M = dL/dt$, we may say that *the external unbalanced torque equals the time rate of change of angular momentum*—a statement equivalent to Newton's second law of motion.

If this torque vanishes, $dL/dt = 0$ i.e. the angular momentum is constant—the rotational analogue of Newton's First Law of motion. Thus if no external torque acts on a rotating body, its angular momentum does not change i.e. is conserved. This is the Principle of Conservation of Angular momentum and ranks with the principles of conservation of linear momentum and energy as one of the most fundamental relations in mechanics.

A gymnast, a dancer spinning on her heels, a diver all utilise this principle for their particular performances. Refer to fig I-5.6(a) where a gymnast on a spinning platform is found to slowly bring in his outstretched hands carrying dumbells close to his body ; he is found

to spin faster. Fig I-6 10(a) shows the second performer, a spinning ballerina, who is found to spin faster as she closes in her outstretched

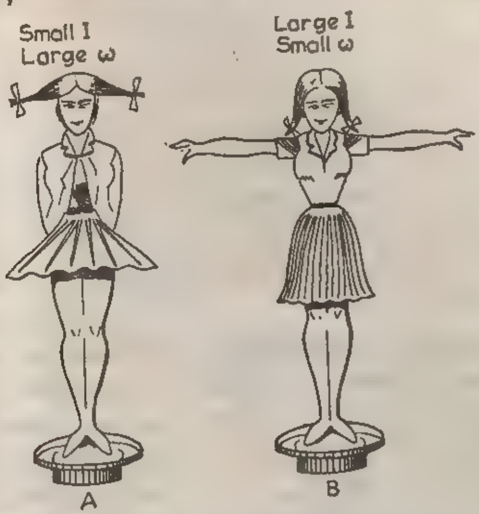


Fig. I-6.10(a)

hands; her flying pigtails indicate her fast spin which however droops as she slows down as she stretches out her hands and palms. Fig. I-6.10(b) shows a diver jumping with his hands and feet extended with slower rotations; his angular speed increases sharply as he doubles himself up. In all these cases with outstretched limbs the moment of inertia ($I = MK^2$) is larger and angular velocity (ω) smaller. As the limbs are drawn in, the radius of gyration (K) and hence I diminishes; so ω has to increase to keep $I\omega$ constant and it does so in all these cases. For the diver, note that however he spins and twists, his C.G. would describe a parabola. We stall return to this point in I-7.10.

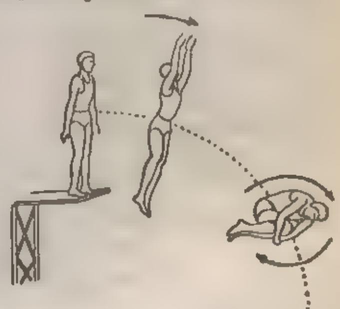


Fig. I-6.10(b)

The earth. spinning on its axis, maintains a constant angular momentum. So long as its moment of inertia about the spin axis does not change, its angular velocity will remain constant. Hence the angular rotation of the earth forms a convenient basis for our time

measurement. If however the earth contracted to half its radius, the day would shorten to 6 hours only to maintain angu'ar momentum constant.

But in the course of aeons new mountains rise above the surface and rivers carry millions of tons of silt towards the equator. Such causes increase the moment of inertia of the earth slightly and reduce its angular speed. Tidal friction dissipates the rotational kinetic energy of the earth into heat and causes it to slow down. It has been estimated that the earth is slowing down at the rate of 0.001 sec per century. The pull of the sun and the moon heaps up water under them thus producing a bulge on the earth's surface, thus raising K and so I and hence diminishing ω or angular velocity and so lengthen the day, very very slowly.

I-6 10. Correspondence between linear and rotational motion In the previous sections we found a striking correspondence between relations governing rotational motion and those governing linear motion. These are such that any equation for linear motion can be converted into the corresponding equation for rotational motion by replacing mass (m) by moment of inertia (I), linear velocity or acceleration by angular velocity or acceleration, and force (P) by torque (T) The correspondence is set forth in the table below.

Linear motion	Rotational motion
Linear displacement (s)	Angular displacement (θ)
" velocity (v)	" velocity (ω)
" acceleration (f)	" acceleration (a)
Mass (m)	Moment of Inertia (I)
Linear momentum $= mv$	Angular momentum $= I\omega$
Force (P)	Torque (T)
$P = mf$	$T = Ia$
Linear kinetic energy $= \frac{1}{2}mv^2$ =	Rotational kinetic energy $= \frac{1}{2}I\omega^2$
Work done $= Ps$	work done $= T\theta$

Note A point of distinction should, however, be remembered. While the mass of a body is constant, the moment of inertia of a body changes with the position of the axis of rolation relative to the body. Corresponding to each position of the axis the body has a different value of the moment of inertia.

I-6 11. Newton's Laws for Rotational motion : Pushing the correspondence further, the following laws taking after Newton's basic laws of motion may be formulated governing rotational motion :

First Law: A rotating body will continue to do so about an *unchanging axis* unless acted upon by an external unbalanced torque.

Second Law: Time rate of change of angular momentum is proportional to the externally impressed unbalanced torque and takes place in the direction in which the torque acts.

Third Law: To every torque applied on a body there is an equal and opposite torque acting on the agent.

Discussion: The first law embodies the principles of *rotational inertia* and *directional inertia* as exemplified by the spinning earth, a top or a gyroscope. They spin without practically any external torque disturbing them.

So long as men have kept records very little change in the length of day and night i.e. rate of *spin of earth* has been noticed. For very long periods travellers and sailors have found an unfailing direction locator in the Pole Star at night because it has never changed its position in the heavens. This constancy is due to the fact that the earth's spin axis passes very close to the pole star. Both the observed phenomena illustrate the principles of rotational and directional inertia.

Look at a *spinning* top. So long as it spins fast its axis remains unchanging in the direction, in which you had planted it on the ground. Only when air resistance slows it down you will observe its axis altering direction i.e. wobbling, technically called *precessing*. The earth is likened to a giant top or gyroscope.

A **gyroscope** is simply a heavy disc revolving fast about a perpendicular axis through its center. As it has directional inertia it will recover direction if the axis is slightly disturbed. This property has been utilised in *gyro-compasses* to maintain steady direction by astronauts in spaceships and sailors anywhere in ocean liners. Its axis is made to point north and south in ships and planes. It also guides a torpedo to the target ship, by having the axis of rotation set towards it.

The barrel of a rifle is so spiralled that a *rifle bullet* leaves it with fast spin. Directional inertia enables the bullet to cover much longer distances in a straight line than that from an ordinary gun.

Moment of inertia also referred to as *rotational obstinacy* which derives its dynamical or physical definition from the second law, is the most important quantity in rotational motion. Large heavy discs rotating about a perpendicular axis called **flywheels** are widely used in various devices in maintaining angular speeds uniform, particularly in face of sudden disturbances in rotating parts of machinery.

Gyroscope (fig I-6.11) consists essentially of a balanced heavy wheel mounted in a gimbal ring so as to spin freely about its horizontal spin axis B_1B_2. When the disc spins very fast it exhibits "gyroscopic stability," i.e. rotational obstinacy or resistance to change of direction of spin axis.

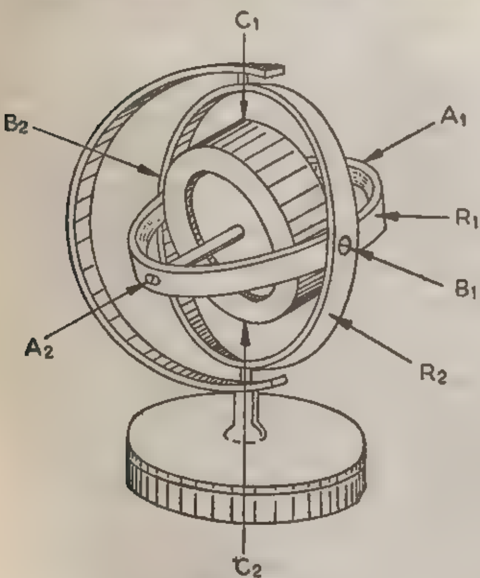


Fig. I-6.11

If it is mounted in three gimbal rings (as shown) and kept spinning fast electrically with small friction at the pivots, then it will maintain its spin axis unchanged in direction. If such a spinning "gyro" is placed on a table with its spin axis horizontal pointing east-west, it will show you that the earth rotates; you will observe that after 6 hours the axis becomes vertical, direction reverses after 12 hrs and it completes one revolution in 24 hours. This is because, the angle between the spin axis and the vertical line changes as the earth rotates; for with the spin of the earth the vertical at the place changes in relation to space but the *gyro* keeps its axis fixed in space.

In the fig. I-6.11, A_1A_2 represents the horizontal, C_1C_2 the vertical while B_1B_2, the spin axis.

I-7

STATICS

I-7 1. Introduction : Statics deals with particles and bodies at rest and in equilibrium. A *system of particles* is a collection of particles that are isolated from *everything else*—the so called closed system. Bodies considered will be *rigid* ones i.e. collection of infinite number of particles of unchangeable separations as we have learnt in the last chapter. A particle or body is said to be *at rest* when it has *no velocity* linear or angular relative to its reference frame i.e. its surroundings. They are *in equilibrium* if there is *no acceleration* i.e. when subjected to balanced forces.

A person in a moving car is at rest w.r.t. his fellow-passengers though he is really in motion. A hanging fan or a car in uniform linear motion is in equilibrium.

Motion may be *linear* or *rotational* or a combination of both. A body *at rest* and *in equilibrium* must have neither. An unbalanced force produces linear motion, an unbalanced torque, a rotational motion in both cases accelerated.

Remember *a particle may be be rest but not in equilibrium* ; e.g. a rising particle when it is at the top of its climb or a pendulum bob at the end point of its swing—there is acceleration but no velocity. Again *a body may be in equilibrium without being at rest* i.e. the above cited passenger in a car in uniform linear motion or the pendulum bob crossing its mean position—there is velocity but no acceleration in either case.

I-7.2 Equilibrium of a body : It occurs when the resultant force i.e. acceleration on it vanishes. It may be (i) at rest (ii) in uniform linear motion or (iii) rotating at a constant speed. For simplicity we consider all the forces to be in the same plane i.e. coplanar.

If a number of coplanar forces act upon a particle, a single resultant may be found (by polygon of forces). When they act on a body they may add up to *either* a resultant **force** through its **center of mass** (where the entire mass may be taken to be concentrated ; see §I-7.10) *or* to a pair of equal parallel unlike forces producing a **torque**. For equilibrium both must vanish. The concept of equilibrium of a force is very important in physics.

Conditions of Equilibrium : (1) The vector sum of all the Forces acting on the body must vanish Then there would be no linear acceleration.

(2) **The torques about any axis within the body or without must be balanced ;** the sum of clockwise torques must cancel out the sum of anticlockwise ones. This latter is also called the **Principle of Moments,** the sums taken being *algebraic* in this case. When this is fulfilled there would be no rotational acceleration.

These are the two general conditions of equilibrium under a set of coplanar forces.

Equilibrant : It is that force or couple that nullifies the action of forces and torques on a body and keeps it in equilibrium. Since it is a *single* force or couple an *equilibrant must oppose and equal the resultant* of coplanar forces or couples acting on the body.

I-7.3. Discussions on the First Condition :

(1) *Equilibrium under two forces* can be obtained (in the light of above discussions) when they are (a) equal in magnitude (b) opposite in direction (coplanar) and (iii) collinear i.e. acting along the same line.

If this last condition is unfulfilled, the system of forces become a couple generating angular acceleration.

(2) *Equilibrium under* **three non-parallel forces** is obtained when they are *coplanar and concurrent* i.e. in the same plane and the line of action passing through the same point.

Since any two coplanar forces are concurrent when non-parallel, and form a resultant (parallelogram law), for equilibrium the third force must be the *equilibrant* i.e. equal, opposite and concurrent to that resultant. Refer to fig I-7 1 (a) where three forces F_1, F_2, F_3 act through the center of mass of a body at O and are in equilibrium. F_1 and F_2 add up to a resultant directed to the right and must be nullified by F_3 acting at the same point opposite in direction and equal in magnitude. Thus the three forces in equilibrium are

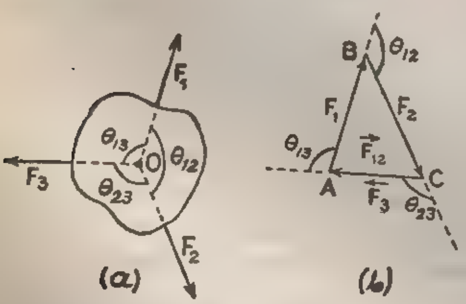


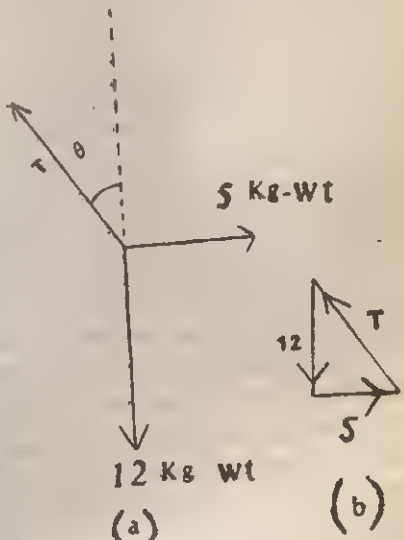
Fig. I-7.1

coplanar and concurrent. In fig. (b) F_{12} represents the resultant and F_3 the equilibrant.

A. **Triangle of Forces rule :** From this discussion as well as fig. I-7.1(b), it follows that *three non-parallel forces in equilibrium may be represented in magnitude and direction by three sides of a tringle taken in order.*

Thus if we lay down the vectors F_1, F_2 and F_3 successively each one being laid in the proper direction from the end point of the vector last drawn we get a *closed triangle.* This is the triangle of forces rule.

Example I-7 1. *A body weighing 12 kg is suspended vertically by a string. Find the tension in the string when the body is pulled to one side by a horizontal force of 5 kg (See fig)*



Solution : The body is in equilibrium under the action of three forces, namely, (*i*) its weight of 12 kg acting vertically downward, (*ii*) the horizontal force of 5 kg. and (*iii*) the tension T in the string. Since the body is in equilibrium, the resultant of any two forces acting on it must be equal and opposite to the third force. Hence T is equal and opposite to the resultant of the perpendicular forces (*i*) and (*ii*). Its magnitude is $\sqrt{12^2+5^2}=13$ kgf Again we may consider the dotted line to be the resultant of T and the horizontal force, which is neutralised by the downward vertical force.

In (b) is shown the triangle of forces, vectors in equilibrium laid out in suceession, one starting from the end point of the last.

B. **Lami's theorem** states that when a body is held in equilibrium by three forces, each force is proportional to the sine of the angle between the other pair. Let a force P replace the pull 5 kg-wt and W, the 10 kg-wt in the above figure. Then from Lami's theorem we shall have

$$\frac{T}{\sin\theta_{WP}}=\frac{P}{\sin\theta_{TW}}=\frac{W}{\sin\theta_{TP}}$$

It follows then $\dfrac{T}{\sin 90^\circ}=\dfrac{P}{\sin(180^\circ-\theta)}=\dfrac{W}{\sin(90^\circ+\theta)}$

or $\dfrac{5}{\sin\theta}=\dfrac{12}{\cos\theta}$ $\therefore$ $\tan\theta=\dfrac{5}{12}$ or $T=13$ kgf

The theorem is a consequence of (*i*) *law of sines* in trigonometry (the ratio of any two sides of a triangle is equal to the ratio of the sines of their opposite angles) and (*ii*) the fact that when three forces on a particle are in equilibrium they form a closed triangle (fig. I-7.1 b). In that figure we see that from the law of sines

$$\frac{F_1}{F_2}=\frac{\sin(180^\circ-\theta_{23})}{\sin(180-\theta_{13})}=\frac{\sin\theta_{23}}{\sin\theta_{13}}$$

whence it follows that

$$F_1 : F_2 : F_3=\sin\theta_{23} : \sin\theta_{13} : \sin\theta_{12}$$

or $$\frac{F_1}{\sin\theta_{23}}=\frac{F_2}{\sin\theta_{13}}=\frac{F_3}{\sin\theta_{12}} \qquad \text{(I-7.3.1)}$$

To summarise : For a body to be in equilibrium under *three non-parallel forces*,

(*a*) the forces must lie in a plane, *i.e.*, be coplanar ;

(*b*) the forces must meet at a point, *i.e.*, be concurrent ;

(*c*) the force polygon must be a closed triangle, *i.e.*, the forces must satisfy Lami's theorem.

C. Yet in another way the conditions of equilibrium proposed above, can be mathematically stated—that of **Resolution of Forces and Algebraical Addition of Components.**

Confining ourselves to *coplanar forces* only, we resolve each force into components F_x and F_y in convenient x and y directions in the plane. Then for equilibrium, since the resultant is to disappear, we must have

$$\Sigma F_x=0, \text{ and } \Sigma F_y=0 \qquad \text{(I-7.3 2)}$$

If T stands for the moment of a force (*i.e.*, torque) about some point in the plane, then condition 2 states that

$$\Sigma T=0 \qquad \text{(I-7.3.3)}$$

Ex I 7.2. *a body weighing 10 lb is suspended from a fixed point by a string of length 5 ft. It is pulled aside by a force of P lb wt, and remains in equilibrium when the string is inclined at 30° to the vertical. Find the value of P and the tension T in the string.*

Solution : (a) Refer to the diagram of the first example ; replace the force 5 kg-wt by a force of P lbf, θ by 30° and draw a vertical downward dotted line from the top of T vector with which it will make an angle of 30°.

The body is in equilibrium under three concurrent forces P, T and 10 lb wt. Resolving vertically we get $T\cos 30^\circ - 10$ lb wt $= 0$, and resolving horizontally $T\sin 30^\circ - P = 0$. So we have

$$T = 20/\sqrt{3} \text{ lb wt and } P = T/2 = 10/\sqrt{3} \text{ lbs wt.}$$

(b) Again from Lami's theorem

$$\frac{T}{\sin 90^\circ} = \frac{P}{\sin(180^\circ - 30^\circ)} = \frac{10 \text{ lbs wt}}{\sin(90^\circ + 30^\circ)}$$

$$\therefore \quad T = \frac{P}{\sin 30^\circ} = \frac{10}{\cos 30^\circ} \quad \therefore \quad P = 10 \tan 30^\circ = 10/\sqrt{3} \text{ lbs-wt}$$

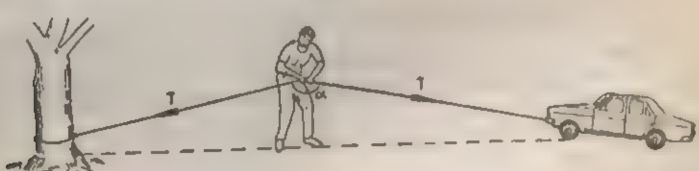
and $\quad T = 10/\sqrt{3}/2 = 20/\sqrt{3}$ lbf

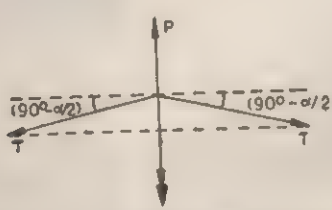
The answers are identical.

Problem : In the above problem what will be the value of P, if in the equilibrium position, there is another force of 2 lbf acting on the body vertically upwards.

(*Hint :* Draw the necessary diagram We have now 4 concurrent forces on the body, namely, 10 lb wt acting vertically downward, 2 lb wt acting vertically upward, P acting horizontally, and T acting upward at an angle of 30° to the vertical. Resolve the foeces vertically and horizontally. The algebraic sum of the vertical forces $= 0$; so also for the horizontal forces. **Ans.** $P = 8/\sqrt{3}$.lb wt.)

Ex I 7.3. *An apparent trick may be applied to pull out a ditched motor car. If a stout tree or pole happenes to be near, a strong rope is bound to both of them, made as far taut possible and pulled by a man at the middle of the rope as shown in the fig. below. A rather small pull suffices to extricate the car. How ?*





If the man pulls with a 100 kgf force and the angle between the segments of the rope is 150° find the tension in the rope.

Solution : (a) Refer to the force diagram. The two components $T\cos(90-\alpha/2)$ along the dotted line cancels out and the two sine components add up to $2T\sin(90-\alpha/2)$. If P the pull exerted by the man just exceeds $2T\sin(90-\alpha/2)$ the car may be pulled up.

$$P \geqslant 2T\sin(90-\alpha/2) \geqslant 2T\cos\alpha/2$$

Larger the value of α, smaller the value of P (for $\cos\alpha/2$ there is small) and it can produce a tension large enough to pull the car out.

(b) From the force diagram $(90-\alpha/2)=15°$. Let the tensions in the two segments be T_1 and T_2. Then resolving the tensions into tangential (horizontal) and normal (vertical in the fig or along y-direction), we must have for equilibrium according to eqn. I-7.2.2

$$\sum T_x = T_1\cos 15° - T_2\cos 15° = 0 \qquad \text{(a)}$$

and $\sum T_y = P-(T_1\sin 15° + T_2\sin 15°) = 0$ (b)

From (a) we have $T_x = T_1 = T_2 = T$ say and
from (b) $P = 100\text{kgf} = 2T\sin 15°$

$$\therefore\quad T = \frac{P}{2\sin 15°} = \frac{100}{2\times 0{\cdot}26} = \frac{50}{0{\cdot}26}$$

$$\simeq 200 \text{ kgf}$$

Thus the tension has been nearly doubled.

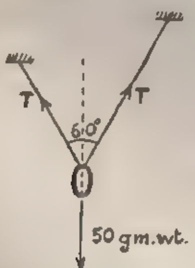
Alternatively, apply Lami's theorem at the point of pull. Then

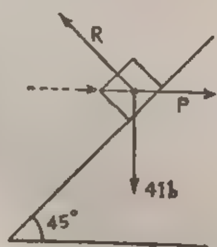
$$\frac{T}{\sin(90+90-\alpha/2)} = \frac{T}{\sin(90+90-\alpha/2)} = \frac{P}{\sin\alpha}$$

or $$\frac{T}{\sin\alpha/2} = \frac{T}{\sin\alpha/2} = \frac{100}{\sin 150°} = \frac{100}{1/2}$$

or $T = 200\times\sin\alpha/2 = 200\times\sin 75° = 200\times 0\,97 \simeq 200$ kgf

Problem : (i) A smooth ring weighing 50 gm is suspended by a light, flexible string passing through it. The ends of the string are





tied to two fixed points. Find the tension in the string if the ring hangs in equilibrium with the two parts of the string inclined at 60°.

(*Hint :* In this case where the ring is smooth and the string is flexible, the tensions in the two strings are equal, and the strings are equally inclined to the vertical. **Ans.** $50\sqrt{3}$ gmf. each)

(2) A body of weight 4 lb rests on a smooth plane inclined at an angle of 45° each the horizontal. Find the magnitude of the *horizontal* force which will keep the body in equilibrium.

(*Hint :* The forces acting on the body are (*i*) its weight 4 lb wt vertically down, (*ii*) force P acting horizontally and (*iii*) the reaction R of the plane *normal* to the plane since the plane is smooth.

Ans. 4 lb. wt.)

Ex. I-7.4 *Show that however small a weight be hung from the mid-point of a string under tension, it is never horizontal.* [S.S Q '79]

Solution : Consider the force diagram in Ex. I-7.3 reversed and replace P by a weight W. Then the downward force W must be balanced by $2T \sin (90-\alpha/2)$. Put $(90-\alpha/2)$ equal to θ. Then

$$\sin \theta = W/2T$$

Since $T \neq \infty$ and $W \neq 0$, $\sin \theta$ *cannot* vanish and so the string will *not* remain horizontal.

(3) *Equilibrium under any number of forces* : This occurs when *all but one* of the forces add up to a resultant which is equal, opposite and collinear to the last one—which actually the *equilibrant* This indicates that when the *force polygon* is drawn including all the forces it must be a *closed one.* Note that this is merely an extension of the triangle of forces rule.

Analytically, the study is simplified by (i) resolving each of the forces into two rectangular components along x and y directions, (ii) adding up all the components along each of the axes and (iii) equating each of the sums to zero. That is what we have already done in equation I-7.3.2 We may write

$$\sum F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + \cdots = \sum_{r=1}^{r=n} F \cos \theta_r = 0 \quad \text{(a)}$$

$$\text{(I-7.3.4)}$$

$$\text{and } \sum F_y - F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + \cdots = \sum_{r=1}^{r=n} F \sin \theta_r = 0 \quad \text{(b)}$$

This represents the vanishing of translational acceleration which produces linear equilibrium.

I-7.4 Discussion on the Second Condition of Equilibrium :

The mathematical expression has been provided in eqn I-7.3.3. In applying the condition we consider moments of the acting coplanar forces F_1, F_2, F_3,$\cdots F_r \cdots F_n$ about any arbitrary point in the same plane which will be

$$L_1 = F_1 \times r_1,\ L_2 = F_2 \times r_2, L_3 = F_3 \times r_3 \cdots$$

Then for rotational equilibrium we must have

$$\sum_{r=1}^{r=n} L_r = L_1 + L_2 + L_3 + L_r + \cdots L_n = 0 \qquad \text{(I-7.4.1)}$$

Principle of moments. Condition of equilibrium stated above is also known as the *principle of moments* and is stated as—

When a body is in equilibrium, the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments, about the same point.

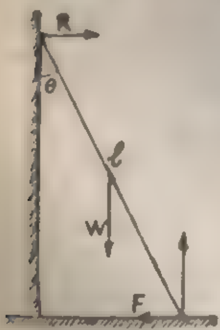
It therefore follows that **when a number of parallel forces are in equilibrium we must have,**

(i) the sum of the forces in one direction is equal to the sum of the forces in the opposite direction ;

(ii the sum of the anticlockwise moments about any point is equal to the sum of the clockwise moments about the point.

Ex I-7 5. *A uniform ladder of length l and weight W rests against a smooth, vertical wall, being inclined at an angle θ to the wall. A force F applied horizontally at the ground level holds the ladder in position. Discuss the equlibrium of the ladder. (For a smooth wall, the reaction on the resting ladder is normal to the wall.)*

Solution : Consider both the conditions of equilibrium. The algebraic sum of the vertical forces must be zero, and so also of the horizontal forces. W is a vertical downward force. There must be an equal upward force. This must be the vertical component of the reaction of the ground. These two forces constitute an anticlockwise couple of moment $W \times \frac{1}{2}l \sin\theta$.



The force F and the reaction R of the wall are the only horizontal forces. They must be equal and opposite. The moment of the couple they form is $Fl \cos\theta$ clockwise.

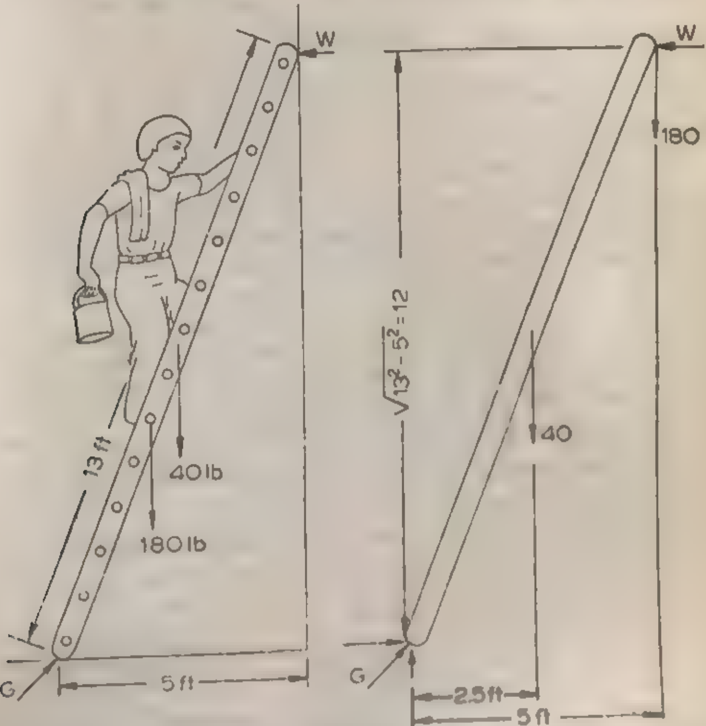
For equilibrium we must have

$$\tfrac{1}{2} Wl \sin\theta = Fl \cos\theta \text{ or } F = \tfrac{1}{2} W \tan\theta.$$

A more complicated but realistic problem follows.

Ex. I-7.6. *A 120 lb paintsman is climbing up a 40 lb, 13 ft long ladder with its base 5 ft away from the base of a vertical but smooth wall. What must be the friction at the base that the ladder does not slip when the man reaches the top?*

Solution: The force diagram is to the right when the man is at the top. The reaction force of the smooth vertical wall is as before



perpendicular to the wall. The wt of the ladder acts at the mid point, downwards. The ground reaction G is along the ladder and broken up into horizontal and vertical components. From the diagram

horizontal component $G_h = W$ (the wall reaction)

and the vertical component $G_v = 40 + 180 = 220$ lbf

We take moments about the base of the ladder and find that both G_v and G_h produce zero moment as they pass through the base point. Then

$$W \times (\sqrt{13^2 - 5^2}) = 40 \times 2.5 + 180 \times 5 \text{ or } W = 83\tfrac{1}{3} \text{ lbf}$$

$\therefore$ $G_h = W = 83\frac{1}{3}$ lbf. Now $\mu = \tan\theta = G_h/N_v = 83\frac{1}{3}/220 = 0.38$

Ex. I 7.7. *Two weights 10 g and 20 g hang vertically from the ends of a rigid horizontal bar of length 12 cm and negligible weight. Find where and how the rod must be supported so as to be in equilibrium.*

Solution : The student is advised to draw the diagram. Let P be the force producing equilibrium. Let it act at a distance x from the 10g weight.

Since $\sum F_y = 0$ we must have $10+20+P=0$ or $P=-30$ g.

The negative sign means that P acts in a sense opposite to that of the weights.

Taking moments about the end of the bar where the 10g weight hangs, we have, since $\sum T=0$ $\quad 10\times 0-30x+20\times 12=0$ or $x=8$.

$\therefore$ P acts at a distance $x=8$ cm from the 10g end of the bar.

Ex. I-7.8. *Two persons carry a load of 100 kg suspended from a rigid bar of length 5 m and of neglible weight, at a distance of 3 m from one end. Calculate the thrust on each. See the diagram with Ex. I-7 12.*

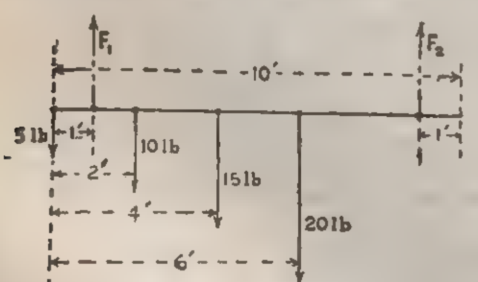
Solution : Let F_1 be the thrust at the end of the bar more remote from the load and F_2 that at the nearer end. Then the resultant of F_1 and F_2 is the 100 kg force. The equilibrant of F_1 and F_2 is equal and opposite to the load.

$\therefore$ From $\sum F=0$, we have $F_1+F_2-100=0$ or $F_1+F_2=100$.

Further, taking moments about the point of application of the load we have $\quad F_1\times 3-F_2\times 2+100\times 0=0$ or $3F_1=2F_2$

Solving for F_1 and F_2 we have $F_1=40$ kg and $F_2=60$ kg.

Ex I 7.9 *A rod 10 ft long and of negligible weight is supported by two strings each 1 ft from the end. It carries loads of 5, 10, 15 and 20 lb at distances 0, 2, 4 and 6 ft respectively from one end. Calculate the tensions in the strings.*



Solution : Let F_1 and F_2 be the tensions as shown in the fig

The loads and the tensions form a system in equilibrium.

$\therefore$ $\sum F=0$, or

$$F_1+F_2=5+10+15+20=50 \text{ lb}.$$

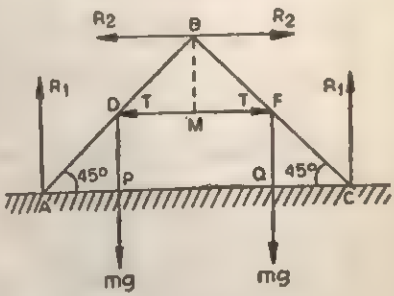
Taking moments about the left end,

$\sum T=0$ or $F_1+9F_2-(5\times 0+10\times 2+15\times 4+20\times 6)=0$

or $\quad F_1+9F_2=200$

Solving for F_1 and F_2 we get $F_1=125/4$ lb. and $F_2=75/4$ lb.

Ex. 17.10. *Two uniform beams AB and CD each of length L and mass M are hinged smoothly together at B and a light inextensible string* ($\sqrt{2}-1$)*L in length is tied between their mid-points D and F. Beams stand in a vertical plane, A and C resting on a smoth horizontal plane. Find the tension in the string.* [I. I. T. '71]



Solution : Refer to the figure and identify the active forces. The beam weights act vertically through their mid-points. Ground reactions R_1 and R_1 act at meeting points on the plane and reactions R_2 at the hinges horizontally. Finally tensions T, T act horizontally.

The system being in equilibrium $2R_1 = 2Mg$ and $2R_2 = 2T$

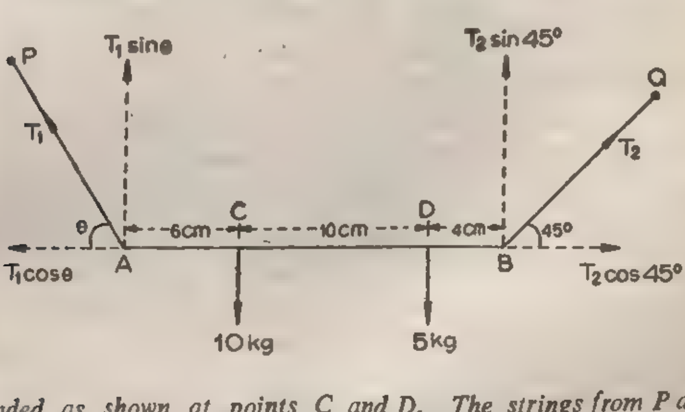
Taking moments about say D, we have

$R_2 \times BM = R_1 \times CQ$

or $R_2 \times BF \sin 45° = R_1 \times CF \cos 45°$ for $\angle FBD = 90°$

$\therefore$ $R_1 = R_2$ and hence $T = Mg$ (the weight of any one beam)

Ex 17.11. *A uniform bar of negligible weight (AB) is suspended by two strings from P and Q. It carries masses of 10 and 5 kg*



suspended as shown at points C and D. The strings from P and Q make respectively 45° and θ° with the horizontal. Find θ, and the tensions in the string when the bar is in equilibrium. [I.I.T. '77.]

Solution : Refer to the fig. above Let T_1 and T_2 be the required tensions. Resolving them parallel to AB and also perpendicular thereto, we take moments about A to get

$T_2 \sin 45° \times (4+10+6)$ cm $= 10$ kgf $\times 6$ cm $+ 5$ kgf $\times (6+10)$ cm

or $(T_2/\sqrt{2}) \times 20$ cm $= 10$ kgf $\times (6+8)$ cm

or $T_2 = 7\sqrt{2}$ kgf

Similarly taking moments about B,

$T_1 \sin\theta \times (6+10+4)$ cm $= 5$ kgf $\times 4$ cm $+ 10$ kgf $(4+10)$ cm

$\therefore \quad T_1 \sin\theta = 8$ kgf ... (i)

The rod being in eqilibrium the horizontal components cancel out.

$\therefore \quad T_1 \cos\theta = T_2 \cos 45° = 7\sqrt{2} \times 1/\sqrt{2} = 7$kgf ... (ii)

Dividing (i) by (ii) $\tan\theta = 8/7$ or $\theta \simeq 49°$

I-7.4. A special application of the conditions of equilibrium : Resultant of parallel forces. When the forces to be added are parallel, the parallelogram law of addition cannot be directly applied. We use the general conditions of equilibrium to get the resultant. A force equal and opposite to the resultant of two forces is the **equilibrant** which the with given forces form a system in equilibrium.

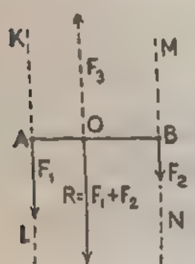


Fig. I-7.2

A. Two like parallel forces In Fig. I-7 2 let the forces F_1 and F_2 act along the parallel lines KL and MN in the same sense. Let the line AB, perpendicular both to KL and MN, intersect them at A and B respectively. Suppose these two forces along with a third force F_3 acting at O keep a body in equilibrium. Then F_3 is the equilibrant of F_1 and F_2, and is equal and opposite to their resultant R.

Since the forces F_1, F_2 and F_3 are in equilibrium, the sum of their components along AB must be equal to zero. F_1 and F_2 have no component in this direction. Therefore F_3 also will have no component along AB and will be perpendicular to AB Further, since the algebraic sum of the components perpendicular to AB is zero, we shall have

$$F_1 + F_2 = F_3 \qquad \text{(I-7.4.1)}$$

Applying the second condition of equilibrium and taking moments about O, we have

$$F_1 \times OA - F_2 \times OB + F_3 \times 0 = 0 \text{ or } F_1 \times OA = F_2 \times OB$$

or

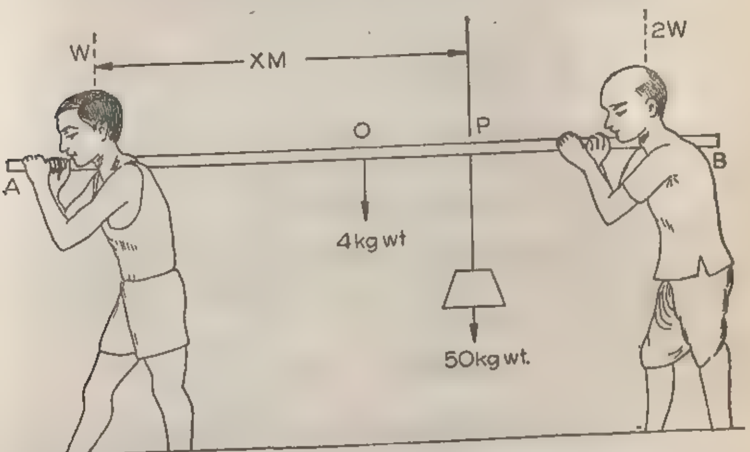
$$OA/OB = F_2/F_1 \qquad \text{(I-7.4.2)}$$

Thus *the point O divides the line AB internally in the inverse ratio of the forces.*

The resultant R of the forces F_1 and F_2 is equal and opposite to F_3. Hence its magnitude is $R = F_1 + F_2$. It acts through O parallel to the forces, where O is a point dividing the distance between F_1 and F_2 internally into two parts inversely proportional to the forces.

Ex I-7 12 *A man and a boy are to carry a 50 kg load slung somewhere on a uniform 4kg pole, 2m long. Where must the load be supported so that the man carries twice as much as the boy ?*

Solution : Let the boy carry the end A and the man, end B of the pole and they carry loads W and $2W$ kg respectively. As the



pole weighs 4 kg the total weight carried by the pair is (50+5 or) 54 kg. Hence the boy is to support 18 kg and the man 36 kg.

Let the required point of suspension be x m from the boy. Then taking moments about the boy's shoulder at A (misplaced in the fig. we have

$$18 \times 0 + 4 \times AO + 50 \times OP - 36 \times AB = 0$$

Now AO = 1m. OP = xm and AB = 2m. So we have from above

$$4 \times 1 + 50x = 36 \times 2$$

$$\therefore \quad x = (68/50)\text{m} = 1\ 34\text{m}.$$

Ex. I-7.13. *P and Q are two parallel forces. If P is displaced through x parallel to itself then show that the resultant displaces itself through Px/(P+Q)* [J. E. E. 80]

Solution : (1) Let P act at A, Q at B and their resultant P+Q through E. Then P/Q = BE/AE. Now adding 1 to each side of the equation

$$\frac{P+Q}{Q}=\frac{BE+EA}{EA}=\frac{AB}{EA}$$

or $EA = AB.Q/(P+Q)$ (a)

(2) P now is shifted through x, say to C, so that the resultant shifts to F. Then

$$P/Q = BF/CF \text{ and } \frac{P+Q}{Q}=\frac{BC}{CF}=\frac{AB-x}{AF-x}$$

Cross multiplying and re-arranging we have

$$(P+Q)AF-Px=Q.AB$$

$$\therefore \quad AF=\frac{Q\,AB+P.x}{P+Q} \qquad \text{(b)}$$

$$\therefore \quad AF-AE=\frac{Q.AB+P.x}{P+Q}-\frac{Q.AB}{P+Q}=\frac{P.x}{P+Q} \qquad \text{(c)}$$

The result (c) is the required displacement.

B. Two unlike parallel forces. When the parallel forces F_1 and F_2 act in opposite senses (fig. I-7.3) an application of the above method shows that the resultant R

(*a*) has a magnitude equal to the difference of the forces,

(*b*) acts in the same sense as that of the larger force and

(*c*) the line of action of the resultant divides the distance between F_1 and F_2 *externally* in the inverse ratio of the respective forces, R lying nearer to the larger force.

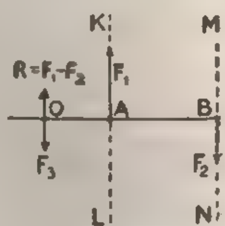


Fig. I-7.3

C. Any number of parallel forces. The same principle may be applied to find the resultant of any number of parallel forces. The system of forces along with its equilibrant keeps a body in equilibrium. Therefore, the given forces along with their equilibrant satisfy the conditions $\Sigma F=0$ (vector sum of forces equals zero) and

$\Sigma T=0$ (anticlockwise torques equal clockwise torques)

From these the magnitude, direction and line of action of the equilibrant may be found out. The resultant is equal and opposite to the equilibrant. For coplanar forces its point of action can be found out as follows :—

Let the perpendicular distances of lines of action of such forces F_1, F_2, F_3,...etc from any arbitrary coplanar point A be x_1, x_2, x_3, ..etc whereas that of their resultant R be $\bar{x}$. Then the sum of their moments about A will be

$F_1x_1+F_2x_2+F_3x_3+\cdots$ etc and $R\bar{x}$. For equilibrium

$$R\bar{x}=F_1x_1+F_2x_2+F_3x_3+\cdots\cdots$$

$$\text{or}\quad \bar{x}=\frac{F_1x_1+F_2x_2+F_3x_3+\cdots}{F_1+F_2+F_3+\cdots}=\frac{\Sigma Fx}{\Sigma F} \qquad \text{(I-7 4)}$$

Ex. I-7 14. *A bar 6 m. long and weighing 20 kg is supported at two knife edges, one (A) at its one end, the other (B) at a distance of 2m from the other end. If a 10 kg wt is now suspended 1m from A find the reactions on the two knife-edges*

Solution : Refer to the figure Let the required reactions at A and B be R_1 and R_2. Since the rod is in equilibrium, the algebraic sum of forces vanish.

$$\therefore\quad \Sigma F=R_1+R_2$$
$$-10\text{ kgf}-20\text{ kgf}=0$$
$$\text{or } R_1+R_2=30\text{ kgf}$$

Taking moments about A we get, as the wt of the bar acts at its mid-point G

$$R_1\times 0+R_2\times \text{AB}-10\times \text{AC}-20\times \text{AG}=0$$
$$\text{or}\quad R_1\times 0+R_2\times 4-10\times 1-20\times 3=0$$
$$\therefore\quad 4R_2=70 \quad\text{or}\quad R_2=17\tfrac{1}{2}\text{ kgf}$$
$$R_1=30-17\tfrac{1}{2}=12\tfrac{1}{2}\text{ kgf}$$

Ex. I-7.15. *A 50 kg beam 10 m long rests on two props at equal distances from the respective ends. Find their maximum seperation if an 80 kg man may stand anywhere on the beam without overturning it.*

Solution : Let the distance of the props from respective ends be x m each. Its weight will act at its C.G the middle point, the rod being uniform.

Now whereever the man stands between a prop and its nearer end the moment of his weight will be in opposition to the moment of the weight of the beam. Obviously if the beam remains unturned with

the man at its one end, then it will also remain so, wherever else he stands. Let him be at one end, distant x from the near prop. Then x will be maximum when

$$80 \text{ kgf} \times x \text{ m} = 40 \text{ kgf} \times (5 - x) \text{ or } 2x = 5 - x \text{ or } x = \tfrac{5}{3} \text{m}$$

$\therefore$ Maximum separation between the props $= 10 - 2 \times \frac{5}{3} = 6\frac{2}{3}$m.

1-7 5. **Common Balance:** This device well-known to you is essentially a lever of the first class with *two equal arms* and works on the principle of balancing equal moments about a central knife-edge, produced by the weights of two scale pans and the bodies they carry.

It consists (fig. I-7.4.) of a symmetrical rigid beam carrying three knife edges as shown. The beam can turn freely about C as fulcrum.

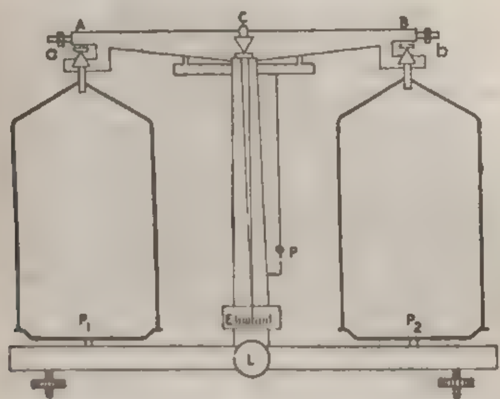


Fig. I-7.4

The fulcrum is central and has the form of a steel knife edge (C) turned downwards and rests on an agate plate. Scale pans (P_1, P_2) are suspended from the upturned knife edges A and B at the ends of the beam. Near the fulcrum a long light pointer is attached to the beam at right angles to its length. It moves over a horizontal scale E. Two screws, (a, b) one at each end are provided with the beam. When the pansare empty the balance beam can be made horizontal with the help of these screws. The balance beam with the two scale pans rests on a support, when it cannot move. But when it is released from the support by means of a lever (L) it becomes free and oscillates. These oscillations are practically simple harmonic, though they are damped by resistance of air and other

causes. The whole arrangement is enclosed in a glass case for protection against airdraughts and temperature variations. The balance case is supported on three levelling screws. A plumb line P is suspended from the stand on which the balance beam rests. When this stands vertical, the beam itself is horizontal.

Working Principle : Let AB (fig I-7.5) represent the balance beam and C its point of support. Let the two arms CA and CB be of lengths a and b and the weights of the scale pan S and S'. Let weights W and W' be placed on the pans. Then the downward force $(W+S)$ will produce an equal and upward force R at C, the two R and $(W+S)$ combining into an anticlockwise couple. Similarly the weight $(S'+W')$ generates another equal and upward reaction R also at C, this couple being clockwise. For equilibrium the beam AB would be horizontal when the *resultant couple vanishes. Taking moments about* C we than have

$$(W+S)a-(W'+S')b=0 \tag{1-7.5.1}$$

In a common balance by design $a=b$ and $S=S'$. Then

$$W(=mg)=W'(=m'g) \text{ or } m=m' \tag{I-7.5.2}$$

Thus when the beam is horizontal with weights on the scale pans the masses are equal. When one of them is a standard mass, the value of the other is equal to it. This is why *masses are said to be compared by a* **common** *or* **beam balance.**

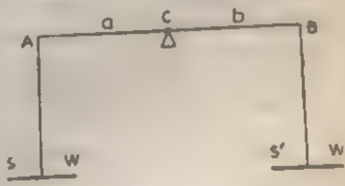


Fig. I-7.5

Requisites of a Good balance are that should be (*i*) *true* (*ii*) *sensitive and* (*iii*) *stable.*

It is said to be **true** when equal masses placed on the scale pans keep the balance beam horizontal. We have just seen that (*a*) balance arms should be equal in length and (*b*) the scale pans equal in weight, to achieve that.

It is said to be **sensitive** when a small difference in weights placed on the scale pans tilts the pointer on the scale appreciably. To achieve sensitivity (*a*) the beam should be light (*b*) its balance arms long and (*c*) C.G. of the beam close to the fulcrum. The time of swing then would be long.

The balance is said to be **stable** when the beam returns to the horizontal position (i. e. the pointer to the zero-mark) quickly. To achieve that, the C. G. of the beam should be considerably below the fulcrum. Note that this requirement goes against that for sensitivity.

In weighing a body accurately, a correction for buoyancy of air (Eqn. II-6.4.1) is necessary.

Weighing by an Untrue Balance : As can be readily understood such a balance may have either (*i*) unequal arms or (*ii*) unequal scale pans. In either case Gauss's method of double weighing is to be used to find the true mass of a body.

(*i*) Let the unequal arm-lengths be a and b and the weight of the body W. It is placed successively on the two scale pans and counter-poised in both cases where the weights are W_1 and W_2. Then

$$Wa = W_1 b \text{ and } W_2 a = Wb \text{ and } W^2 ab = W_1 W_2 ab$$

$$\therefore \quad W = \sqrt{W_1 W_2} \qquad \text{(I-7.5.3)}$$

$$\text{and } \frac{a}{b} = \frac{W_1}{W} = \frac{W}{W_2} \text{ or } \frac{a^2}{b^2} = \frac{W_1}{W_2}$$

$$\text{or } a/b = \sqrt{W_1/W_2} \qquad \text{(I-7.5.4)}$$

Problem : A balance is a rigid rod free to rotate about a point not at its center. It is balanced by unequal weights placed in the pans at each end of the rod. A mass m is balanced by a mass m_1 on the right hand pan and by m_2 placed on the left hand pan. Show that $m = \sqrt{m_1 m_2}$ [**Pat U**]

A dishonest trader using a balance with unequal arms will defraud himself if he is made to weigh half the required quantity on one pan and the other half on the other pan. It comes about this way : let two masses weighing W_1 and W_2 *appear to weigh* the same (W) in the two pans respectively. Then the customer gets a weight of $W_1 + W_2$ in place of $2W$.

$$\therefore \quad (W_1 + W_2) - 2W = W\frac{a}{b} + W\frac{b}{a} - 2W$$

$$= W\frac{a^2 + b^2 - 2ab}{ab} = W\frac{(a-b)^2}{ab}$$

Now the beams being unequal in length, $(a-b)^2$ is always a $+$ve quantity and so the tradesman always loses by this amount

Ex. I-7.1.6. *A tradesman sells his articles with equal quantities weighed alternatively from the two pans of a balance with unequal arms in the ratio of 1.025. Find by how much he cheats himself.* [**Pat. U.**]

Solution : Here $\frac{a}{b}=1.025=\sqrt{\frac{W_1}{W_2}}$, W_1 and W_2 being the weights of apparently equal masses (W) and a, b the lengths of balance arms.

Also W_1+W_2 is the weight he gives out in place of 2W.

$$\therefore \quad W_1+W_2-2W=W\frac{(a-b)^2}{ab}=W\left(\frac{a}{b}+\frac{b}{a}-2\right)$$

$$=W\left(1.025-\frac{1}{1.025}-2\right)=0.0006\ W$$

So he cheats himself by $\frac{0.0004\ W}{2W}\times 100=0.03\%$

(*ii*) If the balance arms are equal in length but the scale pans are of different masses then from I-7.5.1 a and b being equal,

$W_1-W_2=\frac{1}{2}(S_2-S_1)$ and $W=\frac{1}{2}(W_1+W_2)$

Problem : (1) A body is weighed in a balance where scale pans weigh differently. It weighs 10.20g in one and 10.42g in the other. Find the correct weight and difference in weight of the two pans.

(*Ans.* 10.31g, 0.11g) [**Camb**]

(2) Show that when a body weighs W_1 and W_2 in the two pans of unequal weights in a balance of equal arms then the difference in the weights of the pan is $\frac{1}{2}(W_1-W_2)$ [**Raj U.**]

I-7 6. **Centre of gravity.** Any body whatsoever may be considered to be made up of a large number of particles, each of finite mass. Each particle is attracted towards the centre of the earth by the force of gravity which is proportional to the mass of the particle. Owing to the large radius of the earth the forces of gravity on the particles may be taken to be parallel. The weight of a body thus consists of a system of parallel forces (fig. I-7.6) acting upon the forces individual particles which make up the body. The resultant of this system of parallel forces always passes through a point, *fixed* relative to the body, whatever be the

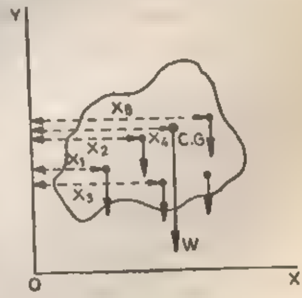


Fig. I-7.6

orientation of the body. This point is called its *centre of gravity* (abbreviated c.g.) of the body. The weight of a body, although actually a system of parallel forces acting upon all parts of the body, can be correctly represented by a single force acting downward at the centre of gravity.

Definition. *The point, fixed with respect to a given body, through which the resultant force of gravity on it acts, no matter how the body is oriented, is called the centre of gravity of the body.*

It follows from the definition of the c.g. that *when a body is supported at its centre of gravity, it will have no tendency to turn under the action of gravity alone.* The position of the centre of gravity is independent of the orientation of the body.

Centre of gravity of bodies of simple shapes can often be located by inspection. For a thin uniform rod the c.g. is at the centre of the rod. The turning moment about the centre due to the weight of any particle is balanced by that of a particle similarly placed on the other side of the centre (See fig. I-7.10). Hence the total turning moment of the rod about its centre is zero. The centre of the rod is therefore its centre of gravity.

The centre of gravity of some uniform, homogeneous bodies having simple geometrical shapes are given below :

Body	*Position of centre of gravity*
Circular lamina	Centre of circle.
Annular disc	Centre of the annulus.
Triangular lamina	Point of intersection of the medians.
Rectangular lamina	Point of intersection of the diagonals.
Sphere	Geometrical centre.
Spherical shell	Geometrical centre.
Cylinder	Midpoint of the axis.
Cone	On the axis at a distance $= \frac{1}{4} \times$ height above the base.

A. Experimental determination of centre of gravity.

(*i*) When a body is suspended by a cord SP from a point P on the body (fig. I-7.7a), it will come to equilibrium when the tension T in the cord is equal and opposite to the weight W of the body. The line of the string, therefore, passes through the

C.G. of the body. By a plumb line this direction PQ may be marked on the body. The body is then suspended from any other point P′ on it (Fig. I-7.7b) and the same procedure followed. Since PQ and P′Q′ pass through the centre of gravity, their point of intersection C gives the position of the c.g.

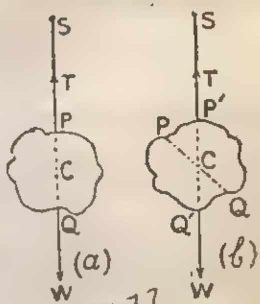


Fig. I-7.7

(*ii*) The above fact suggests that the centre of gravity may be found by balancing the body on a knife-edge, and finding the inter-section of two lines on the body along which it will balance.

Further facts about centre of gravity. *The centre of gravity of a body does not necessarily lie on or within the body.* An annular ring provides a simple example of the centre of gravity lying outside the body. When a sitting stool is suspended from different points in the manner described above, its centre of gravity will be found to lie within the space between the legs.

A set of bricks placed as shown on a horizontal plane will stand or fall according as the vertical line through the centre of gravity meets the plane within, or outside, the base. In the latter case the reaction of the table cannot pass through the centre of gravity (fig. I-7.8) and constitutes along with the weight a couple tending to overturn the body.

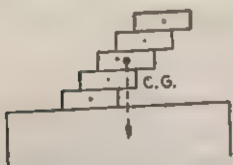


Fig. I-7.8

B. Analytical method of determining the centre of gravity. Let a system of particles lying in a plane have masses $m_1, m_2 \cdots m_n$ and respective weights $w_1, w_2 \cdots w_n$ (fig. I-7.9). Let *OX* and *OY* be two fixed straight lines in the plane at right angles to each other. Let the distances of the particles from *OX* be y_1, $y_2 \cdots y$

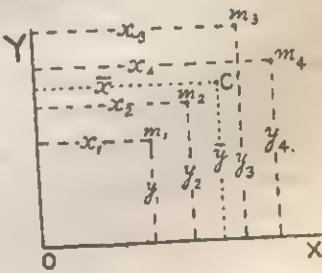


Fig. I-7.9

and from OY, $x_1, x_2, \cdots x_n$. Then the distance of the centre of gravity C from OX is

$$\bar{y}=\frac{w_1y_1+w_2y_2+\ldots w_ny_n}{w_1+w_2+\ldots w_n}=\frac{\Sigma wy}{W} \qquad \text{(I-7.6.1)}$$

and its distance from OY is

$$\bar{x}=\frac{w_1x_1+w_2x_2+\ldots w_nx_n}{w_1+x_2+\ldots w_n}=\frac{\Sigma wx}{W} \qquad \text{(I-7.6.2)}$$

The proof is quite simple. The given weights and their equilibrant acting at the centre of gravity from a system of forces in equilibrium. The magnitude of the equilibrant is the sum of the weights, *i.e*, $w_1+w_2+\ldots w_n$ With the equilibrant acting, the system of particles will be in equilibrium in any position. Hence the sum of the moments of all the forces about any line OX or OY will vanish. Taking moments about the line CX when it is vertical,

$$(w_1+w_2+\ldots w_n)\,\bar{y}=w_1y_1+w_2y_2\ldots w_ny_n.$$

Similarly, taking moments about OY when it is vertical,

$$(w_1+w_2+\ldots w_n)\bar{x}=w_1x_1+w_2x_2+\ldots w_nx_n.$$

Now, $w=mg$ and $W=Mg$, where w stands for the weight of any particle, m for its mass, W for the total weight of the particles and M for the total mass. As g is the same for all particles, g cancels out and above equations become

$$\bar{x}=\frac{\Sigma mx}{M} \text{ and } \bar{y}=\frac{\Sigma my}{M}. \qquad \text{(I-7.6 3)}$$

In three dimensions coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the centre of gravity of a body (or a system of particles) are given by the relations

$$\bar{x}=\frac{\Sigma mx}{M},\ \bar{y}=\frac{\Sigma my}{M},\ \bar{z}=\frac{\Sigma mz}{M} \qquad \text{(I-7.6.4)}$$

Here M is the total mass, m the mass of a constituent particle and x, y, z its position coordinates, the summation extending over all the particles. For a homogeneous body (*i.e.* one of constant density throughout), the summation may be replaced by integrals. If dm is the mass of an elementary volume of the body and $\bar{x}, \bar{y}, \bar{z}$ its coordinates, then

$$\bar{x}=\frac{\int x\,dm}{\int dm} \text{ etc.}$$

C. Properties and Significance of the Center of Gravity :—

(1) *C.G. of a body is unique.* A body can have only one C.G. which can be proved indirectly. The resultant weight of a body acts through its C.G. Were two C.G.'s possible for a body then its weight W would act along the line joining them whatever be the angular position of the body. But that is impossible, for W always acts vertically downwards. So no body can have move than one C.G.

(2) *A body is in equilibrium when supported at its C.G.* As the weight of a body acts through its C.G., a body supported at that point has its *we'ght* and the *reaction* of the support equal and opposite and *collinear* ; hence it will neither move nor rotate. Thus it will be in equilibrium. Hence C.G. is also called the **balancing point** Supported at its C.G., a body of regular geometrical shape as listed in the foregoing table will remain horizontal ; this is the principle of equality of masses by a common balance.

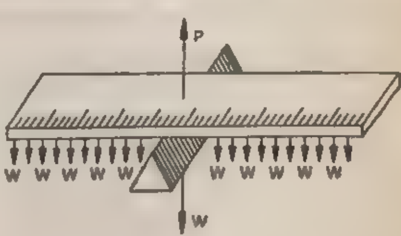


Fig. I-7.10

C.G. of a regular body can then be experimentally located by finding the point of support for which it remains horizontal (fig. I-7.10)

(3) *A body freely suspended at a point has that point collinear with its C.G.* This is the property we have already utilised in locating the *C.G.* of a lamina.

(4) *C.G. of a body may lie outside it* as we have already seen as for an annular disc or a spherical shell.

(5) It is the *C.G. of a projectile* that *describes a parabola* when it is an extended body. The other points of the body may execute any other type of motion e.g. twisting and turning. Refer to fig. I-6.10 (b), the case of a plunging diver doing exactly that, but his C.G. describes a parabola.

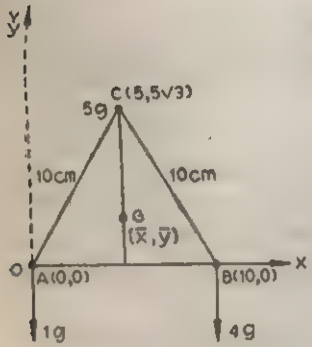
(6) *Mass* of an extended body may be taken to be *concentrated* as a particle *at its C. G.* without altering the system of forces on the body.

(7) The *nature of equilibrium* of a body depends solely on the position of its C. G. and that of the line of the line of action of its weight in relation to its base as we shall presently see.

Note that the *center of gravity* of a body is an effect of the gravity-field of the earth and *has no meaning in gravity-free space.* There the role of C.G. is taken up by another, the **center of mass**, we discuss in §I-7.10.

Ex. I-7.17. *Three particles of masses 1g, 4g and 5g are placed at the vertices of an equilateral triangle of side 10 cm, Find the distance of the C. G. of the system from the 1g particle.* [I. I. T. '70]

Solution : Let the vertices of the triangle be A (1g) B (4g) and C (5g). Let the arm AB be the x-axis of a co-ordinate system with A as the origin (0, 0) ; B then would be at (10, 0) and C at 5, 5 $\sqrt{3}$ Let $\bar{x}$, $\bar{y}$ be the co-ordinates of the C.G. Then from eqn I-7.6.1



$$\bar{x} = \frac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$$

$$= \frac{1 \times 0 + 4 \times 10 + 5 \times 5}{1 + 4 + 5} = 6.5 \text{ cm.}$$

and $$\bar{y} = \frac{1 \times 0 + 4 \times 0 + 5 \times 5\sqrt{3}}{10}$$

$$= \tfrac{1}{2}5\sqrt{3} = 4.32 \text{ cm.}$$

∴ Co-ordinates of the C. G. of the system are 6.50 and 5.32 cm.

∴ Its distance from A (0, 0) is

$$\sqrt{(\bar{x}-0)^2 + (\bar{y}-0)^2} = \sqrt{(6.50)^2 + (4.32)^2} = 7.81 \text{ cm.}$$

Problem : Three particles of masses 1g, 2g and 4g are placed at A (1, 2, 3) B (0, 0, 0) and C (2, 4, 6) in three dimensions find the co-ordinates of its C.G.

[*Hint :* Use eqn I-7.6.4] (*Ans.* $\frac{9}{7}, \frac{18}{7}, \frac{27}{7}$)

Ex. I-7.18 *A uniform circular lamina of radius R has a hole punched out of it such that their two centers are at a separation of d cm. Find the C. G. of the perforated lamina.*

Solution : The C. G. of the lamina as a whole, is at its center C_1. Let the C. G. of the material of the punched circular hole be at C_2 and that of the remaining portion be at C_3. Clearly C_3 will lie on a line through C_1 and C_2. Let the punched out material weigh W_1g

and the remainder of the lamina weigh W_2. Then taking moments about C_1 we have

$$W_1 \times C_1C_2 = W_2 C_1C_3$$
$$\text{or } C_1C_3 = C_1C_2 \cdot W_1/W_2$$

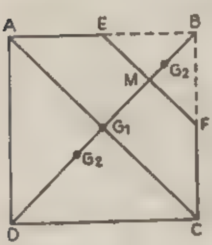
If σ be the mass of the material for unit area then

$$W_2 = \pi(R^2 - r^2)\sigma \text{ and } W_1 = \pi r^2\sigma$$
$$\therefore \quad C_1C_3 = [r^2/(R^2 - r^2)] \times C_1C_2$$

Problem : A uniform circular plate of diameter 56 cm has a circular portion of diameter 28 cm removed form one of its edges. Find the C. G. of its remaining portion. [I.I.T. '80]

Ex I 7.19. *A uniform square plate of side 24 cm has one corner of it removed along the line joining the mid-points of its adjacent sides. Find the C. G. of the remaining portion.*

Solution : Let ABCD represent the square plate of which the portion EBF has been removed. Let the C. G.'s of the original plate, the removed portion and the remainder be at G_1, G_2 and G_3 respectively. Let GG_3 be r which we are to find. The mass of unit area is constant, the plate being uniform.



G_1 by symmetry lies at the intersection of the two diagonals AC and BD. If BM represents a median of $\triangle$BEF we know that $BG_2 = \frac{2}{3}$ BM. To locate the position of G_3 we proceed as follows ; the weight of the plate as a whole acts *downwards* through G_1 and that of the *removed* portion be imag'ned to act *upwards* through G_2 ; their resultant acting through some point on G_2G_1 produced must give the weight of the remainder of the plate. Thus C. G. shifts to G_3 along MD towards D.

From the figure $BG_1 = 12\sqrt{2}$ cm and $G_1M = \frac{1}{2}BG_1 = 6\sqrt{2}$ cm

Again $MG_2 = \frac{1}{3}$ BM $= \frac{1}{3} G_1M = 2\sqrt{2}$ cm

$\therefore \quad G_1G_2 = G_1M + MG_2 = 16\sqrt{2} + 2\sqrt{2} = 8\sqrt{2}$ cm

Area of ABCD $= (24)^2 = 576$ cm^2.

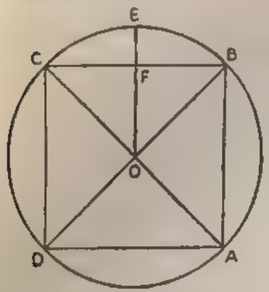
Area of BEF $= \frac{1}{2}$EF.BM $= \frac{1}{2}.12\sqrt{2}.6\sqrt{2} = 72$ cm^2

Taking moments about G_2 we have

$$(24 \times 24)r = (8\sqrt{2} + r).72$$

where L.H.S is proportional to the clockwise moment of the weight of ABCD acting through G_1 and the R. H. S. proportional to

the anticlockwise moment about G_2 of the force acting upwards through G_2.



$$\therefore \quad r = \frac{8\sqrt{2}}{7} \text{ cm.}$$

Problem : A table has a circular table top of mass 20 kg and radius 1m. Four light 1m long legs are fixed at A, B, C, D. Find (i) the max load that may be put and (ii) the area of the top over which it may be put without toppling the table ?

(*Ans :* 48·3 kg ; 2m^2).

I-7.8 A Stable, unstable and neutral equilibrium. The behaviour of a body when slightly displaced from its position of rest; depends upon the position of its c.g.

A body is said to be in **stable equilibrium** if, when slightly displaced, it tends to fall back to its original position. *Equilibrium*

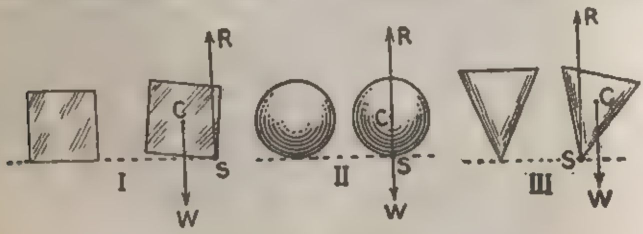


Fig. I-7.11

is stable when the displacement raises the c.g A cone standing on its base (fig. I-7.16a), a cube resting on a face (fig I-7.11 I) a weight suspended by a string, a ball lying in a spherical cup (fig. I-7.12a), a chair or table standing on its legs etc. are examples of stable equilibrium.

A body is said to be in **unstable equilibrium** if, when slightly displaced, it tends towards further displacement. *Equilibrium is unstable when the displacement lowers the c.g.* A chair balanced on two legs, a cube balanced on an edge, a cone balanced on its vertex (fig.

(a) Fig. I-7.12 (b)

(I-7.11 III), an egg balanced with its long axis vertical, a ball on the

top of a sphere (fig. I-7.12b) etc., are examples of unstable equilibrium.

A body is said to be in **neutral equilibrium** if, when slightly displaced, it remains in its new position. *Equilibrium is neutral when the displacement neither raises nor lowers the c.g.* A sphere resting on a horizontal surface (fig. I-7.11 II) a pencil, a cylinder or a cone resting on their respective sides etc, are examples.

B. Statical Equilibrium and Potential Energy. Stability is intimately connected with the gravitational potential energy. Bodies tend to aquire the lowest possible potential energy when left to itself. Refer to fig. I-7.16. When the cone is tilted on its base, its C.G. rises and hence aquires additional potential energy. Left to itself it wi'l tend to minimise that and hence *stable equilibrium is assosciated with minimum potential energy.* When the cone stands on its vertex (fig. I-7.11 II) the C.G. is at its maximum height from the base, it is top-heavy and hence has maximum P.E. When tilted, its C.G. is lowered and as it tends to lower itself further, the cone overturns and falls on its side. When *a body has its C.G. at the highest possible position it is unstable* for it has then maximum P.E. A neutral equilibrium occurs (as for the sphere) when on a little tilting the C.G. is neither lowered nor raised and so P.E remains unchanged. We return to this matter in §I-8 9.

Fig. I-7.13

Further examples : (1) It is very difficult to balance a walking stick on your finger (fig. I-7.13) because it is *top heavy.* When vertical its weight acts through its C.G. on your finger and is neutralised by the re-action of your finger. If your finger gets even slightly disturbed the two forces no longer remain collinear and form a couple tending to topple the stick over. It exemplifies unstable equilibrium.

(2) The music-doll for kids shown in fig. I-7.14 is on the other hand an example of stable equilibrium. Children find it very amusing that whenever the doll is laid on its side and let go, it will rise up

immediately and after a few oscillations tinkling all the time, become vertical.

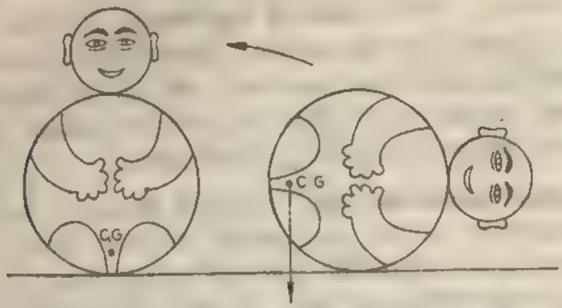


Fig. I-7.14

The mechanism is very simple. The lower part of the doll is filled with lead or any other heavy material so as to lower its C.G. as far as possible. Also it is given such a shape with its upper part very light, that when the doll is laid on its side, the C.G. gets raised. So, as soon as it is let go, the C.G. goes down and the doll sits up.

(3) To achieve stable equilibrium, floating bodies like common or Nicholson's hydrometers or floating test-tubes (for De la Rive's floating battery) are loaded with lead or mercury at the bottom ; so also are boats, ships and ocean liners, with cargo-holds at bottom.

I-7 9 Toppling of a body A body will topple unless the vertical line through its c.g. passes through the base on which it is supported. Try piling up bricks one above the other fig. I-7.8), but slightly displaced to one side. Soon the pile will topple. This occurs when the vertical line through the *c.g* of the pile moves outside the base offered by the lowest brick. Place a cylinder with its axis vertical on a table, and gradually tilt the table. The cylinder will topple as soon as the vertical line through its *c.g.* moves out of the base (fig. I-7.15)

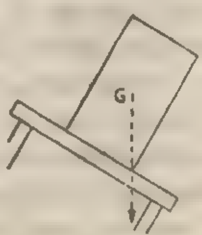


Fig. I-7.15

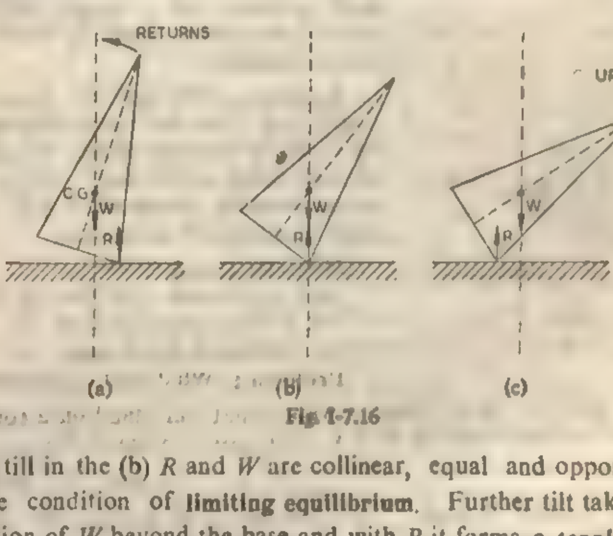
The stability of a body depends on how far it may be tilted without toppling. *Greater stability is obtained by keeping the c.g low*

and making the base large. This is an important point to remember in the design of structures.

Extra passengers are not allowed on the upper deck of double-decker buses. If they are allowed, the *c.g.* will rise, and the risk of the bus toppling over, increases. Racing cars are built with low chassis for the same reason.

Limit of stability : As you have seen above, the law of stability states that *a body acquires a stable equilibrium if the vertical line through its C.G. falls within the base offered by the body.*

We discuss the matter with a cone. In fig. I-7.16 (a) the cone is tilted slightly. Its weight W and reaction by the table R no longer act along the same line and forms a *restoring* anticlockwise couple, the equilibrium being stable. As tilt increases the line of action of W



(a) (b) (c)

Fig. I-7.16

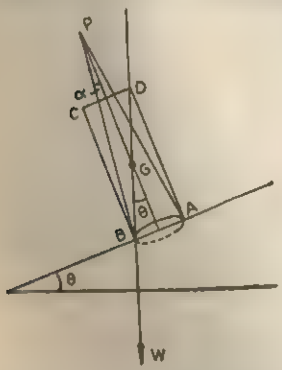
shifts till in the (b) R and W are collinear, equal and opposite. This is the condition of **limiting equilibrium**. Further tilt takes the line of action of W beyond the base and with R it forms a *toppling* clockwise couple and the cone is upset.

Base size and Stability : You must then realise that broader the base of a body greater is its limit of stability. Hence the broader base of modern double-decker buses. Stability of a four-wheeler even a three wheeler is therefore greater than a bicycle. With a heavy load on your head it is better to plant your feet apart. You tilt to the left when carrying a load with your right hand ; for with the load the combined C.G. of the load and yourself has shifted and you must

ensure that the vertical through this shifted C.G passes through the ground in between your feet. If you carry equal heavy loads from your two hands the C.G lowers but does not shift and you can walk erect.

Quadrupeds standing on four legs cover a larger base than bipeds like a man and hence enjoy greater stability ; so it is easier to topple a man than, say a cow. A human child crawls on all fours ; a toddler just learning to walk prefers crawling to walking, for he has more stability on all fours. A human has to learn to walk, to develop reflexes to keep stable whereas a quadruped can walk very soon after birth for its limit of stability is more.

Ex I 7.20. *A solid cone of radius r and height h and semivertical angle α stands on a rough incline. Find the maximum angle of inclination for stability.*



Solution : The limit of stability is achieved when the vertical through the C. G. of the cone passes through an end point of the base. Refer to the adjecant fig. drawn when θ is the angle of minimum inclination.

Now $\tan \alpha = r/h$

and $\tan \theta = \tan \angle OGB = OB/OG$

For a solid cone $OG = \frac{1}{4}h$

$\therefore \quad \tan \theta = 4r/h = 4 \tan \alpha$

Problem : What is the maximum height of a uniform solid cylinder ABCD that can stand on a rough incline of 30° without toppling over ? The base is 8 cm in dia.

(*Ans.* $8\sqrt{3}$ cm)

I-7.10.A. Centre of mass The term centre of gravity will have no significance in free space, *i.e.*, in a gravity-free space. But centre of mass has a significance under all circumstances.

If a body is thrown spinning into the air it will rotate smoothly about its centre of gravity. Out in free space it will spin about the center of mass. *The two points coincide when g is the same all over the body.* Physically, we may say that the **centre of mass** of a rigid body is the point such that if the line of application of a force passes through this point, it will prodcue

only translational motion of the body, but cause no rotation.* The existence of such a point may be easily tested. Place a book on a smooth horizontal table and apply a push with a pencil point (fig. I-7.17). In general it will move and rotate at the same time. But when the line of action passes through the geometerical mid point C of the body, it will move without rotation.

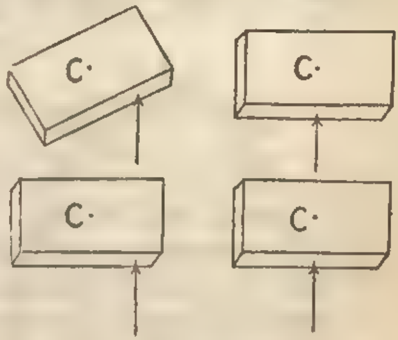


Fig. I-7.17

It is easy to understand how a rigid body may both move and turn when a force is applied to it. Fig I-7.18 shows a body to which a force F is applied. M is its centre of mass. Consider two equal and opposite forces P_1 and P_2, each equal to F, to be applied at M. These two forces do not affect the motion of the body in any way; the three forces are equal to the single force F. F and P_2 form a couple which causes the body to rotate about an axis through M, while P_1 acting at the C.M. causes linear motion. The linear acceleration in the direction of F will be $P_1/m = F/m$ where m is the mass of the body.

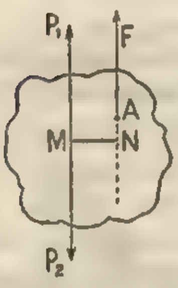


Fig. I-7.18

Motion of C M. In all problems of trsanslational motion of a rigid body, its entire mass may be taken to be concentrated at its center of mass i.e. motion of a body is the motion of its C.M. In fig. I-6.10 (b) we have already seen that when a diver jumps from a spring

* This statement may serve as the definition of centre of mass for a beginner. A rigorous definition is given on different lines as follows ; the centre of mass of a body or a system of particles is that point with respect to wich the vector sum of the mass moments vanishes. (If r_i is the vector distance of the i th particle of mass m_i from the centre of mass, then $\sum m_i r_i = 0$ the summation extending over all the particles. $m_i r_i$ is the vector mass moment of the particle.)

board, regardless of how he twists and turns his *center of mass* describes a parabola. In fig. I-7.19 is shown a small club thrown spinning about a horizontal axis through its C.M. from one performer to another, projectile-fashion; it describes a parabola as a particle similarly thrown, would. In both the cases it is really the C.G. that describes the parabola but in the field of gravity the C.G. coincides with the C.M. This is however not always the case.

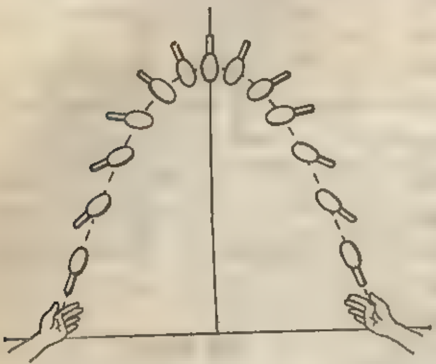


Fig. I-7.19

Internal forces acting on a system of particles or inside a body can change neither the velocity nor the total momentum of the system or the body because of Newton's third law. Hence it is that, if a shell in flight suddenly explodes in mid-air, the fragments may fly off in different directions but its C.M. will continue to move on as if nothing has happened. The gun-shell might have been moving along a parabola or a straight line (fig. I-7 20).

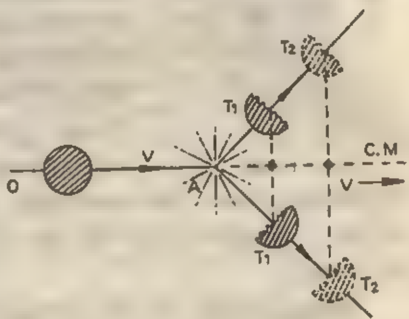


Fig. I-7.20

Importance of the concept. The idea of C.M. is more fundamental than that of C.G. For just as weight has no meaning in gravity-free space but mass has, similarly far out in space, say a space-probe like the Pioneer or the Voyager, has no C.G. but still has a C.M.

For *astronomical bodies* the center of mass is then the only meaningful concept. It greatly simplifies problems. The motion of a star or a galaxy, so immeasurably vast, may be treated as

though it were a particle with the entive mass at its C.M. On or outside the earth, its concept is more universal than that of C.G. ; e.g. whenever we apply Newton's law $F=ma$, we *really* talk about the acceleration of the C.M. of a body.

B Difference between Centre of Gravity and Centre of Mass

C. G.	C. M.
1. It is the point through which the resultant force of gravity passes, whatever the position of the body.	1. It is such a point in the body that if a force acts through this point, it causes only translation but no rotation of the body.
2. If g does not vary over the body, the two points are the same.	
3. In the absence of any force of gravity, c.g. has no meaning, say far out in space.	3. Has its own significance under all circumstances.

The second point above needs elaboration. (1) In discussing a diver or a gun shell fired as a projectile above we have taken the C.G. to coincide with the C.M. That will hold so long as value of g does not vary over their paths ; it will not be so far an ICBM fired from one continent to another far above the range of observation (2) Again if a body is very large the forces of gravity acting on particles near opposite ends no longer remain parallel and its C M. and its C.G. cannot coincide (3) for a very tall structure, such as the massif of Everest soaring upto 8848 metres in the sky, these two points do not coincide for the g value is substantially less at the top than at its base, pulling the C.G. below the C.M. To fix our ideas, let the mass of the massif be replaced by a uniform cylinder when its C.M. will be at the mid-point but the C.G. lower down for the lower part of the cylinder being closer to the center of the earth, is pulled more than its upper part. We had already told you that the two points do not always coincide.

Ex I-7.20. *Find the position of the C.M. of a 40 cm long open-mouthed cylinder half filled with water. The can has a diameter of 10 cm and made of a material of surface-density 10 g/cm²*.

Solution : When empty the C.G. is at R, 20 cm above the base The total weight of the empty can is $\pi r^2.d.g+2\pi h.d.g$.

Again, the weight of water in the can $=\pi.5^2.20.1.g$.

The C.M. of this water is at Q, 10 cm above the base and that of the base is at P, its center. Let the C.M. of the system be at a height H from P.

Taking mass moments about P, of the mass of of the curved surface of the can, that of water and that of the can with water, we get

$$PR\times 2\pi r\,h\,d.g+\pi r^2 h'\times 1\times g\times PQ$$
$$=\pi r^2.d.g.H+2\pi r h.d.g\,H+\pi r^2.h'.g.H$$

or, $$20\times 2.5.40.10+25.20\times 10$$
$$=H(25.10+2.5.40+25.20)$$

$\therefore\quad H=11.3$ cm.

C. C.M. of a Pair of particles. In astronomy it is often necessary to locate the C.M of a pair of celestial bodies like the binary stars or a planet-satellite system ; for they hurtle through space as a unit while rotating about their centre of mass. The earth-moon pair is a case in point. On the other end of the scale, in a hydrogen atom the proton-electron pair does the same. (In refining Bohr's theory of hydrogen spectrum this point had to be taken into account). We deduce below that *the C.M. lies along the line joining the two particles and divides the line in the inverse ratio of their masses.*

Let a pair of particles of mass M and m at distances x_1 and x_2 from an arbitrary origin O (fig. I-7.21) be acted upon by parallel forces proportional to their masses. The C.M. is the point (C) where the resultant of these two forces cut the line joining the two points. Clearly the resultant would be proportional to $K(M+m)$ and act at a distance x from O. Then taking mass moments about O we have

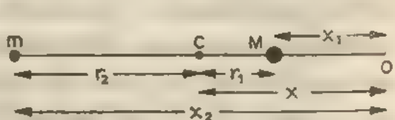


Fig. I-7.21

$$KMx_1+Kmx_2=K(M+m)x$$

or, $$x=(Mx_1+mx_2)/(M+m) \qquad \text{(I-7.10.1)}$$

(Compare the co-ordinates of a point dividing a st. line in a given ratio.)

Again taking moments about C we find

$$KM\times r_1 = Km\times r_2 \quad \text{or,} \quad (M/m)=(r_2/r_1) \qquad \text{(I-7.10.2)}$$

i.e. C.M. divides the line in the inverse ratio of the particle masses.

Ex. I-7.21. The earth is 80 times as massive as the moon and the separation between their centers is 60 times the radius of the earth. Find the distance of the C.M. of the earth-moon system from the centre of the earth taking the radius of the earth as 4000 miles.

Solution : Let r be the required distance and m the lunar mass. Then from I-7.10.2

$$80m\times r = m\times(4000-r), \quad \text{or} \quad r \simeq 3080 \text{ miles}$$

i.e. the point lies within the earth and describes what we call the orbit of the earth.

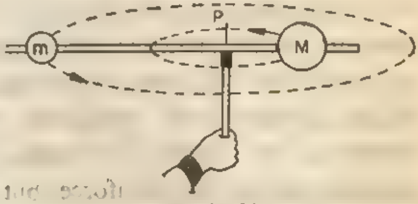


Fig. I-7.22

Fig. I-7.22 shows a model of a *smoothly* rotating pair of unequal masses M and m about a vertical pin P passing through the center of mass of the system. If P passes through any other point there will be *wobbling*. M and m may be thought to represent the earth and the moon respectively.

I-8

WORK, POWER AND ENERGY

I-8.1 Work. The term 'work' in our everyday language means some kind of labour, physical or mental, producing a result. In scientific language the term is used in a special sense. *Work is said to be done when the point of application of a force applied on a body moves in or against the direction of the force.*

Work, so defined, involves two quantities, *viz.*, (i) a force and (ii) a distance *in the direction of the force.* When a man pushes a cart on a level road or lifts a load from the floor, he not only exerts a force, but exerts it through a distance in the direction of the force. Under these conditions he is said to do work. When, however, he supports a bucketful of water in his hand without moving it, he exerts aforce but does not exert it through any distance in the direction of the force. Whatever muscular fatigue

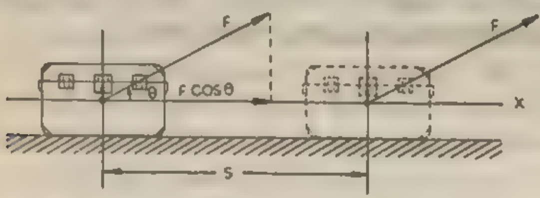


Fig. I-8.1

he may feel in the act of holding the bucket, he does *no work* in the technical sense of the term. Neither is he doing work on a stone which he is whirling at the end of a string (why not ?)

The amount of work (W) done is measured by the product of the force (F) and the distance (s) through which the point of application of the force moves *in the direction of the force.* In symbols,

$$W = F.S. \tag{I-8.1.1}$$

or **Work = Force × Distance.**

When the directions of the force and displacement are not the same (Fig. I-8.1) the component of the force in the direction (x) of displacement is $F\cos\theta$, where θ is the angle between the two. Since

$F \cos\theta$ is the *effective part* of the force in the direction of displacement, the work done is given by

$$W = F \cos\theta \times s = Fs \cos\theta = \mathbf{F.S.} \qquad \text{(I-8.1.2)}$$

or Work = Effective force × distance

Alternatively, we may resolve s into components $s \cos\theta$ and $s \sin\theta$ in the direction of F and perpendicular thereto. The displacement of F in its own direction is $s \cos\theta$; hence the work done by it is $Fs \cos\theta$. For a displacement $s \sin\theta$ no work is done by F. (Why not ?)

You get an example of application of Eq. I-8.1.2 when a body moves down an inclined plane under gravity. The force of gravity and the motion it produces are not in the same direction.

A. **No-work forces :** If no displacement occurs when a force acts or displacement is at right angles to the force, no work is done. For in the first case s is zero in the second so is $\cos\theta$. When a load is hanging from a support or a book is resting on a table no work is done, for the point of application of the force is not being displaced. A man in a moving lift does no work on the brief case he carries nor a man walking on a level road with the same ; in the first, relative position of the case with the man is not changing, in the latter, the motion is at right angles to the pull of the earth.

If we continue pushing a wall or support a heavy load for a long time we do no work, yet may feel tired and spent as if we have done much. Why is it so ? When we push repeatedly against a wall, various muscles move to apply small impulses ; contractions and expansions of muscles thus repeatedly occur leading to their being tired out.

Q. *A man rowing a boat upstream is at rest with respect to the shore. Is he doing any work ?* [S.S.Q.]

Ans. Yes. A force is said to do work when it moves its point of application in its own direction. Here the oar is a lever ; its fulcrum is in the water ; the load is where it is attached to the boat and the effort is applied at the end where the oarsman pulls it. As the man pulls the oar, the point of application of the force moves in the direction of the force. So work is done.

Situation with respect to the shore is immaterial.

A similar problem arises for a man *swimming upstream* but failing to advance. He also is doing work. Had he not exerted, he would have been carried downstream by a distance which may be taken as his displacement ; because, the force of his effort keeps him stationary with respect to the bank. He applies force on the water by thrashing his hand and feet because of which there is relative motion between the swimmer and the flowing water. Again, the position w. r. t. the bank is immaterial.

Problem : Two springs S_1 and S_2 have their *force constants* (i.e. force required to stretch them by unit length) as k_1 and k_2. On which is more work done when they are stretched (*i*) through the same length (*ii*) by the same force ? [*Ans.* (*i*) S_1 (*ii*) S_2.]

Cases however a rise when a force works and a displacement occurs but no work is done of which we have already seen examples. If a body slides over a *frictionless* surface *uniformly* no work is done, for both its weight and the normal reaction are at right angles to the plane. Again, a centripetal force is a no-work force for the same reason, force and motion being normal to each other, $\cos\theta$ in I-8.1.2 vanishes.

B. **Path-Integral of a force :** In the most general case, the force may be variable both in magnitude and direction while the path may be curved (fig. I-8.2). Then the work done as the particle moves from A to B is the sum of small finite amounts

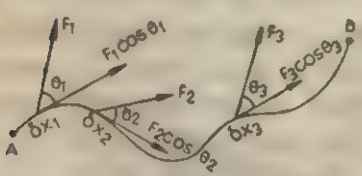


Fig. I-8.2

$F_1 \cos\theta_1.\delta x_1$, $F_2 \cos\theta_2.\delta x_2$, $F_3 \cos\theta_3\delta x_3$ etc

i.e. $W = \sum\delta W = F_1 \cos\theta_1\ \delta x_1 + F_2 \cos\theta_2\ \delta x_2, F_3 \cos\theta_3\ \delta x_3 +$

$$\cdots + F_n \cos\theta_n.\ \delta x_n = \sum_{n=1}^{n=n} F_n \cos\theta_n\ \delta x_n \qquad \text{(I-8.1.3)}$$

If the distances become infinitely small and so indefinitely numerous then the above relation becomes

$$W = \int_A^B F\cos\theta\ dx \qquad \text{(I-8.1.4)}$$

Thus *work becomes* the *path integral of force* as it is integrated over the entire path covered. Recall that, *impulse is the time integral of force.*

C. (i) **Work done by a system of force :** When a number of forces act on a body together and displaces it, then the work done is the *algebraical sum* of the individual work contributions. This is equal to the work done by the resultant of all such forces.

Work is a scalar : Though force (**F**) and displacement (**S**) are both vectors, work is not, it is a scalar. Whatever the direction of F and s, so long as the quantity $F s \cos \theta$ remains unchanged, W remains the same. Work done, is thus independent of direction.

Work is a scalar product of two vectors, **force** *and* **displacement** for their magnitudes are multiplied by the cosine of the included angle. Refer to eqn 1-2.12.2. Later we shall see power, the time rate of doing work, is also a scalar product of two vectors force (**F**) and velocity (**v**).

(ii) **Work done 'by', 'on' or 'against'.** When a body A exerts a force F on another body B making it move through a distance in the direction of F, we say that

(i) A (the agent which applies the force) does work *on B.*

(ii) Work is done *by A* or the force *F on B.*

(iii) When an agent moves a body against an opposing force, work is said to be done *against* the opposing force. Thus when a man lifts a load from the floor, he does work *on* the load *against* gravity. When a body falls under gravity, work is done *by* gravity.

Whenever work is done, it is done *against a resistance*. In lifting a weight, work is done against gravity. In pulling a load over a rough surface, work is done against friction. When an unbalanced force accelerates a body, it does work against the force of inertia. Such work is said to be *positive* work. When the point of application force moves in the direction of force i.e. motion and force are in the same sense, the work done is said to be *negative*

(iii) **Graphical Representation of Work :** If the displacement produced by a force be plotted against the force causing it, the principle of *area under the curve* gives us the work done (**F.s**) just as the distance covered by a moving particle is the area under the curve

on a velocity-time graph (fig. I-1.14). Here we however plot the *component* of the force in the direction of displacement. In fig. I-8.3,

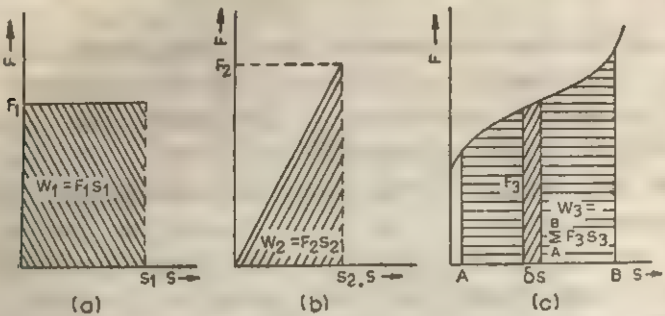


Fig. I-8.3

three cases are shown, work done by (i) a constant force, (ii) force proportional to displacemant (as in SHM or elastic deformation and (iii) a variable force.

I-8 2 A Work done in rotation Let a force F (fig. I-8.4) turn a disc about an axis while acting at a distance r from the axis, e.g a rope wound round the body and the force applied at one end of the rope If the body is turned through θ radians by the action of the force, the point of application of the force moves through a distanec $s=r\theta$, the length of the rope unwound. Hence the work done is

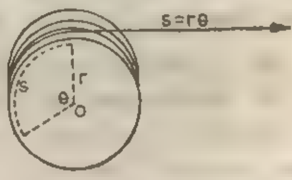


Fig. I-8.4

$$W = Fs = Fr\theta \qquad \text{(I-8.2.1)}$$

But Fr is the moment of the force about the axis of rotation, which is its *torque* (T). θ is the angular displacement (in radians).

$\therefore$ Work = torque × angular displacement = $T\theta$ (I 8.2.2)

It follows then, that when a torque (due to a couple) rotates a shaft, as in an engine, the work done in a given time is equal to the product of the torque and the angular rotation of the shaft in radians. If T is the torque due to the couple (i.e. the moment of the couple) and n the number of revolutions undergone by the shaft, then the work done is

$$W = T\,2\pi n \qquad \text{I-8.2.3)}$$

Problem : Find the work done if 3 turns are necessary in unscrewing a rusted heavy nut by a 10 cm wrench at the end of which the force is to be applied. (*Ans* 1 88J)

B. Work done by a couple : A pair of equal and unlike parallel forces acting at two different points of a finite body form a couple and a couple produces rotation. Its ability to produce rotation increases with the **torque** or the moment of the force, given by the product of one of the forces and the perpendicular distance between the lines of action of the two forces. You know these already.

Note : *Torque is a vector*, for a rotation has direction, clockwise or anti-clockwise. It is measured by the product $\mathbf{F} \times \mathbf{d}$ (Eqn I-6.4.2). Work is also measured by the product $\mathbf{F}.\,\mathbf{d}$. *Numerically torque and work are the same but physically completely different.* The former is a vector product of the two vectors ; it has a sense of rotation and also the vector magnitudes multiplied by the sine of the included angle (Equations I-6.4.2 and I-2.12.3). d in torque is the component of distance measured *perpendicular* to the force ; in work, d is the component of displacement *parallel* to the force.

Suppose you want to open a water tap you place your two fingers near the two ends of the upper crosspiece and press them equally in opposite directions ; the T-piece *rotates* and water begins to flow. This illustrates how a couple produces rotation (fig. I-8.5a). Fig. I-8.5(b) shows the relevant plan where A and B represent the points where your fingers press with equal and unlike forces F_1 and F_2 and rotate the taphead through an angle θ to the displaced position A B'. If AB $= l$, the the moment of the couple T) is $F \times l$. Then the work done by the forces about the axis are as follows :

Fig. I-8 5a

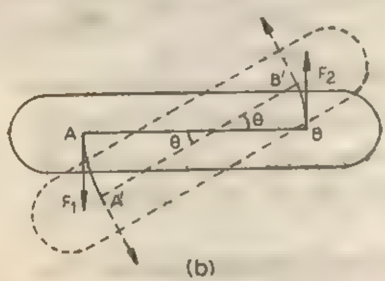


Fig. I-8.5b

F_1 at A $= F_1 \times$ arc AA'
$= F_1 \times \text{OA} \times \theta$
and F_2 at B $= F_2 \times$ arc BB'
$= F_2 \times \text{OB} \times \theta$
But here $F_1 = F_2 = F$ (say)
$\therefore$ Total work done $=$
$F \times \theta \times (\text{OA} + \text{AB}) =$
$F \times \theta \times \text{AB} = F \times l \times \theta$
$\therefore$ $W = T\theta$ same as I-8.2.2

or, *work done by a coup'e = moment of the couple × angular displacement*. Note that, the work done is independent of the position of rotation axis. If the moment of the couple happenes to be not along the rotation axis but inclined to it at an angle ϕ then the work done will be $T \cos \phi . \theta$.

1 8 3. Units of Work :

The unit of work involves the units of force and distance. Force may be expressed in absolute or gravitational units. Besides, there are the metric and the British systems. All these lead to a number of units of work, which however should not have been the case. The more important of these units are listed below.

Dimensions of work is $MLT^{-2} \times L = ML^2T^{-2}$

Nature of unit	*Unit of force*	*Unit of distance*	*Corresponding unit of work*
(1) MKS absolute (SI)	newton	metre	1 newton × 1 metre = 1 newton-metre or 1 **joule** (symbol J)
„ gravitational	kg-wt or kgf	metre	1 kg-wt × 1 metre = 1 kg-m
(2) CGS absolute	dyne	cm	1 dyne × 1 cm = **1 erg**
„ gravitational	gm-wt or gf	cm	1 gm-wt × 1 cm = **1 gram-centimetre** (g-cm)
(3) FPS absolute	poundal	ft	1 poundal × 1 ft = **1 ft-poundal**
„ gravitational	lb-wt or lbf	ft	1 lb-wt × 1 ft = **1 foot-pound** (ft. lb)

A definition in words may be easily provided for each unit of work. The *erg* is the work done when the point of application of a force of one *dyne* moves through a distance of one *centimetre* in the direction of the force.

The **Joule** is the work done when the point of application of a force of 1 *newton* moves through 1 *metre* in the direction of force.

The joule is the practical unit of work in the cgs system, and is equal to 10^7 ergs. But in the mks system or in SI units it is the absolute unit of work.

$$1 \text{ joule} = 10^7 \text{ ergs.} \qquad \text{(I-8.3.1)}$$

What is 'a kilogram-metre' or 'a ton-foot'? The former is the work done when a weight of one kilogram is lifted through one metre against gravity. When a mass m is moved vertically through a distance h, the force of gravity on it is mg and the displacement in the direction of the force, h Hence the work $W = mgh$, is done by gravity when the body falls, but against gravity when the body is lifted. mgh will be the work in absolute units, and mh the same in gravitational units.

[**Note :** In defining all gravitational units the idea gains ground that here work is done only in raising or lowering weights vertically. The impression is incomplete. A *kg-meter* is the work done when a force of 1 kgf i.e. 1 kg-weight (i e, a force generating on acceleration of 9.8 m/s^2 on one kg) acting in *any direction* moves its point of application by 1 m in the direction of motion. Similarly a force of 1 lb, which produces on a mass of 1 *slug** an acceleration of 32 ft/s^2, moves it through 1 ft, the work done is *ft-lb.*]

Conversion of units. We have seen before that the newton may be replaced by its equivalent kg m/s^2, the dyne by g cm/s^2 and the poundal by lb ft/s^2. In converting a unit of work from one system (of units) to another, we shall find this very useful. A few examples are given below.

$$1 \text{ joule} = 1 \text{ newton} \times 1 \text{ metre} = 1 \text{ kg m/s}^2 \times 1\text{m} = 1 \text{ kg m}^2/\text{s}^2$$
$$= 1000 \text{ g} \times (100 \text{ cm})^2/\text{s}^2 = 10^7 \text{ g cm}^2/\text{s}^2 = 10^7 \text{ erg.}$$

The equivalent of the joule is kg m^2/s^2 ; and of the erg, the equivalent is g cm^2/s^2. The foot-poundal $= 1$ poundal $\times 1$ ft $= 1$ lb. ft/s^2 $\times 1$ ft $= 1$ lb ft^2/s^2. The absolute units of work in any system of units is thus formed by the quantity (unit mass) $\times$ (unit length)^2 $\div$ (unit time)^2—ML^2T^{-2}.

We may now try some conversions, remembering that if the quantities are not in absolute units, they must first be so converted.

* As has been noted earlier (§ O-1.6), lb to-day is the unit of force which was formerly called lb-wt or lbf Slug is now the unit of mass formerly called a pound.

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To convert the ft lb into joules.

$$1 \text{ ft. lb} = 1 \text{ lb-wt} \times 1 \text{ ft} = 32{\cdot}2 \text{ poundals} \times 1 \text{ ft}$$
$$= 32{\cdot}2 \text{ lb ft}^2/\text{s}^2 = x \text{ kg m}^2/\text{s}^2 \text{ (say).}$$

Then $x = 32{\cdot}2 \times \frac{\text{lb}}{\text{kg}} \times \left(\frac{\text{ft}}{\text{m}}\right)^2 = 32\ 2 \times 0{\cdot}4536 \times (0{\cdot}3048)^2 = 1\ 356 \text{ J.}$

[N.B. This also shows how the unit symbols are treated as algebraic quantities.]

Example. I-8 1. A man weighing 150 lb ascends a flight of 36 steps each 8 inches high. What is the work he does against gravity ?

Solution : Work done $= 150 \text{ lb-wt} \times \frac{36 \times 8}{12} \text{ ft} = 3600 \text{ ft. lb.}$

Ex I 8.2. A force of 1 megadyne ($=10^6$ dynes) acts on a body weighing 1 kg. Find in joules the work done in 2 seconds.

Solution : To calculate the work done we require the distance over which the force acts.

The acceleration of the mass $= \frac{\text{force}}{\text{mass}} = \frac{10^6 \text{ dyn}}{1000 \text{ g}} = 10^3 \text{ cm/s}^2$

$\therefore$ The distance traversed in 2 s $= \frac{1}{2}at^2 = \frac{1}{2} \times 10^3 \times 4$ cm.

$\therefore$ The work done $=$ force $\times$ distance $= 10^6 \text{ dyn} \times 2 \times 10^3$ cm.

$$= 2 \times 10^9 \text{ erg} = 200 \text{ J.}$$

Ex. I 8 3. How much work was done by the sun in raising water vapour so as to form a cloud which on reaching the earth from one mile high produces a water pool $\frac{1}{2}''$ deep over a square mile ? Pat. U.]

Solution : Volume of water in the pool $= 1$ sq. mile $\times \frac{1}{2}$ in

$$= (5280)^2 \times \text{sq. ft} \frac{1}{2 \times 12} \text{ ft}$$

Mass of that water $= V\rho - \frac{(5280)^2}{2 \times 12} \text{ cu ft} \times 62.5 \frac{\text{lbs}}{\text{cu ft}}$

Work done in raising this mass of water through one mile

$$= (mg)h = Wh = \frac{(5280)^2}{2 \times 24} \times 62.5 \text{ lbs} \times 5280 \text{ ft}$$

$$= \frac{(5280)^3 \times 62.5}{24} \text{ ft-lbs} = 3.8 \times 10^{11} \text{ ft-lbs.}$$

Ex I-8 4. Show that in raising an extended body vertically through a distance, the work done is the product of its weight and vertical rise of its C.G.

Solution : A body is made up of a very large number of particles of weights w_1, w_2, w_3...etc and in raising the body let them rise through h_1, h_2, h_3...etc. Clearly the total weight $W = \sum_{n=1}^{n=n} w_n$

We take the Z-axis of a reference frame in the vertical dir.ction Let the vertical co-ordinates of the points initially and finally be Z_1, Z_2, Z_3 ...and Z_1', Z_2', Z_3'...etc. In raising the particles the total work will be $w_1h_1+w_2h_2+w_3h_3+$...etc$=\Sigma w_nh_n$ Now the initial and final positions of the C.G. will be by eqn. I-7.6.4

$$Z=\Sigma w_nZ_n/W \text{ and } Z'=\Sigma w_nZ'_n/W$$

Thus the vertical displacement of the C.G. will be

$$h=Z'-Z=\Sigma w_n(Z'_n-Z_n)/W=\Sigma w_nh_n/W$$

$$\therefore \quad \text{Total work}=Wh=\sum w_nZ_n$$

Here the body must be raised with uniform velocity for the result to hold.

Problem : 8 stone cubes ($\rho=2.5$ g/cc) each of side 10 cm lie scattered on ground. Find the Work necessary in making a pile placing one neatly above the other (Ans. 58.6J)

[**Hint :** Total height 80 cm. Height of C.G. 40 cm. Height of C.G. of first cube 5 cm]

I-8.4. Power. When work is done, its amount is not the only item of importance. The time in which the work is done is also of great importance. Suppose a 100 gallon tank is to be filled by drawing water from a well, 50 ft deep. 100 gallons of water weigh 1000 lb. Hence the work to be done in drawing it from the well will be $1000\times50=50{,}000$ ft. lb. If the work is done by manual labour a stronger man will be able to do it faster than a weaker one. The former would then have done more *work per unit time* than the latter. If a motor-driven pump were employed, a more powerful motor would complete the job in a shorter time than a less powerful motor. The former does more work per unit time than the latter.

The time rate of doing work is called power, *i.e*, *power is the work done per unit time.*

$$\text{Power}=\frac{\textbf{Work}}{\textbf{Time}}=\frac{\textbf{Force}\times\textbf{Distance}}{\textbf{Time}}=\textbf{Force}\times\textbf{Velocity}. \qquad \text{(I-8.4.1)}$$

$$\therefore \quad P=\mathbf{F}\cdot\mathbf{v}=F\cos\theta\, v \text{ or } F.v\cos\theta \qquad \text{(I-8.4.2)}$$

i.e. power is a *scalar product* of two vectors force and velocity.

When a person does work his sense of fatigue is determined more by the rate at which he does the work than by the total work done.

Suppose a boy weighing 120 lb ascends a flight of steps which carries him 15 ft above the ground. The work he does against gravity is $15 \times 120 = 1800$ ft. lb. When he completes the ascent in 20 seconds he feels little fatigue. But if he tries to do it in 4 seconds, he will feel much more fatigued. In the latter case his rate of doing work *i.e.*, power, is 5 times greater than that in the former case.

Two railway trains of the same weight move at 30 mph and 50 mph respectively. If the friction of the rails and the resistance due to air are the same for both, the latter does more work than the former, because it moves a greater distance in the same time. So its rate of doing work, or power, is greater than that of the former.

The power delivered by a machine may be constant, or it may fluctuate. In the latter case we shall be concerned with the *average power*.

Units of power – Watt and Horse-power. Any of the units of work combined with a suitable unit of time will provide a unit of power. Two practical units of power are, however, very widely used. They are (i) the watt and (ii) the horse-power.

When one joule of work is done per second, the power is one **watt** (symbol W). A power of one thousand watts is a **kilowatt** (KW). In other words, when 1000 joules of work are done per second, the power is one kilowatt. Electric machines are rated in watts and kilowatts. In mks units the watt is the absolute unit of power : in cgs units it is the practical unit. The watt is the SI unit of power, $1\text{W} = 1\text{J/s}$. A million watts is a **megawatt (MW).**

The practical unit of power used in engineering practice in the fps system is the horse-power (hp). When work is done at the rate of 550 foot-pounds per second (or, 33000 ft. lb per minute), the power is called one **horsepower.** James Watt initiated this unit.

To express the horse-power in watts.

$$\begin{aligned}1 \text{ horse-power} &= 550 \text{ foot-pounds/second}\\ &= 550 \times 1.356 \text{ joules/second} = 746 \text{ J/s}\\ &= 746 \text{ W (watts).} \qquad \text{(I-8.4.4)}\end{aligned}$$

A hp is thus approximately $\frac{3}{4}$ kW.

Some units of power and their cnnversion factors are summarised in the table below :

$$1 \text{ watt} = 1\frac{\text{joule}}{\text{second}} = 10^7\frac{\text{ergs}}{\text{second}}$$

$$1 \text{ hp} = 550\frac{\text{ft. lb}}{\text{second}} = 33000\frac{\text{ft. lb}}{\text{min}} = 746 \text{ watts} = 0{\cdot}746 \text{ kW} \quad \text{(I-8.4.4)}$$

$$1 \text{ kW} = 1000 \text{ watts} = 1{\cdot}34 \text{ hp.}$$

$$1 \text{ ft. lb per second} = 1{\cdot}356 \text{ watts.}$$

The kilowatt and the horse-power have given rise to two more *units of work*—the **kilowatt-hour** and the **horse-power hour.** The kilowatt-hour is the work done when power is used at the rate of one kilowatt over a period of one hour. This is the unit on which the cost of electricity is based. It is also known as the B. O. T. (Board of Trade) unit.

Similarly, the horse-power-hour is the work done when power is used at the rate of one horse-power over a period of one hour

Ex. I-8.5. *A 1000 gallon tank at a height of 60 ft above the water level has to be filled with water. If a motor pump rated at ½ hp is used to do the work, find the time required to fill the tank. (Given, 1 gallon of water weighs 10 lb).*

Solution : Weight of 1000 gallons of water = 10000 lb.
the total work to be done = 10000×60 ft. lb.

The motor has a power = ½ hp, *i.e.*, it can do 550/2 ft. lb of work in 1 sec.

$$\text{The time required to fill the tank} = \frac{10000 \times 60 \text{ ft lb}}{\dfrac{550 \text{ ft. lb}}{2 \text{ s}}}$$

$$= \frac{1000 \times 60 \times 2}{550} \text{ s} = 36 \text{ min } 22 \text{ sec (approximately).}$$

Problem : Water is being raised from a well up to a height of 25 ft. by means of a 5 hp motor pump. If the efficiency of the pump be 85%, how many gallons of water will be raised per mininute? (1 gallon of water weighs 10 lb, $g = 32.2$ ft/sec²). [H. S. '79]

Hint : From the wording of the question it appears that 'efficiency' here means 85% of the rated power (5 hp) is available for lifting water.]

Ex I-8 6. *A tractor can exert a horizontal force of 1000 kg. and travel at 5m/s. Find its power in killowats.*

Solution : The work done by the tractor in 1 s = 1000 kg-wt × 5m.
= 1000 × 9.8 newton × 1 metre.

∴ The work done in 1 s = 9800 newton-metres = 9800 joules

∴ Its power = 9800 J/s = 9800 W = 9.8 kW.

I-8 5. Energy. When a body can do work, we say it possesses energy. *Energy of a body is defined as its capacity for doing work.* It is *measured* by the amount of work the body can do. Energy and work are essentially the same kind of quantity, and are measured in the same units. We might characterise *energy* as *latent work.*

Various attempts have been made to define energy—

Energy—the go of things (Maxwell)
—is a measure of price of mass in motion.
—is that which is changed from one form to another when work is done.

A man or a horse can do work ; so he possesses energy. Steam can push the piston within the cylinder of a steam engine ; so it possesses energy. A moving body possesses energy since it can make other bodies move when it collides with them. An elevated body possesses energy since, while falling, it can pull other bodies up. If such a body pulled a mass of 1 kg through a height of 1 m before reaching the ground, we may say its energy was 1 kilogram-metre or 9·8 joules, or that the body lost 9·8J of energy in pulling the weight up.

This entire universe provides the stage for interaction of matter and energy. But matter itself can do no work for it represents inertia It is the energy that imparts to matter its ability to do work. A body moves or vibrates, even warms up when energy is passed into it. Had there been no energy the universe would have been inert, dead. Fortunately the universe is an unbelievably vast storehouse of energy. It has been further established that matter is after all, condensed energy just as ice is condensed water.

Different forms of energy. There are many forms of energy. An electrically charged body attracts lighter bodies, and can make them move. It possesses what we call electrical energy. Magnets possess maghetic energy. Heat of steam makes locomotives move. Heat is a form of energy. Radio waves, light and X-rays are also

forms of energy. An important form of energy is the chemical energy, as is possessed by coal. When coal burns it combines with oxygen releasing chemical energy in the form of heat.

In recent times man has harnessed a vast source of energy. It is derived from the conversion of matter into energy under the action of nuclear forces in the atom, and is called *atomic energy.* The atom bomb the hydrogen bomb and the atomic reactor are devices for the release of atomic energy. This form of energy provides the sun and the stars with their apparently inexhaustible supply of energy.

In mechanics, we are however interested in what we call **mechanical energy**. The definition of energy we gave at the beginning of this section, strictly relates to mechanical energy. In the wider sense, *energy is that which can bring about any change in matter.* A body may possess mechanical enerey due to either one or both of the following two causes, viz, (i) by virtue of its *motion*, and (ii) by virtue of its *position* or *configuration.* The former is called *kinetic energy* and the latter, *potential energy.*

I-8.6. **Kinetic energy** of a body is defined as the energy it possesses because of its motion. It is *measured* by the amount of work a moving body can do against an opposing force before it comes to rest A moving hammer possesses kinetic energy, which enables it to do work in driving a nail into a wall against the resistance. It cannot however do so, if left in contact with the nail.

A. **Translation**. To obtain an expression for the kinetic energy of a body of mass m moving with velocity v, we impress on the body a force F which opposes the motion and finally brings it to rest. The acceleration (or rather the deceleration) produced by the force is $-a=-F/m$. If the body moves through a distance s before it comes to rest, then from the relation that

$$\text{(final velocity)}^2-\text{(initial velocity)}^2 = 2\times\text{acceleration}\times\text{distance}$$

we have $0^2-v^2=-2as$ or $v^2=2(F/m)s$ or $Fs=\frac{1}{2}mv^2$.

Now Fs is the work done by the moving body against the opposing force before coming to rest. Hence this is also the magnitude of its kinetic energy. Since $Fs=\frac{1}{2}mv^2$, the latter expression gives the kinetic

energy of the moving body in terms of known quantities. Note that it does not depend on the opposing force F.

$$\textbf{Kinetic energy} = \tfrac{1}{2}\ \textbf{mass} \times (\textbf{velocity})^2$$

$$K = \tfrac{1}{2} mv^2 \tag{I-8.6.1}$$

Work-Energy theorem. It is easy to see that if a force F acting through a distance s changes the velocity of a body from u to v, then

$$Fs = \tfrac{1}{2}m(v^2 - u^2) = \mathrm{K} - \mathrm{K}_0 = \Delta \mathrm{K} \tag{I-8.6 2}$$

or *work done by the force = change in kinetic energy.*

for $\tfrac{1}{2}m(v^2 - u^2) = \tfrac{1}{2}m.2as = ma\,S = F.s.$

This very important result is known as the **work-energy theorem** for a particle and indicates the relation between *energy expended* and the *work obtained. Kinetic energy* of a moving body is thus *the work it can do before being brought to rest*, as we have noticed already.

Its magnitude can be alternatively and more precisely established as follows. If the point of application of a constant force F moves through a small distance dS against an opposition, the work is $dW = F.dS$.

Then the total work done is

$$W = \int dW = \int_{S_0}^{S} F\, dS = \int_{S_0}^{S} ma.dS = m\int_{S_0}^{S} \frac{dv}{dt} \cdot S$$

$$= m\int_{S_0}^{S} dv \cdot \frac{dS}{dt} = m\int_{u}^{v} dv\, v = \tfrac{1}{2}m(v^2 - u^2) \tag{I-8.6.3}$$

If the body is brought finally to rest $v = 0$ and K.E. $= \tfrac{1}{2}mu^2$.

More general Case : Remember that, F in the last equation is a resultant force, not the applied force. You know that to move a body you have to overcome frictional force. When the applied force on a moving body is equal to the frictional force, the resultant applied force is zero and by Newton's First law there will be uniform motion. That is why you spend petrol and pay for it when you drive a car uniformly along a straight road. To accelerate the car you have to apply a force F greater than the frictional force F'. Then

$$(F - F')s = \tfrac{1}{2}m(v^2 - u^2)$$

where the L. H. S. gives you the work done *on* a body and the R.H.S.

of the equation its consequent rise in kinetic energy. Informations available from the above equation are

(*i*) Work done on a body by a *net* applied force raises its K.E.

(*ii*) If we put the above relation in the form $F.s = F's + \frac{1}{2}m(v^2 - u^2)$, then the first term of the r.h.s gives the work done *against* the opposing force. Thus (*a*) a part of the *applied force overcomes* the oposition and (*b*) the remaining part *increases* the K.E. of the body. With $F'=0$, say on a smooth (were it possible) table, the applied force itself becomes the net force and the entire work it does, becomes the K.E of the moving body.

(*iii*) Work done or energy spent in overcoming opposition is said to be *dissipated*, mostly into *heat* sometimes into light or sound. The factor or agent producing this change (friction, viscosity, electrical resistance) is called the *dissipative factor* or agent.

(*iv*) If $F=0$, then $\frac{1}{2}m(v^2 - u^2) = -F'.S$ which suggests that $v<u$ i.e. K.E. decreases as the distance is described. due to the work done by the body against the opposing force. This is what happenes when you switch off the engine of your moving car and it decelerates. The K.E. that disappears is converted into heat.

B. Kinetic Energy of Rotation : Since motion may be rotational also and it requires a couple to generate rotation, a rotating body also possesses kinetic energy. This energy would, by analogy, be $\frac{1}{2}I\omega^2$ where I is the moment of inertia (analogous to m, the mass) and ω, the angular velocity (analogous to v the linear velocity)

C K. E due to explosion : When an explosion occurs or a nuclear disintegration, the two fragments may move in opposite directions ; the larger mass is said to recoil. It is similar to firing a bullet or a shell from a rifle or a gun If the respective masses be m and M and velocities v and V we have

$$\frac{\text{K.E. of Smaller mass}}{\text{K.E. of Larger mass}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}MV^2} = \frac{mv.v}{MV.V} = \frac{v}{V} = \frac{M}{m} = \frac{1/m}{1/M} \qquad \text{(I-8.6.1)}$$

since moment mv and MV must be equal. So for an explosion, the smaller fragment has a larger K.E, though the momenta of two fragments are equal. This conclusion has been arrived at before (See Example I-3.). K.E. varies inversely as the mass as for collisions or explosions.

If K be the total energy of the two fragments from I-8.6.1

$$\frac{K_m}{K_M}+1=\frac{M}{m}+1=\frac{M+m}{m}=\frac{K}{K_M}$$

$$\therefore \quad K_M=\frac{mK}{M+m} \text{ and } K_m=\frac{MK}{m+M} \qquad \text{(I-8.6.2)}$$

D. Momentum and kinetic Energy : Momentum of a moving body is mv and its kinetic energy is $\frac{1}{2}mv^2$. For a long time there raged a controversy as to which represents the force of a moving body. Among others, Hooke, Newton himself, Descartes, Leibnitz, these giant mathematicians engaged in the debate till D'Alembert solved it. He pointed out that *both represent the accumulated effects of force* but their significances are quite different ; as we have shown above and else where that bodies with same momenta may have different kinetic energies.

If $\mathbf{p}(=mv)$ represents the linear momentum then kinetic energy is

$$K_L=\tfrac{1}{2}mv^2=\tfrac{1}{2}\frac{(mv)^2}{m}=\tfrac{1}{2}(\mathbf{p}^2/m) \qquad \text{(I-8.6.3)}$$

Further, momentum involving $\mathbf{v}$ must be a vector, while kinetic energy (i.e. latent work) involving v^2 is a scalar quantity.

Again, for a rotating body angular momentum $L=I\omega$ and hence it's kinetic energy $K_R=\frac{1}{2}L^2/I$ by analogy.

Ex. I-8.7 A body of mass 10 kg moves with a velocity of 10 metres per second. Find its kinetic energy in ergs and joules.

Solution : 10 kg = 10,000 g ; 10 m/s = 1000 cm/s.

$$\therefore \quad K.E.=\tfrac{1}{2}\times 10^4 \text{ g } (10^3)^2\frac{\text{cm}^2}{\text{s}^2}=\tfrac{1}{2}\times 10^{10}\ \frac{\text{g.cm}}{\text{s}^2}\times \text{cm.}$$

$$=\tfrac{1}{2}\times 10^{10} \text{ ergs}=\tfrac{1}{2}\times\frac{10^{10}}{10^7}=\tfrac{1}{2}\times 10^3 \text{ joules}=500 \text{ joules.}$$

To get the work in absolute units of the cgs system the mass must be in grams and the velocy in cm/s. The values in the problem are in mks units. So kinetic energy $=\frac{1}{2}\times 10$ kg $\times$ 10 m/s$)^2=500$ kg m^2/s$^2=500$ J

Ex. I-8.8 A bullet weighing 4 oz is fired with a speed of 1200 ft/s from a gun weighing 20 lb. Calculate the kinetic energies of the shot and the gun.

Solution : K.E. of the bullet $=\frac{1}{2}\times\frac{4}{16}$ lb $\times(1200 \text{ ft/s})^2$

$=\frac{1}{8}\times 1200\times 1200$ lb. ft^2/s$^2=18\times 10^4$ ft. poundals.

To calculate the K. E. of the gun we must know its velocity. This can be found from the principle of conservation of momentum ;

Now momentum of the shot $= \frac{4}{16}$ lb $\times$ 1200 ft/s $=$ 300 lb ft/s.

$\therefore$ the velocity of the gun $= \frac{\text{momentum}}{\text{mass}} = \frac{300 \text{ lb. ft/s}}{20 \text{ lb}} = 15$ ft/s

$\therefore$ K. E. of the gun $= \frac{1}{2} \times 20$ lb. $\times$ (15 ft/s)^2

$= 10 \times 225$ lb ft^2/s^2 $=$ 2250 ft. poundals.

[Note that the kinetic energies are not equal, but are in the inverse ratio of the masses.]

I–8.7. **Potential energy** is the energy which a body possesses by virtue of its position relative to the surroundings, its condition, or configuration (*i.e.*, the relative position of its parts). It is *measured* by the work the body can do in passing from the given position, condition or configuration, to some *standard* position, condition or configuration. *Potential energy is always measured by the difference from the standard position. The P.E. in the standard position is always taken as zero. We can never find the absolute value of the potential energy.* The choice of the standary position is *arbitrary.*

A **Potential Energy due to Position :** Let a body of mass m be raised to a height h above the ground. In falling it can pull another body up via a pulley. The work it can do, is given by the product of its weight and height above the ground (generally taken as the standard position). This is the measure of its gravitational potential energy.

Gravitational potential energy $= (F.\ d) = mgh.$ (I-8.7.1)

Absolute value of potential energy cannot be determined. Consider the above case. A body raised to a height h above the ground level has a potential energy mgh, if it can come back to the ground level. If the body was on a table of height h', its potential energy relative to the table top would have been $mg\,(h-h')$. So the value of the gravitational potential energy depends on which level we take as the level of zero energy. Now, the choice of this zero energy level is *arbitrary*, that is, we choose it according to our convenience. Any value of potential energy, that we derive depends on our choice of the zero level of potential energy, which is arbitrary. *If the zero level is altered, the value will alter by some constant amount.*

Potential energy of all kinds is always measured with respect to some standard configuration. *We never know* what the potential energy is in the standard configuration. Hence we say *potential energy is undefined to the extent of an arbitrary additive constant.* This constant is the potential energy in the standard configuration. We never know this quantity. In measuring P.E. what we measure is the change in potential energy.*

The arbitrary additive constant can be introduced mathematically as below. Let the body of mass m be displaced vertically through a distance dh. Then the total work done can be found by integrating the quantity $mg.dh$.

$\therefore$ Work done = Potential Energy = $\int mg.dh = mgh + C$ (I-8,7.2)

where C is the constant of integration. It is the work done when $h = 0$, i.e., the potential energy in the standard configuration. We cannot measure it.

B Potential energy of strain, due to change in condition or configuration.

(1) Compressed air can drive machinery. It possesses energy by virtue of its *condition of being compressed.* The standard condition is generally that under normal atmospheric pressure. The work that the compressed air can do in expanding to a pressure of one atmosphere is a measure of its potential energy.

In compressing air, the pump draws air from a large vessel of air at normal atmospheric pressure. Since at normal atmospheric pressure air molecules do not move as a mass it is taken to be the standard configuration. But bringing them together to this condition must have required some unmeasured work which is the P.E at the standard condition, the arbitrary constant.

(2) The main spring of a clock, when wound, drives the hands of the clock. It acquires *potential energy in winding* which changes the relative position of the different portions of the spring. As it unwinds the spring loses energy, ultimately coming to rest. Then its P.E is taken to be zero and it is said to be in its standard configuration. But work must have been done when shaping a metal strip into

* When we measure temperature by a thermometer we measure likewise the temperature difference.

the spring but we never consider that. This is the P.E. at the standard configuration and we do not know it, the arbitrary additive const.

1-8.8. Relation between the work done and the potential energy of a system. Take a body of mass m at rest on the surface of the earth The potential energy of the system consisting of the body and the earth in this condition is *arbitrarily* taken to be zero. When an agent raises the body to a height h, the work done *by* him *against* gravity (*i e.*, against the force operating between the bodies comprising the system) is mgh. The potential energy of the system is also mgh now.

If, on the other hand, the body fell from the height h to the surface of the earth, the work done *by* gravity, *i.e.*, force operating between the bodies comprising the system, is mgh, its loss of potential energy. We therefore find that

(*i*) *When work is done by the forces operating between the bodies comprising a system, the potential energy of the system diminishes exactly by the amount of work done*, while

(*ii*) *the potential energy of a system increases when an external agent does work on it* (*i e*, *against the forces operating within the system*), *the increase of energy being equal to the work done.*

The result applies to gravitational, electric and magnetic cases, dissipative forces of the nature of friction being neglected.

Effect of Altering the Base Plane : The effect is shown in fig. 1-8.6. With reference to the base plane, m at A has a +ve P.E. of mgh_1 and $-$ve potential energy of mgh_2, when at C. If the base plane is shifted to C, then P.E. of m at A will become $mg(h_1+h_2)$.

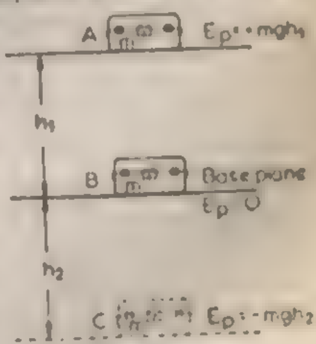


Fig 1-8.6

Heights of mountains are measured from the mean sea-level arbitrarily taken to be at zero level (Water level of the Caspian Sea in Russia is 84 feet below and that of the Dead Sea in Palestine 1291 feet below this level) Everest rises 29,0.8ft above Sea-Level while the greatest oceanic depth known till now, the Challenger Deep in the Pacific Ocean is 34,178 ft below the level. If height were

measured from there, Everst becomes 63,206 ft high ! So *change of base level would alter the measure of optential energy.*

I-8 9. Potential Energy and Equilibrium. Since energy of an agent is its capacity for doing work and potential energy of a body or (system) is the work stored up in it, *every system tends to attain the condition of minimum potential energy.* We have discussed above (I-7.8) the nature of different cases of equilibrium. Here we present the explanation of their behaviour.

The behaviour of bodies in different types of equilibrium are governed by the change in P.E. they undergo, when slightly disturbed. When the potential energy of a body is minimum and it is slightly disturbed its C.G rises and so P.E. increases. The body then tends to reduce P.E. and thus to return to equilibrium. So the equilibrium is *stable.* If P.E. is high the body tends to reduce it and so the equilibrium becomes *unstable.* If however, change of position of the body does not change the position of its C.G. and hence of its P E. the equilibrium becomes neutral.

A ball in equilibrium beside a deep hole rolls down if slightly pushed because of change of base plane has given to it more than minimum P.E. which it had, while lying on the ground.

I 8.9. Conservation of Mechanical energy : Mechanical potential energy as you know are of two types, kinetic and potential and one changes continuously to another when falling or rising whether in a vertical line or an incline or a curved trajectory. In absence of friction their sum total always remains constant. This will be verified in some simple cases.

Note : Potential energy is possible in other forms e.g. chemical potential energy in electric cells, solar energy in fuels like coal or oil, nuclear energy in atomic nucleii, cases outside the scope of our study.

A. The sum of the kinetic and potential energies of a freely falling body remains constant throughout its motion. Let m be the mass of a body falling freely from rest from A at a height h above the ground. Let v be its velocity after it has fallen through x to B. From the relation

(final velocity)^2 – (initial velocity)^2 = 2 × acceleration × distance

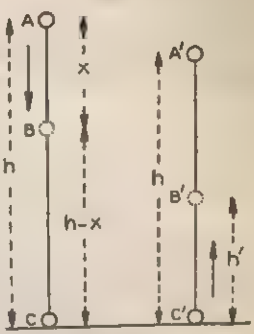
we have $$v^2 = 2gx$$

∴ The kinetic energy of the body at B is $\frac{1}{2}mv^2 = mgx$.

Its potential energy at this height, *i.e.*, $h-x$ above the ground, is $mg\ (h-x)$.

$\therefore$ The total energy = kinetic energy + potential energy

$= mgx + mg\ (h-x) = mgh.$



(a) Fig. I-8.7 (b)

Since x may have any value from 0 to h, we find that the total energy of the falling body remains constant and is *equal to its potential energy at the topmost position.*

It is also clear from the analysis that the *gain in kinetic energy is equal to the loss in potential energy.*

If v_0 is the velocity of the body when it reaches the ground, then $v_0^2 = 2gh$. Hence the kinetic energy when the body just touches the ground at $C = \frac{1}{2}mv_0^2 = mgh =$ its potential energy at the top.

B **Body rising vertically :** Let it start from C′(fig. I-8.7 *b*) with a kinetic energy $\frac{1}{2}mu^2$ and its rise is governed by the relation $h = ut - \frac{1}{2}gt^2$. Now, the maximum height (A′) attained is $h = u^2/2g$ so that $gh = \frac{1}{2}u^2$; multiplying by m we get $mgh = \frac{1}{2}mu^2$.

At a height h' (B′) the velocity v comes from the relation $u^2 - v^2 = 2gh$. Multiplying by $\frac{1}{2}m$ we get $\frac{1}{2}mu^2 - \frac{1}{2}mv^2 = mgh'$ i.e. loss in K. E. equals the gain in P.E. Sum of them at B′, as indeed throughout the upward journey is constant.

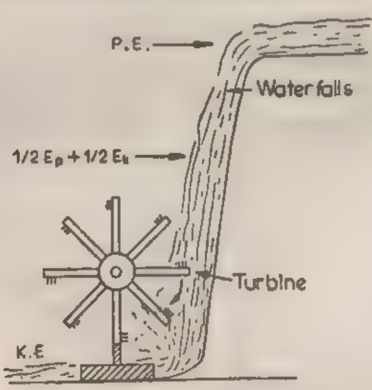


Fig. I-8.8

In fig. I-8.8 we see the transformation of potential energy to kinetic energy in a grand natural sight—the water-falls. At the top the water has P. E. because of its position above the base. As water spills over the edge and falls with ever-increasing speed, its K. E. ($\frac{1}{2}mv^2$) gains and P. E. loses out. At the bottom of the falls

the P. E. approaches zero while K. E. approaches the maximum. A turbine or a paddle wheel placed there will be set rotating and can be utilised in generating electricity (Hydro-electricity).

C. **Body sliding down or climbing up a smooth incline :** Let a mass m slide down a smooth plane inclined at θ to the horizontal.

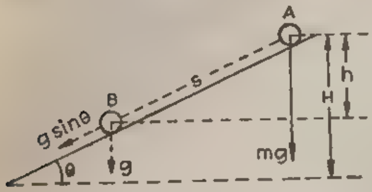


Fig. I-8.9

Its acceleration down the incline is $g \sin\theta$ (i.e. g thereby is 'diluted' as Galeleo did). Let it start from rest at the top (A) and roll down a distance s (B) where it acquires a velocity $v^2 = 2g \sin\theta.s$ and a K.E. $\frac{1}{2}mv^2 = mg\sin\theta.s = mg\ s \sin\theta = mgh$.

But h is the vertical distance through which the mass would have fallen in the same time that the ball has rolled. If the height of A is H form the base-levele the energy is initially wholly potential and mgH.

Hence the potential energy at B is $mg(H-h)$ and kinetic energy mgh. So the total energy at B will be mgH i.e. the initial energy at A, the top. If now the ball is so pushed up the incline that it just gains the top, the initial K.E. is finally converted into P.E. (in absence of friction)

Note that the velocity gained by a body rolling down an incline is independent of the inclination angle and depends only on the height descended for $v^2 = 2g \sin\theta.s = 2gs.\sin\theta = 2gh$.

D **Pendulum :** It is a small ball sus pended by a thread vertically from a rigid support and can oscillate in an arc. We have seen that the movement is caused by g as indeed in all the examples cited above.

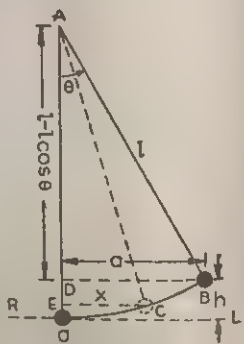


Fig. I-8.10

When the pendulum bob is at rest, it is at its lowest possible position O (fig. I-8.10) and hence has minimum P.E. If pulled up to to B it is at a higher level than the reference level (RL) through O and hence possesses more P.E. than at O.

Let that of B be h, and h' that at C. Then at these points respective P.E. are mgh and mgh'. When released the ball would come down along the arc, gaining in velocity i.e. in K.E. which will maximise at O. At any point C in its path the sum of P.E. and K.E. will be constant.

At B, the maximum diplacement position, the bob is at a horizontal displacement of a from the vertical AO and height h from the horizontal OL.

Now $h = \text{OD} = \text{AO} - \text{AD} = l - l\cos\theta$

$$= l(1-\cos\theta) = l.2\sin^2\tfrac{1}{2}\theta$$

$$= l\,2.\left(\frac{a}{2l}\right)^2 = \frac{a^2}{2l}$$

$\therefore$ $(\text{P.E.})_B = mgh = mg(a^2/2l)$

Similarly $(\text{P.E.})_C = mgh' = mg(x^2/2l)$

Loss in P.E. in coming down from B to C = Gain in K.E. = K.E. at C. In falling through a height $(h-h')$ the velocity acquired will be

$$(v^2)_C = 2g\,(h-h')$$

$$\text{and } (\text{K.E.})_C = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m.2g\left(\frac{a^2-x^2}{2l}\right)$$

$$= \frac{mg}{2l}(a^2-x^2)$$

$\therefore$ Total Energy at C $= \dfrac{mga^2}{2l}$ = Initial P. E. at B.

Ex. I-8.9. *A 20g bullet moving horizontally at 100 m/s embeds itself at the center of a 1 kg wooden block, suspended by a light vertical string 1m long. Find the maximum inclination of the string.*

Solution : From momentum conservation $20\times100=(1000+20)v$ or $v=\frac{100}{51}$ m/s. The K.E. is then $\frac{1}{2}(m+M)v^2$. It raises the mass through say h, gaining a P.E. of $(M+m)gh$. Clearly these two are equal due to energy conservation.

$\therefore$ $h = \frac{1}{2}(v^2/g) = l(1-\cos\theta)$

$\therefore$ $\cos\theta = 1+\dfrac{v^2}{2gl} = 1+\dfrac{(100/51)^2}{2\times9.8\times1} = 0.8038$

$\therefore$ $\theta = 37°$ (from log table).

E. Projectile : Let a particle of mass m be fired at an angle α to the horizontal with an initial velocity u (fig. I-8.11). Its vertical and horizontal components are respectively $u \sin \alpha$ and $u \cos \alpha$; the former raises m while the latter moves it horizontally at the same time. The two together move the particle along a parabola.

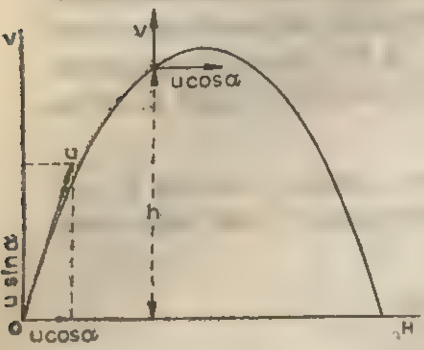


Fig. I-8.11

Let the particle at a given moment be at A, at a vertical height of h and its *vertical* component of velocity at that instant, v. Then from what we have learnt before $(u \sin \alpha)^2 - v^2 = 2gh$, from which we get, after multiplying by $\frac{1}{2}m$,

$$\tfrac{1}{2}mu^2 \sin^2\alpha - \tfrac{1}{2}mv^2 = mgh$$

There, the kinetic energy of the particle is the sum of those due to vertical and horizontal velocities, namely

$$(\text{K.E.})_A = \tfrac{1}{2}mv^2 + \tfrac{1}{2}mu^2 \cos^2\alpha$$
$$\text{and } (\text{P.E.})_A = mgh$$

$$\begin{aligned}\text{So the total energy} &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}mu^2 \cos^2\alpha + mgh \\ &= (\tfrac{1}{2}mv^2 + mgh) + \tfrac{1}{2}mu^2 \cos^2\alpha \\ &= \tfrac{1}{2}mu^2 \sin^2\alpha + \tfrac{1}{2}mu^2 \cos^2\alpha \\ &= \tfrac{1}{2}mu^2 = \text{Initial K.E.}\end{aligned}$$

Hence in absence of friction, mechanical energy of a projectile remains constant throughout its path, for note that A may be any point on it.

Ex. I-8 10 An upward force of 196N raises a 10 kg mass uniformly through 10m. Find the work done by the force and the work done against gravity. The work done by the force is much greater than the gain in gravitational P.E. Show by calculations that the law the of energy conservation is quantitatively satisfied here.

[J. E E '78]

Solution : Work done by $F = F \times h = 196\text{N} \times 10\text{m} = 1960\text{J}$

Work done against gravity $= mgh = 10 \text{ kg} \times 9{\cdot}8 \text{ m/s}^2 \times 10\text{m} = 980\text{J}$

The latter work gets stored up in the body as its P.E.

As $F\uparrow(196\text{N}) > mg\downarrow(98\text{N})$ the body gets an upward acceleration.

The upward resultant force $F_r = F - mg = 196 - 98 = 98$N, and upward acceleration $a = F_r/m = 98\text{N}/10\text{kg} = 9.8 \text{ m/s}^2$

As the body starts from rest with this upward acceleration its velocity at a height of 10m is

$$v^2 = 2gh = 2 \times 9.8 \times 10 = 196 \text{ m}^2/\text{s}^2$$

Hence K.E. there is $\frac{1}{2}mv^2 = \frac{1}{2} \times 10\text{kg} \times 196 \text{ m}^2/\text{s}^2 = 980\text{J}$

Thus at a height of 10m, K.E.+P.E $= 980 + 980 = 1960$J

i.e. there being no change in energy the conservation principle holds.

Problem : A balloon full of hydrogen rises with increasing velocity. Thus with height both its K.E. and P.E. increases How is it consistent with conservation of energy principle ? [J. E. E. '84]

[**Hint :** The force of buoyancy $Vg\ (\rho_{air} - \rho_{hydrogen})$ provides it with an upward force greater than the weight ($V\rho_{air}\, g$) of the balloon.]

I-8.10. **Gravitational P. E. is independent of Path.** Let a particle be taken from A to B (fig. I-8.12) in the earth's gravitational field along ACDB and it gains in P.E. Let CD represent a very small displacement along this path. Then as for an inclined plane, work done on m in shifting it from C to D against gravity is $m.g \sin\theta.$CD $= mg$ CD $\sin\theta = 2mg.$DG $= mg$EF. Then the total work done on m in moving it from A to B will be $\sum_A^B mg \sin\theta\ \ \delta r = mg \sum_A^B \text{EF} = mg$ AB $= mgh$. So work done is independent of path in the gravity field, provided the initial and final points are at the same level difference.

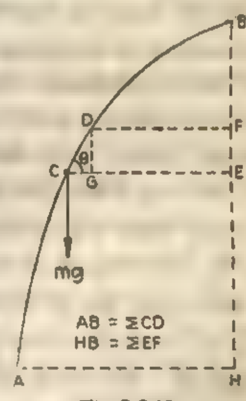


Fig. I-8.12

We have seen the same, when a body is raised along an incline.

If A is above B the same argument applies, only in this case work is done by gravity.

It therefore follows that if a body is raised from one point to another and then returned to the initial point then the change in gravitational potential i.e. the total work done is zero. Hence the

gravitational force (or field) is said to be **conservative** provided there is no friction.

Remember (*i*) When a body falls under gravity, the work done is *negative*; the body moving in the direction of force, work is done *by* gravity.

(*ii*) When a body is raised, displacement occurs *against* gravity and work done is *positive*.

(*iii*) Change in potential energy with raising a body, occurs for the *earth-body system*. For, with raising of the body, the earth also gains in P. E. though that is very small. Potential energy is the common property of the raised body and the earth.

(*iv*) Gravitational potential energy may be considered negative when a body falls below the accepted arbitrary zero level. We shall see in Chapter II-1 that the force of gravity being attractive the potential at any point on or above the earth is –ve, it being zero at infinite distance from the center of the earth.

I-8.10. Elastic Potential Energy: In winding up a watch or a clock, in shooting an arrow from a bow or a stone from a sling, in compressing or elongating a spring, you bring about a deformation of the body concerned. This is to be done against the inherent elastic forces of stress (§ II-3.2) and you have to do work. This work remains stored up in the strained body as its elastic potential energy. On removing these deforming forces, these bodies release this energy to regain their original condition or configuration and hence do work. That is how a watch or clock-hands are made to move, the arrow or the stone gain energy for motion, the spring vibrates.

If we elongate a spring the force opposing it is proportional to the elongation The force required to produce unit elongation is called its *spring constant* or force constant (K). Thus if a spring is elongated from l_1 to l_2, the force opposing that, rises to $K(l_2-l_1)$ from zero. So we take the average force to be $\frac{1}{2}K(l_2-l_1)$. Hence work done on it and the potential energy gained thereby is

$$(\text{P. E})_{\text{Elastic}} = \text{Force} \times \text{distance} = \tfrac{1}{2}K(l_2-l_1) \times (l_2-l_1) = \tfrac{1}{2}K(l_2-l_1)^2$$

Q. Two springs have their force constants as K_1 and $K_2 (K_1 > K_2)$. On which spring is more work done (i) when lengths are increased

by the same amount, (ii) when they are stretched by the same force ? [Ans. (i) First spring ; (ii) Second spring]

1·8 11 Transformation of energy. A falling body provides a very simple example of transformation of energy from one form into another, where potential energy changes into the kinetic form. A body projected upwards shows the reverse transformation. There are various other forms of energy such as thermal, electrical, magnetic, chemical etc. *When we look carefully into any natural event we find that it is nothing but a transformation of energy from one form into another.*

When we rub our hands together, heat is developed. Here mechanical energy is converted into heat. A steam engine converts the heat energy obtained by burning coal into mechanical energy.

When a closed coil of wire is rotated by mechanical means near a strong magnet, electricity flows through the coil. Mechanical energy has been converted into the electrical form. When an electric current passes through an electric motor, as in our electric fans, or in the motors of tram cars, the spinning of the motor illustrates conversion of electrical energy into mechanical energy. Examples of this kind may be multiplied without limit.

Note that **the sun is the ultimate source of energy for all changes** on the earth. Light and heat energy from the sun are necessary for the germination and growth of plants. Animals depend for their food on plants. Coal, one of our main sources of energy, represents stored up sunshine of far-away geologic ages. Oil, another important source of energy, is supposed to have been derived from the bodies of minute organisms which lived aeons ago. The water-cycle, essential for life on earth, is maintained by the solar heat. Recently, we have harnessed atomic energy, which is independent of the solar energy.

Conservation of energy. In all examples of transformation of energy it is to be noted is that *no body or system of bodies can acquire energy except at the expense of energy possessed by other bodies.*

Energy is never obtained from anything that is not energy, nor turned into anything that is not energy. Energy can only change form, or pass from one body into another. Whenever one system loses energy, another system gains it. Wherever measurement is possible it is found that the loss of energy of one system is exactly

equal to the gain in the energy of the other. Hence we believe that energy is an entity of nature which is conserved The law of conservation of energy may be stated as follows :

Energy can neither be created nor destroyed. Hence *the total amount of energy in the universe remains constant.*

A system of bodies which can exchange energy only among themselves, no energy leaving the system nor any energy entering it from outside, is an *isolated system.* The principle of conservation of energy may then be stated as—**the total energy of an isolated system remains constant.**

The law was first formulated by the German physicist Robert Mayer in 1842, and was firmly established by Helmholtz a fellow German. It forms the foundation on which the whole structure of physical science has been built. Since 1932 it has been definitely established that energy can be converted into matter and matter into energy. The Frenchman Lavoisier in 1774 had enunciated that matter can neither be created nor destroyed. The laws of conservation of energy and of matter, therefore, become one, as Einstein had predicted (1905) in his equivalence relation $E=mc^2$.

The law rules out the possibility of constructing a machine that will return more energy than is given to it, i.e., a machine that will create energy. The continual operation of a machine which creates its own energy is called *perpetual motion of the first kind.* The principle of conservation of energy states that such perpetual motion is a delusion. Physically, every change has to be paid for in terms of energy.

•I-8.12. **Dissipation of energy.** Whenever energy is transformed, there is a loss of another kind, though none is destroyed. When a machine runs, energy has to be supplied to it and it does work. The effective work that it does, is always less than the work supplied to it, i.e., *the output is always less than the input.* Part of the energy supplied to it leaks away due to friction etc., mostly in the form of heat, and serves no useful purpose. It must be noted that the loss is not in the total energy but in the *availablity* or *usefulness* of some of it. Lord Kelvin was one of the first to recognise the general principle that *whenever energy is used or transformed some of it leaks away out of our control and becomes for ever dissipated and unavailable.* This

a statement is the **principle of dissipation of energy.** Dissipation cannot be avoided. It is thus not only impossible to construct a machine that will create its own energy but it is also impossible to construct a machine that will return in useful form all the energy put into it (*perpetual motion of the third kind is impossible.*)

Work done against Friction : The most important agent for dissipation of energy is friction and it is ever present opposing motion and producing loss of mechanical energy. Because of friction mechanical energy is **not** conserved ; what we have discussed above about conservation, are simplified idealised cases.

You throw a ball vertically upwards ; it will be resisted by a downward resistance ; when it descends it will again be resisted, but this time upwards. This cuts out the maximum height of ascent H, lengthens the time T to reach it and lessenes the energy with which the ball returns to hand (Is the time of ascent still equal to that of descent ? Why ?) For the projectile also, the time of flight is lengthened and range shortened. The swing of a pendulum becomes progressively shorter, ultimately it stops. Output from a machine is always less then the input of energy.

To investigate the effect of friction on motion we choose the case of a weight mg slipping down a *rough* incline (fig. I-8.13). As we have seen earlier (fig. I-4.) three forces are relevant—$mg \sin \theta$ down the incline, the reaction $R = mg \cos \theta$ acting perpendicular to the surface and frictional force μR up the incline. Let the mass start moving from A from rest and reach B covering a distance S. It slides under a *constant* resultant force

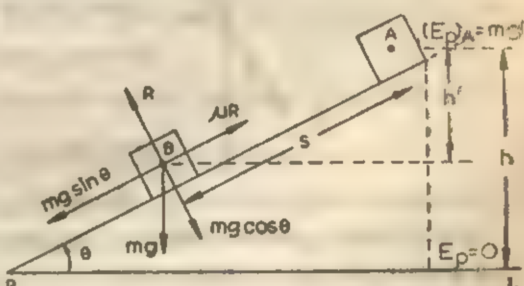


Fig. I-8.13

$$F = mg \sin \theta - \mu R = mg (\sin \theta - \mu \cos \theta)$$

The velocity it gains at B is $v^2 = 2fS = 2S (g \sin \theta - \mu g \cos \theta)$

$$\therefore \quad (K.E.)_B = mg (S \sin \theta - \mu S \cos \theta) \qquad \text{(I-8.12.1)}$$

$$(\text{P. E.})_B = mg\,(h - h') = mg\,(h - S\sin\theta) \qquad \text{(I-8.12.2)}$$

$$\therefore \quad (\text{Total energy})_B = mg\,(h - S\sin\theta) + mg\,(\sin\theta - \mu\cos\theta)S$$
$$= mgh - \mu mg\cos\theta.S$$
$$= mgh - \mu R.S \qquad \text{(I-8.12.3)}$$

Now $\mu R\,S$ represents the work done against friction and the total energy at B is less than mgh by just that amount. So energy dissipated into heat because of friction amounts to $\mu mg\cos\theta.S$

***Friction and Work-energy Principle :** In eqn I-8.6.3 we had established the energy-work relation in presence of friction but for a horizontal motion. If we consider a rough surface up which a body is being dragged by an external agent applying a constant inclined force F (fig I-8.14) greater than friction force μR then it passes the point

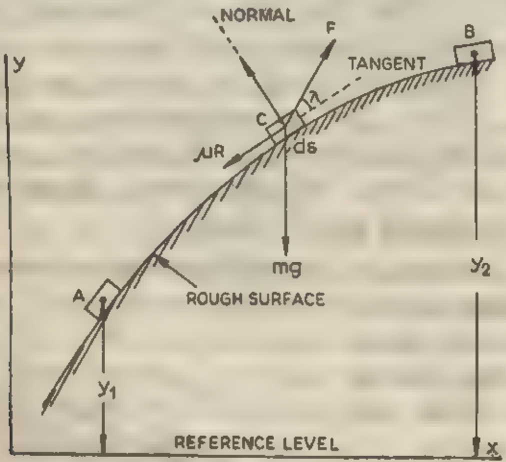


Fig. I-8.14

A at a height y_1 with a velocity u and the higher point B at y_2 with a greater velocity v; then we have

$$\left.\begin{array}{l}\text{gain in K.E.} = \tfrac{1}{2}m(v^2 - u^2)\\ \text{gain in P.E} = mg\,(y_2 - y_1)\end{array}\right\} \text{and work done against friction} = \int_A^B \mu R\,dS$$

So the total work done =

$$\int_A^B F\cos\lambda\,dS = (\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2) + (mgy_2 - mgy_1) + \int_A^B \mu R.dS \qquad \text{(I-8.12.4)}$$

* For the inquisitive student.

where λ is the angle between the direction of force F and tangent to dS, the small displacement containing the point C. Thus

Total work done by the external agent = Change in K.E.

+ Change in P.E. + work done against friction

or $\quad W_F = (K_2 - K_1) + (V_2 - V_1) + W_f \qquad$ (I-8.12 5)

(*ii*) If there is no friction $W_f = 0$. Then

$$K_F = (K_2 - K_1) + (V_2 - V_1).$$

Thus the work done by the outside agent is equal to the total change in mechanical energy.

If the motion is on a horizontal frictionless plane $W_F = K_2 - K_1$.

(*iii*) Without an outside agent W_F is zero. And if there is no friction, W_f is also zero. Then

$$0 = (K_2 - K_1) + (V_2 - V_1)$$

or $\quad K_1 + V_1 = K_2 + V_2.$

i. e. *in absence of friction and of an outside agent* applying a force, the motion will be such that the sum of the kinetic and potential energies remains constant. This is the **principle of conservation of mechanical energy**.

(*iv*) When friction is present, but no outside agent is doing work, part of the mechanical energy is wasted in doing work against friction. This work appears as heat at the points of contact. The total mechanical energy diminishes gradually.

(*v*) If the outside agent lifts the body *at constant velocity* against gravity, then

$$W_F = V_2 - V_1.$$

This means that the work done by the outside agent is equal to the increase in gravitational potential energy of the body.

Ex. I-8.11. *A mass of 1g slides down an incline of 60° to the horizontal against a frictional force of 0·2R through a distance of 1m. Find its acceleration and change in the sum of K.E. and P. E.*

[J.E.E. '82]

Solution : Change in the sum of K.E. and P.E. = Loss of mechanical energy = $\mu mg \cos\theta\, S = 0{\cdot}2 \times 1 \times 980 \times \frac{1}{2} \times 100 = 9800$ ergs.

Its sliding acceleration $f = g \sin\theta - \mu g \cos\theta$

$$= g\,(\sin 60° - 0{\cdot}2 \cos 60°)$$

$$= 0{\cdot}76g \text{ cm/s}^2.$$

I 8 13. Conservative and dissipative forces. When an outside agent lifts a body vertically he does work. The work done is equal to the increase in gravitational potential energy of the body. Such an agent also does work when he pulls the body along a rough horizontal plane; but here the potential energy of the body does not increase.

The reason for this difference can be understood if we consider what happens when we restore the body to its initial position. In being lowered, the body can do work at the expense of the gravitational potential energy it had acquired. The amount of this work is, in the ideal case, equal to the energy it had gained. So, in this case the work spent on the body in lifting it, is *recoverable.*

In the other case, when we reverse the motion of the body to restore it to its initial position on the rough surface, the force of friction is also reversed. So, the same work is done against friction during the return motion as in the forward motion. This work is *not recoverable.*

We may express the difference in another way also. In the first case (case of gravity), as much work will be done by the body in falling from a level A to a level B as will be done on it in lifting it from B to A. In still other words, *total work for a round trip is zero.* But not so in the presence of friction; work is done by the outside agent for both trips.

When a force is such that the work done against it is recoverable, it is called a **conservative force.** In such a case potential energy increases. When the work done against a force is not recoverable, as in the case of friction, the force is called a **dissipative force.** The work done in such a case appears mostly as **heat.**

The mechanical energy of an isolated system is conserved only when all the forces acting on it are conservative. When dissipative forces are present as they always are, the *total energy of the system is conserved, but not the mechanical energy.* Some of the latter is generally converted into heat.

EXERCISES

[illegible]

[illegible]

[illegible]

I-1. [Kinematics]

[A] Essay type questions :

1. What do you mean when you say a body is at 'rest' or in 'motion'? Why is 'rest' or 'motion' considered relative?

2. Explain the term 'acceleration' with suitable examples. Establish the equation $v=u+at$, algebraically and graphically.

3. (i) Distinguish between speed and velocity.

(ii) Distinguish between 'average value' and 'instantaeous value' of a quantity. When are they the same? Which one is more important in the laws of mechanics?

4. What is meant by a reference frame? What are space co-ordinates?

5. Derive the equations $S=ut+\frac{1}{2}at^2$ and $v^2-u^2=2aS$, algebraically and graphically, clearly explaining the meanings of the symbols.

6. A particle has an initial velocity and a uniform acceleration. From the time-velocity graph how can you deduce (a) average velocity, (b) the distance travelled in a given time. Also explain how can you obtain the instantaneous velocity?

[B] Short answer type questions :

7. Why is 'time' mentioned twice in the unit of acceleration?

8. What is the difference between the distance travelled in t seconds and that travelled in the t-th second?

9. When the velocity is constant, does the average velocity over any time interval differ from the instantaneous velocity at any instant?

10. The initial velocity of a body is u and the final velocity is v. If the average velocity is $(u+v)/2$, can the acceleration be varying?

11. Does the speedometer of an automobile register speed?

12. What happens to our kinematic equations under the operation of time reversal, that is, replacing t by $-t$?

13. Can a body have (i) an acceleration with zero velocity? (ii) velocity without acceleration? (iii) an acceleration if its speed is constant? (iv) constant velocity and still varying speed? (v) constant speed and still a varying velocity?

14. Can a body have a northward velocity while experiencing a southward acceleration?

15. Can the direction of the velocity of a body change when its acceleration is constant?

16. A man goes round a circular garden of perimeter 100 m and comes back to the starting point. What is his displacement ?

17. What is the nature of the motion expressed by the equation $S=S_0+ut+\frac{1}{2}at^2$?

18 A train is moving along a striaght track. Explain what would be the nature of its velocity-time graph in the following cases—(i) It is moving with a uniform acceleration. Its acceleration is (ii) increasing, (ii) decreasing ?

19. A man standing on a tower throws a ball straight up with initial velocity u. He throws another ball down with the same velocity. Which of them strikes the ground with greater velocity ?

20. A man on the observation platform of a train moving with constant velocity drops a coin while leaning over the rail. Describe the path of the coin as seen by (i) the man on the train, (ii) a person standing near the track, (iii) a person in a second train moving in opposite dircetion to the first train on a parallel track.

21. If a particle is released from rest with a constant acceleration a, then according to the equation of motion it is at a distance s at two different times $\pm\sqrt{2s/a}$. What is the significance of the negative root ? When can it happen ?

22. The velocity-time graph of a particle is a straight line parallel to the time axis What would be the time-displacement curve ?

23 The time-displacement curve of a partcle is a straight line parallel to the time-axis. What do you guess about its velocity ?

[C] Numerical Problems :

24. A particle moves with an acceleration of 5 cm s^{-2}. If its nitial velocity is 100 cms^{-1}. What will be its velocity after 20 s ? iHow far will the particle move in that time ? [200 cms^{-1}, 30 m]

25. A car has a velocity of 20 ms^{-1}. It is decelerated at 5 ms^{-2}. How far will it move before stopping and how long will it take to do so ?

What would be the retardation of the car if it is to be stopped in (a) one-fourth the distance, (b) one-fourth the time.

[4 s, 40 m, 20 ms^{-2} in both cases]

26 Two particles move along the x-axis uniformly with speeds of 8 ms^{-1} and 4 ms^{-1}. At the initial moment the first point was 21 m to the left of the origin and the second 7 m to the right of the origin. When will the first point catch up with the second ? Where will this take place ? [7 s, 35 m]

27. A car left a city travelling uniformly at a speed of 80 km h^{-1}. It was followed 1·5 hours later by a motor cycle whose speed was 100 km h^{-1}. How much time passed after the car left

the city before the motor cycle caught up with it ? Where did this take place ? [7·5 h, 600 km]

28 A car travelled the first third of a distance at a speed of 10 km h^{-1}, the second third at 20 kmh^{-1} and the last third at 60 km h^{-1}. Find the mean speed of the car over the entire distance. [18 km h^{-1}]

29. A motor-boat travelling upstream met a raft floating downstream. An hour after this the engine of the boat stalled. It took 30 minutes to repair it and during this time the boat freely floated downstream. When the engine was repaired, the boat travelled downstream with the same speed relative to the current as before and overtook the raft at a distance of 7 5 km from the point where they had first met. Find the river current, taking it as constant. [3 km h^{-1}]

30. An engineer works at a plant out of town A car is sent for him from the plant every day that arrives at the railway station at the same time as the train he takes. One day the engineer arrived at the station an hour before his usual time and without waiting for the car, started walking to work. On his way he met the car and reached his plant 10 minutes before the usual time. How long did the engineer walk before he met the car ? [55 minutes]

31. A train moves 40 km due west and then 30 km due south in 6 hours. Calculate its average speed and velocity. [3·24 ms^{-1}, 2.31 ms^{-1} due south of west]

32. A car starts from rest with a constant acceleration of 9 ms^{-2}. Find its (i) instantaneous speed at the end of 10 s, (ii) the average speed for a 10 s interval and (iii) the distance covered in 10 s from rest. (P. U.) [90 ms^{-1}, 45 ms^{-1}, 450 m]

33. A train, starting from rest, acquired an acceleration of 3 ft s^{-2} in 6 seconds. Thereafter, the train moved with uniform velocity for half a minute and applying brakes came to rest in 5 seconds. Find *graphically* the greatest velocity attained by the train and the distance covered by it. (Bihar) [12·3 mile h^{-2}, 639 ft]

34. A particle describes 72 cm in the 12th second and 96 cm in the 16th, of its motion. Calculate its initial velocity and acceleration. How far will it move in 20 seconds ? [3 cm s^{-1}, 6 cm-s^{-1}, 1260 cm]

35. A man on the road is 9 m behind the entrance door of a train when the train begins to take motion from rest with a uniform acceleration of 2 ms^{-2}. The man immediately starts and runs with uniform speed to get into the train. He is just able to get in. Find the speed of the man. (J. E. E. '72) [6 ms^{-1}]

36. A particle moving with uniform acceleration described in its last second of motion one quarter of the whole distance. If it started

from rest, how long was it in motion and through what distance did it move, if it described 10 m in the first second ? [7·464 s, 557·1 m]

37. A body moving in a straight line with uniform acceleration describes three successive equal spaces in time intervals t_1, t_2 and t_3 respectively. Show that

$$\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{3}{t_1+t_2+t_3}.$$

38. A car A is travelling on a straight level road with a uniform speed of 60 km/hr. It is followed by another, B, moving at 70 km/hr. When the distance between them is 2·5 km, the car B is given a deceleration of 20 km/hr^2. After what distance and time will B catch up with A ? (I. I. T. '86) [32·5 km, 0 5 hr.]

39. A body travels 200 cm in the first two seconds and 220 cm in the next four seconds. What will be the velocity at the end of seventh second from the start ? (I. I. T. '64) [10 cm s^{-1}]

40. Two trains, each at a speed of 30 mi/hr, move towards each other on the same straight track. A bird that can fly 60 mi/hr flies off one train when they are 60 miles apart and heads directly for the other train. On reaching it, it flies directly back to the first train, and so forth. Find (i) how many trips can the bird make from one train to the other before they crash ? (ii) What is the total distance the bird travels ? [infinite, 60 mi]

41. A man walking with a uniform velocity v passes under a lantern hanging at a height H above the ground. Find the velocity with which the edge of the shadow of the man's head moves over the ground if his height is h. [$Hv/(H-h)$]

42. A car was suspected to exceed the speed limit of 30 mi/hr. It made a skid-mark of 19·2 ft on the ground after the driver made an emergency stop. Assuming reasonably that the maximum deceleration of the car would not exceed the acceleration of a freely falling body, investigate whether the driver was speeding.

[not speeding, 22 mi/hr]

43. A driver on the average takes 0.7 s to bring the car to rest after seeing the red signal and applying the brakes. If a car can produce a retardation of 4·88 ms^{-2}, find the distance travelled before it stops if it has an initial speed of 40 km/hr. [8·14 m]

44. A train stopping at two stations 2 miles apart take 4 minutes, on the journey from one station to the other. Assuming that its motion is first that of uniform acceleration f_1 and then that of retardation f_2, show that $\frac{1}{f_1}+\frac{1}{f_2}=4.$

45. Four points are at the four corners of a square of side a. They begin to move at the same time with a constant speed v, the

first point always moving towards the second, the second towards the third etc. Will the points meet and if they do in what time ?

$\left[\text{yes; } \frac{a}{v}\right]$

46. Three ants are located at the vertices of an equilateral triangle whose side equals l. They all start moving at the same time with a constant speed v, the first ant always moving towards the second, the second towards the third and the third towards the first. How long will the ants take to meet ? $\left[\frac{2l}{3v}\right]$

47. At the instant the traffic signal turns green, an automobile starts with a constant acceleration of 6ft s^{-2}. At the same instant a truck travelling with constant speed of 30 ft s^{-2} overtakes and passes the automobile. Find (i) how far beyond the starting point will the automobile overtake the truck and (ii) how fast will the car be travelling at that instant ? [300 ft, 60 ft s^{-1}]

48. A bus is running along a highway at a speed of 16 ms^{-1}. A man who can run at a speed of 4 ms^{-1}, is at a distance of 60 m from the highway and 400 m from the bus. In what direction should the man run to reach any point of the highway at the same time as the bus or before it ? At what minimum speed should he run to be able to catch the bus ? In what direction ?

[between 36°45′ and 143°15′, 2·4 ms^{-1} perp. to the initial line joining the man and the bus.]

49. A particle moves in a straight line. At a distance x from a particular point on the line the velocity of the particle is proportional to $\sqrt{(a^2-x^2)/x^2}$. Find how the acceleration vary.

$[-a^2/x^3.]$

50. A particle starting from rest moves with uniform acceleration for a time t_1. Then it covers a distance d, moving with uniform velocity for time t_2. Afterwards, it moves with uniform retardation and come to rest after time t_3. Show that the total distance travelled by the particle is

$$d\left(1+\frac{t_1+t_3}{2t_2}\right).$$

51. The relation between the distances traversed by a body and time t is given by the equation $S=a+bt+ct^2$ where $a=3$ m, $b=2$ m s^{-1} and $c=1$ ms^{-2}. Determine the average velocity and

average acceleration of the body during the first, second and third seconds of motion. [3, 5, 7 ms^{-1} ; 2 ms^{-2}]

52. The relation between the distance and time is given by $S = a + bt + ct^2 + dt^3$ where $c = 14\ ms^{-2}$ and $d = \cdot 01\ ms^{-3}$. Find (i) in what time after motion begins will the acceleration of the body be equal to 1 ms^{-2} ? (ii) What is the average acceleration the body acqires during the time ? [12 s, 0·64 m/s^2]

53. A rocket is fired vertically from the ground with a resultant vertical acceleration of 80 ms^{-2}. The fuel is finished in 1 minute and it continues to move up. What will be the maximum height reached ? $g = 9{\cdot}8\ ms^{-2}$. (I. I. T. '75) [36·4 km]

54. If a body falls freely from rest, calculate (i) its acceleration, (ii) distance it falls in 4 s, (iii) its velocity after falling 90 m, (iv) time required for its veloity to become 98 ms^{-1}, (v) time of fall through 44·1 m.

[(i) 9·8 ms^{-2}, (ii) 78·4 m, (iii) 42 ms^{-1}, (iv) 10 s, (v) 3 s.]

55. A stone is projected vertically upwards with a speed of 30 ms^{-1} from a height of 40 m. Find (i) the maximum height attained. (ii) the time required to strike the ground. (iii) the speed with which it strikes the ground.

[(i) 85·9 m, (ii) 7·24 s, (iii) 41 ms^{-1}.]

56. A body is thrown vertically upward with a velocity of 100 fts^{-1}. When will it be at a height of 80 ft from the ground ? Explain the double answer. ($g = 32\ fts^{-2}$) [0.94 s and 5·3 s]

57. A stone is thrown horizontally with a speed of 400 fts^{-1} from the top of a house 400 ft high. When and where will it strike the ground ? [5 s later, 2000 ft away]

58. A stone is dropped from a height of 19·6 m. What distances it falls during (i) the first 0·1 s, (ii) the last 0·1 s of its motion ?

How long does the stone take to travel (i) the first metre, (ii) the last metre ? [4·9 cm, 1·9 m, 0·45 s, 0·05 s]

59. A stone is dropped from the top of a tower 90 m high and at the same time another stone is projected vertically upwards from the foot of the tower with a speed of 30 ms^{-1}. When and where will the stone pass each other ? [3 s, 45·9 m from ground]

60. From the top of a tower, a stone is thrown vertically upwards with a velocity of 30 ms^{-1} and 4 s later another stone is dropped from the same point. Both the stones reached the ground simultaneusly. Find the height of the tower and the time of fall of the second stone. [80 m, 4 s].

61. A stone is dropped from a balloon at an altitude of 300 m.

What time is required for the stone to reach the earth if the balloon is (i) stationary, (ii) ascending at 5 ms^{-1}, (iii) descending at 5 ms^{1-} ? [7·8 s, 8.4 48, 7·3 s]

62. A stone dropped from a balloon strikes the earth's surface in 30 s. Find the height of the balloon (i) if it is at rest, (ii) if it is ascending with a constant speed of 100 cms^{-1}, when the stone was released. [4410 m, 4380 m].

63. Two stones are projected upwards simultaneously. One ascended 112 ft more than the other and reached the ground 2 s later. Find the velocities of projection of the stones. [128 fts^{-1}, 96 fts^{-1}]

64. A lift is ascending with an accelration of 2 m^{-2}. When its velocity is 8 ms^{-1} a piece of iron from the ceiling of the lift falls. If the height of the lift box be 3 m, find the time after which the piece will reach the floor of the lift. What height will it rise or fall ?

($g = 9·8$ ms^{-2}) [0·7 s, 3·2 m rise.]

65. A stone falling from the top of a tower has descended x ft when another is let fall from a point y ft below the top. If they fall from rest and reach the ground together, show that the height of the tower is $(x+y)/4x$ ft.

66. A wooden block of mass 10 g is dropped from the top of a cliff 100 m high. At the same time a bullet of mass 10 g is fired from the foot of the cliff vertically upwards with a velocity of 100 ms^{-1}. When and where will they meet ? If the bullet after striking the block gets embeded in it, how high will it rise above the cliff before it starts falling ? (I. I. T. '73) [1 s, 95·1 m, 77·55 m]

67. The acceleration due to gravity at two places are g_1 and g_2. From same height at two places if a body be dropped, it takes t s less to reach the ground in the second place than the first but the velocity on reaching the ground in the second place is greater by v than in the first place. Show that $g_1 g_2 = v^2/t^2$.

68. A juggler is maintaining four balls in motion, making each in turn rise to a height of 90 cm from his hand. With what velocity does he project them and where will the other three balls be at the instant when one is just leaving his hand ?

[420 cms^{-1} ; 67·5 cm, 90 cm, 67·5 cm.]

69. A block of ice starts sliding down from the top of an inclined roof of a house, inclination being 30° with the horizontal, along a line of maximum slope. The highest and the lowest points of the roof are at heights of 8·1 m and 5·6 m respectively from the ground. At what horizontal distance from the starting point will the block hit the ground ? (I. I. T. '72) [9 m]

70. Water drips from the nozzle of a tap on to the floor 3·24 m below. The drops fall at regular intervals of time, the first drop

striking the floor at the instant the fourth drop begins to fall. Find the location of the drops when a drop strikes the floor.

[3 24, 1·44, 0·36, 0 m from the top]

71. A rifle with a muzzle velocity of 1500 ft s^{-1} shoots a bullet at a small target 150 ft away. How high above the target must the gun be aimed so that the bu let will hit the target ? [1 9 in]

72. A dog sees a flowerpot sailing up and then back down past a window 5 ft high. If the total time the pot is in sight be 1 s, find the height above the window where the pot rises. ($\frac{1}{16}$ ft .)

73. The acceleration of a body is given by $a = 2t^{\frac{1}{3}}$. Find (i) instantaneous velocity after 4 s and (ii) distance travelled after 4 s.

[10·66, 17·06.]

74. A bullet fired into a target loses $\frac{1}{n}$th of its velocity after penetrating x metres into the target ; how much further will it penetrate ? $\left[x\frac{(n-1)^2}{2n-1} \right]$

75. A particle moves along a straight line, and at a distance x from a fixed point O on the line its velocity is $\mu\sqrt{\frac{C-x}{x}}$. Prove that its acceleration is directed towards O and is inversely proportional to the square of the distance.

76. A particle moves from rest at a distance C from a fixed point O, with an acceleration μ/x^2 away from O at a distance x. Show that its velocity when it is at a distance 2C from O is $\sqrt{\mu/C}$.

77. A particle of mass m moves in a straight line under an acceleration n^2x towards a fixed point on the line when at a distance x from it. If it is initially projected with a velocity v towards the point from a distance a from it, prove that it reaches the point after a time $\frac{1}{n}\tan^{-1}(na/v)$.

78. The distance between the two stations is 1·5 km. The first half of the distance is covered by a train with a uniform acceleration and the second half with uniform retardation. The maximum speed of the train is 50 km hr. Find (i) the acceleration, taking it to be numerically equal to the retardation, (ii) the time taken by the train to travel between the stations. [0 13 ms^{-2}, 3·6 min]

79. A body moving with a uniform acceleration has velocities of 20 ms^{-1} and 30 ms^{-1} when passing two points in its path. Find its velocity midway between the points. [25·5 ms^{-1}]

83. Drops of water fall from the roof of a building 16 m high at regular intervals of time, the first drop reaching the floor at the same time the fifth drop starts its fall. Find the distance of the individual drops from the roof in the air at the instant the first drop reaches the earth. [1 m, 4 m, 9 m, 16 m.]

I-2. [Vectors]

[A] **Essay type questions :**

1. What is a vector quantity ? In what way is it different from a scalar quantity ? Give 3 examples of each.

How can a vector be represented by a line segment ?

2. State the law of addition of vectors. How can one vector be subtracted from another ? Draw diagrams. What is meant by the principle of independence of vectors ?

3. Vectors P and Q are inclined to each other at an angle θ. Find their resultant (Give both the magnitude and direction.) Find also the magnitude of their difference, and also its direction. What will be the result if the vectors are mutually perpendicular ?

4. What do you mean by resolution of a vector ? How will you resolve a single vector in two mutual perpendicular components ? When is the resolved part the same as the component in its direction ?

5. Explain the composition of a number of vectors by (i) the process of resolution and (ii) the polygon rule.

6. What is relative velocity ? How can you determine the relative velocity of two particles when (i) they are moving along the same line, (ii) moving along two different lines ?

7. How can you represent a vector by co-ordinates in a three dimensional space ? Discuss the method of finding the resultant of two vectors by this method.

[B] **Short answer type questions :**

8. Does it make any sense to call a quantity a vector when its magnitude is zero ?

9. Can two vectors of different magnitudes be combined to give a zero resultant ? Can three vectors ?

10. Can a vector be zero if one of its components is not zero ?

11. Is the value of a scalar quantity dependent on the reference frame chosen ?

12. Is time a vector quantity ? If not, why not ?

13. Can a scalar product be a negative quantity ?

14. Can the magnitude of the resultant of two vectors be smaller than the magnitude of either ?

15. Can a vector have a component at right angles to itself ?

16. Work is the product of force and displacement of the point of application of force. Both force and displacement are vectors. Is work then a vector or a scalar ?

17. During the rains, the rain drops falling vertically appear to come down obliquely to a person sitting in a running train. Explain.

18. A school-boy holding an umbrella runs with a velocity equal to that of the rain falling vertically. At what angle should he hold the umbrella in order to protect himself best ?

19. Drops are falling vertically in a steady rain. In order to go through the rain from one place to the other in such a way as to encounter the least number of rain drops, should one move with the greatest possible speed, the least possible speed or some intermediate speed ?

20. A train moves over a straight track with an acceleration f. A man in the train drops a stone. What is the acceleration of the stone with respect to the train and the ground ?

21. An elevator moves with an acceleration f. A man in it drops a coin. Find the acceleration of the coin with respect to the floor of the lift if (i) the lift is going up (ii) going down.

22. Show that the instantaneous acceleration at any point in the velocity-time graph equals the slope of the tangent to the graph at that point.

[C] Numerical Problems :

23. What is the angle between two vectors so that the resultant is (i) $\sqrt{2}$ times (ii) $\sqrt{3}$ times each vector ? [90°, 60°]

24. Show that two vectors of equal magnitude has a resultant which bisects the angle between the component vectors.

25. In the parallelogram ABCD, $\vec{AB} = a$, $\vec{AD} = b$. Find $\vec{AC}$, [illegible], [illegible] in terms of a and b. [$a + b$, $a - b$, [illegible]]

26. In the regular hexagon ABCDEF, $\vec{AB} = a$, $\vec{BC} = b$. Find [illegible]. [$a + b$, $b - a$, [illegible]]

27. The position vector of a point P is $3i + 2j + 6k$. Find the length of $\vec{OP}$. [7]

28. If the vertices of a triangle are the points $i - j + 2k$, $2i + 3j + 4k$, [illegible], what are the vectors determined by its sides ? i, j, k are unit vectors parallel to the axes of coordinates.

29. A vector P is the resultant of two vectors X and Y which makes angles 30° and 60° respectively with P on opposite side. Find X and Y. [$0{\cdot}866P$, $0{\cdot}5P$]

30. Find the horizontal force and the force inclined at 30° to the vertical, where the resultant is a vertical force of X N.

[$X/\sqrt{3}$N, $2X/\sqrt{3}$N]

31. If the resultant of the two forces is equal in magnitude to one of them, and perpendicular to it, find the magnitude and direction of the other force. [$\sqrt{2}$ times the other force and inclined to 135° to it.]

32. Three forces act on a particle in the same plane. One is 100 dynes due east, the second is 200 dynes due north and the third is 200 dynes acting north-east. Resolve the forces into components and find their resultant. [418·1 dyn]

33. A boat is towed by two ropes, each inclined at 30° to the stream, but on opposite sides of it. If the pull along each rope is 50 kg force, find the magnitude and direction of the resultant.

[86·6 kg-force, 30°]

34. Two equal forces, inclined at 60°, act on a particle. Resolve the forces along the bisector of the included angle and perpendicular thereto. Hence find the magnitude and direction of the resultant.

[$\sqrt{3}$ times of either force, along the bisector of the included angle.]

35. Three coplanar forces of 1, 2 and 3 gf respectively act on a particle, the angle between the first and the second is 60°, the angle between the second and the third is 30°, and the angle between the first and the third is 90°. Find the magnitude of the resultant and the angle it makes with the first force. [[illegible]·14 gf making an angle of [illegible] with the first force.]

36. Forces of 3 gf and 4 gf act at a point. Find the angle between their lines of action if they have a resultant of (a) 7 gf, (b) 6 gf, (c) 5 gf, (d) 4 gf, (e) 1 gf.

[0°, 62°42′, 90°, 131°48′, 180°]

37. A man who swims at 3 [illegible] in still water, wishes to cross a river [illegible] wide flowing at the rate of 5 [illegible]. Find the direction in which he should swim in order to cross the river. How long will he take to cross and how far will he be carried down stream?

[at right angles, [illegible], [illegible]]

38. Two swimmers leave from the same point on one bank of the river to reach a point lying right across on the other bank. One of them crosses the river along the straight line while the other swims at right angles to the stream and then walks the distance that he has

been carried away by the stream. What was the velocity of his walking if both the swimmes reached the destination at the same time ? Given that the stream velocity is 2 km/hr and the velocity of each swimmer is 2 5 km/hr. [3 km/hr.]

39. Two forces P and Q acting at a point have a resultant R; if Q be doubled, then R is doubled. Again if Q be reversed in direction then also R is doubled. Prove that $P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$.

40. A man rows directly across a flowing river in time t_1, rows an equal distance down the stream in time t_2. If u be the speed of the man in still water and v that of the stream, show that

$$t_1 : t_2 = \sqrt{u+v} : \sqrt{u-v}.$$

41. On a rainy day when a man is walking at the rate of 4 miles an hour, he is struck by the rain vertically and when he increases his velocity to 8 miles an hour, the rain strikes him at an angle of 45°. Find the magnitude and direction of the velocity of the rain.

[$4\sqrt{2}$, 45° with the vertical.]

42. A ship steams due west at the rate of 15 km/hr when the river is flowing at 6 km/hr due south. What is the relative velocity of the ship to that of a train running due north at 30 km/hr. ?

[39 km/hr ; in a direction east of north at an angle of $\tan^{-1} \frac{5}{12}$.]

43. A river 240 m broad is flowing at 1·8 km/hr and a swimmer who can swim at 3·6 km/hr wishes to reach a point just opposite. Along what line must he strike out and how long will he take in reaching the point ?

[60° with the bank, 4·62 min.]

44. A steamer is going due north with a velocity V, the smoke from the chimney points θ degrees south of east. If the wind be coming from the west, find its velocity assuming that the smoke loses the velocity of the steamer as soon as it leaves the chimney and moves with the velocity of wind. [V cot θ.]

45. Rain water is falling vertically with a velocity of V ms^{-1} when the velocity of wind is zero and water is collected at a certain rate in a vessel in the rain. What will be the change in the rate of collection of water when the velocity of wind is W ms^{-1} in a direction perpendicular to V ?

(J. E. E. '82) [No change.]

46. A steamer is travelling due east at a rate of u mi/hr. A second steamer is travelling at 2u mi/hr in a direction θ north of east, and appears to be travelling north-east to a passenger on the first steamer. Prove that $\theta = \frac{1}{2} \sin^{-1} 3/4$.

I-3 [Newton's laws of method]

[A] Essay type queitions :

1. (a) State and explain Newton's laws of motion.

(b) Give examples of inertia of rest and inertia of motion.

(c) Establish the equation $F=ma$, explaining the meanings of the symbols.

2. (a) 'Newton's first law introduces the concepts of inertia and force'—Explain the statement.

(b) Explain the statement that Newton's second law indicates how force and inertia can be measured. What is inertial mass ?

3. Explain the terms dyne, newton, gram-weight, kilogram-weight. What symbols do you use for them ? Express the last three quantities in dynes Take $g=980$ cm^{-2}.

4. What do you understand by the impulse of a force and an impulsive force ? Find how impulse and momentum are related.

5. State the principle of conservation of linear momentum, and explain how can you arrive at it from Newton's laws.

Give three examples of conservation of linear momentum. Discuss the motion of a rocket on this basis.

6. What is an elastic collision ? Establish the momentum and energy equations for an elastic collision in a straight line.

[B] Short answer type questions :

7. 'Newton's first and second laws relate to the same body ; but the third law introduces another body'. Explain this statement with examples.

8. If action and reaction are equal and opposite, why should a body move at all under the action of a force ?

9 If we accept the principle of conservation of momentum as a basis, we can get Newton's first law from the third. Can you say how this can be done ?

10. A passenger sits in an aeroplane. State the conditions in which the action and reaction between the two will be (i) equal to the weight of the passenger ; (ii) greater than the weight of the passenger ; (iii less than the weight of the passenger ; (iv) zero.

11. A boy sitting in a railway compartment moving with a constant velocity throws a ball straight up into the air Will the ball fall (i) behind him (ii) in front of him (iii) into his hands ? Explain your answer. If the train accelerates while the ball is in air, what happens to the ball ?

12. A mass is suspended by a thread X. From the bottom of the mass, another identical thread Y is fixed. If a steady pull is applied at the end of the thread Y, the thread X snaps. But if a jerk is given, the thread Y breaks. Explain with reasons.

13. We fall forward when a moving bus stops and fall backwards when a bus starts moving. Why ?

14. A meteorite burns out in the atmosphere before it reaches the earth's surface. What happens to its momentum ? To its energy ?

15. Can a boat be propelled by air blown at the sails from a fan on the boat ?

16. Why is it easier to pull a roller than to push it ?

17. Reconcile with the conservation of momentum the cases of (i) a handball bouncing off a wall (ii) a body falling freely.

18. 'A system of unbalanced forces produces acceleration while a system of balanced forces causes deformation'—Explain the statement.

19. Show that the Newton's first law of motion can be derived from the second law.

20. 'Mass is a measure of intertia of body'—Explain the statement.

21. Distinguish between elastic and inelastic collisions.

22. A bird is in a wire-cage hanging from a spring balance. Is the reading of the balance when a bird is flying about (i) greater than, (ii) less than or, (iii) the same when the bird sits in the cage ?

23. A man standing on a large platform of a spring scale notes his weight. He then takes a step on this platform and the scale reads less than his weight at the end of the step. Explain.

24. A monkey and a mirror having the same mass are hanging at the ends of a rope passing over a pulley, both at the same height with respect to floor. Can the monkey get away from his image seen in the mirror by (i) climbing up the rope ? (i) climbing down the rope ? (iii) releasing the rope ?

25. According the Newton's law, only an outside force impressed by another body can alter the state of motion of a given body. Then what outside force brings a car or any other self-moving vehicle to a stop under braking ?

26. Jet planes generally fly at higher altitudes, while propeller planes fly at lower altitudes. Why ? [I. I. T. '76]

27. Two men stand facing each other on two boats in still water a short distance apart. A rope is held at its ends by both. The two boats are found to meet always at the same point whether

each man pulls separately or together. Why ? Will the time taken to meet be same in both cases ?

28. Two teams having a tug-of-war must always pull equally hard on one another. The team that pushes harder against the ground wins. Explain.

29. How would you choose the relative masses of a target particle, and a probe particle, so that the target particle should recoil with i) the greatest speed (ii) greatest momentum (iii) greatest kinetic energy.

30. Two balls of different masses have same kinetic energy. Which ball will possess greater momentum ?

31. Apply the laws of conservation of energy and momentum to show that for elastic collision of a body of mass M with another of mass KM, the energy lost by the former for a head-on collision is a maximum when $K = 1$. [J. E. E. '79]

32. Why an athlete in the long jump or high jump event always runs through a little distance before jumping ?

33. It is not possible for a person sitting on a chair to lift the chair by its handle unless he lets his feet touch the ground. Explain.

34. Why is a man hit harder when he falls on a paved floor than when he falls on soft earth from the same height ?

35 When a man jumps from a certain height, he feels more pain if he does not flex his knees on landing then when he does. Why ?

36. Can a man sitting inside a car move it by pushing it from inside ?

37. When a body falls freely in air, it is weightless—Explain.

38. Two identical weights are connected by a spring. At the initial moment the spring is so compressed that the first weight is tightly pressed against the wall and the second weight is retained by stop. Describe the motion of the system when the stop is removed.

39. Let the only force acting on two bodies be their mutual interaction. If both the bodies start from rest, show that the distances each travels are inversely proportional to their respective masses.

40. How could a 100 kg object be lowered from a roof using a cord with a breaking strength of 87 kg without breaking the rope ?

41. Explain why the load on the back wheels of a motor car increases when the vehicle is accelerating.

42. Before the start of a race, each runner has momentum which differs from that which he has during the race. Does this represent a violation of the law of conservation of linear momentum ?

43. A man in a boat facing the bank with its stern walks to the bow. How will the distance between the man and the bank change ?

44. Is those any force acting on a vehicle when it is moving with uniform velocity on a rough road ?

45. In a tug-of-war, should all members of a team pull in a straight line or zig-zag fashion ? Explain your answer.

46. 'To clean garments of dust particles, it is suddenly set into motion'—explain why ?

47. In which case the tension in the rope will be greater (i) two forces of magnitude F is applied in the opposite directions (ii) one end is tied and a force of magnitude if is applied at the other end ?

48 To smooth a garment by hot iron it is better to push the iron than to pull it—Explain.

49. Is is possible to keep a rope horizontal by placing a load on the rope ? Why not ?

50. A heavy body and a lighter body moves with same momentum. Which one would travel a greater distance when equal opposing forces are applied to stop them ?

[C] Numerical Problems :

51. A ball weighing 150 g was moving with a speed of 10 ms^{-1}. I. is stopped in 0.2 s. What force was applied to stop it ?

[7.5×10^5 dyn]

52 The barrel of a gun is 50 cm long. A bullet weighing 10 g is fired from the gun with a velocity of 400 ms^{-1}. What is the average acceleration of the bullet inside the barrel ? What is the average force on it ? [$1 6 \times 10^7$ cms^{-2}, 1.6×10^8 dyn]

53. A car moving initially at a speed of 50 mi/hr. and weighing 3000 lb is brought to a stop in a distance of 200 ft. (a) Find the braking force and the time required to stop. (b) Assuming the same braking force, find the distance and time required to stop if the car were going 25 mi/hr. initially. [1300 lb, 5.5 s ; 50 ft., 2.7 s]

54 A train with a mass of 500 tons is uniformly retarded by applying its brakes Its velocity drops from 50 km/hr. to 28 km/hr. in one minute. Find the braking force.

55. A body with a mass of 0.5 kg is in rectilinear motion. The relation between the distance s travelled by the body and the time t is given by the equation $s = A - Bt + Ct^2 - Dt^3$, where $C = 5$ m/s² and $D = 1$ m/s³. Determine the force which acts on the body at the end of the first second of motion. [2 N]

56. Under the action of a constant force F = 1 kgf a body so moves in straight line that the relation between the distance Stravelled by the body and the time t is described by the equation

$S=A-Bt+Ct^2$. Find the mass of the body if the constant $C=1\ ms^{-2}$. [4.9 kg.]

57. A molecule with the mass $m=4.65\times10^{-26}$ kg flying at a velocity of $v=600\ ms^{-1}$ strikes the wall of a vessel at an angle of 60° to the normal and rebounds from it elastically at the same angle without losing its velocity. Find the impulse of the force received by the wall during the impact. [2.8×10^{-23} Ns.]

58. A jet of water with a cross section of 6 cm^2 strikes a wall at an angle of 60° to the normal and rebounds elastically from the wall without losing its velocity. Find the power acting on the wall if the velocity of the water in the jet is 12 ms^{-1}. Density of water 1000 kgm^{-3}. [86 W.]

59. What angle to the horizon will be formed by the surface of petrol in the tank of a motor vehicle moving horizontally with a constant acceleration of 2.44 ms^{-2}. Take $g=9.8\ ms^{-2}$ [14°]

60. A ball is suspended on a thread from the ceiling of a tramcar. The braks are applied and the speed of the car changes uniformly from 18 km/hr to 6 km/hr during 3 seconds. By what angle will the thread with the ball deviate from the vertical. Take $g=9.8\ ms^{-2}$.

[6° 30′]

61. Two weights 2 kgf and 1 kgf are linked by a thread passing over a weightless pulley. Find (i) the acceleration with which the weights move, (ii) the tension of the thread. Neglet friction on the pulley. Take $g=9.8\ ms^{-2}$. [3.27 ms^{-2}, 13 W.]

62. A weightless pulley is attached to the top of an inclined plane forming an angle of 30° with the horizon. Equal weights are linked by a thread passing over a pulley. Find (i) the accleration when the weights move (ii) the tension on the thread Neglect friction in the pulley and between the weight and inclined plane. $g=9.8\ ms^{-2}$. [2.45 ms^{-2} 7.35 N]

63. A weightless pulley is attached to the apex of two inclined planes forming angles 30° and 45° with the horizon. Equal weights are linked by a thread passing over the pulley. Find (i) the acceleration with which the weights move, (ii) the tension in the rope Neglect friction of the pulley and between weights and inclined planes. Take $g=9.8\ ms^{-2}$. [1.02 ms^{-2}, 5.9 N]

64. A ball flying at 15 ms^{-1} is thrown back by a racket in the opposite direction with the velocity 20 ms^{-1}. Find the change in the momentum of the ball if its kinetic energy changes by 8.75 J.

[−3·5 kg. ms^{-1}]

65. A bullet with a mass of 5×10^{-3} kg flies at a velocity of 600 ms^{-1} from a rifle with a mass of 5 kg. Find the velocity of the rifle kick. [0.6 ms^{-1}]

66. A flat car with a weight 10 tonf stands on rails and carries a cannon weighing 5 tonf from which a shell is fired along the rails. The weight of the shell is 100 kgf and its initial velocity with respect to cannon is 500 ms^{-1}. Determine the velocity of the flat car at the first moment after the shot if (i) the flat car was standing, (ii) the car was moving with a speed of 18 km/hr and the shell was fired in the direction of motion, (iii) the car was moving with a speed of 18 km/hr and the shell was fired in the direction opposite to its motion.

[-12 km/hr, 6 km/hr, -3 km/hr]

67. An automatic gun fires 600 bullets a minute. The mass of each bullet is 4 g and its initial velocity is 500 ms^{-1}. Find the mean recoil. [19.6 N]

68. A grenade flying at 10 ms^{-1} bursts into two fragments. The larger one having 60 per cent of the entire weight of the grenade continues to move as before but with an increased velocity equal to 25 ms^{-1}. Find the velocity of the smaller fragment. [-12.5 ms^{-1}]

69. A body with a weight of 1 kgf moves horizontally with a velocity of 1 ms^{-1}, overtakes another body weighing 0.5 kgf and collides with it inelastically. What velocity is imparted to the bodies if (i) the second body was at rest, (ii) the second body was moving at a velocity of 0.5 ms^{-1} in the same direction as the first one, (iii) the second body was moving at a velocity of 0.5 ms^{-1} in direction opposite to the motion of the first one ?

[0.67 ms^{-1}, 0.83 ms^{-1}, 0.5 ms^{-1}]

70. A body weighing 2 kgf moves with a velocity of 3 ms^{-1} and overtakes another body weighing 3 kgf moving at 1 ms^{-1}. Find the velocities of the bodies after collision if (i) the impact is inelastic, (ii) the impact is elastic. The bodies move in a straight line and the impact is central.

If the first body stops after elastic impact what should be the ratio between the masses of the bodies ?

[1.8 ms^{-1} ; 0.6 ms^{-1}, 2.6 ms^{-1} ; 1 : 3]

71. A body weighing 49 N strikes an immobile body weighing 2.5 kgf. The kinetic energy of the system of these two bodies becomes 5J directly after the impact. Assuming the impact to be central and inelastic, find the kinetic energy of the first body before the collision. [7.5 J]

72. Two balls are suspended on parallel threads of the same length so that they touch each other. The mass of the first ball is 0.2 kg and that of the second 100 g. The first ball is deflected so that its centre of gravity rises to a height of 4.5 cm and is then released. To what height will the balls rise to after the collision if (i) the impact is elastic, (ii) the impact is inelastic ?

[5×10^{-3} m, 0.08 m ; 2×10^{-2} m]

73. A bullet flying horizontally strikes a ball suspended on a very light rigid rod and gets stuck in it. The mass of the ball is 1000 times greater than that of the bullet. The distance from the point of suspension to the centre of the ball is 1 m. Find the velocity of the bullet if the rod with the ball deviates by an angle of 10° after the impact. [550 ms^{-1}]

74. A bullet flying horizontally strikes the ball suspended fom a vertical light rigid rod and gets stuck in the ball. The mass of the bullet is 5 g and that of the ball 5 kg. The velocity of the bullet is 500 ms^{-1}. For what maximum length of the rod will the ball rise to the top of a circle as a result of the impact ? [0.64 m]

75. A wooden ball is dropped vertically from a height of 2 m with an initial velocity of zero. When the ball strikes the floor, the coefficient of restitution is 0.5 Find (i) the height which the ball rises to after striking the floor, (ii) the amount of heat evolved during the impact. The mass of the ball is 100 g. [0.5 m, 1.48 J]

76. A steel ball falls from a height of 1.5 m onto a steel plate and rebounds from it with the velocity 0.75 times the velocity of approach. (i) What height does the ball rise to ? (ii) What time passes from the moment the ball begins to move, to its second impact with the plate ? [0 84 m ; 1.4 s]

77. A neutron (mass m_0) strikes an immobile nucleus of (i) a carbon atom (mass 12 m_0), (ii) an uranium atom (mass 235 m_0) Assuming the impact to be central and elastic, find the part of the velocity lost by the neutron during the impact. [2/13, 2/236]

78. A plastic ball falls from a height of 1 m and rebounds several times from the floor. What is the coefficient of restitution during its impact with the floor if 1.3 s pass from the first impact to the second one ? [0.94]

79. A body weighing 2 kg is placed on a smooth plane inclined to the horizontal at 30°. what is the magnitude of the force tending to slide it down the plane ? What acceleration does it give to the body ? ($g = 9.8$ ms^{-2}) [9.8 N, 4.9 ms^{-2}]

80. A body is pulled simultaneously with a force of 50 kg acting due east and a force of 20 kg acting at an angle of 60° east of north. Find the magnitude and direction of the resultant.

[62.4 kg making an angle of tan 1/2 $\sqrt{3}$ with east.]

81. The resultant of two forces makes angles 30° and 45° with the forces. If the resultant is 1000 dynes, what are the magnitudes of the forces ? $\left[\frac{1000\sqrt{2}}{\sqrt{3}+1}, \frac{2000}{\sqrt{3}+1} \right]$

82. A particle of mass 2g is acted on by a force of 26 dynes at an angle θ north of east where tan $\theta = 5/12$. How far will this force move the particle towards the east in 5 s ? [150 cm.]

83. A rocket burns 0.25 kg of fuel per second and ejects it as a gas with a velocity of 1 km/s. Calculate the force exerted by the ejected gas on the rocket. [250 N.]

84. Three particles of equal mass move with equal speed V along the medians of an equilateral triangle. They collide at the centroid of the triangle. After collision a particle comes to rest, another retraces is path with a speed V. What is the speed of the third particle ? (I. I. T. '82) [Retrace the path with speed V.]

85. A body of mass 1 kg initially at rest, explodes and breaks into three fragments of masses in the ratio 1 : 1 : 3. The two pieces with equal mass fly off perpendicularly to each other with a speed of 30 m/s each. What is the velocity of the heavier fragment ? (I. I. T. '81) [14.14 ms^{-1} directed opposite to the bisector of the right angle.]

86. Two bodies of equal mass are, at rest, side by side. One of the bodies starts moving under a constant force F while the other, at the same instant, receives an impulse of I. Show that the bodies again come side by side after a time $\frac{2I}{F}$.

87. A train of mass M is travelling with a uniform velocity on a level line ; the last carriage, whose mass is m, becomes detached. The driver discovers it after travelling a distance l and then shuts off steam. Show that when both the parts come to rest, the distance between them is $\frac{Ml}{M-m}$, if the resistance to motion is uniform and proportional to the weight and pull of the engine is constant.

88. An engine of mass 200 tonnes is pulling a train of 10 carriages, each of mass 60 tonnes. The engine is exterting its maximum pull of 2×10^5 N when the train is moving at a constant speed of 36 km/hr. Due to the disengagement of the coupling between the 5th and 6th carriages, the rear five carriages get separated and the engine hauls the first five carriages exerting the same pull. Assuming that the tractive resistance is proportional to weight, find how far ahead the front half of the train is when the rear half comes to rest ? [400m.]

89. A metal disc is kept in equilibrium at a certain height by throwing marbles directly upwards. If the collision be elastic and 50 marbles strike the disc per second, find the mass of the disc. Each marble has a mass of 2 g and strikes the disc with a velocity of of 30 ms^{-1}. [6.12 g]

90. A cricket ball weighing 5.5 oz rises to a height of 68 ft vertically and drops into a fielder's hand 4 ft above the ground. The fielder brings the ball to rest in 1/10 s. Calculate (i) the speed

of the ball as it reaches the fielder's hand, (ii) the average force in lb-f that must be applied in stopping it. (16 OZ = 1 lb)

[64 fts^{-1}, 6.88 lb-f.]

91. A mass m g is acted upon by a uniform force of F dynes under which in t s it moves a distance S cm from rest and acquires a velocity of v cm s^{-1}. Show that $S=\frac{1}{2}\frac{mv^2}{F}$ and $t=\frac{mv}{F}$.

92. A load W is raised by a rope from rest to rest through a height h. The greatest tension which the rope can safely bear is nW. Show that the least time in which the ascent can be made is

$$t=\sqrt{\frac{2nh}{(n-1)g}}.$$

93. Sand drops vertically at the rate of 2 kg s^{-1} on to a conveyor belt moving horizontally with a velocity of 1 ms^{-1}. Calculate (i) the extra power needed to keep the belt moving (ii) the rate of change of kinetic energy of the sand. [0·02 watt, 0.01 watt]

94. Three blocks, A, B, C are connected by means of a string on a horizontal, frictionless table. The masses of the blocks are 10, 20 and 30 kg respectively. Tension between A and B is T_1, that between B and C is T_2 and the whole system is pulled by a force of 60 N. Find the tensions T_1 and T_2. [10N, 30N.]

95. A goat weighing 10 lb is standing on a flat boat so that it is 20 ft from the shore. It walks 8 ft on the boat towards the shore and then halts. How far is the goat from the shore ? The boat weighs 40 lb and assume that there is no friction between water and and the boat. [13·6 ft]

96. A boat on a lake is perpendicular to the shore and faces it with its bow. The distance between the bow and the shore is 0.75 m. Initially the boat was stationary. A man steps from its bow to its stern. Will the boat reach the shore if it is 2 m long ? If not what will be lhe length of the boat so that it will reach the shore ? The mass of the boat is 140 kg and that of man 60 kg.

[No ; more than 2.5 m.]

97. A 100 km/hr wind blows normally against one wall of a house having an area of 50 m^2. Calculate the force exerted on the wall if the air moves parallel to the wall after striking it and has a density $1·134\times10^{-3}$ gm cm^{-3}. (J. E. E. '81) [4.4×10^4 N.]

98. By what fraction is the kinetic energy of a neutron of mass m_1 decreased in a head on collision with an atomic nucleus of mass m_2 initially at rest ? [$4m_1m_2/(m_1+m_2)^2$.]

99. A railway truck of mass 4×10^4 kg moving at a velocity of 3 ms^{-1} collides with another truck of mass 2×10^4 kg which is at rest. The couplings join and the trucks move off together. What

fraction of the first truck's initial kinetic energy remains as the kinetic energy of the two trucks after collision ? [2/3.]

100. Two trolleys P and Q of masses 0.50 kg and 0.30 kg respectively are held together on a horizontal track against a spring which is in a state of compression. When the spring is released, the trolleys separate freely and P moves to the left with an initial velocity of 6 ms^{-1}. Calculate (i) the initial velocity of Q, (ii) the initial total kinetic energy of the system, (iii) also the initial velocity of Q if the trolley P is held still when the spring under same compression as before is released. [10 ms^{-1}, 24J, 4 $\sqrt{10}$ ms^{-1}]

I-4 (Friction)

[A] **Essay type questions :**

1. Friction is said to be a 'self-adjusting' force. What does this statement mean ? A force is said to have direction. What is the direction of the force of friction ? Is the direction fixed ?

2. Define coefficients of Static and dynamic friction. Which of these two is the more important in everyday life ?

State and explain the laws of friction.

3. 'Friction is useful in many ways though it is wasteful in other ways'—Discuss.

Write briefly the methods of minimising friction.

4. Deduce the eqution of motion of a body on a rough surface.

Define angle of repose and angle of friction and establish a relation between them.

Find a relation between angle of repose and coefficient of friction.

[B] **Short answer type questions :**

5. 'Friction is a self-adjusting force'—Explain.

6. Why are the roads made not very smooth ?

7. Why should the tyres of motor cycles and cars have rough surfaces ?

8. Why are the brake shoes made rough ?

9. 'We grip a stopper with a piece of sand-paper when there is difficulty in removing the stopper from a bottle, or put a drop of oil where the stopper enters the neck of the bottle'—Explain.

10. Why is it inconvenient to write on unglazed or overglazed paper ?

11. If the rails are oiled the wheels of an engine go on turning but the engine does not move forward. Putting some sand on the rails makes the engine move—Explain with reasons.

12. A locomotive with a heavy load has difficulty in starting. This is remedied by putting sand on the wheel—Why ?

13. Why should one take short step rather than long when walking on ice ?

14. Friction being independent of area, explain why are broad belts used in driving heavy mechinary and why are automobile brake shoes of large area desirable ?

15. 'When a body is resting on a rough horizontal plane, no force, however great, applied towards the plane at an angle with the normal less than the limiting angle of friction, can push the body along the plane'—Explain.

16. Take a ruler and place it on your index fingers. Move the hands closer and closer. Note that the ruler will slide alternately on the two fingers, say, first over the right and then over the left, again over the right and then over the left, and so on. Explain why.

17. Two similar roller spin in opposite directions, the right one anticlockwise. A metre-scale is supported horizontally by the rollers with its centre of gravity between the rollers. The scale will be seen to exceute a to-and-fro motion. Explain why.

18. 'When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion'—Is the statement true ? Give reasons in brief.

[I. I. T. '81]

19. 'In the absence of friction, a nail can't be fixed on the wall of a room'—Is it true ? Explain.

20. Machine bearings are often made of one metal while their rotating shafts are made of another. Why ?

21. While pulling a roller, a large force is required to start its motion but once it begins to roll, comparatively smaller force is found to be sufficient to maintain its motion. Explain with reasons.

22. Why are ball-bearings employed in some machine in place of sleeve-bearings ?

23. 'A block is placed on a horizonal table and is pulled weakly by a string but it does not move'—how do you explain this ?

24. Why wheels are used in carriages ?

25. To shift a drum of oil from one place to another, it is easier to roll it than pull it. Why ?

26. A man can not walk in the absence of friction. Is it true ? Explain.

27. A chair is resting on the floor. When will the force of friction act between them ? Where does this force act ? Is the magnitude of the force is constant ?

28. When a lorry get bogged in mud, it fails to move forward though its wheels go on rotating. Explain why ?

29. Space-crafts are so designed as to have a special heat shield. Why?

30. Why are 'racing car' built in streamlined shape?

31. How could a person completely at rest in frictionless ice cross a pond to reach the shore, rolling, jumping or kicking his feet?

32. There is a limit beyond which further polishing of a surface increases rather than decreases frictional forces. Why?

[C] **Numerical problems :**

33. A box weighing 80 kg can be moved on a rough floor when a horizontal force of 20 kg is applied. A man weighing 60 kg finds that he cannot push the box because his leather shoe-soles slip on the floor. When he changes to rubber-soled shoes, he can push the box along. Explain.

How much load should he put on himself to push the box without changing shoes if the coefficient of friction between the floor and his shoe-soles is 0·25? [20 kg]

34. You are pressing on a stone floor with a stick. When the stick is inclined at more than 10° with the vertical it slips. What is the coefficient of friction? [0·18]

35. A body of weight W is held in equilibrium on a rough inclined plane by a horizontal force P. Show that P must lie within the limits W tan $(\theta \pm \alpha)$ where θ is the angle of inclination of the plane to the horizontal and α is the angle of friction.

36. A body of weight W placed on a rough inclined plane of angle of inclination θ is just prevented from sliding down by the application of a force P at an angle λ to the inclined plane. Show that $P = W\dfrac{\sin(\theta-\alpha)}{\cos(\lambda+\alpha)}$, where α = angle of friction.

When the body is on the point of moving up the plane show that the force to pull it will be least when the angle between the force and the plane is equal to the angle of friction.

37. An incline rises 3 in a distance of 5. A force of 16 kg acting parallel to the plane, just prevents a load of 40 kg from sliding down. Find the coefficient of friction. [0·25]

38. A mass of 100 lb rests on a rough inclined plane of angle 30°. If the coefficient of friction is $1/\sqrt{3}$, find the greatest and the least forces, which acting parallel to the plane in both cases, can just maintain the mass in equilibrium. [100 lb and 0]

39. A box weighing 150 lb is moved at constant speed across a horizontal floor by dragging it by means of a rope attached to the front end. If the rope makes an angle of 30° with the floor and the coefficient of friction between the box and the floor is 0·4, find the force exerted by the pulling rope. [56·3 lb]

40. A body of weight 100 lb is pulled up a rough plane inclined at an angle of 45° to the horizontal by a force parallel to the plane. If the coefficient of friction is 0·2 find (i) the normal reaction of the plane on the body, (ii) the force parallel to the plane necessary to just overcome the frictional resistance, (iii) the resolved part, down the plane, of the weight of the body, and (iv) the minimum force required to move the body up the plane.

[70·6 lbf, 14·1, lbf, 70·7 lbf, 84·8 lbf]

41. A body of mass 10 lb is maintained at rest on a plane inclined at 30° to the horizontal, by a string making an angle of 45° to the plane. If the coefficient of friction is 0·10, what is the minimum tension necessary in the string ? [6·5 lbf]

42. A wooden block of weight 10 lb is placed on a plane at 30° to the horizontal. If the coefficient of friction between the block and the plane is 0·30, what upward force at an angle of 20° to the plane will just cause the block to move up the plane ? [7·29 lbf]

43. A uniform ladder rests in equilibrium against a vertical wall. If the coefficient of friction between the wall and the ladder, and the ground and the ladder is 0·10, find the maximum angle the ladder can make with the wall. [11°25′]

44. At what angle to the vertical will a uniform ladder be on the point of slipping if placed against a vertical wall ? The coefficient of friction between the wall and the ladder is 0·10, and that between the ladder and the ground is 0·20. [22°12′]

45. A uniform ladder, 30 ft long and weighing 40 lb, rests against a wall with its base 10 ft from the wall. A man weighing 200 lb climbs to a point 17·5 ft from the bottom before the ladder starts to slip. Find the coefficient of friction between the ladder and the ground, and the ladder and the wall, assuming them to be equal.

[0·18]

46. A man, weighing 180 lb, climbs a 40 lb, 13 ft ladder with s base 5 ft out from a smooth vertical wall. What must be the coefficient of friction between the foot of the ladder and the ground if the man is to be able to climb the top of the ladder ? [0·38]

47. A rectangular block with a square base of 4 inches edge rests on a rough table. A force is applied to it perpendicular to one of the vertical faces. It is found that if the point of application is less than 5 inches above the table, the block slides ; if higher it tips over. Find the coefficient of friction. [0·4]

48 A piece of ice slides down a 45° incline in twice the time it takes to slide down a frictionless 45° incline. What is the coefficient of knietic friction between the ice and the incline ? [0·75]

49. A rope so lies on a table that part of it hangs over. The rope begins to slide when the length of the hanging part is 25% of

the entire length. What is the coefficient of friction between the table and the rope ? [0·33]

50. A force of 10 kgf parallel to a plane of inclination 60° is required to keep a 10 kg block moving up the plane with constant speed. Find the coefficient of kinetic friction. [0·268]

51. The minimum force required to move a block up a rough plane inclined at an angle of 30° to the horizontal was found to be five times the minimum force needed to move the block down the plane If the forces act parallel to the plane, find the coefficient of friction between the plane and the block. [0·866]

52. A sledge weighing 200 kg is drawn along a rough horizontal floor by a 100 kg force making an angle of 60° with the vertical. How far will the sledge move from rest in 10 s. Given that the coefficient of kinetic friction = 0.2. [138·67 m]

I-5 (Uniform circular motion)

[A] Essay type questions :

1. Explain the terms angular displacement, angular velocity. periodic time and frequency in angular motion.

For motion in a circle, how are the first two related respectively to the linear displacement and linear velocity ?

2. What kind of force is necessary to make a particle move in a circle with a constant angular velocity ? What is the name of this force ? Find its magnitude and direction.

3. Distinguish between centripetal and centrifugal force. Why do we call the former a true force, and the latter a 'pseudo' or 'fictitious' force ?

Distinguish between a centrifugal force and a centrifugal reaction, giving examples of each.

4. Prove that a constant force directed towards the centre of a circle is required to make a particle move with a constant speed in the circle.

Calculate the magnitudes of the centripetal force and the centripetal acceleration.

How will the particle move when the centripetal force ceases to act ?

5. (a) Why is a road made sloping at a sharp bend ?

(b) In the case of a curved railway track, the line away from the centre of the curve is slightly raised. What advantage does it give ?

(c) Explain how a cyclist gets the necessary centripetal force on taking a bend on a level road.

6. Show that the minimum horizontal velocity with which a particle, hanging freely at the end of a string of length l must be projected so as to describe completely a vertical circle is $\sqrt{5gl}$ where g is the acceleration due to gravity.

What is a speed governer ?

7. How can we express a simple harmonic motion by the projection of a point moving with unifrom angular velocity along the circumference of a circle of reference on any diameter ?

8. Obtain a mathematical expression for the displacement of a particle executing S. H. M. What is the nature of the time-displacement curve ?

9. Obtain an expression for the velocity, acceleration, time period and force in the case of a S. H. M. Under what conditions will they assume maximum and minimum values.

10. Prove that the motion of a simple pendulum is simple hermonic and hence find its time period.

[B] **Short answer type questions :**

11. Centripetal force is a real force and centrifugal force is a 'pseudo' force—Justify the statement. (H. S. '80)

12. Does the centripetal force do any work ? Explain with reasons.

13. If a heavy body on the end of a string is rotated rapidly enough, the string will break. What force causes the string to break ?

14. Can the centripetal force be greater than the centrifugal reaction ?

15. Explain why a motor-cyclist in a circus 'well of death' can ride around the walls of the well without falling down. (I. I. T. '67)

16. A stone is whirled in a vertical circle by means of a rope. In what positions of the stone is the tension in the rope greatest and least. Justify your answer.

17. The driver of a truck travelling with a velocity v suddenly notices a wall in front of him at a distance r. Is it better for him to apply brakes or to make a circular turn without applying brakes in order to just avoid crashing into the wall ? why ? (I. I. T. '77)

18. When a bus takes a sharp bend, the passengers inside the bus seem to be thrown outwards. How is it explained by a passenger in the bus and a person standing on the road-way ?

19. On a cinema screen a cart moving from left to right appears to audience to have the spokes move in opposite direction. Explain.

20. What is the angular speed of the minute and second hands of a watch ?

21. In a centrifuse particles can be separated from the fluid. Explain the action.

21. A cylindrical chamber is rotating about a vertical axis. At a certain speed of rotation a man seems to be pinned up against the wall. Explain.

23. Is it possible to rotate a bucket of water in a verticle plane without spilling the water ?

24. A cork is put on a gramophone disc. The disc starts rotating but before the final speed is reached, cork flies off. Explain.

25. Why does the cream separates from the milk, when it is charned ?

26. A cyclist rounding a curve must lean inwards otherwise he may fall outwards. Why ?

27. Why does a small chain set rotating at a high speed roll along a floor as though it were a rigid metal loop ?

28. When a particle moves in a circular path with uniform speed, does its acceleration remain unaltered ?

29. The earth moving about the sun in a circular orbit is always acted upon by a force and hence work must be done on the earth by this force'—Do you agree with this statement ? (I. I. T. '73)

30 A boy sitting in a railway carriage moving with constant velocity throws a ball vertically upward. Where would the ball fall when the carriage goes round a curve when the ball is in air ?

31. Why is the earth taken the shape of an oblate spheroid ?

32. In rectilinear motion a body may move without any acceleration, but in circular motions it would always have an acceleration. Is it true ? Explain.

33. Mention the nature of the forces with give rise to certripetal force in the following cases—(i) a satellite moving round the earth, (ii) electron moving round the nucleus, (iii) railway trucks banked on curve, (iv) rotation of a stone tied to thread in the horizontal plane, (v) the motion of string when its mid-point is plucked.

34. Under what conditions a particle will execute S. H. M. ?

35. What property of matter takes the bob of a swinging simple pendulum across the mean position to the other side ?

[C] **Numerical problems :**

36. The minute hand of a clock is 50 cm long. What is its linear speed ? [0·087 cm s^{-1}]

37. If a body revolves 42 times in a minute along the circumference of a circle, what will be its angular velocity ? [4·4 rad.]

38. A flywheel rotates at 1200 rpm. Find its (i) frequency, (ii) angular speed, (iii) time period and (iv) time required to turn through 72°. [20 rps, 125·6 rad s^{-1}, 0·05s, 0·01s.

39. Calculate the angular speed of the earth around the sun assuming the orbit to be circular. (1 year = 365 days.)
[$1{\cdot}99 \times 10^{-7}$ rad s^{-1}]

40. A string snaps under a tension of 10^7 dyne. A stone weighing 500 g is tied to it and made to revolve in a horizontal circle of radius 50 cm. At what frequency of rotation will the string snap ?
[$10/\pi$ s^{-1}]

41. If the speed of the cycle is 4 ms^{-1} and its coefficient of friction with the road is 0·3, what will be the shortest diameter of the circle the cyclist can take ? ($g = 980$ cm s^{-2}) What will be his angle of tilt ?
[10·9 m, 16°42′]

42. A motor car moving with a speed of 20 ms^{-1} takes a bend of 30 m radius. If the driver weighs 72 kg what is the centrifugal force acting at him ? How many times is this force greater than his weight ?
[$9{\cdot}6 \times 10^7$ dyn, about 1·4 times]

43. What should be the angular speed of diurnal rotation of the earth if a person on the equator is to have no weight ? How long will a day be then in hours ? (Earth's radius = 6400 km)
[$1{\cdot}24 \times 10^{-3}$ rad s^{-1}, about 1·4 hour]

44. A boy is sitting on a horizontal platform of a joy wheel at a distance of 15 feet from its centre. The joy wheel begins to rotate and when the angular speed exceeds 10 rpm, the boy just slips. What is the coefficient of friction between the boy and the platform.
($g = 30$ ft s^{-2}) (Delhi) [0·5]

45. A body of 1 kgf suspended by a thread deviates through an angle of 30°. Find the tension of the thread when it passes through the position of equilibrium.
[12·4 N]

46. What should be the velocity of a cyclist turning over circular track of 100 m radius such that his inclination to the vertical is 30° ?
($g = 980$ cm s^{-2}) [23·79 m s^{-1}]

47. A mass m on a frictionless table is attached to a hanging mass M by a cord through a smooth horizontal table. If m is spinning with speed v along a horizontal circle of radius r such that the mass M is at rest, then show that $v = \sqrt{Mg\, r/m}$.

48. A stone weighing 20 g is tied to a sting 10 cm long and set revolving in a vertical plane. If the velocity of the stone at the top of the circle is 1 ms^{-1}, calculate the tension in the string when the stone is (i) at the top of the circle, (ii) at its bottom and (iii) at a level with its centre.
[0 004 N, 1·18 N, 0·592 N]

49. A smooth circular tube is held firmly in a vertical plane. A partical which can slide inside the tube is slightly displaced from rest at it highest position in the tube. Find the pressure between the tube and the particle in terms of it mass m and its angular displacement θ from its highest position.
[$mg\,(3\cos\theta - 2)$]

50. A ball of mass M hangs on a string of length l. A bullet of mass m flying horizontally hits the ball headon and sticks to it. At what minimum velocity must the bullet travel so that the ball will make a complete revolution in a vertical circle ? $\left[\frac{(m+M)}{m}\sqrt{5gl}\right]$

51. A 2 kg ball is swung in a vertical circle with a constant angular speed of 60 rpm by means of a wire of length 50 cm. Find the tension in the wire (i) at the top and (ii) at the bottom of the circle. [19·84 N, 59·04 N]

52. A small body is tied to a point by an inextensible sting of negligible mass and is rotated in a circle of radius 500 cm in the vertical plane. What is the minimum speed that it must have at the uppermost point of the circle, so that the string does not slacken ? What would be its speed and angular velocity at the lowermost point of the circle if it has the above minimum speed at the uppermost point. (I. I. T. '67) [700 cm s^{-1}, 1565·2 cm s^{-1}, 3·132 rad s^{-1}]

53. A small bob of mass 1 g is attached to one end of a string 20 cm long, the other end of the string being fixed to a point. The bob is so projected that it describes a horizontal circle of radius r about a point vertically below the fixed point. Calculate the speed of the bob if the string makes an angle of 45° with the vertical.

[198 cm s^{-1}]

54. A motorcycle with its rider has mass of 250 g and travels round a curve in a road of 36 m radius. Snow and ice on the road reduce the coefficient of friction between the tyre and road to 0·2. If the curve in the road is banked to 15° in the motor-cyclist's favour, find (i) the maximum velocity attainable without slipping, and (i) the angle the rider must make with the road surface at this velocity, assuming that the rider and motor-cycle remain in one plane.

[19·5 ms^{-1}, 78° 41′]

55. A car moves in a curve of radius of curvature R. The width beween the wheels is b and the height of the centre of gravity from the ground is h. With what velocity must the car move in order that the vertical force on the inside wheels shall be reduced to zero ?

[$\sqrt{Rgb/2h}$]

56. A bucket-full of milk is being rotated in a vertical circle of 66 cm radius. What should be the minimum speed of rotation so that the milk is not spilled even when the bucket gets inverted ? $g = 980$ cms^{-1}.

[252 cms^{-1}

57. An arched bridge is to be so designed that cars may pass over the bridge safely at speeds upto 180 km/hr without jumping off the road. What should be the minimum or maximum value of the radius of curvature of the arch ? (J. E. E. '75) [255 m]

58. A simple pendulum is suspended from the celling of a railway carriage. The carriage passes a curve of radius 65 m at a speed of 12 km/hr. Through what angle will the bob deviate ? [1°]

59. A car having a mass of 100 kg travels round a horizontal circle of radius 30 m. The height of the centre of gravity is 320 mm above the road surface and the truck width between the wheels is 1·25 m. The coefficient of friction between the tyre of the verticle and the wet road surface is 0·4. What is the maximum safe velocity for rounding the bend ? If this velocity is exceeded, will the car overturn or slip sideways ? [23·95 ms^{-1}, overturn]

60. Calculate the acceleration of a particle at the equator of the earth due to its rotation. The radius of the earth is 6600 km and its period of rotation is 24 hour.

[0·035 ms^{-2}]

61. Two disks are mounted 40 cm apart on an axle. The axle with disks rotates with uniform angular velocity of 3000 rpm. A bullet flying parallel to the axle pierces both the disks. The hole in the second disk is displaced with respect to that in the first one by an angle of 18°. What was the velocity of the bullet ?

[400 ms^{-1}]

62. A small body of mass m is placed on the top of a smooth sphere of radius r. If the body slides down the surface of the sphere, at what point does it flies off the surface ? [At a point which makes an angle of $\cos^{-1}(\frac{2}{3})$ with vertical radius]

63. The bob of a simple pendulum weigh m g. It is pulled from its position of equlibrium through an angle θ and than released. Show that the tension of the thread when the bob passes through the position of equilibrium is given by mg $(3-2\cos\theta)$.

64. A heavy particle at the end of a tight string of length 20 cm, the other end being fixed, is allowed to fall from a horizontal position of string. When the string is vertical it encounters an obstruction at its middle point and the particle continues its motion in a circle of radius 10 cm. Find the height the particle will attain before the string slackens.

[16·66 cm]

65. A body weighing 100 g vibrates in S.H.M. with a period of 5 s. and an amplitude of 1·5 cm. Find its velocity, accleration (i) at the mean position, (ii) at the end of the path and (iii) at a point 1 cm from the mean position. [1·89 cms^{-1}, 0 ; 0,2·38 cms^{-2} ; 1·41 cms^{-1}, 1·58 cms^{-2}]

66. A particle of mass 5 g is executing S.H.M. The effective force on it is 1280 dyne when its displacement is 25 cm. What is the time period ? [1·96 S]

I·6 (Accelerated rotation)

[A] Essay type questions :

1. Explain what is meant by angular acceleration. For motion in a circle how it is related to the linear acceleration.

2. What is meant by angular momentum of a partical about a point ? Find how it is related to the moment of the force acting. What is the importance of the moment of a force ?

3. What do you understand by the term torque ? What is the moment of a force about a point ? What is the importance of a torque in angular motion ?

4. Explain the terms couple, moment of a couple, arm of a couple. Give examples of couples. How can a couple destroy the action of another ? What is meant by the axis of a couple ?

5. Explain the principle of conservation of angular momentum with suitable examples.

[B] Short answer type questions :

6. The force applied to a point near to the rim of a wheel produces greater amount of motion—Explain.

7. Why do we prefer long arm wrench to remove a tight bolt ?

8. It is easier to open or close a door by pushing or pulling near it's outmost edge than by pushing or pulling near the hinges—Explain.

9. Why does a diver, diving from a high platfrom, get a good number of somersaults if he curbs his body more ?

10. At what point should a force be applied to a wheel at rest in order that it produces no rotational motion ?

11. A man stands on a rotating turntable with his arms outstretched. What will happen when the man draw his hands towards his chest ?

12. A cannon is at the centre of rotating platform, rotating about a vertical axis. A shell is fired in a horizontal direction along the radius of a platform. Will the velocity of rotation change ?

13. Will the angular velocity of earth decrease if a heavy meteor hits the earth from the sky ?

14. A heavy weight is fitted at one end B of a light rod AB. When will the rod fall faster, when pivoted at A or when at B ?

15. When a torque is applied on a rigid body free to rotate about an axis, the angular acceleration produced depends not only upon the size and the shape of the body but also upon the distribution of its mass with respect to the axis of rotation—Explain.

16. Prove that if the earth suddenly contract to half its radius, the length of the day will be shortened from 24 hours to 6 hours.

17. Moment of inertia is the rotational analogue of mass of a body—Explain.

18. Why is barrel of a rifle spiraled ?

19. Why pole-star appear to be fixed relative to earth ?

[C] **Numerical Problems :**

20. A fly-wheel of radius 50 cm increases its speed from 720 rpm to 1440 rpm in 1minute. Calculate (a) its angular acceleration in rad/s^2 and (b) linear acceleration in cm/s^2 of a point on its rim. [1·256 rad/s^2, 62·8 cm/s^2]

21. A wheel is uniformly retarded by braking and its velocity of rotation drops from 300 to 180 rpm in one minute. The moment of inertia of the wheel is 2 kgm^2. Find (i) the angular acceleration of the wheel, (ii) the braking moment, (iii) the number of revolutions completed by the wheel during this minute, (iv) the work done in braking. [0 21 rad/s^2, 0·42 N m, 240 rev, 630 J]

22. A fan rotates with a velocity of 1500 rpm. When its motor is switched off, the fan has uniformly retarded rotation and makes 150 revolutions before it stop. In how much time the fan stops after it has been switched off ? [12 s]

23. An engine having wheels of radius 40 cm changes its speed from 25 to 50 km/hr in 5 s. Calculate the angular acceleration of the wheel. [3·47 rad/s^2]

24. A uniform circular disc of mass 50 g and of radius 10 cm is acted upon by a torque consisting of two equal and opposite forces applied tangentially to the rim from the extremities of a diameter. The disc is free to rotate about an axis passing through its centre perpendicular to its plane. What should be the magnitude of each force so that the disc may rotate with an angular acceleration of 20 rad/s^2 ? What is the kinetic energy transferred to the disc after 2 seconds ? [40 rad/s, 0·2J]

25. A light horizontal bar is 10 m long. A 3 kg force acts vertically upward on it 25 cm from the right hand end. Find the torque about each end. [7·35 Nm (clockwise), 286·6 Nm (counter clockwise)]

26. A circular disc of mass 100 g and radius 10 cm revolves at the rate of 30 rpm about a vertical axis passing though its centre. A piece of wax of mass 20 g is slowly dropped on the disc at a point 8 cm from the centre. What will be the rate of rotation of the disc now ? [32 rpm.]

27. A flywheel has moment of inertia 0·1 kgm^2. What constant unbalanced torque is required to increase its speed from 3 rpm to 9 rpm in 18 revolution ? [1·256 Nm]

28. A wheel is rotating about its axis with constant angular acceleration of 2 rad/s^2. It turns through an angle of 500 radians

in 5 second. It is started from rest, what was its angular velocity at the beginning of the 5 second interval ? [95 rad/s.]

29. The driving shaft of an engine revolves 1800 times per minute and transmits 200 hp. Find the torque exerted. [583·6 ft lb]

30. A boy spins a top by pulling on the sting wound round the top at an average distance of 1 cm from the axis. His pull is 0·15 kg. His hand holding the string accelerates at the rate of 2 m/s^2. What is the M. I. of the top ? [735 gm cm^2]

34. A cylindrical fly-wheel of 500 g and radius 8 cm is caused to rotate by a 200 g mass attached to one end of a string which passes round the fly-wheel, the other end being attached to the wheel. Find the tension in the string and the angular acceleration of the fly-wheel. [185 gf, 9 1 rad s^{-1}]

32. A fly-wheel is mounted on a horizontal axle of radius 3·0 cm. It is let in motion by a falling mass of 200 g which is attached to a light string passing round the axle and fastened to it. The mass falls a distance of 120 cm to the ground in 12 s. If the wheel continues to rotate for 10 revolution after the mass has reached the ground, find (a) the moment of inertia of the wheel, (b) the tension in the string. [$6{\cdot}5 \times 10^5$ g cm^2 199·4 gf]

1-7 (Statics)

(A) **Essay type questions :**

1. Under what condition will a number of forces acting on a body fail to produce an acceleratron in it ?

Under what condition will a number of forces acting on a body fail to produce a rotation of the body ?

2. What is the difference between a body being at rest and being at equilibrium ?

State the conditions of equilibrium of a body.

3. Find the relation between three coplanar forces which, acting on a particle, keep it in equilibrium.

4. Distinguish between centre of gravity and centre of mass of a body. What characteristic propert.es do these two points have ? When are these two points the same ? When is it not proper to speak of a centre of gravity ?

5. Explain how the centre of gravity of a body distinguishes the stable, neutral and unstable equilibrium of a body.

6. What are like and unlike parallel forces ? How can you find the resultant of a number of unlike parallel forces ?

7. What is a couple ? What is the torque of a couple ? What is the effect of couple on a body ?

[B] **Short answer type questions :**

8. A parachute falling with constant velocity. Is it in equilibrium ?

9. When will three concurrent forces produce equilibrium ?

10. What is meant by the equilibrant of forces ?

11. What is the difference between centre of gravity and centre of mass ?

12. A body is held on an inclined plane. What factors would determine whether it would remain at rest or get upset ? Explain.

13. When do the centre of gravity and centre of mass of a body coincide ?

14. A picture hangs from a wall by two wires. What orientation should the wires have to be under minimum tension ?

15. Is it possible that the centre of gravity of a body may lie outside the body ?

16. Why is it easier to stand on two legs than on one leg ?

17. 'For body executing a pure translational motion its mass may be assumed to be concentrated at the centre of mass'—Explain.

18. Why bipeds require training to walk but a quadrupeds can not ?

19. A tradesman uses a false balance, the lengths of the two arms of the balance are l_1 and l_2 respectively. He weighs out two customers w kg of tea leaf as indicated by his balance But in serving one of the customers he puts the weights in one pan while in serving the other he puts them on the other pan. How much do he gain or lose by this ?

20. Show that for a body to be in equilibrium under the action of three non parallel forces, the forces must be coplanar and concurrent.

21. Where is the centre of gravity of the system made of the earth and the moon ?

22. A ladder is at rest with its upper end against wall and the lower end on the ground. Is it more likely to slip when a man stands on it at the bottom or at the top ?

23. Can two coplanar and unequal forces acting on a body keep it in equlibrium ?

24 A homogeneous beam whose weight is G lies on a floor. The coefficient of friction between the beam and the floor is k. Which is easier for two men to do—turn the beam about it centre or move it translationally ?

25. Three forces are acting on a body simultaneously, two of them being of equal magnitude. If these two forces act in opposite

directions at different points, the body cannot remain in equilibrium. Why ?

26. While walking on a rope in a circus, why acrobat swings to and fro ?

27. If a force acting on a body be so directed that its line of action passes through (i) the centre of mass of the body, (ii) any point other than the centre of mass of the body, what would be the nature of motion of the body in each case ?

28. A homogeneous rectangular brick lies on an inclined plane. What half of the brick exert a greater pressure on the plane ?

29. A ping-pong ball is floating on the top of a vertical water jet. Is it in stable, unstable or in neutral equilibrium in the vertical direction ? (I. I. T. '73)

30. Why is it dangerous to stand on the upper deck of a double-decker bus ?

31. A body freely suspended from a fixed point rests with its C. G. vertically below the point of suspension. Explain.

32. A stick balanced on the finger remains in neutral equlibrium. Explain why ?

33. At what coefficient of friction will a man not slip when he runs along a straight hard path ? The maximum angle between a vertical line and the line connecting the man's centre of gravity with the point of support is θ.

34. Do the centre of mass and centre of gravity coincide for a building ? For a lake ?

35. Which is better—Carry load in one hand or in two hands ?

36. Is the centre of gravity of a body change if the orintation of the body changes ?

37. A man buys 3 kg of rice. To check that the weight is correct he collect a metre scale and a spring balance reading upto 1 kg. How can he check the weight of the rice with this equipment ?

[C] **Numerical problems :**

38. Forces of 30, 20, 60 and 60 gf acts on a particle in directions south, east, north and west respectively. Find the magnitude and direction of their resultant. What is meant by the 'equilibrant' of these forces ? [50 gf inclined at an angle $\tan^{-1}4/3$ with the north towards the west.]

39. A body weighing 500 g is suspended from the middle of a straight horizontal string 200 cm long. As a result the centre of the string is depressed by 25 cm. What is the tension in the string with the load on ? [about 2060 gf.]

40. A 3 metre long plank is supported horizontally at the ends by two strings. A man weighing 72 kg sits on the plank at a distance

of 1 m from one end. What are the tensions in the strings ? Ignore the weight of the plank. [48 kgf in the nearer string and 54 kgf in the other.]

41. A lever of 2 m long and weighing 4 kg has its fulcrum 50 cm from one end. If a weight of 16 kg is suspended from the end nearer the fulcrum, what force at the other end will be required to keep the lever in equilibrium ? [4 kgf]

42. Two unequel forces inclined at a certain angle act on a particle. Show that the resultant is nearer the greater force.

43. A non-uniform beam 16 m long rest on two pegs 9 m apart with the centre midway between them. The greatest mass that can be suspented in succession from the two ends without disturbing the equilibrium are 4 kg and 5 kg respectively. Find the weight of the rod and the position of the point at which its weight acts.
[3·5 kg, 0·5 m from the centre.]

44. ABCD is a square, each side 2 m long. Forces of magnitude 1, 7, 24, 4, 10 $\sqrt{2}$ and 13 $\sqrt{2}$ kgf are acting along AB, BC, CD, DA, AC and BD respectively. Find their resultant. [36 kgf]

45. A load weighing 50 $\sqrt{3}$ kg is supported in equilibrium by two ropes, one horizontal and the other in a direction 30° with the vertical. Find the tension in each rope. [100 kgf and 50 kgf]

46. Two men are carrying a load of 45 kg placed on a rod whose ends are resting on the shoulders of the men. If the load be 3 m away from one man and 5 m from the other, find the weight carried by each man. [16 9 kg, 28·1 kg]

47. If C_1 and C_2 are the centres of gravity of two portion of a body of weights W_1 and W_2 respectiverly and C, the centre of gravity of the entire body show that

$$CC_1 = \frac{W_2}{W_1+W_2} C_1C_2 \text{ and } CC_2 = \frac{W_1}{W_1+W_2} C_1C_2.$$

48. A unifrom horizontal beam AB 40 cm long and weighing 15 kg is hinged at A to a vertical wall and supported by a wire from B to a point on the wall 30 cm vertically above the hinge. A load of 30 kg hangs from the beam at a distance of 30 cm from the wall. Find (i) the tension in the wire, (ii) the push of the beam (iii) the reaction at the hinge. [50 kgf, 40 kgf, 42·72 kgf at an angle of 20°33′ with the horizontal.]

49. A beam weighing 100 lb stands inclined to the horizon at an angle of 30° with one end resting against a smooth vertical wall and the other end on the ground. The lower end of the beam is tied to the wall by a horizontal rope so that it may not slip. If the C. G. of the beam divides the length of the beam in the ratio 3 : 5, calculate the tension in the rope. [21·65 lbf]

50. Two force P and Q—one parallel to the length and the other

23

parallel to the base of an inclined plane can separately keep a body of weight W at rest on the inclined plane. Show that $\frac{1}{P^2}-\frac{1}{Q^2}=\frac{1}{W^2}$.

If R and S are respectively the forces of reaction acting on the weight, prove that $RS=W^2$.

51. A unifrom ladder weighing W resting against a rough vertical wall and on a rough floor. It is inclined at an angle 45° with the horizon. If μ_1 and μ_2 be the coefficients of friction at the floor and the wall respectively then show that the minimum horizontal force that can move the lower end of the ladder towards the wall is given by $\frac{W(1+2\mu_1-\mu_1\mu_2)}{2(1-\mu_2)}$.

52. A uniform iron beam is carried by three men, one is holding the beam at one end and the other two support it by means of a cross-bar placed underneath. At what point of the beam must the bar be placed so that each may carry one-third of the weight of the beam ? [1/4 length from free end.]

53. A uniform plank 6 m long and weighing 60 kg leans against a smooth vertical wall and stands on the level concerte floor having co-efficient of sliding friction equal to 0·3. Find the distance of the plank from the wall at which it will just start to slip. (J. E. E. '79)
[3·09 m]

54. Two like parallel forces P and Q acting on a rigid body has a resultant R. If the forces interchange their points of application, the resultant R is displaced through a distance x. Show that $x=\frac{P-Q}{P+Q}y$ where y is the distance between the points of application of the forces P and Q.

55. A wheel of radius 4o cm rests against a step of height 20 cm. What is the minimum horizontal force F, which if applied perpendicular to the axle will make the wheel climb up the step. The mass of the wheel is 2 kg. (I. I. T. '76) [2·46 kgf.]

56. A heavy homogeneous sphere is suspended from a string whose end is attached to a vertical wall, The point at which the string is fastened to the sphere lies on the centre of the sphere. What should the coefficient of friction μ between the sphere and the wall be for the sphere to remain in equilibrium ? [$\mu>1$]

57. A dumbell with spherical masses 1360 8 g and 2268 g are connected by a wire of length 20·96 cm. It is set to 180 rpm and thrown up. Find the tension in the wire. (I. I. T. '69) [18·78 kgf]

58. A table has a heavy circular top of radius 1 m. and mass 20 kg. It has four light legs of length 1 m fixed symmetrically on its cicumference. Then (i) What is the maximum mass that can be placed anywhere on this table without toppling the table ? (ii) What

is the area of the table top over which any weight may be placed without toppling it ? (I. I. T. '74) [48·3 kg, 2 m^2]

59. A flexbile chain of weight w hangs between two fixed points A and B, at the same level. Find the tension in the chain at the lowest point and the force exerted by the chain on each end point.

[$\frac{1}{2}$ w cot θ, w/2 sin θ tangent to the chain.]

60. A metal square of uniform thickness and side 18 cm is divided into four equal triangles by drawing the diagonals and one of the triangles is then cut out. Find the position of the C. G. of the remainder. [2 cm away from the vertex of the removed traiangle.]

61. A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of 42 cm is removed from one edge of the plate. Find the position of the centre of mass of the remaining portion. (I. I. T. '80) [9 cm away from the centre of the removed portion.]

62. Calculate the maximum height of uniform cylinder of diameter 8 cm that can be placed on its base on a rough inclined plane of angle 30° without the cylinder toppling over. [13 856 cm.]

63. A rectangular beam of thickness t is balanced on a curved surface of a rough cylinder of radius r. Show that the beam will remain in stable equilibrium if $r > t/2$,

64. A square hole has been punched out of a circular sheet in such a way that the radius of the circle is a diagonal of the square. If d be the diameter of the circle, show that the C. G. of the remaining position of the sheet is at a distance of $\frac{d}{8\pi - 4}$ from the centre of the circle.

65. A wheel of mass M can revolve in a vertical plane and slide down on inclined plane inclined at an angle ϕ. Show that if a mass of m be attached to the rim of the wheel, it will attain stable equilibrium if $\sin\phi < \frac{m}{M+m}$. The slipping along the plane may be ignored.

I-8 (Work, Power and Energy)

[A] **Essay type questions** :

1. Define work. Distinguish between work done by a force and against a force, giving two examples of each.

2. What is meant by energy ? What is mechanical energy ? Prove that kinetic energy of a body of mass m moving with a velocity v is $\frac{1}{2}mv^2$.

Calculate the increase in kinetic energy of a body which moves a distance s in the direction of the applied force F.

3. What is potential energy ? What do you understand when it is said that the gravitational potential energy is mgh ?

Prove that a body moving freely under gravity has a constant mechanical energy (that is, the sum of its kinetic and potential energies is a constant). Consider the case of both rise and fall.

4. Explain what is meant by 'Conservation of energy'. What is an isolated system ?

5. Explain the equation $W = Fs \cos \theta$ relating to work done by a force. What willl be the interpretation of the equation if θ is greater than 90° ?

6. Define the terms joule, erg and watt. How are they related ? What is a killowatt-hour ?

Distinguish between momentum and kinetic energy.

7. What do you mean by work done by a couple ? Find an expression for it.

State work-energy principle.

8. (a) Justify the principle of conservation of energy with referenc to a body sliding down an inclined plane.

(b) Deduce an expression for the total energy at any instant of a body sliding down an inclined plane.

9. What are meant for a conservative and dissipative force ? Prove that the gravitational force is conservative but the frictional force is dissipative.

10. (a) Justify the principle of conservation of energy with reference to a swinging pendulum.

(b) Prove that a projectile obeys the law of conservation of energy.

(c) Obtain an expression for the elastic potential energy of a stretched spring of unstretched length l_0.

[B] Short answer type questions :

11. In a tug-of-war, team A is defeated by team B. Which team did work against the other ? Explain your answer.

12. A man is rowing a boat upstream, but is unable to advance relative to the bank. Explain whether the man who is in the boat is doing work or not.

13. Does an applied force always do work ?

14. When a gun fires a bullet, who does work-the powder or the bullet ? What kind of transformation of energy that occuss in this case ?

15. A motor car is moving with a constant velocity on a level road. In such a case there will be no unbalanced force on the car. Is work being done anywhere ? Explain your answer.

16. 'The power of an engine is 10 hp'—What is the meaning of the statement ? (H. S. '80)

17. What is meant by 'No-work force'. Explain with example.

18. Under what condition the work done by a force is positive, negative and zero.

19. Give an examples of a force acting on a moving body, but doing no work.

20. When a watch is wound, what type of energy is stored in it.

21. A force acts on a body whose velocity changes thereby. Prove that the change of kinetic energy is equal to the work done by the impressed force.

22. Is it possible for a body to prossess negative value of the potential energy ? Explain your answer with a suitable example.

23. What is meant by conservative system ? Is it ever possible that such system can exist in nature ?

24. In the motion of a simple pendulum, show that the work done by the tension in the string is zero.

25. Does the work done in raising a body to a certain height depend on how fast it is being raised ? Is power ?

26. Deduce the relation between kilowatt and horse power.

27. What is the relation between kilowatt hour and joule ?

28. A man rises in a lift carrying a bucket of water. Explain (a) if any work is done by the man on the bucket of water, and (b) if the energy of the bucket of water would remain unaltered. (H.S. '78)

29. Differentiate between gravitational potential energy and elastic potential energy.

30. A man is swiming upstream in such a manner that he is stationary with respect to the bank. Is he doing any work ?

31. When a body moving in a circular orbit it is acted upon by a force. Is that force doing any work on the body ?

32. When two boys play catch on a train, does the kinetic energy of the ball depend on the speed of the train ?

33. Two identical springs, one of steel, the other of copper, are stretched with identical forces. On which operation must more work de expended ? (Young's modulus of steel is greater than that of copper.)

34. What is the change in the potential energy of a body raised through a height h in water ? The density of the body is d and that of water is d_0, where $d > d_0$. The volume of the body is v.

35. How do you explain the fact that when a stone is dropped on the ground, the change in the momentum of the earth is equal to that of the stone, while the change in the kinetic energy of the earth is neglected ?

36. How should the power of a pump motor change for the pump to deliver twice as much water in unit time through a narrow orifice ? Disregard friction.

37. A lift loses potential energy in comming down from the top of a building to a stop at the ground floor. What happens to its potential energy ?

38. Can a body have energy without momentum and momentum without energy ?

39. If a body of mass m possesses kinetic energy E, show that its momentum is $\sqrt{2mE}$.

40. A body falls freely from rest under gravity acquire a velocity v by losing its potential energy V. Find the mass of the body.

41. A heavy body and a light body possess equal momentum. Which one will have a larger kinetic energy ?

42. A lorry and a car moving with the same kinetic energy are brought to rest by the application of brakes which provide equal retarding forces. Which of them will come to rest in a shorter distance ? (I. I. T '75)

43. When a gas-filled balloon rises up, its gains both kinetic and potential energy. How does the principle of conservatio energy hold in this case ?

44. Is the resistive force of air a conservative one ?

45. When a compressed air is allowed to do some work, what type of energy is stored in it ?

46. 'Work is always done against friction, work is not done by friction'—Explain.

47. A meteorite burned completely in the atmosphere before it reaches the earth's surface. What happened to its energy ?

48. Explain why a falling body becomes hotter when it strikes the ground.

49. 'A system of bodies has always a tendency of remaining in a position where the potential energy is minimum'—Explain.

50. A boy tries to raise a bucketful of water but fails to do so. What is the amount of work done ?

51. The acceleration due to gravity on a planet is 196 cm s^{-2}. If it is safe to jump from a height of 2m on earth, find the corresponding safe height at the planet. (I. I. T)

52. Two similar safely-pins, one open and the other closed are put into two separate cups containing the same acid of equal volume. Both the pins get dissolved in the acid. Which cup will be at higher temperature ?

53. The driver of an automobile travelling at a speed of v suddenly sees a wall at a distance d dircetly in front of him. To avoid crashing, is it better to slam on the brakes or to tern sharply away from the wall ?

54. A light and a heavy body posses equal kinetic energies Which one will have a greater momentum ?

55. The work done by a resultant force is always equal to the change in kinetic energy. Can it happen that the work done by one of the component forces alone will be greater than the change in kinetic energy ? It so, give example.

56. Explain, using work and energy ideas, how a child pumps a swing up to large amplitude from a rest position.

57. Two disks are connceted by a stiff spring. Can one press the upper disk down enough so that when it is released it will spring back and raise the lower disk off the table ? Can mechanical energy be conserved in such a case ?

58. An object is dropped and observed to bounce to one and one-half times its original height. What conclusion can you draw from this observation ?

59. Moutain roads rarely go straight up the slope but wind up gradually. Explain why.

60. A boy of mass M can throw a stone of mass m with a horizontal velocity v_1 leaning against a wall. With what velocity can he throw the stone putting on sketting shoes on the ice ? Will the boy do work in both the cases at the same rate ? What is the relative velocity of the stone with respecct to the boy in the second case ?

[C] Numerical problems :

61. What is the K.E. of a 100 g bullet fired with a velocity of 400 m/s ? [8000 J]

62. How much work is required to build a column 12 ft high of 4 marble-blocks each 2 ft thick and weighing 500 lb ? [15,000 ft-lb]

63. A train weighing 10^5 kg starts from rest and acquires a speed of 12 m/s in one minute. How much work has been done on it ? If the engine can pull the train with a constant speed of the above value, what is the power of the engine in killowatts ? [72×10^5 J, 240 kw]

64. A cloud 5 km about the ground condenses into rain which collects 1 cm deep over an area of 10^4 m^2. What is the loss of potential energy of the cloud ? [$9{\cdot}8 \times 10^8$ J]

65. A boy weighing 60 kg ascends 32 steps, each 25 cm high, in 10 seconds. What power in kilowatts did he expend ? [0·59 kw]

66. A 5 kw motor lifts water to a height of 10 m above the water level. If the efficiency of the pump is 80%, how much water will be lifted per minute ? [2570 litre]

67. Forces of 1 kg each act on masses 10 kg and 40 kg. Find the ratio of the times required to give them (a) the same momentum, (b) the same kinetic energy. [Equal times, 1 : 4]

68. A pile driver drops a ball weighing 250 kg from a height

of 5 m on a pile, the pile sinks 2·5 cm into ihe ground. What is the average resisting force of the ground ? [5×10^4 kgf]

69. A stone weighing 2 kgf fell from a certain height during 1·43 s. Find the kinetic and potential energy of the stone at half the height. ($g = 9{\cdot}8\ ms^{-2}$) [98 2 J (both)]

70. A stone is thrown horizontally with the velocity of 15 ms^{-1} from a tower with a height of 25 m. Find the kinetic and potential energy of the stone in one second after motion begins. The mass of the stene is 0·2 kg. ($g = 9{\cdot}8\ ms^{-2}$) [32·2 J, 39·4J]

71. A stone is thrown at an angle of 60° to the horizon with a velocity of 15 ms^{-1}. Find the kinetic, potential and total energy of the stone (i) in one second ofter motion begins, (ii) at the highest point of the trajectory. The mass of the stone is 0·2 kg. ($g = 9{\cdot}8\ ms^{-2}$) [6·6 J, 15·9 J, 22·5 J ; 5·7 J, 16 8 J, 22·5 J]

72. The work spent to put a shot at an angle of 30° to the horizon is 216 J. In how much time and how far from the point of throwing will the shot fall to the ground ? The shot weighs 2 kgf. ($g = 9{\cdot}8\ ms^{-2}$) [1·5 s, 19·1 m]

73. A material particle with a mass of 10 g moves along a circle having a radius of 6·4 cm with a contant tangential acclerations. Find this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J by the end of the second revolution after motion beings. [0·1 ms^{-2}]

74. A body with a mass of 1 kg slides down an inclined plane 1 meter high and 10 m long. Find (i) the kinetic energy of the body at the base of the plane, (ii) the velocity of the body at the base (iii) the distance travelled by the body over the horizontal part of the route until it stops. Assume the coefficient of friction to be constant over the entire route and equal to 0·05. [4·9 J, 3·1 ms^{-1}, 10 m]

75. A motor vehicle with a mass of 2 tons runs up a grade of 1 in 25. The coefficient of friction is 8 per cent. Find (i) the work performed by the vehicle engine over a distance of 3 km, (ii) the power developed by the engine if this distance was covered in 4 minute. [7×10^6 J, 29 4 kw]

76. Find the power developed by the engine of a vehicle with a mass of 1 ton if it moves at a constant speed of 36 km/hr, (i) over a level road, (ii) up a grade of 1 in 20, (iii) down the same grade. The coefficient of friction is 0·07. [6·9 kw, 11·8 kw, 1·98 kw]

77. The constant force resisting the motion of a car, of mass 1500 kg, is equal to one-fifteenth of its weight. If, when travelling at 48 km/hr, the car is brought to rest in a distance of 50 m by applying brakes, find the additicnal retarding force due to tbe brakes (assumed constant) and the heat developed in the brakes. [1667 N, 83330 J]

78. A 100 lb block of ice slides down an incline 5 0 ft long and

3·0 ft high. A man pushes up on the ice parallel to the incline so that it slides down at constant speed. The coefficient of friction between the ice and the incline is 0·10. Find (i) the force exerted by the man, (ii) the work done by the man on the block, (iii) the work done by gravity on the block, (iv) the work done by the surface of the incline on the block, (v) the work done by the resultant force in the block, and (vi) the change in kinetic energy of the block. [52 lbf, – 260 ft-lb, 300 ft-lb, – 40 ft-lb, 0, 0,]

79. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1 m/s and then has the same kinetic energy as the boy. What were the original speeds of man and boy ? [2·4 ms^{-1}, 4 8 ms^{-1}]

80. Show from consideration of work and kinetic energy that the minimum stopping distance for a car of mass m moving with speed v along a level road is $v^2/2\mu_s g$, where μ_s is the coefficient of static friction between tyres and road.

81. A fielder at 'cover point' throws a cricket ball with an initial speed of 60 ft/s Wicket keeper at the same level catches the ball when its speed is reduced to 40 ft/s. What work was done in overcoming the resistance of the air ? The mass of a cricket ball is $5\frac{1}{2}$ 0Z. [10·74 ft. lb]

82. What power is developed by a grinding machine whose wheel has a radius of 8 inch and runs at 2 rps when the tool to be sharpened is held against the wheel with a force of 40 lb ? The coefficient of friction between the tool and the wheel is 0·32.

83. A body of mass m accelerates uniformly from rest to a speed v in time t. Show that the work done on the body as a function of time T, in terms of v and t is $\frac{1}{2}$ m $\frac{v^2}{t^2}$ T.

84. A wicket-keeper catches a cricket ball of mass 100 g moving horizontally at a speed of 40 ms^{-1}. If his glove moves back a distance of 20 cm while bringing the ball to rest, what average force was exerted on his hand ? [400 N]

85. A crane lifts a 1500 lb steel beam to a height of 44 ft in 10 s. Find the horse-power developed. (H S. '67) [12 hp.]

86. What should be the hp of an engine which is intended to pump 250 gallons of water per minute to a height of 40 yards ? (1 gallon of water weighs 10 lb) (H. S. '61) [9·1 hp.]

87. A lift weighing 2000 kg rises from rest in the ground floor to the fifth floor, a height of 20 m. As it passes the fifth floor, it has a speed of 4 ms^{-1}. There is a constant frictional force of 10 kgf. Calculate the work done by the lift. [$4 \cdot 1 \times 10^5$ J]

88. A particle is falling down a rough inclined plane. Assuming the frictional resistance to be 0·2 times the normal reaction and the

angle of the inclined plane to be 30°, calculate the acceleration of the particle. Calculate also the change in the sum of the kinetic and potential energies as the particle falls through a length of 1 m along the inclined plane. The mass of the particle is 1 g. Take $g=980$ cms^{-2} (J. E. E. '82) [320·26 cm s^{-2}, 1666 erg.]

89. A bullet of mass 2 g and moving at 500 m s^{-1} pierces a plate and moves on with a velocity of 100 ms^{-1}. With what velocity would it emerge if the plate had only half its thickness ? It may be assumed that the work done in piercing a plate is proportional to its thickness. [360·6 ms^{-1}]

90. A ball flying at a velocity $v_1=15$ ms^{-1} is thrown back by a racket in the opposite direction with a velocity $v_2=20$ ms^{-1}. Find the change in momentum of the ball if the kinetic energy changes by $\Delta E=8{\cdot}75$ Joule (J E. E. '84) [3·5 kg ms^{-1}]

91. An engine working with constant rate draws a train of total mass 600 tonnes up a plane whose inclination to the horizontal is $\sin^{-1}\left(\frac{1}{100}\right)$ If the frictional resistance to the motion is 5 kgf per tonne and the greatest speed up this incline is 10 ms^{-1}, calculate the power of the engine in killowatt. (J. E. E. '81) [735 kw.]

92. A horse pulls a cart with a force of 50 lbf at an angle of 30° with the horizontal and moving along with a speed of 6 mile/hr. How much work does the horse perform in 10 minute ? What is the power output of the horse in practical unit ? (J. E. E. '62) [$2{\cdot}287\times10^5$ ft lb, 0·693 hp.]

93. A vertical shield is made of two plates of wood and iron respectively, the iron plate being 3 cm and the wooden plate 6 cm thick. A bullet fired horizontally goes through the iron plate first and then penetrates 3 cm into the wood. If the shield is reversed the same bullet with same velocity goes through the wood first and then penetrates 2 cm into the iron. Find the ratio of the average resistances offered by the iron and wood. [3 : 1]

94. A water-fall discharges 10^3 cu.ft/s from a height of 200 ft. If 75% of the available energy at the bottom of water-fall is converted into power, calculate hp developed. Take 1 cu.ft of water = 62·5 lb.

If this power is utilised to raise water from a tube-well of depth 600 ft, find how much water can be raised per second. (J. E. E. '69) [$17{\cdot}046\times10^3$ hp, 250 cu.ft]

95. A vehicle weighing 7 metric tons with its engine shut-off moves down an incline 1 in 35 at a constant speed of 20 km/hr. What amount of engine power in hp, would be required to drive the vehicle up the same incline at the same speed, assuming frictional resistance to be the same in each case ? (J. E. E. '80) [29 2 hp.]

96. An ideal massless spring can be compressed 1 m by force of 100 N. The same spring is placed at the bottom of a frictionless inclined plane, inclined at an angle of 30° to the horizontal. A 10 kg

mass is released from rest at the top the incline and is brought to rest momentarily after compressing the spring 2 m. Calculate (i) the distance through which the mass slides before it reaches the spring, (ii) the velocity of the mass just before it reaches the spring. ($g = 10\ ms^{-2}$.) [4m, 4·47 ms^{-1}]

97. A force of 5 lb is found to stretch a door spring 6 inch. What is the potential energy of the spring when opening the door stretches it 18 inches ? [11·3 ft lb]

98. Water is to be pumped to a tank 27·8 ft above a reservoir. The bottom of the tank measures 5 ft × 5 ft. What work is to be done in pumping water to a depth of 4·4 ft in the tank. If the power of the pump is 0·5 kw, what time will be required for the work ? (J. E. E. '76) [$2·06 \times 10^5$ ft-lb, 17 min. 18 s]

99. 400 kg of air moving at 20 ms^{-1}, impinge on the vanes of a windmill every second. At what rate in kw is the energy arriving at the windwill ? What is the maximum mass of water that could be pumped each second through a vertical height of 5m ? (Take $g = 10\ ms^{-2}$) (Oxf. Univ.) [80 kw, 1600 kgs^{-1}]

100. The human heart forces 60 cm^3 of blood at each beat against an average pressure of 12 cm Hg. If the pulse frequency is 72 per minute, calculate the power of the heart in watt. (Density of mercury = 13·6 g cm^{-3}). [1·15 W.]

101. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest from the rim of the truck. The cylinder slips on the semicircular frictionless truck. How far has the block moved when the cylinder reaches the bottom of the truck ? How fast is the block moving when the cylinder reaches the bottom of the truck ? (I. I. T. '83) $\left[\dfrac{m(R-r)}{m+M},\ \dfrac{m}{M}\sqrt{2g(R-r)}\right]$

102 An upward force F = 196 N is applied upon a body of 10 kg till it is raised vertically upwards by a distance of 10 m. Calculate the work done by F and the work done against the gravity. Here the work done by F is much greater than the gain in gravitational potential energy. Show using clear calculations that the law of conservation of energy is quantitatively satisfies here. (J. E. E. '78) [1960 J, 980 J]

103. A bullet of mass m, travelling horizontally with a velocity v struck a heavy wooden block of mass M and got struck into it. The combined mass then began to move in the same direction. What fraction of the kinetic energy of the bullet will be retained as mechanical energy after the collision. What will happen to the remaining part ? [m/(m + M), will be converted to sound and heat energies.]

104. A ballet of mass 20 g travelling horizontally at 100 ms^{-1}, embeds itself in the centre of a block of wood of mass 1 kg which

is suspended by a light vertical string 8 m in length. Calculate the maximum inclination of the string to the vertical. $\cos 37° = 0.8038$.

[37°]

105. A test tube of mass 15 g, closed with a cork cf mass 1 g contains some volatile liquid. The test tube is suspended by a string of length 8 cm. What is the minimum speed with which the cork must fly off on heating the test tube so that the tube may describe a full vertical circle about the point of suspension? Assume that the string always remain taut. [26·56 ms^{-1}]

106. A hemisphere of radius 1 ft and weighing 12 lb is placed on a table with its circular base in contact with the table. How much work has to be done in turning the hemisphere upside down? [3 ft-lb]

107. A uniform rectangular parallelopipied of sides l, 2l and 4l lies on a horizontal plane on each of its three different faces, in turn. What is the potential energy of the parallelopied in each of these positions? Which position is the most stable?

[2 mgl, mgl, $\frac{1}{2}$ mgl ; when lies on the large face]

108. A shell of mass M is moving with velocity V. An internal explosion generates an amount of energy E and breaks the shell into two fragments whose mass are in the ratios $m_1 : m_2$. If the fragments continue to move in the original direction of motion, show that their velocities are $V + \sqrt{\dfrac{2\,m_2 E}{m_1 M}}$ and $V - \sqrt{\dfrac{2\,m_1 E}{m_2 M}}$.

109. A shell of mass 10 kg moving vertically upwards explodes into two pieces when its velocity is 22·5 ms^{-1} and is at a height of 23·6 m above the ground. The lower piece of mass 2 5 kg returns to the ground in 1·5 s after the explosion. Find how much higher the upper piece of mass 7·5 kg will rise after the explosion. Find also the energy of explosion. [54·86 m, 1588·47 J.]

110. A ball with a radius of 10 cm floats in water so that its centre is at a height of 9 cm. above the surface of water. What work should be done to submerge the ball upto the diametral plane?

[0·74 J.]

111. A cork 0·5 m long is drawn slowly from the neck of a bottle, the force exerted at any instant being proportional to the area of the cork in contact wlth the bottle. If initially the whole of the cork is in the bottle and the pull at starting is 0·45 kgf, find the work done in drawing the cork out. [5·56 J.]

112. An acrobat jumps on to a net from a height of 8 m. At what minimum height should the net be stretched above the floor so that the acrobat will not hit it when he jumps? If the acrobat jumps down from a height of 1 m, the net depresses by 0·5 m. [1·23 m]

PART II

Properties of Matter

PART II

PROPERTIES OF MATTER

GRAVITATION AND GRAVITY

II-1. Introduction.

We have stated before that one of the fundamental properties of matter is that *two pieces of matter always attract each other.* The property by virtue of which this happenes is called **gravitation.** Matter anywhere in our limitless universe has this property.

When the attracting body is our earth the force of attraction is called **gravity.** *Gravity is thus the pull of the earth on any body.* Theoretically this pull extends to infinity, but practically disappears at a distance equal to the diameter of the earth from its center or the radius from its surface.

II-2. Law of Gravitation :

This law first enunciated by Newton (1687) is one of the most important in Physics and provided the bed-rock of that fascinating branch of science, Astronomy. This law, called the **universal law** of gravitation, states that

Every material particle in the universe attracts every other with a force F which varies

(*i*) directly as the product of their masses (m_1, m_2)

(*ii*) inversely as the square of their separation (r) and

(*iii*) acts along the line joining the two particles.

In symbols we may write

$$F \propto m_1 m_2/r^2 \quad \text{or} \quad F = G m_1 m_2/r^2 \quad \text{or} \quad F = G(m_1 m_2/r^2)\mathbf{r} \qquad \text{(II-1 I)}$$

where G is the variation constant called the **universal constant of gravitation** and $\mathbf{r}$ the unit vector along the line joining them.

Note that (1) the law is one of force and not of motion. It specifies the magnitude of the gravitational force irrespective of whether the

pair is in motion or not, or even how they are moving; (2) the forces are mutual action and reaction equal in magnitude, measuring in fact the *strength of interaction.* This mass symmetry means that if the earth pulls the moon or any other body, it will be

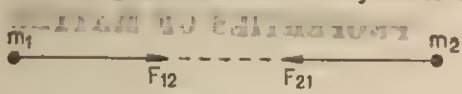


Fig. II-1.1

pulled by an equal and opposite force. That the earth does not move towards the falling apple, is because of its far greater mass and hence very small accelerat on, as we have noticed before. The equal and opposite interaction is illustrated in fig II-1.1.

Newton's Verification of the Inverse Square Law: Newton had arrived at this law from Kepler's laws of planetary motion (II-1.15). To verify this, he introduced the then novel idea, that the centripetal accelration moving the moon round the earth is the force of gravity, and found the pull of the earth on the moon. He *assumed* * that (1) Masses of earth and moon are concentrated at their respective centers and (2) the Lunar orbit is circular. The moon circulates round the earth in 27.3 days and their centers are apart by about 60 t mes the earth's radius. So he argued that according to the inverse square law, the pull of the earth on a body on the lunar surface would be about $(1/60)^2$ of the pull on the same body on the earth surface. That pull provides the centripetal accelera- tion of the moon towards the earth, which is

$$\omega^2 r=(4\pi^2/T^2)r=[4\pi^2/(27.3\times 86400)^2]\times 60\times 6.37\times 10^6 \text{ m}=0.00271 \text{ km/s}^2$$

Here to remember: 1 hour = 86400s and radius of earth 6.37 million metres. Now value of g on the surface of the earth is 9 8 m/s². Hence

$$g/60^2=9.8/3600=0.00272 \text{ km/s}^2 = \text{the earth's pull on the moon.}$$

The identity of the two values justified Newton's assumption. Thus was unified the terrestrial rotation of a stone at the end of a string from your finger and the rotation of an astronomical body the moon, round the earth.

II-1.2. Definition of G: If in the expression for the law of gravitation we put $m_1=m_2=1$ and $r=1$, we get $F=G$. Thus G may be defined as *the force of attraction between two unit point masses, unit distance apart.* Its numerical value depends upon the system of units in which the masses and their separation are measured.

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* He later mathematically proved that (1) mass of a sphere, solid or hollow may be taken to be concentrated at the center and (2) the orbits of moon and other planets are *very nearly* circular: Their ellipticites are very small, less than about 0.02.

Values of G : In the cgs system we shall have two point particles of mass 1 g each at a separation of 1 cm, and they would be attracting each other with a force of 6.67×10^{-8} dyn. In the mks system, the *point particles* of mass 1 kg each when separated by 1 m would be attracting each other with a force of 6.67×10^{-11} N. In the fps system two point masses 1 lb each at a separation of 1 foot would be attracting each other with a force of 1.069×10^{-9} poundals. Thus

$$\begin{aligned} G &= 6.67 \times 10^{-8} \text{ dynes-cm}^2/\text{g}^2 = 6.67 \times 10^{-11} \text{ Newtons m}^2/\text{kg}^2 \\ &= 1.069 \times 10^{-9} \text{ pdl-ft}^2/\text{lb}^2 = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1}\text{s}^{-2} \\ &= 6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \end{aligned}$$

Though thus G appears to be a very small quantity it is the force of gravitation that keeps the moon moving round the earth, the latter round the sun and the last round the center of the Milky Way, our galaxy. This can happen only because of the enormous masses involved. If an iron cable can support 10 metric tonnes ($=10^4$ kgf) it will require many more than a thousand billion ($=10^{15}$) such cables to hold the moon to the earth, should by chance, gravitation cease to act.

Dimension of G : From the equation II-1.1 we note that

$$G = \text{Force} \times (\text{distance})^2/(\text{Mass})^2 = MLT^{-2} \times L^2/M^2 = M^{-1}.L^3T^{-2}$$

Remember, G is a scalar quantity.

Ex. II-1.1. *Two spheres of masses 160 and 20 kg. when at a separation of 40 cm between their centres attract each other with a force of 0.14 mg-wt. Find the value of G*

$$G = \frac{Fr^2}{mm'} = \frac{(0.14 \times 10^{-3} \times 980)(40)^2}{160 \times 10^3 \times 40 \times 10^3} = 6.86 \times 10^{-8} \text{ cgs units}$$

Ex. II-1.2. *A hydrogen atom has a proton ($m_p = 1.67 \times 10^{-24}$g) as its nucleus round which revolves an electron ($m_e = 9.1 \times 10^{-28}$g), the diameter being 1 Å or 10^{-8} cm. Find the forces of gravitational and electrostatic attraction between them and their ratio. Charge on both particles $= 4.8 \times 10^{-10}$ e.s.u units*

$$\frac{F_G}{E_E} = \frac{G\,mm/r^2}{q\,q'/r^2}$$

$$= \frac{(6.67 \times 10^{-8} \times 1.67 \times 10^{-24} \times 9.1 \times 10^{-28})(10^{-8})^2}{(4.8 \times 10^{-10})^2/(10^{-8})^2}$$

$$= \frac{12.77 \times 10^{-60} \text{ dynes}}{23.02 \times 10^{-20} \text{ dynes}} = 0.55 \times 10^{-40}$$

This is why *gravitational attraction is a weak, while the electrostatic one is a strong force* but the range of the former is incomparebly larger.

Prob : (1) Suppose a man is measuring a force taking gravitational attraction between unit masses at unit separation as the unit. What then will be the value of G according to this new unit ? (*Ans.* 1.5×10^7). [J. E. E. '76]

(2) 0.1 mg wt. of attraction acts between two spheres of masses 40 and 15 kg when their centers are 20 cm apart. Find G. (*Ans.* 6.54×10^{-8} cgs units.)

Determination of G : The value of G quoted above has been experimentally determined by many methods though the quantity is so very small. We describe below in short the first such determination by Cavendish (1798) using a torsion balance (Fig. II-1.2)

A pair of small gold spheres (m, m) are fixed to two ends of a long light horizontal rod. The rod is suspended at the mid point by a long thin quartz fibre from a rigid support. Two large lead spheres M-M are placed on two sides of m, m such that the attractions produce a torque on the rod and twists the fibre. The rod comes to rest when the restoring torque by the twisted fibre equals the deflecting gravitational torque on the rod. A lamp and scale arrangement measures the angle of twist from which G can be determined. Very careful experiments (repeated 23 times) gave to Cavendish, $G=6.72 \times 10^{-8}$ cgs units.

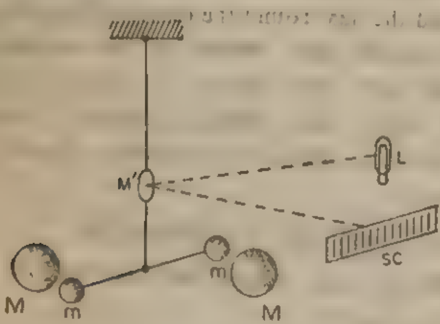

Fig II-1.2

II-1.3. Universality of the Law of Gravitation :

This law is applicable anywhere and everywhere and at all times. Among the very few universal constants like h (Planck's constant) or c (velocity of light in vacuum) or electronic charge e, the Gravitational constant G is one. For

(1) The law holds for all bodies, big or small, terrestrial or astronomical, material or even immaterial (like photons, neutrinos etc) ; no exceptions have been noted. It truly embraces all masses from an electron, to the sun, stars, galaxies.

(2) For separations from atomic to stellar magnitudes, the inverse square law has been found to hold accurately.

(3) The attractive force and G is independent of time.

(4) Change of separating medium does not affect the magnitude of the force nor the value of G. On both counts, it differs from the other inverse square laws, the Coulomb forces in electro- and magnetostatics where the nature of medium changes both the force and the medium constants K and μ. We see thus that *G is independent of* **permeability** *of the separating medium.*

(5) The material of the attracting bodies, their shape, volume, state of agregation (solid, liquid or gaseous), chemical composition, temperature, pressure, in fact no external or internal factor can affect the magnitude of the force or G, *i.e.*, they are *independent of* **susceptibility** *in any form of matter.*

(6) Gravitational force has *no* **directivity** or **anisotropy** *i.e.* matter may be crystalline or amorphous or the forces may be measured in any direction, without G changing.

We so conclude that the **gravitational force depends only on the masses of interacting bodies and the separation of their centres of mass.**

Newton derived his law of gravitation (1687) from Kepler's laws of planatery motion (1609-18) and satisfied himself as we have seen above, of the validity of his law of inverse square by calculating the value of g on the lunar surface. Later astronomers applied it successfully in explaining the perturbations of smaller planets in their orbits by the attractions of giant planets like Jupiter and Saturn They were also successful in calculating the orbits and hence appearance times of comets*. The planet Uranus had been discovered accidentally in 1802 ; but the farther planet Neptune was located by Adams and Leverrier independently (1846) by calculations from the perturbation it produces on the motion of Uranus in accordance with the inverse square law—an awe-inspiring triumph of the law of gravitation.

* You must be aware of the appearance of Halley's comet early this year (1986) which had come 76 years before in 1910. The Chinese astronomers had recorded it almost a thousand years back.

The most outstanding technological achievement of to-day, the artificial satellite had already been foretold by Netwon 300 years back, based on this law of gravitation.

Discrepancies : In a very few and highly specialised cases only these have been noticed ; for example,

(*i*) At separations of less than atomic diameters ($<10^{-9}$ m) the law fails.

(*ii*) In a very very strong gravitational field e.g. close to the sun the law fails. There is no explanation of a slow change of the plane (technically, *precession*) of the orbit of Mercury, the planet closest to the sun.

(*iii*) Its universality fails for particles moving verv fast i.e. close to the velocity of light, for then their masses are no longer invariant (constant).

(*iv*) According to Newton gravitation spends no time in reaching the farthest point. But we know that nothing can move faster than light ($=3\times10^{8}$ m/s) [If the sun suddenly loses its gravitation we shall know of it more than 8 minutes later]. So gravitation propagates with a finite velocity.

These discrepancies are very small very specialised, very few. compared to the credit side, which number legion. All of these except the first have been satisfactorily accounted for by Prof. Einstein's Theory of Relativity which includes Newtonian mechanics as a very satisfactory first approximation.

No satisfactory explanation has yet been put forward to explain the phenomenon of gravitation. Quantum mechanics has explained how and why electric and magnetic forces work but could not explain gravitation which is said to be a **fundamental force*** for its idea is not forthcoming from any other force. The search is still on for the so-called Unified Field theory.

II-1 4. Gravitational and Inertial mass : Mass as defined in Newton's second law ($m=F/a$) is called the inertial mass—for it is a measure of inertia. Mass as it appears in Newton's law of gravitation is called the gravitational mass.

A force of push or pull (F) produces an acceleration a on a mass m independently of whether gravity acts on it or not. The force W on a body of mass m with which the earth attracts it is $W=mg$. This relation enables us to measure the gravitational mass of a body by

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* In nature there are only three other fundamental forces—electromagnetic, molecular, nuclear. Of the four, gravitation is the weakest yet ranges the farthest. Successive forces grow progressively stronger but operates over progressively shrinking distances. Molecular forces range over 10^{-9} m while the nuclear over 10^{-15} m only. All the four fundamental forces appear to be independent of each other.

balances, either spring or common. Inertial mass is measured by finding the ratio of the force applied on a body and the acceleration generated—a far more difficult task. It struck Newton whether the two masses are equivalent. He utilised a hollow pendulum bob to test the point.

Now, a pendulum bob displaced from its equilibrium position is acted upon by a component $W \sin \theta$ ($= m_G g \sin \theta$) towards that position From Newton's second law we say that force is $m_I a$. Hence equating the inertial force to the restoring force we have

$$m_I a = m_G g \sin \theta \simeq - m_G g\,(x/l) \quad \text{[Eqn, 1-5,10.1 derivation]}$$

$$\therefore \; a = -(m_G g/m_I l)\,x = -\omega^2 x$$

$$\text{or,} \quad T = 2\pi \sqrt{m_I l/m_G g} \qquad \text{(II-1.4.1)}$$

Now Newton filled up the hollow bob successively with equal *weights* ($m_G g$) (as determined by a balance) of different substances. As one material replaced another, any change in T could be due only to difference in m_I, the inertial mass. Newton found no change and perforce had to conclude that they are equal i.e. equivalent. Eotvos in 1909 and Dicke in 1964 could detect no difference between inertial and gravitational masses to an accuracy of 1 in 10^{11}. Classical physics attached no significance to this equivalence but it provided a clue to the emergence of the theory of relativity. They are not distinguishable though gravitational mass is meaningless in gravitation- free space where however inertial mass ($= F/a$) would exist.

II-1 5. Gravitational attraction between Extended bodies : In formulating the law of gravitation Newton envisaged two particles i.e. masses that are small enough to be represented by geometrical points. Material bodies can hardly be so small. But there is a way out ; if their *dimensions are small compared to their separation* the bodies can be taken as point masses. Two bodies of dimension 1 cc, when separated by 100m can be taken as particles, but not when they are say, 5 cm apart. For then, *each* particle of one body attracts *each* particle of the other along lines joining each particular pair. Thus the total attraction between two such bodies is the resultant of a very large number of forces of different magnitudes and directions. To find that resultant is almost a hopeless task.

Attraction between two spheres : But in one case such is not the case. If the bodies are both spheres, calculation of attraction becomes very easy, whatever be their separation. For then, their

whole mass may be taken to be concentrated at their centers, and the attraction equal to that between these concentrated point masses.* Even if the spheres be big and in contact, this relation therefore still holds. It also holds whether the sphere is solid or hollow, the material be homogenesus or heterogeneous.

Let there be two spheres of radii a and a' and densities ρ and ρ'. Then their masses would be $M=\frac{4}{3}\pi a^3\rho$ and $M'=\frac{4}{3}\pi a'^3\rho'$. Now whatever be these radii their minimum separation is $a+a'$. If the sphere centers be at this separation or any value R greater than this, the gravitational attraction between them would be $F=GMM'/R^2$.

This result is used to find *the force of gravitation between the sun and the planets, planets and satellites where they are taken as homogeneous spheres, their masses concentrated at their centers.* If the attracting bodies be shaped otherwise, their masses would be taken concentrated at respective *centres of mass* so that they become particle masses, provided however their separation much exceeds their dimensions. If a small body rests on a sphere, calculations are carried out by taking the mass of the sphere as concentrated at its center and that of the body at its center of mass.

Newton had arrived at *two important conclusions* when finding the gravitational forces for a sphere, namely.

(1) For a solid sphere or a spherical shell, at a point outside, its entire mass can be taken as concentrated at its center. The same holds if the sphere is made up of *concentric* shells of materials of different densities.

(2) Inside a homogeneous hollow shell no attraction exists at any point. In other words inside a hollow spherical shell gravitational field does not exist and so potential is constant.

These conclusions hold also for charged and magnetised spheres.

II-1.6 Gravitational Field and Potential.

Any body is attracted by a nearby body. The range over which this attractive force is felt is called the gravitational field of force.

* It is said that Newton had arrived at the law of gravitation in 1666 when he was only 23, but did not publish his results for more than 20 years for it took him that long to prove to his own satisfaction that a spherical mass can be replaced by concentrating the mass at its center.

Bigger is the mass of a body more extended is its gravitational field. For example, all the planets and their satellites in our solar system move in the gravitational field of the sun. The moon and other artificial satellites are subject to the gravitational field of the earth, distinguished as the **gravity field**. As the gravitation force is very weak, the gravitational fields of earthly bodies are very limited in extent.

We can introduce the idea of gravitational potential as follows: If a very small mass m is kept near a heavier mass M then the former feels attraction; if m moves under it towads M gravitational force *does work on m*; if it is to be moved away, an external agent must apply a force and do work on *m against gravitation.* **Potential is then that property of the gravitational field because of which work is required to move a very small mass near a static mass.**

The intensity of the gravitational field or field (in short) at a point is measured by the attractive force acting on a unit point mass placed at that point. The potential at that point is measured by the work done in removing a point mass from that point to infinity. In symbols, they are

$$(F_G) = -G\frac{M\times 1}{R^2} = -GM/R^2 \qquad \text{(II-1.6.1)}$$

$$\text{and } U = \int_R^\infty dW = -\int_R^\infty \frac{GM}{R^2} \cdot dR = GM/R \qquad \text{(II-1.6.2)}$$

negative sign undicates an attraction.

Gravitational, Electric and Magnetic forces: The introduction of the ideas of gravitational field and potential comes from those of electric and magnetic cases. In all of them, forces act from a distance and hence they are field forces. But in mechanics forces are not effective till the bodies touch, e.g. frictional or impulsive forces. These latter are contact forces, as we have already noted.

Physics deals with and seeks to analyse the two fundamental forces, those of electromagnetism on one hand and gravitation on the other; for they embrace most of the phenomena we can readily perceive. You shall later learn that Coulomb laws of electric or magnetic forces between point charges or poles are formally very similar to Newton's law of gravitation—

$$F_E = \pm qq'/kr^2,\ F_M = \pm mm'/\mu r^2 \text{ and } F_G = -GMM'/R^2$$

—each of them being of inverse square law type. The same law governs rotation of planets round the sun and that of an electron round an atomic

nucleus—both in elliptical orbits. Just try to grasp the difference in scales involved !

They differ, however in details e.g. (1) Gravitation is only attractive (indicated by the −ve sign) while the other two may be repulsive (i.e. +ve) as well ; (2) Gravitation is a weak force while the others are much stronger ; (3) Range of gravitation includes the known universe but that of the other two are much restricted ; (4) G is a ***universal constant*** independent of permeability directivity and susceptibility of the medium (making gravitational shielding impossible) whereas K and μ are constants depending on these properties of the separating media ; (5) G appears in the numerator while μ and K in the denominator of the expression for the force.

II-1 7. Force of Gravity or Gravitational attraction of the earth : In accordance with the law of universal gravitation the earth attracts all bodies on its surface with a force proportional to the mass m of a body. Considering the earth to be a homogeneous sphere of radius R and mass M, the gravitational attraction W it exerts on a body on its surface, is given by

$$W=G\frac{Mm}{R^2} \qquad \text{(II-1.7.1)}$$

This force, the **force of gravity** is directed towards the centre of the earth and is *often called the weight of the body.* More correctly, *the weight of a body is the force it exerts on anything that supports it.* We came across this idea in the investigation of accelerated lifts. Later we shall return to it, in discussing weightlessness in satellites.

We have also seen that a body falling freely under gravity, does so with a constant accelerration, *acceleration due to gravity* denoted by g. Hence by Newton's second law of motion, the weight W of a body of mass m is given by $W=mg$.

$$\therefore \quad mg=GMm/R^2 \text{ or } g=GM/R^2 \qquad \text{(II-1.7 2)}$$

This relation tells you why g is a constant acceleration. It is so as g depends on G, a universal constant and mass and radius of the earth, both constants.

Mass of the Earth : This can be calculated easily from eqn. II-1.7.2 as we know the values of g ($=9.81 m/s^2$), G ($=6.67\times10^{-11}$ mks units) and R ($=6.37\times10^6 m \simeq 6400$ km) we obtain the mass of the earth to be $\mathbf{M_e=5\ 96\times10^{24}}$ **kg.**

If ρ_m be the mean density of the earth, it being taken as rigid sphere* then $M_e = \frac{4}{3}\pi R^3 \rho_m$. Substituting values we get $\rho_m = 5.5 g/cm^3$ or $5.5 \times 10^3 \ kg/m^3$. But the density of rocks that make up most of the earth's crust ranges from 2.5 to 3.5 g/cm^3. Hence the interior of the earth must be made up of some heavier stuff.** The earth thus is not really homogeneous nor exactly a sphere.

g on the Moon : Relation II-1.7.2 applies to any attracting sphere of mass M and radius R, for example the moon. The mass of the moon is 1/81 $(=1.23 \times 10^{-2})$ of the earth's mass and its radius is 1.98×10^6m. Then

$$g_m = G\frac{M_m}{R_m^2} = 6.67 \times 10^{-11} \ \frac{1.23 \times 10^{-2} \times 5.96 \times 10^{24}}{(1.98 \times 10^6)^2} \simeq 1.64 m/s^2$$

which is about 1/6th that of g_e. Hence if a person clears a high jump of about 2 metres or 6 ft on earth, he will easily clear a building about 36 feet (nearly 3 stories high).

Problem : If acceleration due to gravity on a planet is 196 cm/s² find what height is safe to jump down there, if on earth the safe height is two metres (*Ans.* 10 m). [I. I. T. '72]

Ex. II-1 3 : *Find the point where on the line joining the earth and moon the resultant pull on a probing rocket would vanish. Given that earth is 81 times as massive as the moon and their centres 0.384 million kilometres apart.*

Ans. Let the required point be r million km away from the centre of our earth. Then the field intensities there due to the earth and the moon are equal and opposite. An astronaut bound for the moon enters the lunar gravity field as he crosses the point. So

$$\frac{GM_e}{r^2} = \frac{GM_m}{(0.384-r)^2}$$

$$\therefore \quad \frac{(0.384-r)^2}{r^2} = \frac{M_m}{M_e} = \frac{1}{81} \text{ or } \frac{0.384}{r} - 1 = \pm\frac{1}{9} \text{ or } 0.384/r = 10/9$$

$$\therefore \quad r = 0.384 \times 9/10 = 0.346 \text{ million km.}$$

---

* In discussing uniform circular motion we have noticed that it is not so and why. It was then regarded as an *oblate spheroid.* Investigations or gravity survey by satellites has revealed that it is not that either. It is now called a *geoid,* meaning shaped like the earth.!

** Scientists surmise that the core of the earth is a small solid iron-nickel sphere surrounded by a much thicker molten iron-nickel belt. As they contain free electrons moving in circles with the spinning earth, the magnetic field of the earth arises. Except Jupiter none of the planets boast of a magnetosphere as our earth does.

Problem : The sun has a mass of 2×10^{30} kg and earth 6×10^{24} kg. Their centers are 1 A. U. (Astronomical unit, the mean radius of earth orbit $=1.50\times10^{8}$ km) apart. Find the distance of null (nil) gravitation on the radius vector joining them. [*Ans.* 0.263 million km from Earth centre].

Note that the sun-earth null-gravity point is closer to the earth than that for the earth-moon pair.

II-1 8. Motion under Gravity :

Gravity is the force with which the earth attracts any body towards its center. Aristotle (384 B.C-332 B.C.) the private tutor to Alexander and the greatest man of science in ancient Greece, taught that heavier bodies fall to the earth faster than lighter bodies—a fact of observation. This was disproved by Galileo (1589) nearly 2000 years later when he let fall from the uppermost balcony of the Leaning Tower of Pisa, two pieces of stone, one heavier than the other and the two reached the ground together. He declared that all bodies when falling from rest fall through the same distance in the same interval of time. In proving this quantitatively, he *diluted* gravity ($g \sin\theta$) by letting spheres roll down gentle inclines and timing their descents with water clocks.

Galileo's (1564-1642) work was extended by Newton (1642-1728). He devised the well known Guinea and Feather experiment. In it a heavy coin and a light feather was allowed to fall through a long glass tube from which air had been pumped out. With no air inside the two were found to fall and reach the bottom together. But the fall of the feather slowed down on admitting air inside. This proves that if a falling body is not retarded by air, all bodies light or heavy will fall together under gravity. Hence bodies falling under gravity come down with the same constant acceleration. This follows from the relation II-1.7.2.

A. Laws of falling bodies From a study of falling bodies Galileo concluded about the nature of their motion which are known as the **law of falling bodies**. These may be stated as follows :

When a body falls *freely* *from *rest* under gravity

(*i*) *all bodies traval equal distances in equal times,*

(*ii*) *the velocity of a body is proportional to its time of fall*, and

---

* A *freely falling body* is one that is acted on by no force other than gravity. The resistance of the medum through which the body is falling, is ignored.

(*iii*) *the distance travelled in a given time is proportional to the square of the time.*

B. **Freely falling bodies have the same constant acceleraton :** From law (*ii*) above, for a given body $v \propto t$. Therefore v/t is constant. But v/t is the time rate of change of velocity, i.e., the *acceleration.* Thus we conclude from law (*ii*) that the acceleration of a freely falling body is constant. We have now to show that this constant has the same value for all bodies.

Since the body starts from *rest* and moves with a constant acceleration we can put $u=0$ in the equation $s=ut+\frac{1}{2}at^2$ and get $s=\frac{1}{2}at^2$. This shows that the distance traversed by the body in a given time is proportional to the square of time. This is law (*iii*)

From law (*i*) we know that for a given s, t is the same for all bodies. Hence from the relation $s=\frac{1}{2}at^2$ of the previous paragraph, a will be the same for all bodies, *i.e. all bodies falling freely under gravity have the* same *constant acceleration* (g).

II-1.9. Gravity Field and Intensity : When the attracting body is the earth, the force is that of gravity. It is just a special case of gravitational field. Since the earth attracts any body in its neighbourhood, we *postulate* that a gravity field surrounds the earth. The moon, the artificial satellites, the shooting stars or meteorites, all move in the outer regions of this field whereas balloons, aeroplanes, bodies moving up and down nearer home, travel in its nearer regions. Theoretically however like the gravitational, the gravity field extends to infinity.

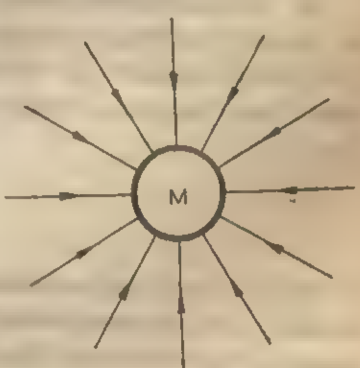


Fig. II-1.3

Intensity of this field at any point is measured by the pull our earth exerts on unit mass. If we put $m=1$ in the eqn. II-1.7.2 we find that the pull becomes $g=GM/R^2$. Thus g stands for both *acceleration due to gravity* as well as *intensity of gravity*— one an acceleration, the other a force. The ideas do not contradict, for from Newton's second law

acceleration comes out to be a force on unit mass. The idea of *intensity of gravity* is derived from those of electrical, magnetic or gravitational fields. Difference lies in the fact that, whereas different charges, poles or masses may create different intensities at a given point, intensity at a given point in the gravity field does not change, for only the earth creates this field. So g in the vicinity of the earth has a constant value at the same distance from the center. The lines along which a body falls to the earth can be taken as *lines of force* always directed towards the center of the earth. (Fig II-1.3)

Mass and Weight : These two widely used terms are very often synonymously used but they are fundamentally different quantities. The former is an intrinsic quantity while the latter an extrinsic one.

Mass	Weight
(1) Quantity of matter in a body is its mass. It is denoted by m.	(1) The pull on it by the earth is the weight of the body. It is denoted by mg.
(2) Mass is a scalar.	(2) Weight is a vector.
(3) Mass measures inertia of a body. It tends to oppose starting or changing any motion.	(3) Weight being a force tends to produce motion.
(4) It is an *intrinsic* property of matter remaining constant everywhere and under all conditions, *unless they move very very fast.*	(4) It is an externally generated property and changes with value of g. At the center of or a an appropriate distance from the earth, in far-away space and under suitable conditions—freely falling frames, floating bodies artificial satellites, it vanishes.
(5) It may be measured *statically* in a common balance against a standard mass or *dynamically* by collision with a known mass.	(5) It can be measured by a spring balance, *statically* only.

II-1 10. Variation of g :

But perfectly valid causes exist which change the value of g, though by a small amount both *on or near about the earth's surface both below and above.* They are detailed below.

A. On the Earth's surface : The variation is due to fact that the earth is neither a true sphere nor is it homogeneous ; *g* is found to—

(1) increase regularly with **increasing latitude** for the polar radius $R_P = 6357$ km, is shorter by about 21 km to the equatorial radius $R_E = 6378$ km. *g* therefore has the lowest value at the equator (9.78 m/s) and the highest at the poles (9 83 m/s), roughly about 0.5 part in 100 more. In connection with uniform circular motion we have already learnt why the carth is so deformed.

(2) change *abruptly* at some regions. These pockets of *gravity anomalies* have been deduced from sudden dips in the orbits of satellites overflying these areas. One such prominent anomaly lies just to the south of Indonesia. It is surmised that it is due to a very large undersea deposit of iron.

(3) Increase smoothly again with latitude because of *diurnal* (daily) *spin of the earth.* The change can be understood with reference to the fig II-1.4. On the representation of the globe a great circle parallel to the equaor (EFQF′E) said to be a parallel of latitude, has been indicated. It is the base of a cone with vertex at the center (O) of the earth. All lines drawn from any point to the circle MGPG′M make equal angles with its corresponding equatorial radius. That

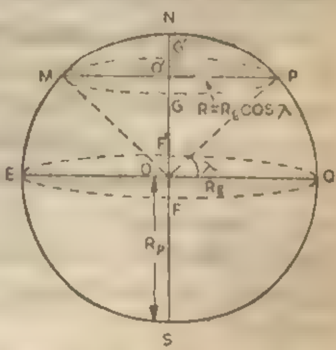


Fig. II-1.4

angle is said to be the *angle of latitude* (λ). Let P be a point on the surface of earth and PO the line joining it to the center of the earth. The angle POQ represents its latitude λ.

Now the points P and Q are both spinning once in 24 hours. Since their radii OQ and O′P differ, their angular speeds must differ,

Q spinning faster. Now P is rotating along a circle of radius $R' = R\cos\lambda$ and Q along a circle of radius R_E where $R_E > R'$. Remember, part of the pull of the earth mg on a body goes to supply the required centripetal force at any point. Hence the weight of a body at any point is the difference between the pull of the earth mg and the centripetal force $m\omega^2 r$ i.e. if g_T is the true g (i.e. if the earth be at rest) we have $mg = mg_T - m\omega^2 r$.

$$\therefore \quad (mg)_{Eq} = mg_T - m\omega^2 R_E \quad \text{and} \quad mg_\lambda = mg_T - m\omega^2 R'\cos\lambda$$

$$= mg_T - m\omega^2(R_E \cos\lambda)\cos\lambda = m\omega^2 R_E \cos^2\lambda$$

$$\text{or} \quad g_\lambda = g_T - \omega^2 R_E \cos^2\lambda = g_T\left(1 - \frac{\omega^2 R_E}{g} \cdot \cos^2\lambda\right) \qquad \text{(II-1.10 1)}$$

Hence at the equator ($\lambda = 0$) the g-value is minimum and at the pole it is true g, for the point is not spinning.

More than half the variation in g due to the latitude effect is contributed by the spin of the earth.

The above analysis is more readily understandable if you consider centrifugal forces at Q and P acting radially outwards.

On a ship steaming fast due east along the equator, g is found to be *slightly less* than if it reverses direction ; for as the earth spins west to east, ship-speed is added to that of the earth, this increasing the effective ω, while it is that much diminished when the ship reverses track.

B g above the surface of the earth. As a body is raised from the surface of the earth its distance from the center increases and hence g diminishes. When it is at a height h from the surface of the earth it is $(R+h)$ away from the center. If g_h and g_0 be the intensities at a height h and on the surface of earth we have

$$\frac{g_h}{g_0} = \frac{GM/(R+h)^2}{GM/R^2} = \frac{R^2}{R^2(1+h/R)^2} = (1+h/R)^{-2} = (1-2h/R) \quad \text{(II-1.10.2)}$$

(expanding by the binomial and neglecting the higher terms for $h << R$)

Alternatively, $g = GM/R^2 = GM.R^{-2}$

$$\therefore \quad dg = GM.\, d(R^{-2}) = GM.\ (-2R^{-3}dR) = (GM/R^2)(-2dR/R) =$$

$$= g_0(-2dR/R)$$

$$\text{or} \quad dg/g = -2dR/R = -2h/R \qquad \text{(II-1.10.3)}$$

$$\text{But} \quad dg = g_h - g_0 \text{ and } dR = h. \quad \text{So}$$

$$g_h = g_0\,(1 - 2h/R)$$

—a result identical with above. See then that at a height equal to the radius of the earth ($2h=R$) intensity of gravity vanishes and a body becomes weightless.

Prob: (1) Assuming the earth to be a sphere of radius 6400 km find the height at which g becomes 1% of its surface value of 9.8 m/s². (*Ans.* 3167 km).

(2) A pendulum clock that beats seconds on the surface of the earth is taken up in a balloon to a height of $1\frac{1}{2}$ miles above the earth's surface. Assuming the radius of the earth to be 4000 miles, calculate how many seconds the clock will gain or lose in a day. (*Ans.* lose nearly 33.5 s)

C. Variation of g with depth. Consider a body at a depth x below the surface (fig. II-1.5). The spherical shell of thickness x (the unshaded portion in the figure) does not contribute to the central

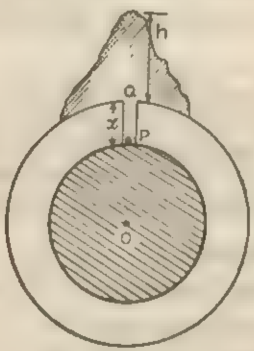


Fig. II-1.5

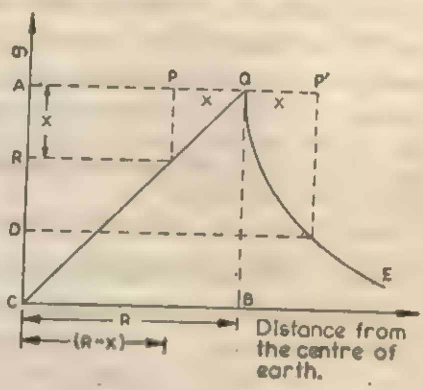


Fig. II-1.6

attraction on the body. Let R be the radius of the earth and $x<<R$. Attraction on the body is due to the shaded portion of the sphere. Its mass M' which may be taken to be concentrated at the centre, is $\frac{4}{3}\pi(R-x)^3\rho$ where ρ is the density of a homogeneous earth.

Then the acceleration at P would be given by

$$g_x=\frac{GM'}{(R-x)^2}=\tfrac{4}{3}G\pi(R-x)\rho$$

and at Q, $\quad g_0=\dfrac{GM}{R^2}=\tfrac{4}{3}G\pi R\rho$

$$\therefore \quad g_x=g_0\left(\frac{R-x}{R}\right)=g_0\left(1-\frac{x}{R}\right) \qquad \text{(II-1.10.4)}$$

Alternatively $g_0 = \frac{4}{3}\pi\rho RG$. Now $dg = \frac{4}{3}\pi\rho G(-dR)$ as R diminishes.

$$\therefore \quad \frac{dg}{g_0} = -\frac{dR}{R} \quad \text{or} \quad dg = g_0\left(-\frac{dR}{R}\right) \qquad \text{(II-1 10.5)}$$

But $dg = g_x - g_0 \quad \therefore \quad g_x = g_0\left(1 - \frac{dR}{R}\right) = g_0\left(1 - \frac{x}{R}\right)$

Note that diminution in g occurs both above and below the surface of earth and at some height above, the decrease is twice as much as at same depth below. Fig. II-1.6 shows graphically the variation of g with distance from the center of the earth (C) where it is zero. Now CA = CB = Radius of earth. CA = QB represents value of g at the surface (g_0). AR = g_x = PQ. The rise in g is linear along CQ (eq. II-1.10.5) but it falls away along QE. If we take a point P′ at a height of x above Q, note that its g value is CD and see that AR the diminution in g for a depth x, is half of AD that for the same rise x from the earth's surface.

Prob : Assuming that the value of g inside the earth is proportional to the distance from the earth's centre, at what depth below the earth's surface would a pendulum, which beats seconds at the earth's surface, lose 5 minutes in a day. Earth's radius = 4000 miles. (*Ans.* 28 miles nearly)

[*Hint :* $g'/g = R/R$. $dg/g = dR/R$. Apply Eq. II-1.14.3B $dn = 300$. Find dR.]

II-1.11. Simple Pendulum : A. Description :

The simplest yet quite accurate method of measuring g is by using a simple pendulum. With it also we may indirectly find the height of a hill as well as depth of a mine by equations II-1.10.2 and II-1.10.4.

Any body that can vibrate about a horizontal axis under gravity is a pendulum, more particularly a **compound pendulum** (§ II-1.4ii). The time taken by it in between successive transits across any point in its path in the same direction is called its **period of oscillation** or **periodic time** (T). The distance between an extreme position of the pendulum from its position of rest is called its **amplitude**.

To study the relation between the period, the dimensions of the pendulum and the intensity of gravity we imagine a **simple** or **mathematical pendulum** defined as *a heavy point mass suspended from a rigid support by a weightless inextensible perfectly flexible thread.* It can be realised *approximately* by hanging a small metal ball by a thin cotton thread in the laboratory.

Let AO in fig II-1.7. represent such a simple pendulum where O is the *point of suspension* and A the *bob* of which C is the centre. AC is $(l+r)$ the *effective length* of the pendulum. The bob moves between B and D and the time it takes in moving from B to D and then back to B is the *period.* That motion is said to be an *oscillation* and half of it from C to D and back to D a *beat* or *vibration* The distance OC or OD is the amplitude. The number of vibrations of the bob in a second is the *frequency* and the angle COD or COB is the *angular amplitude.*

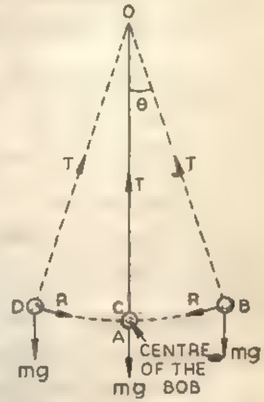


Fig. II-1.7

B Time-Period of a Simple Pendulum : To find this the most important quantity assosciated with a pendulum, we *surmise* that it should depend upon (i) the mass of the bob (ii) the length of the pendulum and (iii) the pull of the earth on it represented by the value of gravity in the laboratory.

Applying the method of dimensions we have deduced before in O-I.9.1 and I-5.11.1 that

$$T = 2\pi \sqrt{l/g} \qquad \text{(II-1.11.1)}$$

provided the angular amplitude does not exceed 4°.

From this result we arrive at the *laws of pendulum.*

C. Laws of Simple Pendulum :

Laws of simple pendulum. Certain statements are generally known as the laws of simple pendulum. They are all included in eqn. II-1.11.1. The statements are as follows : **Provided the angular amplitude is small,**

(i) Law of isochronism. *(*isos*—same, *chron*—time.) At a given place, the oscillations of a simple pendulum of given length are executed in equal time intervals.

* Galileo, then only 17, is reputed to arrived at this conclusion by timing the swings of long candelabras at the Cathedral of Pisa by his pulse beats, there being no watches or clocks at that time (1581).

(*ii*) *Law of length.* The periodic time of a simple pendulum varies as the square root of its length at a given place. ($T \propto \sqrt{l}$ at a given place). Here g is const.

(*iii*) *Law of gravity.* For a given pendulum the periodic time varies inversely as the square root of acceleration due to gravity at the place ($T \propto 1/\sqrt{g}$)

(*iv*) *Law of mass.* The periodic time of a simple pendulum does not depend on the mass or material of the bob. Newton experimentally found so.

Discussions : The derivation of the expression for time period of a simple pendulum *assumes* that (*i*) Anglular amplitude is small ; (*ii*) the length of the pendulum is the distance from the point of suspension (must be well-defined) to the centre of the bob which is its C.G. (*iii*) Temp and place of experiment does not change. Remember these three points carefully.

(*a*) Since the C.G of a solid sphere and a hollow sphere lie at the centre, time period will *not* change if the solid bob is replaced by a hollow sphere of the same diameter, mass or material being of no consequence.

(*b*) But changes would occur if the hollow sphere is *partially* filled up with something, say a liquid. Then the C.G. of the ball will be lowered, the effective length having increased and time-period would increase. Now if a hollow sphere full of water is suspended by a long thread and made to oscillate and a fine hole is made at the bottom, the time of oscillation would be found to slowly increase at first and then as slowly decrease back to its original value. Why this happenes ? The oscillating system is a pendulum, effective length reaching to the centre of the water-filled ball. While oscillating, water slowly trickles out, making the lower part of the bob heavier *i.e.*, lowering its C.G. Thus the effective length increasing, the time period increases to a maximum when the bob is just half empty (Remember, mass plays no part in influencing the time-period). Beyond, as the lower half gradually empties, the C.G. mounts back to the centre of the bob when the bob is totally empty.

(*c*) If the pendulum suspension experiences a change in temperature its length changes, and so does its time-period. Again as g diminishes up a hill or down a mine, period of a pendulum rises. We can hence find their height or depth or even the prevailing room temperature with a pendulum. These are bonuses obtained from pendulum experiments.

II-1.12. Determination of g in the laboratory. A. Pendulum

A given pendulum is not only a very good time-marker because of constancy in the value of g at a given place but also a simple but

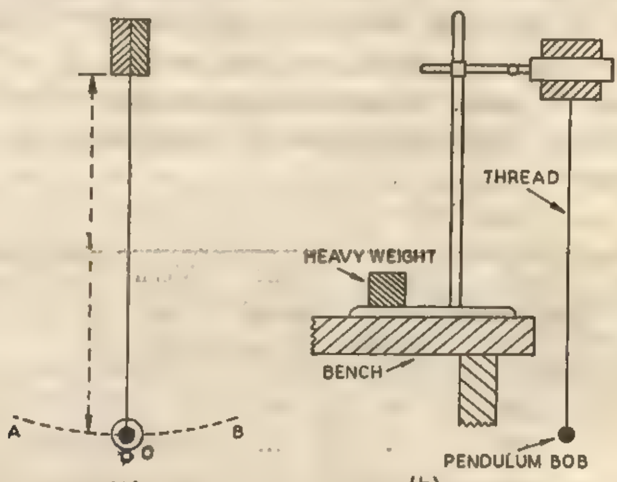


Fig. II-1.8(a) Fig II-1.8(b)

accurate device of measuring g. To approximate as far as possible a simple pendulum, we take a small metal ball with a small hook and suspend it from a sturdy support by a long thin **unspun** cotton thread. Before starting the experiment lay down the pendulum with the thread beside a meter scale. Put the zero mark of the scale as nearly as possible at the middle of the bob hold the string taut and put ink marks at say 90, 95 and 100 cm mark on the string. When you hang the thread with any of these ink marks just at the point of support you get straight away the appropriate effective length of the pendulum (*Measuring the radius of the bob with a pair of slide*

callipers is unnecessary as an error in measuring length by at most 1mm in a length of 1m is smaller than that in measuring the time-period). The arrangement is shown in fig II-1.8(a) and (b).

Now hang the pendulum from the support, pull the bob a little to one side and let go. See to it that the bob does not spin, nor does it move in a circle or ellipse. This displacement of the bob should not exceed one-tenth of the length chosen. After a few oscillations, start the stop-watch when the bob just stops at the end of a swing. An oscillation is completed when the bob next returns to the same point. Count 25 oscillations and stop the watch. Total time taken divided by 25 gives you the period. Find the period next from 30 oscillations and then again from 35 oscillations. Though the number of oscillations vary, time-period in all the three cases should be equal. Change the length and find the time period thrice as before. Repeat the whole for the third length and record as follows :—

Effective Length (L) (cm)	No of oscillations	Time taken (in sec)	Time period (in sec)	Mean time period (T)	L/T^2	mean L/T^2
...	25	...	...	...	...	
	30	...	...			
	35	...	...			
...	"	"	"	"	"	...
...	"	"	"	"	"	

At Calcutta, mean L/T^2 should come out near about 24·9 cm/s²

$$\therefore \quad g = 4\pi^2 (L/T^2) \text{ cm/s}^2$$

Draw a *mean* graph plotting L against T^2 and find the value of L/T^2 from there. *That the graph is a straight line through the*

origin verifies the law of length. Fig II-1.8 (c) shows you the nature of the relevant graph It is a *straight line* passing through the origin. From the coordinates of any point $P(x,y)$ you obtain $x/y = L/T^2$ and putting that value in the above equation find g.

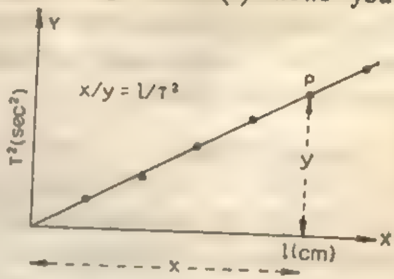


Fig.- II-1.8(c)

Limitations of the simple pendulum method of determining g

The method cannot give an accurate result for the following reasons :

(*i*) The formula $T = 2\pi\sqrt{l/g}$ is derived for a simple pendulum. It is an ideal pendulum which we cannot make in practice. No real string is perfectly flexible nor inextensible or weightless. Since these conditions are not fulfilled, there is bound to be some unknown error in the value of g that we get.

(*ii*) The formula assumes that the amplitude is infinitely small. As the amplitude increases, T also increases. Keeping the linear amplitude within about $\frac{1}{12}$th the length of the pendulum confines the error in T to about 1 or 2 parts in 1000. This is difficult to achieve for shorter lengths.

(*iii*) There is difficulty in accurately measuring the distance between the point of suspension and the centre of gravity of the bob. The measured value of l has therefore some error in it.

(*iv*) An error also occurs in the measurement of the time for a given number of oscillations. We cannot start or stopa stop-clock at the exact moments. So the measured value of T will have an error in it. Besides, the stop-clock may not be running correctly.

All these factors lead to an error in the calculated value of g. If the error could be confined to within 1%, the result should be considered good. Since the correct value is about 980 cm/s^2, an 1% error means values in the approximate range 980 ± 10, i. e., between 970 and 990 cm s^2. Any value in this range should be

considered equally acceptable and of equal merit. Special emphasis should *not* be given to values near 980 cm/s². A better result can be obtained with a **compound pendulum**. But the experiment is extremely long and tedious.

B Atwood's Machine : g can be determined by this piece of apparatus where its action is more evident, namely vertical motion. In it g is 'diluted' by one weight pulling up another just as in motion of connected systems. See Chap I-3.

Principle : Two weights m_1 and m_2 are carried at the two ends of a string passing over a smooth weightless pulley. Like in the pendulum the string is inextensible and weightless. We take $m_2 > m_1$ (Fig II-1. 9*b*) when the latter will be pulled up as the former goes down, their common acceleration a.

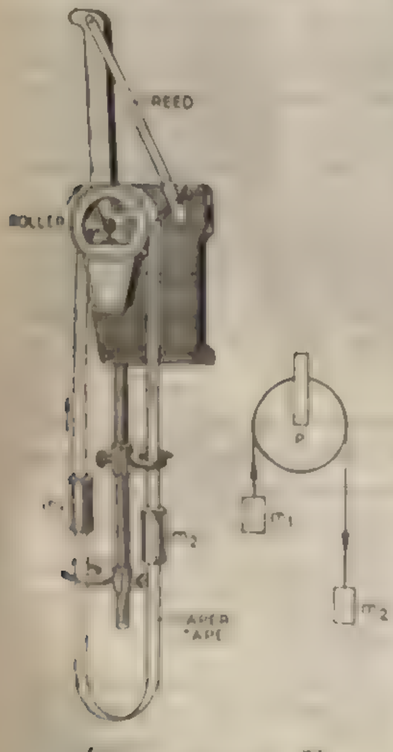
The pair of opposite forces on m_1 are $m_1 g$ and T and on m_2, they are $m_2 g$ and T. Then from Newton's 2nd law of motion

$$a = \frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$= \frac{m_2 - m_1}{m_1 + m_2} \cdot g$$

(by componendo-dividendo)

Description : Refer to Fig II-1 9 *a*). Here the string is replaced by a paper tape on which an inked stylus vibrating at the end of a metal reed can trace a wavy curve as in Fletcher's trolley. As there, here also the velocity and acceleration of the system of moving masses can be found.



(a (b)

Fig. II-1.9 b)

Fletcher's trolley can also provide the value of g. If M be the mass of the trolley, m that of the descending load and a' the horizontal acceleration of M then clearly

$$a' = mg/(m+M)$$

In both these experiments friction at the pulley and between the tape and stylus introduce inaccuracies.

C. **Inclined planes :** g can also be determined by 'diluting' i.e. lessening its value by timing a small metal ball rolling down a smooth incline as Galeleo first did (fig II.1.10). If the ball rolls over a length l of the incline then

$$l = \tfrac{1}{2} g \sin\theta . t^2$$

If h be the vertical height of the incline then $\sin\theta = h/l$ $g = \dfrac{2l^2}{ht^2}$

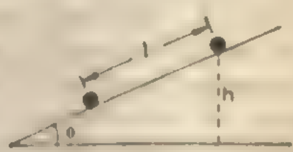


Fig. II-1 10

The length l of the incline, its vertical height h and the time taken t by the ball to roll down are measured for different inclinations ; l remains constant h and t being the variables.

D. **Verification of the Laws of pendulum :** In the above pendulum experiment the value of L/T^2 is found to be reasonably constant. This verifies the *law of length* that $T \propto \sqrt{l}$ at a given place. Next replace just the brass bob you have used with those of iron and lead bobs of same diameter, solid or hollow. The time period in each case would be the same substantiating the *law of mass*. During measuring the time-period, you time different oscillations, say the 5th, 12th the 18th or the 30th. You will find them very nearly equal establishing as Galeleo did, the *law of Isochronism*. In the same laboratory you cannot obviously verify the law of gravity.

II-1.12. Second's Pendulum :

If a pendulum bob takes *one second* for one swing or half an oscillation it is said to be a second's pendulum. Obviously its length

would depend on the local value of g, the period everywhere being 2 seconds. So its length is given by

$$l_s = gT^2/4\pi^2 = g/\pi^2 \qquad \text{(II-1.12 1)}$$

Thus at the N. pole	$l_s = 983.22/\pi^2$
at the equator	$l_s = 978.03/\pi^2$
at Calcutta	$l_s = 978.82/\pi^2$
at London	$l_s = 981.19/\pi^2$

In the FPS system average value of g is taken to be 32.2 ft/s². So the length of a second's pendulum would be 3.26 ft.

As g on moon is almost 1/6th that on earth, a second's pendulum ($g = 980$ cm/s²) taken from earth to the moon would have its time period increased $\sqrt{6}$ times i.e. would take nearly 4.90 s to complete an oscillation. Incidentally, as the moon has no air, the pendulum would continue to oscillate far longer than it would on earth where air-friction damps out the oscillations. On the sun g is 27 times as large as on the earth and hence, were it possible for a second's pendulum to operate there, time period would be reduced to $2/3\sqrt{3}$ s i.e. it would oscillate much faster.

Ex. II-1.4 *The mass and diameter of a planet are both twice those of the earth. Find the time period of a pendulum if it is a second's pendulum on earth.*

(I. I. T. '73)

Solution : Remember $g = GM/R^2$

$$\therefore \quad \frac{g_E}{g_P} = \frac{M_E}{M_P}\left(\frac{R_P}{R_E}\right)^2 = \frac{1}{2}\cdot\frac{4}{1} = 2$$

$$\text{Again} \quad \frac{g_E}{g_P} = \frac{4\pi^2 l/T_E^2}{4\pi^2 l/T_P^2} = \left(\frac{T_P}{T_E}\right)^2 \qquad \therefore \quad T_P = T_E\sqrt{2} = 2.8 \text{ s}$$

Problem : You take a second's pendulum and a synchronised watch driven by an oscillating spring to the moon. Explain what happenes to the times kept. (Neglect temperature effects).

II 1.13. Accelerated Pendulum :

Pendulum bobs are made to vibrate in a vertical plane by the component of g perpendicular to the suspension when the suspension is not vertical. Additional accelerations may be imparted on the bob by (*i*) making it describe a horizontal circle with uniform speed (*ii*) accelerating the support in a vertical direction (*iii*) accelerating the same in a horizontal direction.

A. Conical Pendulum : If the bob (P) of a simple pendulum of length l is so projected that it moves uniformly in a circle we have three forces acting on it (fig. II-1.11)—the weight mg acting vertically downwards, the centripetal force mv^2/r along PN horizontally towards the center and tension S acting along the suspension PO. For the bob oscillating in a vertical plane, the second force is zero but not so here.

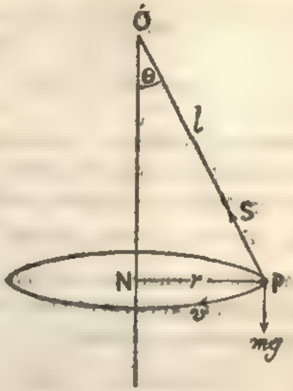


Fig. II-1.11

Resolving S horizontally and vertically we see that

$$m\omega^2 r = S \sin\theta \text{ and } mg = S\cos\theta$$

$$\therefore \quad \tan\theta = \omega^2 r/g = \frac{4\pi^2}{T^2}\cdot\frac{r}{g} = \frac{4\pi^2}{T^2 g}\cdot l\sin\theta$$

$$\therefore \quad T^2 = \frac{4\pi^2 l\sin\theta}{g\tan\theta} \quad \text{or} \quad T = 2\pi\sqrt{\frac{l\cos\theta}{g}} \qquad \text{(II-1.13.1)}$$

So the centripetal acceleration effectively reduces the pendulum length. Faster the bob rotates greater is the angle θ and higher its plane of rotation. Conversely, for a slow rotation θ would be very small and it would behave as a simple pendulum. Compare the action of Watt's Steam Governor in Chap. (I-5).

Ex. II.1.5 *A large mass M and a small mass m hang at the two ends of a string that passes through a smooth tube as shown in the figure. m moves in a circle in a horizontal plane. The length of the string from m to the top of the tube is l. Find the frequency of rotation of m which keeps M steady.*

(I. I. T. '78)

Solution : m here is the bob of a conical pendulum, the pull of the support O being replaced by the downward pull of M.

So the tension F of the string would be $m\omega^2 r/\sin\theta$ as per above analysis.

$$\therefore\quad F = Mg = \frac{m\omega^2 r}{\sin\theta} = \frac{m\omega^2 l \sin\theta}{\sin\theta}$$

$$\therefore\quad \omega^2 = 4\pi^2 n^2 = Mg/ml \quad \text{or} \quad n = \frac{1}{2\pi}\sqrt{\frac{Mg}{Ml}}$$

Problems: (1) A sphere of mass 1 kg hung by a string 1m long is rotating once a second in a horizontal circle. If $g = 9.8\ m/s^2$ how far the ball will be raised from its position of rest and what will be the pull on it?

[*Hints* : $h = l - l\cos\theta = l - gT^2/4\pi^2$, $T = 2\pi\sqrt{l\cos\theta/g}$]

(2) Find for a sphere of mass m suspended by a thread from a point and describing a horizontal circle of radius r with an angular velocity ω, the tension of the thread and the time period.

Hints: The system is a conical pendulum. Refer to fig. II-1.11 and see that

$$S^2 = m^2\omega^4 r^2 + m^2 g^2 = m^2(g^2 + \omega^4 r^2)$$
$$\therefore\quad S = m\sqrt{g^2 + \omega^4 r^2} = m\sqrt{g^2 + v^4/r^2}$$

[*To find the time-period let us note that the bob is subjected to two accelerations at right angles to each other g aud $\omega^2 r$. So there resultant is*

$$f = \sqrt{g^2 + \omega^4 r^2} \text{ nad hence } T = \frac{2\pi}{f} = 2\pi\sqrt{\frac{l}{g^2 + \omega^4 r^2}} \Big]$$

B. Pendulum accelerated parallel to g: If the pendulum-support accelerates vertically (up or down) the vertical component of the tension F on the string will be $F\cos\theta = m(g \pm f)$. [Recall the case of reaction in an accelerating lift]. The horizontal component is $F\sin\theta$, θ being the inclination of the string to the vertical. For a simple pendulum we have to take θ to be small so that

$$F = m(g \pm f) \text{ and } F\theta = F(x/l) = m(g \pm f).x/l$$

The restoring force per unit displacement is $m(g \pm f)/l$ and hence

$$T = 2\pi\sqrt{l/(g \pm f)} \qquad \text{(II-1.13.2)}$$

The $+ve$ sign applies to acceleration upward as in a similarly rising balloon or lift. If the balloon or lift attains a steady velocity $f = 0$ and the period of pendulum identical with that in a stationary case which is $T = 2\pi\sqrt{l/g}$.

Ex. II-1.6 *A simple pendulum hangs from the top of a stationary lift. Compare the time period with those when the lift is (a) ascending with unifoom velocity of 8 ft/s (b) accelerating upwards at 8 ft/s² (c) desending with a uniform velocity of 8 ft/s (d) accelerating downwards at 8 ft/s²*

Solution : When *stationary* and *rising or falling with constant velocity f* is zero and g is the only effective acceleration and the time-periods will be equal.

But when accelerating upwards effective acceleration is $g+f$, here $g+g/4$; when accelerating downwards the effective acceleration will be $g-g/4$. So the ratios of time periods will be $1:1, /(2/\sqrt{5}), 1/(2/\sqrt{3})$.

Ex. II-1.7. *An oscilating pendulum is just dropped from the top of a tower. Find its time-period. Also find the period of a pendulum in a small satellite.*

Solution : In both cases the pendulum stops oscillating. In the first case the pendulum is falling freely i.e. its accelaration is g i.e. effective acceleration $g-g=0$. Thus the time period is infinity.

In a freely falling lift one is weightless ; so he is in an orbiting satellite for we shall see soon that it is also a freely falling body. So the pendulum stops.

C. Pendulum accelerated perpendicular to g : Let a simple pendulum hang from the roof of a railway carriage accelerating at f m/s² forwards. This acceleration would be transmitted through the string to the bob from the support. The string and the bob would *incline backwards* at say θ, to the vertical ; because it is subjected to a pseudo-force in the accelerated frame. As in other cases $F\cos\theta = mg$ and $F\sin\theta = mf$ where m is the mass of the bob, F the tension along the string. Then

$$\tan\theta = f/g \text{ and } F^2 = g^2 + f^2 \qquad \text{(II-1.13.3)}$$

for the two accelerations are at right angles to each other. If the pendulum is now allowed to oscillate, it would do so about the inclined position but *without change of period.*

If a plane moves in a horizontal circle a hanging pendulum in it gets inclines just as above, away from the center as the sine component of the tension of the string provides the centripetal force. The passenger can, by noting the direction of deflection, find which way the plane is turning. It is the opposite.

II-1.14. Change of Period of a Pendulum :

Time-period of a simple pendulm depends only on its *effective length* and the *effective acceleration* acting on the bob. Change of any one of these quantities would change the period. If the *period*

increases the pendulum is said to ***go slow*** while it ***goes fast*** when the period *decreases.*

A discussion with a second's pendulum would clarify the idea. A swing of such a pendulum takes 1 sec and in a day it would swing 86400 times, there being so many seconds per day. If the time period increases, in a day there would be less than 86400 swings and thus the clock would go slow. If the period diminishes there would be more than 86400 swings in a day and the clock would run fast.

A. Now *change in length occurs with change of temperature.* We shall return to the topic in the Chapter IV-3.8. For the present know that, if a solid rod of length l, is heated through a temperature rise of t then its length increases by $l\alpha t$ where α, the coefficient of linear expansion, is a characterstic of the solid used as the suspension of the bob in clock pendulums. With rise in temp in summer the effective length increases, so does T (the time period) and the clock runs slow. Reverse occurs during winter.

B. Change in the value of g also occurs with *height above and depth below* the surface as well as *change in latitude* on the earth. On other planets g is different Again *at the same place however, g can be changed artificially.* We consider two such cases.

(*a*) Let the bob be of iron and just below its mean position let there be the pole of a strong magnet. Magnetic attraction added to gravity obviously increases the downward acceleration, say by f and the time-period becomes $T=2\pi\sqrt{l/(g+f)}$ and thus the pendulum will swing quicker, clock running faster.

(*b*) Above we have considered a pendulum, in a plane or a car moving in a circle. Then the support has a centripetal accleration in addition to that due to gravity and at right angles to each other. So the effective acceleration increases from g to $\sqrt{g^2+\omega^4r^2}$ (see problem 2 on page 28). Hence the time period becomes

$$T=\frac{2\pi}{f}=2\pi\sqrt{\frac{l}{g^2+\omega^4r^2}}=2\pi\sqrt{\frac{l}{g^2+v^4/r^2}} \qquad \text{(II-1.14.1)}$$

C. Loss or gain of time by a pendulum clock. Since T depends on l and g, a change in T can be obtained from differentiating its expression.

$T=2\pi\sqrt{l/g}$ Now $\log T=\log 2\pi+\frac{1}{2}\log l-\frac{1}{2}\log g$.

Differentiating each variable w.r.t itself we get

$$\frac{dT}{T}=\frac{1}{2}\left(\frac{dl}{g}-\frac{dg}{g}\right)$$

Again if N be the number of swings a day we know

$$NT=86400\text{ s}=\text{const}$$

$$\therefore\quad N.dT+T.dN=0$$

$$\text{or}\quad \frac{dN}{N}=-\frac{dT}{T}=\frac{1}{2}\left(\frac{dg}{g}-\frac{dl}{l}\right) \qquad \text{(II-1.14.3)}$$

From this result we can very easily find how many swings a *second's pendulum* would lose or gain per day due to change in length or acceleration due to gravity.

From the above formula we find three cases—

(*a*) If g is const $dg=0$. Then $dN=-43200\ dl/l$. The pendulum loses. (II-1.14.3A)

(*b*) If l is const $dl=0$. Then $dn=\frac{1}{2}N.dg/g=43200\ dg/g$. The pendulum gains. (B)

(*c*) If both l and g change dN may be +ve or −ve.

Ex II-1 8. *A Second's pendulum loses 5 s a day. How and by how much is the length to be altered so that it may keep correct time?* [H.S.' 71]

Solution: We must use (A) above. Here $dn=5$. $N=86400$

$$\therefore\quad \frac{dN}{N}=-\frac{1}{2}.\frac{dl}{l}\quad\text{or}\quad \frac{dl}{l}=-2\frac{dN}{N}=-\frac{10\times 100}{86400}\%=0.0116\%$$

Ex. II-1.9. *A pendulum beating seconds is taken from Calcutta ($g=978.82$ cm/s²) to London (981.19 cm/s²). How many seconds will it gain per day?*

Solution: Here $dg=983.19-978.82\text{ cm/s}^2=4.37\text{ cm/s}^2$ $g=981\text{ cm/s}^2$

$$\therefore\quad \text{From (B) above } \frac{dN}{N}=43200\times\frac{4.37}{981}=19.24\text{ s}$$

Ex. II-1.10. An iron pendulum (second's) keeps correct time at sea-level and at 20°C temp. It is taken to the top of Everest 8848 m high and at a temp of −30°C. How many seconds change would occur in a day?

Solution: Diminution in temp leads to a shortening in length which diminishes T while height diminishes g leading to an increase in T. The former makes the pendulum swing faster, the latter slower. Now

$$N=\frac{1}{T}=\frac{1}{2\pi}\sqrt{g/l}\qquad\text{or}\quad \frac{dN}{N}=\frac{1}{2}\left(\frac{dg}{g}-\frac{dl}{l}\right)$$

We know from eqn II-1.10.3 that $dg/g = -2h/R$. Again α, the coefficient of linear expansion with temperature is defined as

$$\alpha = \frac{\text{Increase in length } dl)}{\text{Original length } (l) \times \text{rise in temp } (dt)} = \frac{dl}{l.dt}$$

$$\therefore \quad \frac{dN}{N} = \frac{1}{2}\left(\frac{dg}{g} - \frac{dl}{l}\right) = -\frac{h}{R} - \frac{1}{2}\alpha.\,t = \frac{8848}{6367 \times 10^{3}} - \frac{1}{2}12 \times 10^{-6} \times (-50)$$

The radius of the earth is taken to be 6367 km and α for iron 12×10^{-6} °c and dt is $-$ve as the temperature has fallen, $N = 86400$ s

$$\therefore \quad dN = \left(3 \times 10^{-4} - \frac{8848 \times 10^{-3}}{6367}\right) \times 86400$$

$$= \left(0.03 - \frac{884.8}{6367}\right) \times 864 = -69.1 \text{ s}$$

So the clock will go slow by ***nearly*** 1 min. 10 s a day.

Prob. Find the change in seconds of a second's pendulum per day if its length is (i) increased by 1% (ii) diminished by 0.1% [*Ans.* Loss 432 s gain 43.2 s]

D. Use of a Pendulum to find the height of a hill, depth of a mine, latitude of a place :

The basic fact in all these cases is that g diminishes with height and depth and increases with latitude and the time-period varies as $1/\sqrt{g}$.

We have seen in the article II-1.10 that in successive cases

$$(g/g_0) = \left(1 - \frac{2h}{R}\right),\ (g/g_0) = (1 - d/R) \text{ and } (g/g_0) = \left(1 - \frac{\cos^2 \lambda}{288}\right)$$

$$\left[\text{for } (\omega^2 R_E/g_E) = \frac{1}{288}\right]$$

$$\therefore \quad T/T_0 = \left(1 - \frac{2h}{R}\right)^{-1/2} = \left(1 + \frac{h}{R}\right);$$

$$T/T_0 = \left(1 - \frac{D}{R}\right)^{-1/2} = \left(1 + \frac{D}{2R}\right);\ \frac{T}{T_0} = 1 + \frac{\cos^2 \lambda}{576}$$

$$\text{Now, } \frac{n}{n_0} = \frac{T_0}{T} = \frac{1}{1 + h/R} = 1 - \frac{h}{R};\ \frac{n}{n_0} = \left(1 - \frac{1}{2}\frac{D}{R}\right);\ \frac{n}{n_0} = 1 - \frac{\cos^2 \lambda}{576}$$

$$\therefore \quad \frac{n - n_0}{n_0} = -\frac{h}{R};\ \frac{n - n_0}{n} = -\frac{1}{2}\frac{D}{R} \text{ and } \frac{n - n_0}{n_0} = -\frac{\cos^2 \lambda}{576}$$

(II-1.14.4)

Prob. (1) A second's pendulum correct at sea-level loses 10 s a day at the top of a hill. Find is height, if $R = 6400$ km. (*Ans.* 740 m)

[***Hint ;*** $dn = 10 n_0 = 86400$]

(2) The same pendulum in a mine loses 5 s a day. Find depth. (*Ans* 740 m)

(3) A second's pendulum is taken at sea-level from the equator to the Tropic of Cancer. How many seconds will it lose a day ?

[*Hint :* $\lambda_{EQ}=0°$, $\lambda_{T\cdot C}=23\frac{1}{2}°$ N. $\cos 23\frac{1}{2}°=0.917$]

(4) A pendulum beats seconds on the top floor of a high rise building. At its basement it gains 2.7 s a day. How high is the building ? (*Ans.* 200 m)

II-1.15. Motion of Planets :

From the dawn of history man has gazed with awe, wonder and very often with profit at the sky, the sun, the moon, stars and planets. Ancient Sumerians and Egyptians nearly 6000 years ago kept surprisingly accurate track of the stars. The Egyptians synchronised the rise of the Dog Star or Sirius in their late summers with floods of the Nile that gave life to their land. The Chinese and Indians showed great awareness of the motion of heavenly bodies and the correlation with change of seasons. The Greeks coined the term 'planets' meaning wanderers and developed complicated theories about their orbits that lasted from the days of Aristotle (384 B.C.-322 B.C) to Copernicus (A.D. 1469 to 1543). The later overthrow of their geocentric (*geos*-Earth) system led to initiate the heliocentric (*helios*-Sun) system which grew quickly at the hands of Tycho Brahe (1546-1601), Kepler (1571-1630) Galeleo (1564-1642) and Newton (1642-1728) into the great science of Astronomy we know to-day.

A. Kepler's laws of planetary motion : Tycho Brahe a Danish astronomer, was the first among the moderns to make reasonably accurate records of the positions of planets in their motion across the sky. He had no telescope then to help him. From a close study of these records Kepler enunciated (1609-1618) three empirical laws on the motion of planetary bodies.

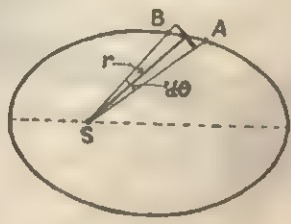


Fig. II-1.12

(1) *Each planet moves in an ellipse around the sun, which occupies one focus of the ellipse.*

(2) *The line joining the centers of the sun and the planet sweeps out equal areas in equal times.*

(3) *The squares of the periods of revolution of the planets round the sun are proportional to the cubes of their mean distances from the sun.*

B Discussions : (1) The first law follows from the geometrical analysis, that if a particle is to move in a closed path under an inverse square force, it must follow an elliptical path. Thus it is a simple corollary from the Newton's law of gravitation. Kepler's law was published (1609) long before Newton's (1687).

The shape of an ellipse is determined by its *eccentricity*, which is the ratio of the distance between the foci to the major axis of the ellipse. Mercury has an eccentricity of 0.2. That of Pluto is also high. But other planets have values ranging from 0.007 (Venus) to 0.09 (Mars). That for earth is about 0.017. For such small eccentricities we may treat the orbits as circular.

(2) This law is known as the law of *constancy of areal velocity* and follows from the principle of conservation of angular momentum.

Consider fig. II-1.12 P is a planet of mass m moving in an elliptic orbit around the sun. The sun is at a focus S of the ellipse. If in time dt, P moves from A to B, the area of the sector it describes is $\frac{1}{2}r^2 d\theta$, and the areal velocity is $\frac{1}{2}r^2 d\theta/dt$. Now, the angular momentum of P around S is the product of its moment of inertia $I(=mr^2)$ and angular velocity $\omega(=d\theta/dt)$. Since the angular momentum of an isolated system is constant, $mr^2\, d\theta/dt$ is a constant.

Because of the constancy of m, $r^2 d\theta/dt$ *will be a constant* and also the areal velocity. But because of this constancy, velocity at perihelion *exceeds* that at aphelion. Fig. II-1.13 shows the path of the earth round the sun approximately at different months of the year. Note that we are nearest to the sun (r_1) during our winter and farthest in our summer (r_2)

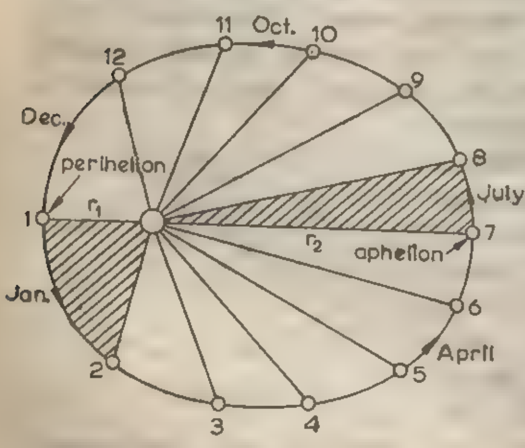


Fig. II-1.13

Hence to keep the area swept out equal, the arc length 1 to 2 is much greater than that between 7 and 8 on the orbital path of the earth. So it moves the fastest in January and the slowest in July.

(3) This law can be directly deduced from the law of Gravitation by taking the planetary paths to be *circular*. This we achieve by equating the gravitational pull to the centripetal force required to move the planet in a circular orbit.

Let M = mass of the sun, m = mass of a planet, v = orbital speed of the planet and r = the distance between them. Then, if T is the periodic time of the planet.

$$GMm/r^2 = mv^2/r \text{ or } GM/r = v^2 = (2\pi r/T)^2 \qquad \text{(II-1.15.1)}$$

$$\therefore \quad T^2/r^3 = 4\pi^2/GM \quad \text{a constant} \qquad \text{(II-1.15 2)}$$

This relation is independent of the mass of the planet, and hence is the *same for all planets*, a very important conclusion.

Prob : Calculate the mass of the sun, given that the radius of the earth's orbit is 1 Astronomical unit ($1{\cdot}5 \times 10^{13}$ cm) and $G = 6{\cdot}67 \times 10^{-8}$ cgs unit.

[*Hint :* Take T = 1 year = 365 × 86,400 seconds and apply the above eq. Ans. 2×10^{33} g nearly.]

C. Simple deduction of the law of gravitation from Kepler's laws. Kepler's laws give a simple and fairly accurate description of planetary motion without offering any explanation. To interpret these laws Newton discoverd the law of gravitation. A rigorous derivation is beyond our scope. We use the simplifying assumption that a planet moves in a circle round the sun.

To keep the planet moving in a circle round the sun requires the application of a centripetal force given by

$$F = m\omega^2 r = \frac{4\pi^2 r}{T^2} m.$$

This force must be directed towards the sun.

For two planets, distinguished by subscripts 1 and 2, the ratio of F_1 to F_2 is given by

$$\frac{F_1}{F_2} = \frac{m_1}{m_2} \cdot \frac{r_1}{r_2} \cdot \frac{T_2^2}{T_1^2}$$

But Kepler's third law states that $T^2 \propto r^3$ or that

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3} = \text{the same constant for all planets.}$$

Hence $$T_2^2/T_1^2 = r_2^3/r_1^3 \tag{II-1.15.3}$$

$$\therefore \frac{F_1}{F_2} = \frac{m_1}{m_2} \cdot \frac{r_1}{r_2} \cdot \frac{r_2^3}{r_1^3} = \frac{m_1}{m_2} \cdot \frac{r_2^2}{r_1^2}$$

or $$\frac{F_1}{m_1/r_1^2} = \frac{F_2}{m_2/r_2^2} = \text{a constant} = k \text{ (say)} \tag{II-1.15.4}$$

$\therefore$ For any planet k must have the same value.

$$F = k\frac{m}{r^2} \tag{II-1.15.5}$$

k having the *same value for all planets* in accordance with Kepler's third law. The force of attraction, therefore, varies as the inverse square of the distance from the sun, and is directly proportional to the mass of the planet.

From the law of equality of action and reaction, it is easy to argue that k will contain the mass of the sun. For the force F will also be the force with which the planet attracts the sun. Since F involves the mass of the planet, the reactional force should involve the mass of the sun. Thus k should be of the from GM where G is a constant and M the mass of the sun. Hence the force of attraction between the sun and a planet is of the form $F = GMm/r^2$. (II-1.15.6)

Ex.II-1.11 *At perihelion Mercury is 2.86×10^7 mi from the sun and has a speed of 35 mi/sec. At aphelion it is 43.4×10^6 mi from the sun. What is its speed at aphelion ?*

(*Note :* Perihelion is that point in a planet's orbit at which it is nearest to the sun. At aphelion it is farthest from the sun).

Solution : At perihelion and aphelion, the path of the planet is perpendicular to the radius vector drawn from the sun. See fig. II-1.13. Hence the rate at which the radius vector sweeps out areas is $\frac{1}{2} \times$ length of radius vector $\times$ velocity, By Kepler's second law this remains constant throughout the motion. If v be the speed at aphelion in mi/sec,

$\frac{1}{2} \times 28.6 \times 10^6 \times 35 = \frac{1}{2} \times 43.4 \times 10^6 \times v$, whence $v = 23.1$.

Ex. II-1 12. *Distance of a planet. Calculate from the following data the distance of the Mars from the sun*

Radius of earth's orbit $(r_1) = 149.5 \times 10^6$ km.

Earth's year $(T_1) = 365\frac{1}{4}$ days. Mars' year $(T_2) = 687$ days.

From Kepler's third law $\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$. $\therefore$ $r_2^3 = \frac{T_2^2}{T_1^2} \cdot r_1^3$.

Solution : Substituting values we find $r_2 = 227.8 \times 10^6$ km.

Prob. (1) Mars has two moons. Deimos, the larger, orbits at a mean distance of 6.9 Martian radius from the centre of the Mars, and its period is about 30 hours.

Phobos, the smaller moon, has a period of about 7·6 hours. How far, in Martian radii, is Phobos from the centre of Mars ? (*Ans.* 2.77)

(2) Neptune goes round the sun in 165 years. Show that the radius of its orbit is about 30 times that of the earth, both being taken as circular.

II-1.16. Motion of satellites The gravitational attraction of a planet on its satellite makes the satellite move round the planet. Let its orbit be circular. The gravitational attraction supplies the necessary centripetal force. This applies equally to natural and artificial satellities.

Let M = the mass of the planet, m = the mass of the satellite, r = the radius of the circular orbit of the satellite, T = the time in which the satellite goes once round the planet (that is its periodic time), v = the speed of the satellite in its orbit. Then since the gravitational attraction = centripetal force, we have

$$GMm/r^2 = mv^2/r \text{ or } GM/r = v^2 = (2\pi r/T)^2$$
$$\text{or} \quad T^2/r^3 = 4\pi^2/GM \qquad \text{(II-1.16.1)}$$

Note that this is the same equation as for circulation of planets round the sun. **T^2/r^3 will have the same value for all satellites, real or artificial, of a given planet.**

Problem : An artificial satellite circles the earth *near* its surface. Find the periodic time, given that the radius of the earth = $6{\cdot}4\times10^8$ cm and $g = 980$ cm/s^2.

[*Hint* : $g = GM/r^2$. Hence $T^2 = 4\pi^2 r^2 \cdot r/GM = 4\pi^2(r/g)$ or $T = 2\pi\sqrt{r/g}$.]

(*Ans.* 1 hour 25 minutes nearly.)

A. Orbital Velocity and Period of revolution of Planets and satellites

Taking the planetary orbits round the sun and those of satellites natural or artificial, to be circular these quantities can be very easily found. (See the hint to the above problem.) We take as above Centripetal force = Gravitational pull of the sun.

$$\text{or} \quad mv_0^2/r = GMm/r^2$$
$$\text{or} \quad v_0 = \sqrt{GM/r^2 \cdot r} = \sqrt{gr} \qquad \text{(II-1.16.2)}$$
$$\text{and} \quad m\omega^2 r = GMm/r^2 \quad \text{or} \quad (4\pi^2/T^2) = GM/r^3$$
$$\therefore \quad T = 2\pi\sqrt{r^3/GM} = 2\pi\sqrt{\frac{r^2}{GM}\cdot r} = 2\pi\sqrt{\frac{r}{g}} \qquad \text{(II-1.16.3)}$$

Note that the time period is equal to that of a simple pendulum of length equal to the orbital radius. Remember however that in both

of the last expressions g refers to *acceleration produced by the attracting bodies*, the sun for the planets and planets for satellites. Note that in the problem above M and g refer to those of the earth.

Orbital velocities of any planet can be very easily determined however, if we know its time-period i.e. the year. For example, the earth is 93 million miles from the sun and takes $365\frac{1}{4}$ days to go once round the sun. So its obital velocity would be

$$(v_0)=\frac{2\pi r}{T}=\frac{2\times3.14\times93\times10^6}{365\frac{1}{4}\times86400}\frac{\text{mi}}{\text{s}}=18.5\ mi/s\simeq29.6\ \text{Km/s}.$$

B. Geo-stationary or Parking Orbits : It is one of the latest triumphs of space technology to place a satellite at such a height that (*i*) it has the same period as the spin time of the earth and (*ii*) moves in the same sense from west to east. Then there would be between that satellite and the earth no apparent relative motion and it would appear to be stationary in the heavens like the pole star, though for an entirely different cause. Such a satellite is said to be *Geo-synchronous* (*Syn*-same, *chron*-time) or *Geo-stationary* and their orbits *Parking* or geo-stationary. They generally orbit in the plane of the equator as shown in the Fig II-1.14. Their height (H_P) follows from Kepler's 3rd law. We have seen above that

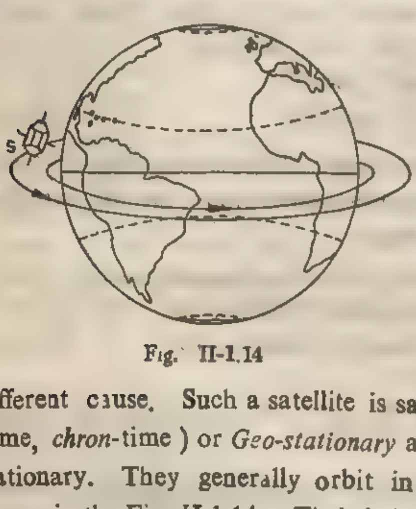


Fig. II-1.14

$$\frac{T^2}{r^3}=\frac{4\pi^2}{GM}\quad\text{and}\quad m\omega^2r=\frac{GMm}{r^2}$$

$$\therefore\quad \omega^2=\frac{4\pi^2}{T^2}=\frac{GM}{r^3}=\frac{GM}{(R_E)^2}\cdot\frac{(R_E)^2}{r^3}=g\frac{R_E^2}{r^3}$$

Now then $$T^2/r^3=\frac{4\pi^2}{GM}=\frac{4\pi^2}{g(R_E)^2}\qquad\text{(II-1.16.4)}$$

$$\therefore\quad r=\left[\frac{gR_E^2T^2}{4\pi^2}\right]^{1/3}=\left[\frac{9.8\times6367\times10^3\times24\times3600}{4\pi^2}\right]^{1/3}$$

$$\simeq42400\ \text{km}.$$

Hence its height would be $H_p = r - R \simeq 36000$ km.

Some countries including our India have placed (April 1982) geo-synchronous satellites (INSAT—1) in orbit They reflect short radio waves back to earth over a wide area beneath it. Radio and TV programmes including educational items and sports events are transmitted *live* with their help. In addition Insat—1 is monitoring growth of cyclones and moonsoons, weather and condition of glaciers, the source of our life-giving rivers on the high Himalayas, continuously, instantaneously and transmitting all these informations cheaply.

C. **Artificial earth satellites** The world was taken by surprise when, early in October 1957, Russia successfully put the first artificial earth satellite in orbit at the first attempt. Subsequently there have been more of them and bigger too, America vying with Russia in the field. Fig II-1·15 shows a satellite* in orbit, its velocity changing direction as at points (1) and (2) under the pull of the earth.

Let us examine the circumstances in which a body can continue to move round the earth more or lesss like the moon. It is not difficult to imagine that the body must be moving at a high speed to avoid being drawn to the earth. If such a fast motion takes place in the atmosphere, friction with the air will reduce the speed and, burn up the body by the heat generated, as for a meteor. Hence, for the motion to continue, at least for a reasonable period the orbit of the body should be beyond the atmosphere.

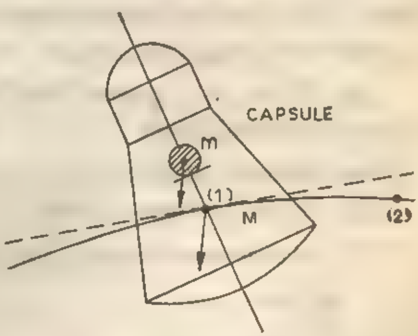


Fig. II-1.15

We know that the atmosphere gets thinner and thinner as we go up, its density diminishing exponentially (fig. in § II-6.15). At a height of 100 miles the density is very low, of the order of 10^{-13} g/cm^3. An artificial satellite should move at heights of 100 miles or

* Our India have put several satellites in orbit, the first being Aryabhatta 1—more than a decade ago (1975).

more above the surface of the earth if it is to continue in its orbit for a resonable time.

If M = mass of the earth ($= 6 \times 10^{24}$ kg), R = radius of the earth ($= 6.4 \times 10^6$ m.), G = universal gravitational constant ($= 6.6 \times 10^{-11}$ mks units), it may be shown that a body projected with a speed v from the earth will move as stated below :

(*i*) When $v^2 < GM/R$ (i.e., v less than 4·9 miles per sec), the path is an ellipse and the body is drawn back to the earth.

(*ii*) When $v^2 = GM/R$ (i.e., $v = 4.9$ mi/sec. or about 18,000 mph.), the body moves in a circle around the earth.

(*iii*) When v^2 is greater than the above value but less than twice the value, i.e., $2GM/R$, the body moves outside the earth in an elliptic path with the centre of the earth as one of the foci of the ellipse. (v between 4.9 and 6.96 mi/sec).

(*iv*) When $v^2 = 2GM/R$, or $v = 6.96$ mi/sec or about 25,000 m.p.h., it moves in a parabolic path and escapes from the earth. The earth's attraction can no longer hold the body. This value of v is called the *escape velocity* v_e.

(*v*) When v is greater than the above speed, the body escapes along a hyperbolic path.

For artificial satellities we are concerned with case (*iii*) above, i.e., v lying within the range, 18,000 to 25,000 miles per hour.

To visualize what happens, following Newton, imagine a mountain rising beyond the atmosphere and a powerful gun placed there. Suppose it can fire horizontally shells with speeds we desire (fig. II-10.16).

Fig. II-10.16

When the speed of a shell is less than $\sqrt{GM/R}$ (i.e. 18000 mi/hr), the earth's pull causes the shell to crash on its surface at some point depending on the actual speed. A larger speed,

(but less than $\sqrt{GM/R}$ will cause the shell to crash at a remoter point. When the speed is equal to $\sqrt{GM/R}$ (i.e., 18,000 mi/hr) the shell moves along a circle The earth pulls the shell continually towards its centre, but the speed of the shell just keeps it from falling into the earth. It will then behave as a satellite.

If the speed is greater than 18,000 mi/hr, but less than the escape velocity of 25000 mi/hr, the orbit of the satellite is an ellipse with the centre of the earth as one of its foci. As the satellite moves round the earth in an elliptic orbit, its distance from the earth varies from a minimum to a maximum (i.e., from perigee to apogee).

D. **Launching of Earth satellites:** We do not have such a mountain so we arrange to raise the satellite to the required height and there discharge it just parallel to the earth's surface there with a fantastic speed of between 18 to 25 thousand mph. The raising is done by very powerful multistage rockets. Whereas the satellite alove may weigh say a thousand kg, the rocket system goes up to several thousand tonnes; for the satellite has to be raised at least about 100 miles. The system is fired vertically and at first rises slowly so as to avoid burn-out by friction at the lowermost and densest part of the atmosphere. With height friction falls off and the rocket speeds up. At a designated height, one stage of the rocket falls off and suddenly the speed boosts up due to loss of a large mass, linear momentum being conserved. Small retro-rockets slowly tilt the trajectory till at the desired height the last of the rockets disengages itself, spewing out the satellite or capsule in the desired direction and speed. It moves out in the direction the earth spins so as to include the considerable velocity of the earth in its motion. The freed capsule in its flight path is shown in fig. II-1.15.

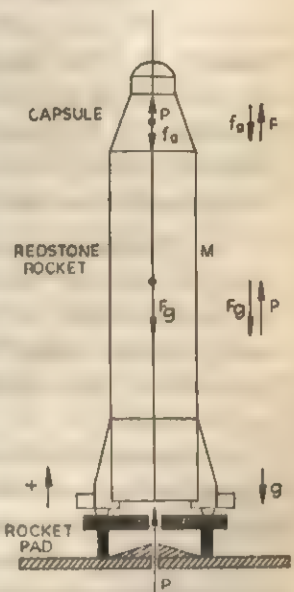


Fig II-1.17

Fig. II-1.17 shows the forces acting, which are $F_g = Mg$ downwards and P the reaction of the rocket pad upwards. When rising the forces are upthrust P, downward pull F_g so that the upward force is their difference $F = P - F_g = Ma$ producing acceleration a.

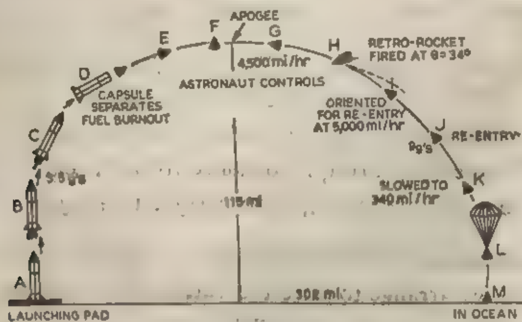


Fig. II-1.18

In the figure f_0 is the weight of the astronaut. Fig. II-1.18 shows the parabolic flight path of the first American astronaut. *It is assumed* that over the whole flight path g remained sensibly constant for at the maximum height of about 115 miles, it diminished by less than 5%

B Weightlessness in Artificial earth satellites. In an orbiting earth-satellite the whole of the earth's gravitational pull is spent in giving the satellite the necessary centripetal pull. It is actually a **freely falling body,** falling towards the centre of the earth with the acceleration due to gravity. Its orbital speed prevents it from crashing on the earth.

Our sense of weight arises out of the reaction which the support exerts on us. A man seated in such a satellite does not experience any force exerted by the seat on him, as he does when seated on a stationary body on the earth, since both are falling freely. He therefore feels weightless. The situation is very much the same when one descends in a lift. As the lift accelerates downwards the reaction of the floor on a person standing in the lift diminishes. The person feels lighter. If the lift descended with the accleration of a free fall under gravity, the reaction of the lift on the person will be zero. The person will feel weightless. It is this kind of thing that happens in an orbiting earth satellite.

An *alternative view* is also to the point. Since the orbiting satellite is accelerated centripetally it provides an accelerated frame of reference to the astronaut inside. Hence a pseudo-force would act on him away from the center, (the centrifugal force) equal to the centripetal force. Hence the resultant force on him towards the earth vanishes and the astronaut feels weightless This is a very unusual condition and the astronaut with everything inside would be just floating about. The physiological functions accustomed as they are to gravity, get completely upset. Hence the long training of prospective spacemen in simulated "weightless" condition on earth. To minimise these inconviniences the orbiting satellites are now-a-days made to spin about an axis so as to restore some of the sensation of weight.

In a gravitational field, if on a *moving small* body no other external force acts except the gravitational, the state of weightlessness would exist. In a freely falling lift, spaceships or artificial satellites such conditions hold. The phenomenon is independent of material medium. But a solid floating in a fluid medium (liquid or gas) is also weightless ; because its weight is balanced by the upward acting force of buoyancy due to Archemedes Principle to be discussed under Hydrostatics.

[**N. B.** We ordinarily define the weight of a body as the pull of the earth on it. But this pull is always present. Actually our *sense of weight comes from the reaction to this pull.* When we hold a book on our palm, the weight of the book presses on the palm. The reaction which the palm exerts on the book gives us a sense of the weight of the book. As we stand on the floor, the reaction which the floor exerts on our feet gives us the sense of our weight. In an artificial earth satellite, the floor does not exert any reaction on one standing on the floor. Both are acted on by the force of gravity, but the force on each is fully spent in supplying the necessary centripetal force. No force is left to produce a thrust by one on the other.

(In water, a man feels almost weightless. This gives you some idea of what weightlessness is like.)]

II-1.17. A. Escape velocity. The gravitational pull of a larger body can hold a smaller body *captive* in its gravitational field if the

speed of the latter is not too large. The minimum velocity which a smaller body must have to escape from the gravitational attraction of a larger body is called the **escape velocity** v_e.

A. **Velocity of escape from the surface of the earth.** Consider a body of mass m at a distance x from the centre of the earth. If x is greater than the radius R of the earth, the force f with which the earth pulls the body is given by $f = GMm/x^2$, where G = gravitational constant and M = mass of the earth. If the body is displaced by an amount dx against this attraction, the work done will be $f.dx$.

To pull the body out of the earth's attraction from its position on the surface will therefore require an amount of work

$$W = \int_R^\infty f dx = \int_R^\infty \frac{GMm}{x^2} dx = \frac{GMm}{R} \qquad \text{(II-1.17.1)}$$

If a body on the surface of the earth be projected outwards (in any direction) with a kinetic energy greater than this value, it will escape. The minimum *escape velocity*, v_e, is therefore given by

$$\tfrac{1}{2} m v_e^2 = \frac{GMm}{R} \quad \text{or} \quad v_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{2GMR/R^2} = \sqrt{2gR} \qquad \text{(II-1-17.2)}$$

This is also the velocity a body would acquire if allowed to fall to the earth from infinity.

Prob. (1) Calculate the velocity of escape from the surface of the earth, given that the mass of the earth $= 5.98 \times 10^{24}$ kg, its radius $= 6.37 \times 10^6$ m, and $G = 6.67 \times 10^{-11}$ mks unit [*Ans* 11.2 km/sec.]

(2) The mass of Mars is 0.108 times the earth's mass, and it has a radius 0.532 times the radius of the earth. What is the escape velocity from the Mars ? [*Ans.* 5.04 km/sec.]

(3) The mass of Mercury is 0·045 time the earth's mass, and its radius is 0·39 earth radius. What is the escape velocity ? [*Ans.* 3·8 km/sec.]

B. **Rarity of certain gases in the Atmosphere :**

The escape velocity from the earth, as we see above, is about 7 miles/sec. This is more than thirty times the velocity of sound in air at 0°C, and about 6 times the velocity of molecules at the same temperature (IV-6 6). Even if the average velocity of the molecules of a gas is 25% or so of the escape velocity, an appreciable fraction of the molecules will have velocities above the escape velocity. Such fast molecules will soon be lost by escape from

the earth's atmosphere. The earth was very hot in the past and the gas molecules in its atmosphere had higher velocites. The lightest gases, hydrogen and helium, had the highest velocities. They gradually escaped from the earth while it was hot. This explains the rarity of hydrogen and helium in the earth's atmosphere.

If the moleculer speed is one-fourth the escape value, half the number escapes in several thousand years. If it is one-fifth, the half-life is of the order of several hundred million years.

C. Absence of atmosphere on Moon and Mercury :

The moon has a much smaller mass than the earth ($v_e = 2.4$ km/s) ; and so also has the planet Mercury. They could not have retained any gaseous atmosphere about them because of the weakness of the pull. There is evidence to support this view. The same thing applies to satellites of all planets. But Titan of Saturn has an atmosphere.

D. Trajectories : The following table shows what the path of a body will be if it is projected from a point on the earth's surface with various velocities v Theminimum escape velocity, v_e, tis $2GM/R$.

Velocity	Nature of path
A. $v^2 > v_e^2$	Hyperbola. The body will escape from the earth.
B. $v^2 = v_e^2$	Parabola. The body will escape.
C. $v^2 < v_e^2$, but $> v_e^2/2$	Ellipse, with the earth at one focus (cf. motion of earth round the sun)
D. $v^2 = v_e^2/2 = GM/R$	Circle.
E. $v^2 < v_e^2/2$	Ellipse.

Prob. Show that the escape velocity of a body from a planet is 1.41 times that of its orbiting velocity close to the planetary surface.

II-1.18. Energy considerations in the Motion of Satellites :

Refer to eqn II-1.6.2 for the potential of a unit mass on the surface of the earth ; it is GM/R. If the mass of the satellite be m and it rotates in a circular orbit of radius r then the potential energy of the system is

$$U = -GMm/r. \qquad \text{(II-1.18.1)}$$

[As force of gravity is attractive and so $-ve$, potential is zero at infinity and as the body moves towards the earth in the direction of the force the work done and hence the potential energy is $-ve$ (see § 1-6.1)].

Now the K. E. of the revolving satellite which must always be $-ve$ is

$$K=\tfrac{1}{2}m\omega^2r^2=\tfrac{1}{2}m\omega^2r.r=\tfrac{1}{2}\frac{GMm}{r^2}.r$$

$$\therefore\quad \tfrac{1}{2}\omega^2r^2=\tfrac{1}{2}\frac{GM}{r}\quad \text{or}\quad K=\tfrac{1}{2}m\omega^2r^2=\tfrac{1}{2}.\frac{GMm}{r} \qquad \text{(II-1.18.2)}$$

$$\therefore\quad \text{Total energy}\quad E=P.E+K.E=-\frac{GMm}{r}+\tfrac{1}{2}\frac{GMm}{r}=-\tfrac{1}{2}\frac{GMm}{r}$$

(II-1.18.3)

The total energy is this constant but $-ve$ indicating that the system is a closed one, the satellite or the planet being always bound to the attracting planet or the sun and never escaping. Fig. II-1.19 shows the relation between the energy and the separation, further the satellite, less $-ve$ is its energy.

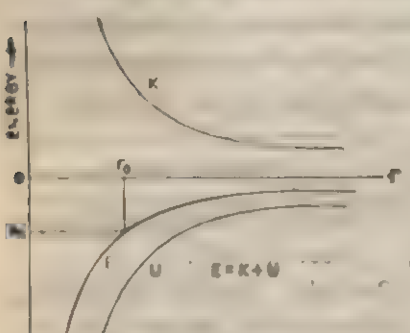


Fig. II-1.19

Note : The same picture holds for the electron revolving round the proton in a hydrogen nucleus.

Because of air friction energy of a satellite gradually diminishes and it falls to a lower orbit, the respective energies being $-GMm/2r_1$ and $-GMm/2r_2$. As $r_1>r_2$ it follows that $GMm/2r_2$ $>GMm/2r_1$ i.e. from II-1.18.3 diminution in PE is twice the increase of $K.E$ i.e. there *occurs* a loss of energy on the whole.

II-2

*STRUCTURE AND PROPERTIES OF MATTER

II-2.1. Three States of Matter :

Day to day experience tells us that matter exists in three forms or states solid, liquid and gasesus. Scientists recognise a *fourth* state of matter the *plasma*, matter totally ionised at a very very high temperature showing none of the properties of matter we know. It is the stuff stars and our sun are made of.

Solids have definite shape and volume, a liquid has volume but no shape, gases neither. A liquid takes up the shape of its container while a gas both the shape and volume of the same. We may put their differences this way—a *solid* is to be supported *from below* to prevent its fall, a *liquid* is to be supported *from its sides in addition*, while a *gas* is to be limited *from all the directions*. As liquids and gases can flow, they are called *fluids*.

II 2.2. Common Properties of Matter :

All types of matter exhibit the properties detailed below—

(i) Inertia : This, the most basic of all properties of matter, relate to the facts given in Newton's first law of motion, that matter can (*i*) neither start moving nor when moving, change (*ii*) its speed or (*iii*) its direction of motion *by itself*. They have been discussed in detail in connection with Newton's laws of motion.

(ii) Gravitation : Any two particles or bodies attract each other anywhere in the universe. This has been the subject matter of the previous chapter.

(iii) Extension : Each atom must occupy some space, however small. A repulsive force comes into play when they are closer to each other than a critical separation. So matter occupies volume. This property of matter is called its extension.

(iv) Porosity : The above point indicates that there must exist empty spaces in between atoms of matter. This separation in solids

* For the more inquisitive student. May be omitted by others.

and liquids is of the order of 1 Angstorm unit ($=10^{-10}$ m). Hence matter must be porous, This is the property responsible for osmosis and diffusion in liquids and gases. Very slow diffusion occurs in solids also.

(v) **Compressibility :** Because of porosity, matter may be compressed to more or less extent, inspite of atomic repulsive forces. For solids and liquids compressibility is very small, while gases are highly compressible. Compression is always opposed and that happens because of the property of elasticity.

(vi) **Elasticity :** All matter tends to maintain its shape, size or volume ; for under that condition only the *potential energy* in it *is minimum*. Compression or dilatation requires work to be done and hence storing up of potential energy. Elasticity is that property of matter by virtue of which any tedency to deform the shape size or volume is opposed and it regains its original condition when the balanced system of deforming forces is removed.

(vii) **Impenetrability :** Two different pieces of matter cannot occupy the same space. Drop a stone in water and it displaces its own volume of water to take its place. To some extent it is opposite to porosity but note that the latter is a small scale (*microscopic*) while the former a large scale (*macroscopic*) property of matter.

(viii) **Divisibility :** Matter, particularly solids can be broken down into smaller and yet smaller parts ; you can take out from a container small amounts of liquids or gases. In no such case, the physical or chemical properties do change. You know that these remain unchanged till you reach the molecule or for elements, atoms.

(ix) **Cohesion and Adhesion :** Water sticks to glass, paints stick to wood, glue adheres to paper, tin to brass—all these happen due to the property of adhesion. When attraction is found to occur between molecules of different substances, adhesion is said to be active.

Cohesion is said to be responsible for molecules of any given substance sticking together. It is due to cohesion that a piece of solid maintains its shape and resistance offered to breaking it down.

II-2.3. Molecular structure in relation to states of aggregation of Matter :

As the smallest piece of any matter is its molecule, it should be possible from the atomic or molecular theory of matter, to describe

and derive the three states of matter and their properties. Atoms or molecules of matter are always in motion, the magnitude of which depends upon temperature. In fact, the temp. of a gas is proportional to the sum total of the kinetic energy of all its molecules (Chap. IV-6).

A. The solid state : A solid piece of matter has a definite shape and size and it resists strongly any effort to change either. It has a definite boundary surface all around indicating very small molecular movements.

To explain these characteristics, molecules of a solid are assumed to be confined in small intermolecular spaces. Strong forces of attraction and repulsion keep them so bound, to their individual locations. We may take them to be bound by small springs (fig. II-2.1). A solid thus behaves often as if it is a microscopic bed spring, the molecules being held together by elastic forces. The neighbouring parts are strongly attracted ; hence increasing molecular separation face strong resistance ; same happens in efforts to diminish the separation (springs require strong forces to tense or to compress). This explains the *stress* or internal resisting forces set up when deforming a solid. This behaviour leads to elasticity (to be developed in the next chapter). This *model* also clears up the properties of compressibility porosity, divisibility.

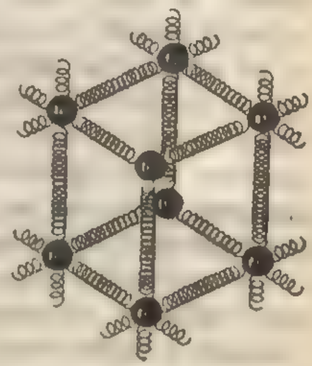


Fig. II-2.1

The molecules however are not at rest but vibrating, amplitudes being restricted by the springs. At all temperatures other than the absolute zero, the molecules vibrate, amplitudes increasing with temperature. This fact explains increase in length or volume of a solid with rise in temperature. The molecules possess both kinetic and potential energies (like a pendulum) because of such vibrations.

A heated solid at some temperature turns into a liquid and begins to flow But a sharp line between the two states cannot always be definitely drawn ; for many (non-crystalline) substances the change of state it not sudden but gradual. Take a stick of sealing wax and give it a sharp blow , it breaks apart into sharp pointed fragments as a solid should. But take such a long heavy stick and keep

it horizontally supported at the two ends. After a long time you will find it sagging in the middle as if flowing very sluggishly. A thin glass tube is fragile but hold it over a flame and it will soften, bend and sag, you can draw it into a capillary. Sharp glass edges held over a flame is found to soften, curl on itself and turn blunt. A large piece of asphalt appears solid and brittle but put it over a sloping tin roof in the sun ; soon you will find it very slowly flowing. A piece of cork buried in pitch is found to surface up, of course a long time later. Vast glaciers on mountain sides, those fields of solid ice have been found to flow downslope an inch or two, a year. These plastic or non-crystalline substances possess *viscosity* a property characteristic of liquids and hence behave thus.

Crystalline solids do not however do so.

B The Liquid state : These have definite volumes but no definite shapes. A liquid shows a free surface but assumes the shape of the container beneath. It *strongly resists normal forces* (perpendicular to the surface) that tend to compress but *very weakly resists shearing forces* (parallel to the surface). Such a force therefore can move the liquid surface so that it would flow. In all liquids inter-surface frictional forces oppose such flow or relative motion between two adjoining layers, due to its property of **viscosity** Plastic solids described above, are in fact very viscous liquids.

When a liquid is poured into a large vessel it easily flows sideways until halted by the container walls so as to take up the condition of minimum potential energy. Hence it is that a liquid assumes the shape of its container and has a free surface.

To explain the transition from the solid state to the liquid and vice versa the following model is assumed to hold. As a solid is heated up the molecules vibrate with rising amplitude. At a critical temperature (normal melting point) characteristic of the material, the restraining springs suddenly snap, all at a time as it were, and the molecules collect in numbers of close-knit swarms which move about at random with speeds characteristic of temperature. Within a swarm, molecules are held together by forces of *weak* intermoleculer attraction but kept at definite separation by *strong* repulsive forces, also intra-molecular. This is why a liquid has a definite volume and is very nearly incompressible. Again with fall in temperature this thermal motion slows down. They collect at last in local groups and a definite geometrical shape. This shape is such as to hold the maximum number of molecules These groups may or may not be in relative motion. Again, below a critical temperature (the normal freezing point the groups stop moving, coalesce together and freeze into a solid. Molecules occupy fixed positions and move to and fro relative to that position.

C. The Gaseous State : A gas resists compression weakly but expansion not at all. It occupies the entire interior of a closed

space and its various portions can be held in position only by applying external forces. The molecules are quite free to move, do so quite fast and exert on the container walls perpendicular forces.

To explain such behaviour of a gas in a vessel it is considered as a collection of fast-moving particles with large separations. They move at random, collide with each other and container walls, collisions being elastic and very short-lived. Gas pressure results from the change in momentum due to these collisions. Later detailed discussions in this regard will be provided in Chapter IV-6.

The large inter-molecular separation produce little attraction between the flying molecules and they continue to rush on till rebounding from the container walls and thus fill up the entire space. Diminishing the volume leads to rise in molecular density and number of collisions. So the gas pressure on the walls rise and hence opposes the compression of a gas but not its expansion.

The motion of gas molecules depend primarily on their masses and temperature of the space. At a given temperature heavier molecules move slowly but with rise in temperature all molecules move faster. At the freezing point of ice, a hydrogen molecule would cover a mile each second.

It is the change in attraction between molecules with change of state, that is responsible for change in molecular motion and that again with change in separation. It is remarked that, molecules in the liquid state retain some memory (i.e. intra-molecular attraction) of the solid state, but in the gaseous state, not at all.

D. **Plasma :** Though it is regarded as the fourth state of matter, nothing of what we understand as matter is retained by it. At millions of degrees of temperature as in the cores of sun and stars, all atoms and molecules are stripped of electrons and get ionised. Naturally in the crowds of nucleii, protons, or electrons we cannot expect to find properties obtaining in well-knit atoms or molecules. Plasma is this ionised gas which because of the charged nature show properties totally alien to what we find in uncharged gas molecules Though very strange to learn, most of matter in the universe is in the plasma state and very little in the three states we know so well.

II-2.4. Particle-nature of Matter :

Millions of particle go to form any piece of matter. You know that (i) the smallest particle of any substance is a molecule (ii) that of an element is an atom and (iii) an atom is a stable configuration of protons, neutrons and electrons. The particle nature of matter is certified beyond doubt by the phenomena of Brownian motion in liquids and gases, the various conclusions from the kinetic theory of gases and the diffraction-spots of X-rays scattered by solids ; the last phenomena is absolutely impossible for a continuous medium.

We now try to estimate the separation between molecules in the three states of matter. From the spreading of oil on water it has been surmised that the diameter for an oil molecule cannot exceed 0.5Å (1Å = 10^{-8}cm). Diffraction experiments with X-rays clearly indicate that inter-atomic separation inside a crystal is about 3Å only. In solids and liquids such separations are nearly the same and the maximum range of intra-molecular attraction is of the order of 10Å only. We now calculate this separation in a gas. At N.T.P. a mole of a gas occupies a volume of 22.4 litres and contains 6.02×10^{23} molecules, as you know from chemistry. Taking gas molecules as spheres of diameter d and volume V we have

$$V = \tfrac{1}{6}\pi d^3 = \frac{22.4 \times 10^3 \text{ cc}}{6.02 \times 10^{23}}$$

$$\therefore \quad d = \left(\frac{6\pi \times 22.4 \times 10^3}{6.02 \times 10^{23}}\right)^{\frac{1}{3}} \simeq 33 \times 10^{-8} \text{ cm} = 33\text{Å}$$

This diameter is the closest possible separation between centers of gas molecules—11 times as much as the separation between solid molecules and more than thrice that of the range of molecular attraction. No wonder that the gas molecules lose all memory of the solid state.

II 2.5. Intra-molecular Force :

That the range of inter-molecular forces should be very small is intelligible for if two pieces of matter be placed quite close to each other no force other than that of gravitation is found to act between them. Yet we have seen above that the macro or bulk behaviour of

matter in all the three states may be *roughly* explained by assuming these forces to be acting. As we have noticed above, these forces range over no more than a few atomic diameters. An atomic diameter is estimated not to exceed 1Å.

Intermolecular forces originate from *two* main sources—

(1) Because of interaction between a molecule and its neighbours, *potential energy* is developed. This interaction arises from electrical and *not* from gravitational effects and

(2) The thermal motion arises from *kinetic energy* of the molecules.

Their ratio it is, that governs the states of matter and their characteristic properties.

Molecular Potential energy and Force : Though even a small piece of matter contains millions of molecules we shall choose a pair of them only to discuss their mutual potential energy (U) and force (F) where $F=(-dU/dr)$.

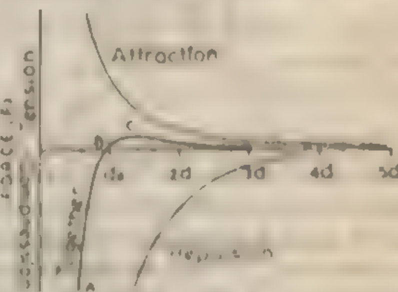


Fig. II 2.2.

Normally they should be so separated that there will be no force exerted between them i.e. no mutual potential energy. That separation is *equilibrium spacing*. This spacing at absolute zero is taken to be d_0. If the separation exceeds this critical spacing, attractive force comes into play while at less separation the force turns repulsive. To bring about either change, work is to be done which remains stored up as the potential energy. In fig. II 2.2 variation of the forces of both types with separation is shown separately as well as jointly. In the next figure II-2.3 is shown how the potential

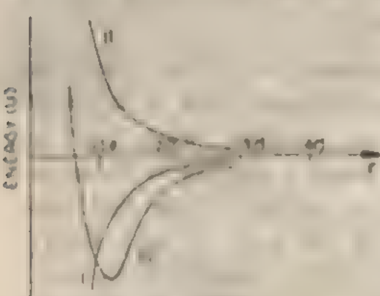


Fig. II-2.3

energy (U) aquired by changing separation varies with intermolecular spacing (r).

Finally in fig II-2.4 are depicted together the variation with separation (r) of both mutual force (F) and potential energy V by the curves MENQ and ABCD. Force and P. E. are related quantities.

Observe that at equilibrium spacing (E) the interaction force is zero and the mutual potential energy (EC) negative and a minimum, It is so along BCD i.e. most of the time. The force curve is similar but flatter. Zero P. E. and zero

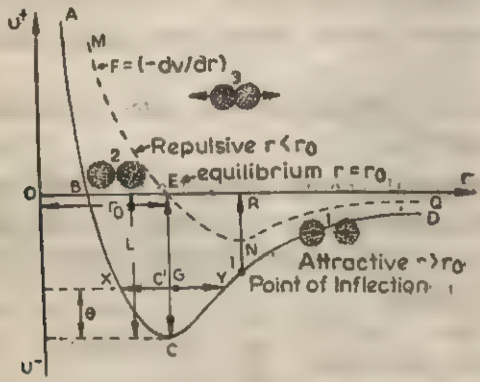


Fig II-2.4

mutual force do not occur for the same separation. Also note that attractive force (conventionally −ve) and P. E. for $r > r_0$ rises for more gradually than does repulsive force (conventionally +ve) and P. E. in the region $r < r_0$. Repulsive force is therefore far stronger for a given separation than the attractive force for same separation. It follows also that a molecule or atom displaced beyond its equilibrium spacing would be vibrating for the force is always directed towards the equilibrium position. Position of and force-directions on the pair of molecules are shown for $r < r_0$, $r = r_0$ and $r > r_0$.

As in the previous case $r = r_0$ represents equilibrium separation between the particles. As the separation between them is increased corresponding to tension in the last diagram, work is to be done on them and the total potential energy is negative as shown by the curve I(II-2.3). When the particles are pushed closer than their equlibrium spacing (represented by d_0 the point of zero force) work is again done and this is positive for it corresponds to compression in the force-separation diagram. This is shown in curve II. Adding these two graphs we obtain curve III the general pattern.

Note that at the equilibrium separation mutual force is zero but not the potential energy ; there, it is at a negative maximum, i.e. lowest possible value.

Hence there appears a clear asymmetry in the P.E.—Separation curve (ABCD) as also for the force-separation curve (MENQ). We conclude (i) a weak attraction force and a much stronger repulsive force exist ; their mutual effect (ii) produces an oscillation of solid molecules about their mean positions. These conclusions, we shall, later find would help us to understand (a) elasticity, Hooke's law breaking strain (b) thermal expansion (c) latent heat of vaporisation.

II-2.6. Atomic and Molecular bonds :

In solids and liquids atoms and molecules are bonded together. Bonds may be of different types. But all of them arise from the Coulomb force of attraction between the +vely charged nucleus and the revolving −vely charged electrons, in atoms.

Generally speaking, the distance between the nucleus and the electron is no more than O.5Å. Charges being equal and opposite an atom is electrically neutral and the resultant electrical effect goes little beyond the atom. When, say two hydrogen atoms (H H′) are a little apart (1 in fig. II-2 4) there appears a *weak* attraction —for protons and electrons do not coincide here But as in the uppermost (3), if their centers are less then 1Å apart they cannot interpenetrate and develope a very strong repulsive force ; at this separation the force experienced by the atoms no longer retains the simple coulomb form ; for each of the two protons and two electrons affect each other, developing two pair of forces of differing magnitudes. It is the weak attraction that forms a hydrogen molecule and still weaker molecular force is responsible for liquefaction and solidification at much low temperatures.

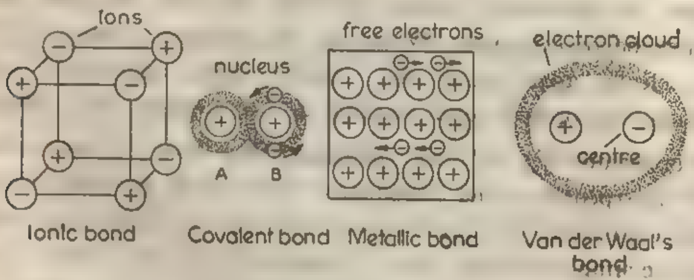
It is very remarkable that the interacting forces between atoms that go to build up molecules, have some common characteristics —of which strong repulsion at short separation and weak attraction at larger separation is the chief—though atoms can be of any number in a molecule, two in Hydrogen to millions of them in a human DNA molecule and of any type from hydrogen to uranium.

Four types of bonds namely ionic, covalent, metallic and Van der waals type, are discussed below.

A. **Ionic Bond :** In crystalline solids like NaCl, the Na^+ and Cl^- ions take up the positions as shown in fig. II-2.5a. Na donates one electron very easily to Cl and they turn into oppositely charged ions. In a crystal they are held together by coulomb attraction. *Ionic bonds are quite strong.*

B. **Covalent Bond :** One electron moves from one atom of a Hydrogen atom to the other very easily. Then the donor atom becomes +vely and the acceptor atom becomes −vely charged, and hence attraction appears between the two (Fig. II-2.5b).

Note that the same electron is shared between two atoms but not transferred from one atom to the other as in the ionic bond. It is the covalency between neighbouring carbon atoms that gives such large strength to diamond. The stablity of hydrogen molecules and high melting point of diamond show the *covalent bond* to be *a strong one.*



(a) (b) (c) (d)
Fig. II-2,5

C. **Metallic Bond** : Every atom in a metal has one or two free electrons that move about inside the metallic crystal lattice at random thus making the atoms steady +ve ions (Fig. II-2.5c). This bond resembles the covalent one but a free electron remains bound to no atom in particular ; *this bond is a weak one* and is responsible for the solid state of most metals. Free electrons conduct heat and electricty. Wieddmann and Franz found the ratio of these conductivities to be constant.

D. Van der waals' Bond : If we consider a long enough interval, we may take that the center of the electron cloud would be at the nucleus of an atom although the electrons are in continuous motion. But at a given instant more of these electrons may chance to be on one side of the nucleus when their center will no longer be at the nucleus but will shift somewhat, forming a dipole* (fig. II-2.5d). This dipole would then attract more electrons from neighbours forming more dipoles. Between these dipoles exist weak attraction. This bond is named after Van der waals who had assumed similar weak attraction between gas molecules

Summarising the above discussions we conclude—(1) Strong ionic bond arises out of strong attraction between oppositely charged ions in a crystal ; (2) Sharing of the same electron between two atoms produce strong covalent bond. (3) Randomly moving free electrons in a metal piece are responsible for weak metallic bond and (4) The weak Van der waals bond arises from the interaction between an atomic nucleus and its electron cloud.

* A *dipole* is a pair of point charges at a small separation. It is very much like a molecular magnet envisaged by Weber.

II—3
ELASTICITY

II-3·1. Elasticity

In mechanics we have almost always spoken of particles. Whenever we have introduced a body it has been characterised as perfectly rigid i.e. such bodies are not deformed however large may be the force applied on them ; no forces can alter the separation between their constituent molecules. But none of them, a particle or a rigid body actually occur in practice, they are idealised and simplifying assumptions. All bodies even very small ones have *some dimension* and under the action of equal and opposite (unlike) forces *suffer deformation*, i.e. change in shape or size. On removing the forces they regain their original shape or size. They are said to be elastic bodies and their behaviour is due to one of the fundamental properties of matter, *elasticity*. In mathematics, *elastic deformation* is treated as the *mechanics of deformable bodies.*

In the last chapter we have learnt that **elasticity is a property of matter because of which bodies oppose any tendency to change its shape or size and restores the body to its original shape of size whenever the deforming forces are removed** As instances consider *i*) pressing a rubber ball enclosed between your two palms ; it gets squeezed which changes its volume (*ii*) pulling a rubber cord with your two hands when it gets lengthened i.e. its length is changed (*iii*) twisting the free end of a long thin rope hanging from the ceiling, which changes its shape. In all cases, if you have not deformed any one of them much, they would come back to their original configuration when you stop applying the be deforming forces.

Fig II-3.1

Remember very carefully that while unbalanced forces produce motion, *balanced forces acting on a body produce deformation.* Consider a spiral spring (fig. II-3.1) supported rigidly at its upper end and weighted

at the lower. If the lower end is pulled downwards the spring is acted on by two forces which balance, namely, (*i*) the applied downward pull, and (*ii*) an equal and opposite pull exerted by the support on the spring The result of these two forces is an elongation—a deformation—of the spring. Very often we leave out of our consideration the force exerted on the spring by the support and speak of the elongation as being due to the pull exerted on it In § II-2.5 we have seen why it so behaves.

In the above example, the hand that pulls the spring experiences an upward force. This upward force, brought into play by the deformation of the spring, resists the deformation. It is larger, the greater the applied pull. When the pull is removed, the spring regains its original length. This simple observation shows (*i*) When a body (here the spring) is deformed, the body resists the deformation and

(*ii*) When the deforming force is removed, it tends to go back to its original condition.

Let us consider the above example a little more in detail. Suppose at the free end of the spring we hang a load. The attached load starts coming down because of gravity ; but while the upper end of the spring remaining immovably attached, its lower end moves increasing the separation between its turns and hence its length. A little later after a definite elongation the load stops descending Clearly two forces, reaction of the spring at its clamped upper end and the weight of the load downwards, must have been equal unlike and collinear so as to (*i*) increase the separation between its turns, (*ii*) thereby increasing the length of the spring and (*iii*) bring the system to equilibrium and rest. A body in uniform linear motion is also in equilibrium.

If you further elongate the spring by pulling down the load further you will feel the spring also pulling your hand upwards. This is the force opposing deformation. Harder you pull the load down, greater is the force you feel upwards, just according to Newton's 3rd law. Note that the spring and your hand form an *isolated or closed* system.

II-3.2 Deforming Forces and Deformation.

Robert Hooke in 1678 had enunciated the fundamental law of elasticity in the form that *deformation produced is proportional to deforming force.* The statement applies to all deformations *provided they are small.* The statement that goes by the name of Hooke's law now, was enunciated by Thomas Young in 1802. We discuss it in detail later.

If the force F has brought about an extension of δl in the spring then according to this law $\delta l \propto F$ and the opposing i.e. elastic force F_e is equal and opposite to it and we may write

$$F_e = -k.\delta l \qquad \text{(II-3.2.1)}$$

The variation constant k, the elastic force developed by unit extension is called the *spring constant* already discussed in introducing S.H.M. In fig. II-2.4 note that near E the equilibrium point ($r = r_0$) the graph F vs r is approximately a st. line i. e. extension proportional to applied force. k is the negative gradient ($-dF/dr$) of tangent to the curve at E. Thus is Hooke's law physically explained.

Elastic Potential Energy. Elongation by the load against elastic forces means work done against a force; this generates potential energy that is stored up in the lengthened spring. For producing an elongation of l against the direction of F_e, the work done will be $-kl$ and the extra work done in further elongating by dl will be

$$dW = -kl(-dl)$$

So the total work done for an elongation of l will be

$$W = \int_0^l kl\,dl = \tfrac{1}{2}kl^2 = \tfrac{1}{2}kl.l$$

$$= \tfrac{1}{2} \text{ Applied force} \times \text{extension} \qquad \text{(II-3.2.2)}$$

Note that work done must be +ve as it involves a squared directed quantity (l). This means that same work is done if l is compression instead of elongation.

Example. II-3.1. *The force constant of a spring of length L is K. It is cut into two parts of lengths l and l' where l = nl', n being an integer. Find k and k' the force constants of the two parts.*

Solution: Here $L = l + l' = l + l/n = l(n+1)/n$

and similarly $L = l + l' = (n+1)l'$

Now for a given spring $k \propto 1/l$ or lk = const

$\therefore$ $kl = KL = Kl(n+1)/n$ or $k = K(n+1)/n$

and similarly $k' = K(n+1)$

Ex II-3.2. *A 20 ton railway engine struck a wall and each of its two buffers was compresed by 10 cm. Find the energy stored up in them if the force constant of the spring is 1 ton-wt per cm [1 ton = 1000 kg]*

Solution : By Hooke's law compression is proportional to force. So the force on each buffer $= 10\times 1$ ton-wt $= 10\times 1000\times 9.8\text{N} = 9.8\times 10^4\text{N}$

$\therefore$ P.E. stored in the two buffers $= 2\times\frac{1}{2}$ Applied force $\times$ compression

$$= 2\times\tfrac{1}{2}\times 9.8\times 10^4\times 10^{-1}$$
$$= 9.8\times 10^3\,\text{J}$$

Problem. 50 kg-wt stretches a wire by 1 cm. How much P.E. will it aquire if stretched by 2 mm ? (*Ans.* 1J)

II-8.3. Some Definitions : The concepts elaborated here have been introduced in the context of Collisions in Chapter I-3.

The material of a body is said to be **perfectly elastic** if on the removal of deforming forces it regains completely its former shape and size.

When the material retains its deformed shape and size unchanged, after the deforming forces are removed, the material is said to be **perfectly plastic.**

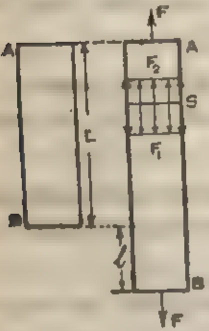
A body is said to be **perfectly rigid** when no amount of applied forces can deform it.

All of these three are idealised abstractions only, they do not exist in nature. All real bodies suffer more or less deformation when deforming forces are applied ; also they return more or less to their initial dimensions when such forces are removed. They are said to be **partially elastic,** We shall discuss only such bodies. They would all be solids. In the discussions that follow *a force must be taken as a balanced system of forces.* Deformed bodies take some time to regain their original dimensions. This behaviour is said to be due to **elastic hysteresis** (*hystere*—to lag behind) which of course changes with materials. Joule had observed the glass bulb of a sensitive thermometer to go on contracting for 40 years after it had been strongly heated. Further, we shall consider only *homogeneous* (same density everywhere) *isotropic* (same property in all directions) bodies under *isothermal* (unchanged temperature) conditions only. Crystals are anisotropic ; and change in temperature affects elastic properties of a material. Remember, under a balanced system of forces, a body does not move as a whole

but its parts suffer relative displacement and it is said to be strained.

Strain. If the shape or size of a body changes under the action of a pair of balanced forces, the body is to be *strained.* The change in shape or size that it undergoes is its *deformation.* The fractional deformation or the relative change so produced, is its *strain.*

Stress. When a body in equilibrium is strained, forces applied to it are transmitted throughout the body, setting up reactional forces within the body. Then *if we consider an imaginary surface* (s) *within the body, the part of the body on one side of the plane will exert a force across the surface on the other part. Such a force per unit area of the surface is called* **stress.** Stress is measured by the force per unit area. The force can be divided into two components: One perpendicular to the surface and the other parallel to the surface. The perpendicular component is called the **normal stress,** and the parallel component, the **tangential stress.**



(a) (b)

Fig. II-3.2

Fig. II-3.2(a) shows a rod AB of original length L. A longitudinal pull due to a force F has increased its length by l (fig. II-3.2b). If we consider any imaginary plane S anywhere perpendicular to F, the portions of the rod both above and below S are in equilibrium. Consider the equilibrium of the upper portion. The upward force F on it must be balanced by an equal and opposite force (F_1) acting across its lower surface. Similarly, the lower portion must be acted on by a force F_2 upward across S. As there is equilibrium, we must have $F_1 = F_2 = F$. The lower portion exerts the force F_1 across S on the upper portion. The upper portion exerts the force F_2 on the lower portion, also across S F/S is the longitudinal stress. Had we taken S parallel to F, the stress across it would be zero.

Many authors prefer to define stress as the applied force/unit area, as $F_1 = F_2 = F$, internal forces equal to external ones.

Units of strain and stress. Let a piece of wire, rigidly

supported at its upper end, be stretched by a load at its lower end As the load increases, so does the elongation. If

e = the elongation for a load F,
l = the initial length of the wire,
a = area of cross section of the wire,

then strain $= e/l$ and stress $= F/a$. Strain, being a ratio of two similar quantities, is expressed as a *pure number*. F is in reality a force. Hence stress is to be expressed as a *force per unit area*. Note that this is also how we shall express a pressure. Often F is expressed in mass units, like kg-wt, lb-wt, ton-wt etc.

In cgs, mks and fps systems units of stress are respectively dyn/cm^2, Newtons/m^2 (nowadays called pascal) and lbf/in^2.

The dimension of stress will be F/A or $MLT^{-2}/L^2 = ML^{-1}T^{-2}$

Ex. II-3.3. *A wire, 2 metres long, elongates by 1 mm under a load. What is the strain ?*

Solution : Strain = elongation/length = 1 mm/2 metres = 1/2000 = 0.0005.

Ex. II-3.4. *What must be the elongation of a wire 5 metres long so that the strain is 0.1 of 1% ?*

Solution : 0.1 of $1\% = \frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000}$.

$\therefore$ Elongation = strain × length $= \frac{1}{1000} \times 5$ metres $= 5$ mm.

Ex. II-3.5. *If a wire has a cross-section of 1 mm^2 and is stretched by a load of 10 kg what is the stress ?*

Solution : Stress = Load/area = 10 kg-wt/1 mm^2 = 1000 kg-wt/cm^2 or 10^6 gm-wt/cm^2 = $10^6 \times 980$ dynes/cm^2 = 98×10^6 N/m^2.

Elastic limit. In the case of a stretched wire, we find that the wire regains its initial length on removal of the load if the load does not exced a certain limit. If the limit is exceeded a permanent increase in the length occurs This limit is known as the *elastic limit*. Such a limit exists for all kinds of strain.

If we plot the extension of the wire under different loads, we get a curve as shown in fig. II-3.3. The graph is a straight line upto a certain point. The load corresponding to this point divided by the cross section of the wire gives the elastic limit. Upto the elastic limit, the extension is proportional to the load i.e. stress proportional to strain.

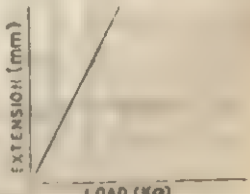


Fig. II-3.3

II-3.4 A Hooke's Law. The fundamental law relating to elasticity was discovered by Robert Hooke. As modified by Young it states that **so long as the elastic limit is not exceeded the stress is proportional to the strain.**

$$\text{Stress} = \text{a constant say, } E \times \text{strain.}$$

The constant E has a value characteristic of the material of the elastic body, and is called the *modulus of elasticity*. There are different moduli of elasticity depending on the nature of the strain.

The modulus depends upon the material and nature of the body, the nature of stress and temperature. Generally it diminishes with rise in temperature. Small amounts of impurities also affect the modulus which is sometimes deliberately introduced to attain a required goal. By the nature of stress we recognise three, namely *longitudinal*, *volume* and *shear*. When a wire is pulled or a thin rod pushed with equal forces from both sides, the stress is longitudinal. When a body is compressed or tensed from all sides, the stress is one of volume. When a pair of unlike opposite parallel forces or couples change the shape of a body, a shear stress is said to be applied.

B. Verification of Hooke's law. To verify Hooke's law we can use a spiral spring. We take its elongation under a load as the strain, and the load itself as the stress.

Fig. II-3.4

Clamp a metre scale vertically with the zero mark uppermost (fig. II-3.4). Suspend the spring from a rigid clamp. Attach a long pin to its lower end so that the tip of the pin moves over the vertical scale. Attach a weight to the bottom of the spring so that any kinks in the spring are removed. Note the scale reading (l_0). Gradually increase the load W at the end of the spring. (The kink-removing load will always be there, but it should not be included in W.) For each load, note the length l of the spring from the scale.

Plot graphically the load W along the x-axis and the corresponding elongation $(l - l_0)$ along the y-axis. The graph may be a straight line all along (fig. 11.3.3). This shows that stress is proportional to strain and the elastic limit has not been exceeded. Or, you may find that the graph, after running straight upto a certain distance, slowly bends (fig. 11.3.8). The point of bending marks the **elastic limit of the spring**. The load W corresponding to this point or the corresponding elongation $(l - l_0)$ may be taken as the elastic limit of the spring.

C. **Spring balance measures a force.** The above diagram illustrates the *principle* of a spring balance widely used to measure a weight. Hooke had first discovered that the extension of a spring is proportional to the load it carries, an observation that ultimately led to the law associated with his name. It holds whenever a spring fixed at one end is pulled axially in any direction. The ratio force/extension is the spring constant, we have already come across.

Fig. 11.3.5

A spring balance (fig. 11.3.5) has a spiral spring attached firmly to the body of the instrument carrying a hook at the lower end for suspending weights. Known loads are hung and the elongations graduated in terms of weights, on the scale S. A pointer slides along it and reads off the unknown weight suspended at the hook. The figure gives you the appearance, working and principle.

The instrument measures the weight (mg) of a body i.e. the force with which the earth attracts it. Since mass of a body remains unchanged it can be used to measure g at different places on, above or below the surface of the earth.

D. Vibration of a weightless spring carrying a point mass at the end [I. I. T. '78]

Consider the spring to be of length L and negligible weight hanging vertically from a fixed support. Let it carry a mass m at its lower end. The spring extends a little, the resultant increased tension supporting the load.

Let the load be pulled down by a small distance a from this position (Fig. II-3.6). The spring will then exert a restoring pull on the load which, for small extensions, is proportional to the extension. The additional upward pull on the load will therefore be kx, where k is the pull per unit extension of the spring. The equation of motion of the load will be given by

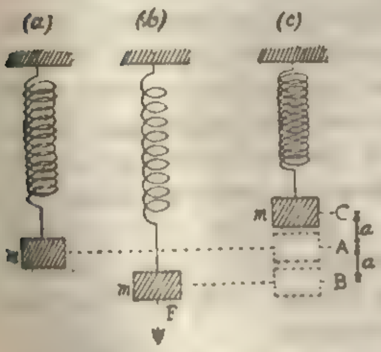


Fig. II-3.6

$$mf = -ka$$

or $f = -(k/m)a = -\omega^2 a$.

Acceleration being proportional to a the motion of the load is simple harmonic with a periodic time

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad \therefore \quad n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{k/m}$$

Next we find the force constant k in this case. From Hooke's law, here

Stress $= Y \times$ (Strain) where Y is Young's modulus of the material

or $F/s = -Y.a/l$ where F ($= mf$) is the restoring force, l the length and s the area of cross-section of the wire.

$\therefore \quad f = -(Ys/ml)a$ or the force const $k = f/a = Ys/ml$

$$\therefore \quad n = \frac{1}{2\pi}\sqrt{Ys/ml} \qquad \text{(II-3.4 1)}$$

II-3.5. Modulus of Elasticity :

The more important amongst them are (1) Young's modulus (2) Bulk modulus (3) Modulus of Rigidity and (4) Poisson's ratio. The last however is a ratio of two strains in perpendicular directions and hence a pure number. Besides these, axial and dilatational elasticities are also recognised but we shall not consider them.

Solids possess all these moduli while fluids (liquids and gases) only the bulk modulus.

In considering elastic deformations we *assume* that

(i) *Strains occur isothermally.* Actually slight thermal changes do occur. So you should wait a little before taking readings after loading a wire

(*ii*) *The elastic behaviour of a body is independent of its past history* i.e. it carries no trace of its earlier stresses and strains. Actually it may do so, because of *elastic hysteresis*. This is another reason for you to wait a little before taking readings after loading or unloading a wire.

(*iii*) *Deformations take place freely always*, without any opposition.

(*iv*) *Strains are small enough* for their higher powers or products to be negligible.

(1) **Young's modulus** $= \dfrac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$ or $Y = \dfrac{F/A}{\delta l/l}$ (II-3.5.1)

where the axial force applied is F on an area of cross-section A, δl is the change in length on a wire or thin rod of length l. Force and strain act a long the same direction.

(2) **Bulk modulus** $= \dfrac{\text{Volume stress}}{\text{Volume strain}}$ or $K = \dfrac{p}{-\delta v/V}$ (II-3.5.2)

Here p is the pressure applied normally on a body of volume V, producing a *diminution* of δv of it

(3) **Rigidity modulus** $= \dfrac{\text{Shearing stress}}{\text{Shearing strain}}$

or $n = \dfrac{F/A}{\tan \theta}$ or $\dfrac{\text{Couple } (c)}{\text{Twist } (\theta)}$ (II-3.5.3)

where F is the tangential force on an area A of a rectangular solid and θ its shift from the vertical or twist from the original direction. Recall that $\tan \theta \to \theta$ when θ is small.

From material to material their values differ but always are of the order of 10^{11} dyn/cm^2 or 10^{10} pascal (i.e. N/m^2).

(4) **Poission's ratio** $= \dfrac{\text{Lateral strain}}{\text{Longitudinal strain}}$, $\sigma = -\dfrac{\delta r/r}{\delta l/l}$ (II-3.5.4)

where δr is the change in radius r of a wire when its length changes by δl. These two changes occur in opposite directions. Hence the $-$ve sign, i.e. increase in length leads to a decrease in radius and vice versa.

Inter-relations exist between these constants for a solid. They are

$$Y = \frac{9kn}{3k+n} = 2n(1+\sigma) = 3k(1-2\sigma) \text{ and } \sigma = \frac{3k-2n}{6k+2n} \qquad \text{(II-3.5.5)}$$

The three moduli of elasticity have the same dimension as stress namely $ML^{-1}T^{-2}$ as strains in all cases are pure numbers.

The elastic constants determine the behaviour of homogeneous, isotropic elastic bodies under different stresses. They are very important in engineering practice. Elongation and contraction of beams and struts as well as bending of beams under load are determined by Young's modulus, and twist of wires and rods by the modulus of rigidity.

Distinction between solids, liquids and gases on the basis of the three elastic moduli. Solids have all the three moduli of elasticity—Young's modulus, bulk modulus and shear modulus. Liquids and gases have bulk modulus only. The difference between liquids and gases lies in the large value of the bulk modulus for the former, and small value for the latter. The reciprocal of bulk modulus is *compressibility*. Liquids have very little compressibility (an ideal liquid is taken to be incompressible). Gases are highly compressible. The bulk modulus of a gas at a fixed temperature comes out to be equal to its pressure.

The fact that liquids and gases have no shear modulus is also expressed by saying that *they cannot resist a shearing force.* Any tangential force, however small, causes them to flow. Hence liquids and gases together are known as *fluids.*

Elastic moduli and molecular structure. In *solids*, atoms and molecules are arranged in a regular geometrical pattern. We have seen (fig. II-2.4) that both attractive and repulsive forces act between them. The equilibrium position of atoms and molecules is determined by these forces. For closer approach than the equilibrium distance, there will be repulsion Any force tending to increase this distance, brings attractive forces into play. A solid will therefore resist any force tending to alter the relative positions of its constituent atoms or molecules. It will resist any change of length, volume, and also of twist or shear. It will therefore have all the three moduli of elasticity.

In *liquids*, the molecules are free to move, but the mean distance between neighbouring molecules remains constant at a given temperature. So a liquid can resist only a change of volume, but not any

change of shape. Due to mobility of molecules it takes up the shape of the container. It thus has bulk modulus, only, but no rigidity modulus or Young's modulus. Application of a horizontal force to a liquid molecule will cause it to move continuously under the force. But in a solid, such a molecule will be displaced by only a finite amount as restoring forces due to rigidity come into play. *The fact that a liquid cannot resist a shear, is of basic importance*; it explains the hydrostatic behaviour of liquids.

Gases are like liquids. Their molecules are much farther apart than in liquids and are much more free to move than liquid molecules. The expansibility of gases is due to the very weak attracting forces between molecules. Gases have bulk modulus only; but it is much smaller than that of liquids.

Some examples of elastic moduli in practical life. Young's modulus (Y) and shear modulus (n are of great importance in structural engineering. The ability of pillars and struts to support loads is determined by the Young's modulus of their material. Horizontal beams supporting loads are both elongated and compresed; they are also sheared. Hence for them, both Y and n are of importance. Compression or extension of springs is determined mainly by Y when the turns are very close. But when the turns are more open n also plays an important part.

Thermal expansion or contraction can exert a large force. The force is determined by the temperature change, area of cross-section and Young's modulus of the material. (See II-3.6C).

The performance of rotating axles in machinery in transmitting power is determined by its modulus of rigidity (n).

The high value of the bulk modulus (K) of liquids makes them a suitable substance for absorbing shocks, such as recoil of guns.

Examples of this kind may be multiplied without limit.

II-3. 6 Longitudinal Deformation.

A Young's modulus : In introducing elasticity and developing relevant concepts, we have relied mainly on wires or light springs fixed at the upper end and loaded at the lower. We may use as well, thin long rods as the experimental bodies. They may be pulled away at the two ends or pushed inwards from both ends.

In fig II-3.2(a) we see a long elastic bar AB of initial length l and cross-section α ; in (b) the bar has been stretched by a pair of equal unlike axial forces FF when the bar is subjected to **tension**. Let the bar be extended by δl. We consider a plane at right-angles at S through which forces of stress act in opposite directions and we study the equilibrium above and below. Considering the equilibrium of AS we have *external* force F upwards and an equal *internal* force acting downwards. The same happens for equilibrium of the portion BS. The forces act all over the cross-section as shown. The same will happen if a **compression** occurs when F, F are reversed in direction. These internal forces resist the effort of externally applied forces to elongate the rod and tend to restore the rod to the initial length. The rod is said to be in a state of longitudinal stress under the action of such forces in its interior. The entire rod is under such stress, for S was any arbitrarily chosen plane. This stress is given by F/α while the strain is $\delta l/l$.

Their ratio, if temperature remains constant, called the **Young's modulus** (Y, is experimentally found to be independent of sizes of the bar of the same material and a constant within elastic limit. It is a property of the material as the constant value changes with material.

Tensile and compressive strain The extension of a wire or a spring under a load represents the kind of strain known as *tensile strain* The corresponding stress is called *tensile stress.* If the force is one of compression along a direction instead of extension as above, the corresponding strains and stresses are called *compressive strains* and *compressive stresses.* If we load a vertical pillar at the top it will be compressed.

When a beam supported at the ends is loaded in the middle, the depression is proportional to the load so long as a certain limiting load is not exceded. A portion of the beam is compressed and a portion elongated. Both tensile and compressive strains and stresses appear in the beam together.

In fig II-3.7 is shown a bar AB supported at or near two ends. When loaded at the middle only the central strand ob maintains its

original length ; those above are compressed while those below are extended along their lengths This behaviour is attended to in bulding cantilever bridges and setting long iron beams under roof tops.

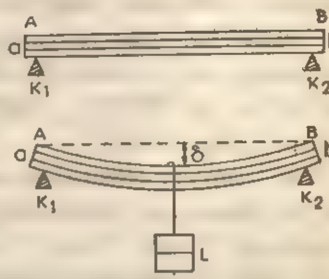


Fig. II-3.7

Thus Young's modulus of the material of a bar or wire under tension or compression is the ratio of the longitudinal stress to longitudinal strain, that bar remaining at constant temperature and strained within elastic limit and is a constant given by the relation

$$Y = \frac{F/a}{\delta l/l} \qquad \text{(II-3.6.1)}$$

As fluids can have no length, Y is the elastic modulus peculiar to solids only.

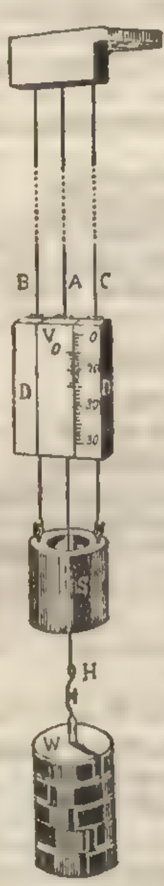


Fig. II-3.8(a)

B. Determination of Young's modulus. Suppose the experimental material is available in the form of a uniform wire. We take three pieces of it, each about 3 metres long and suspend them from a common support (fig.II-3.8). One of the wires A carries a finely etched vernier (V_o) and is kept free of kinks by a weight at its lower end. Two wires B and C carry a scale beside C which can slide along V_o. They are also kept free of kinks by a suitable load (S). attached to the lower end The vernier reading is taken with no other load. Additional known loads are then placed on W and the vernier reading taken for each additional load. After getting about 5 or 6 such reading we calculate the extension for each load. Measuring the diameter of the wire at several places we can get its mean diameter and hence its cross-section a. The length l of the wire from its

point of attachment at the top to that with the vernier is measured. We now have the stretching force $F = Wg$ (where g is the acceleration due to gravity), the cross-section a, the length l and the elongation e for each load. Plotting extension against load we get a st line fig. II-3.8(b). Find l' and W corresponding to the point P and apply eqn. II-3.6.1

Two wires are required to neutralize the effect of change of length due to change of temperature, as also of the sag of the support due to loading.

Ex. II-3.6. *An iron wire 2 metres long and of diameter 1 mm, stretches by 1 mm when a stretching load of 8 kg is applied to it. Find the strain, stress and Young's modulus.*

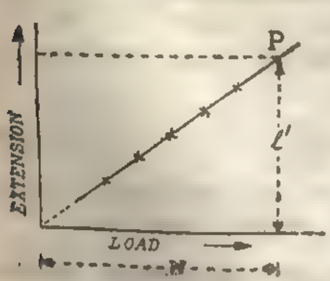


Fig. II-3.8(b)

Solution : Stretching force = 8 kg-wt = 8000 × 980 dynes. Area of cross-section $= \pi r^2 = \pi \times (0.1/2)^2 = 3.142 \times 0.0025$ cm^2.

$$\therefore \quad \text{Stress} = \frac{8000 \times 980 \text{ dyne}}{3.142 \times 0.0025 \text{ cm}^2} = 9.984 \times 10^8 \text{ dyn/cm}^2.$$

Original length = 2 metres = 200 cm.

Increase in length = 1 mm = 0.1 cm.

$\therefore$ Strain = 0.1 cm/200 cm $= 5 \times 10^{-4}$.

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{9.984 \times 10^8 \text{dyn/cm}^2}{5 \times 10^{-4}} = 1.997 \times 10^{12} \text{ dyn/cm}^2$$

(Try using MKS units from the beginning $Y = 1.997 \times 10^{11}$ N/m^2).

Ex. II-3.7. *Find the load in kg required to elongate a steel wire 628 cm long and 2 mm diameter through 1 mm. $Y = 2 \times 10^{12}$ dyn/cm^2, $g = 980$ cm/s^2.*

[H.S. '64]

Solution : Given data in MKS units would be

$L = 6.28$ m (Note that it is 2π), $Y = 2 \times 10^{11}$ N/m^2, $g = 9.8$ m/s^2, $d = 2 \times 10^{-3}$ m $l = 10^{-3}$ m

$$\text{Now} \quad Y = \frac{F/\alpha}{l/L} = \frac{4mgL}{\pi d^2 l} \quad \therefore \quad m = \frac{Y\pi d^2 l}{4gL}$$

$$= \frac{2 \times 10^{11} \times 3.14 \times 4 \times 10^{-6} \times 10^{-3}}{4 \times 9.8 \times 6.28} = \frac{10^2}{9.8} = 10.2 \text{ kg}.$$

Ex. II-3.8. *A sphere of mass 25 kg and radius 10 cm is hung by a steel wire from a roof top which is 5.21 m from the floor. When the sphere swings like a pendulum bob its lowest point just scrapes the floor. Find its velocity then.*

$Y = 20 \times 10^{11}$ dyn/cm^2. Initial length of the wire = 5m, its radius = 0.05 cm.

[I. I. T. '72

Solution : At the lowermost point the extension of the wire is $[521 - (500 + 20)]$ cm. This elongation produces a stress (T) in the wire, tensile in nature which

balances the weight of the ball mg and also supplies the centripetal force (mv^2/r) required to make the sphere describe the circular arc.

$$\therefore \quad T = mg + mv^2/r$$

Again from elastic deformation $T = \dfrac{Y\pi r^2 l}{L} = \dfrac{2\times10^{12}\times\pi\times(0.05)^2\times1}{500}$ dyn

$$\therefore \quad 25\times10^3\times980 + \frac{25\times v^2\times10^3}{(500+10+1)} = \frac{2\times10^{12}\times\pi\times25\times10^{-4}\times1}{500}$$

$$\text{or} \quad 25\times10^3\left(980+\frac{v^2}{511}\right) = \pi\times10^7$$

$$\therefore \quad v = 3.75 \text{ m/s}$$

Ex. II-3.9. *A body of mass 2 kg and density 2.7 g/cc is suspended by a steel wire 1 m long and 1 mm in radius. What change in length occurs when it is fully immersed in water? $Y = 2\times10^{12}$ dyn/cm^2 and $g = 9.8$ m/s^2*

Solution : When the hanging body dips in water it appears to lose a part of its weight by Archemedes principle. Now volume of displaced water = volume of the body = 2000/2.7 cc. So buoyancy on the body = (2000 × 980)/2.7 gf acting up wards.

$\therefore$ Apparent wt of the body $= mg - V.1.g = 2\times1000\times980-(2000/2.7)\times1\times980$

$$= 2000\times980(1-1/2.7) \text{ dyn}$$

$$= 2\times(1.7/2.7)\times98\times10^4 \text{ dyn}$$

So with the body in water from $Y = \dfrac{F}{\alpha}\times\dfrac{L}{l'}$ we have

$$l' = \frac{F\times L}{\alpha\times Y} = \frac{2\times(1.7/2.7)\times98\times10^4\times100}{\pi\times2\times10^{12}\times(0.1)^2} = \frac{(1.7/2.7)\times98\times10^{-4}}{\pi} \text{ cm}$$

Again with the body in air we have

$$l = \frac{2\times98\times10^4\times100}{\pi\times(0.1)^2\times2\times10^{12}} = \frac{98\times10^{-4}}{\pi} \text{ cm}$$

$$\therefore \quad (l-l') = \frac{98\times10^{-4}}{\pi}\left(1-\frac{1.7}{2.7}\right) = \frac{98\times10^{-4}\times1}{\pi\times2.7} = 1.15\times10^{-4} \text{ cm}.$$

Ex. II-3.10. *A steel wire 7 metres long and 1 mm in diameter is subjected to a tension of 30 kg, and its elongation is observed to be 1.21 cm. Find the strain, the stress, and Young's modulus for the specimen.*

Solution :

Elongational strain $= \dfrac{\text{Change in length}}{\text{Original length}} = \dfrac{1.21 \text{ cm}}{700 \text{ cm}} = 0.00173$

Stress = Applied force per unit area $= \dfrac{30 \text{ kg}}{\pi(0.5 \text{ mm})^2} = 38.2$ kg/mm^2.

$\therefore$ Young's modulus $= \dfrac{\text{Stress}}{\text{Strain}} = \dfrac{38.2}{0.00173}$ kg/mm^2 $= 2.208\times10^4$ kg/cm^2.

Ex. II-3.11 *A load of 60 tons is carried by a steel column having a length of 24 ft and a sectional area of 10.8 in^2. What decrease in length will this load produce if the elastic modulus is 30×10^6 lb/in^2?*

Solution : Let the decrease in length be x in.

Strain $= \dfrac{x \text{ in}}{24\times12 \text{ in}}$. Stress $= \dfrac{60 \text{ ton}}{10.8 \text{ in}^2} = \dfrac{60\times2240}{10.8}$ lb/in^2.

Now, elastic modulus $=\frac{\text{Stress}}{\text{Strain}}$. Hence 30×10^6 lb/in^2 $=\frac{60\times 2240}{10.8}$ lb/in^2 $\div \frac{x}{24\times 12}$

$\therefore \quad x=\frac{60\times 2240\times 24\times 12 \text{ in}}{10.8\times 30\times 10^6}=0.112$ in.

Ex. II-3.12 *A st el wire of length 2 metres and diam ter 0 8 mm is stretched horizontally between rigid supports attached at its ends. When a load is hung from the mid-point of the wire a depression of 1 cm is produced. Calculate the load, g ven that $Y=2\times 10^{12}$ dyn/cm^2.*

Solution : Draw the necessary diagram. The load is supported by the vertical components of the tens ons in the two segments of the depressed wire. Half-length of the stretched wire $=\sqrt{100^2+1}=100.005$ cm. Its extension $=0.005$ cm. If θ is the angle the stretched wire makes with the vertical line through the point of attachment of the load, $2T\cos\theta=$ load, where T is the tension in the wire, $\cos\theta=$ depression/half-length of stretched wire $=1/100$ very nearly.

$\therefore$ Tension $T=50\times$ load. The stretching force per unit area or stress $=T$/area of cross-section $=50\times$ load$/(\pi\times 0.04^2 \text{ cm}^2)$. Strain $=0.005/100$.

$\therefore \quad 2\times 10^{12}$ dyn/cm^2 $=\frac{50\times \text{load}}{\pi\times 0.04^2 \text{ cm}^2}\div\frac{0.005}{100}$ whence load $=10.3$ g-wt.

Ex. II-3.13. *A material aquires a permanent set if strained beyond $\frac{1}{1000}$. If its material has $Y=10^7$ lbs/in^2 find the maximum load that a wire of the material of diameter 0.04 in can support without crossing the elastic limit.* [J.E.E. '71]

Solution : Maximum stress $=Y\times$ maximum strain

$$=10^7\times\frac{1}{1000}=10^4 \text{ lb/in}^2$$

Applied Load = max. stress $\times$ area of cross-section

$$=10^4\frac{\text{lb}}{\text{in}^2}\times\pi\times 4\times 40^{-4} \text{ in}^2=4\pi \text{ lb-wt.}$$

Ex. II.3.14. *What force will be required to stretch a steel wire 1 sq. cm in cross-section to double its length? Y for steel $=2\times 10^{12}$ cgs units.*

Ans. By definition, Young's modulus

$$Y=\frac{\text{applied force }(F)/\text{area }(\alpha)}{\text{elongation }(e)/\text{original length }(l)}.\quad \text{Here } \alpha=1 \text{ cm}^2 \text{ and } e=l$$

$\therefore \quad F=Y\times(e/l)\times\alpha=2\times 10^{12}\times 1\times 1$ in cgs units $=2\times 10^{12}$ dynes.

(*Note.* A wire snaps long before its length has been doubled. Besides, with elongation, the cross-section diminishes)

Problems. (1) A steel wire ($Y=2\times 10^{12}$ dyn/cm^2) one meter long and one mm radius supports a 5 kg load. Find its length when the load is removed.
(*Ans.* 99.92 cm) [H. S. '78]

(2) A 200 cm long wire of 1.22 mm diameter is tightly stretched horizontally. A load at its middle point makes it sag by 2 cm. Find the load.
Given $Y=12.3\times 10^{11}$ dyn/cm^2. (*Ans.* 115 g)

(3) A load of mass 5 kg elongates a vertical wire 200 cm long and 1 mm radius by 0.07 cm. Given $g=10$ m/s^2 find Y of its material. [J.E.E. '76]
(*Ans.* 5×10^{11} N/m^2)

C. Thermal stress : A heated rod expands. If held between solid supports the expansion presses against them. The stress so genearated is called the thermal stress.

If a rod of initial length l and temp. coefficient of its material for expansion α is heated through a temperature difference of t°C then its expansion $l\alpha t$. Then we shall have for its Young's modulus

$$Y=\frac{\text{Long. stress}}{\text{Long. strain}}=\frac{F/A}{l/L}=\frac{F/A}{\alpha t} \tag{III-3.6.1}$$

So thermal stress is $F/A=Y\alpha t$ [Note that it is independent of the dimensions (length and area of cross-section) and dependent only on the material of the rod (Y and α)]. Now the force developed is

$$F=YA\alpha t \tag{III-3.6.2}$$

which is independent of length but depends on the area of cross-section. This force may be considerable.

II-3.15. *A steel rod 25 cm long has a cross-sectional area of 0.8 cm^2. What force would be needed to stretch it by the same amount as expansion produced in it by raising the temp through 10°C? $Y=2\times10^{12}$ dyn/cm^2 $\alpha=10^{-6}$/°C.* [I.I.T '71]

Solution : Thermal expansion of the rod $\delta l=l\alpha t=25\times10^{-6}\times10$ cm.

This is the stretching of the rod by the required force which then will be

$$F=AY.\delta l/l=AY.\alpha t=0.8\times2\times10^{12}\times10^{-6}\times10=16\times10^{7}\text{ dyn}$$
$$=16\times10^{2}\ N=16.1\text{ kg-wt}$$

Ex. II-3.16. *A wire of diameter 1 mm supports a load which keeps it straight. If the temperature falls by 20°C what additional load will be required to keep the length of the wire unchanged? ($Y=2\times10^{12}$ cgs units ; coefficient of linear expansion $=1\times10^{-6}$ per °C)*

Solution : Let l be the initial length of the wire. Its contraction due to fall in temperature $=l\times10^{-6}\times20$. The extra load must produce this much of elongation. Hence the strain should be $l\times10^{-6}\times20\div l=20.10^{-6}$.

The cross-section of the wire $=\pi\times(0.05)^2=0.0025\pi$ cm^2. If the extra load is W g-wt, the stress is $980\ W/0.0025\pi$ dyn/cm^2.

$$\therefore\quad 2\times10^{12}=\frac{980W/0.0025\pi}{20\times10^{-6}},\text{ whence } W=3.2\times10^{3}\text{ (g-wt)}=3.2\text{ kgf.}$$

Problem : A steel wire of cross-sectional area 0.5 mm^2 is held just taut at 20°C. Find the tension when it cools to 0°C. $Y=2.1\times10^{11}$ N/m^2 ; $\alpha=12\times10^{-6}$/°C (*Ans.* 25.2N). [I. I. T. '73]

D Work done in stretching a wire : In stretching a wire its free end must be pulled against the resisting and restoring force of stress. Then work has been done by the external agent. The work

becomes the stored up potential energy of strain. This is a particular case of what we discussed in II-3 2. When the external force is *suddenly* removed the wire regains its original length when the *strain energy is released as heat.*

(1) If a tensile force F applied at the free end of a wire clamped at the other end of length L and area of cross-section A, lengthens by l we have $F = YAl/L$. Let the wire lengthen by a further small amount dl, then the small amount of work done is

$$dW = F.\, dl = (YA/L)\, l.dl$$

Then the total work done in stretching the wire through l will be

$$W = \frac{YA}{L} \int_0^l l \cdot dl = \tfrac{1}{2} \cdot \frac{Ys}{L} l^2 = \tfrac{1}{2} K l^2 \qquad \text{(II-3.6 3)}$$

formally similar to the eqn II-3.2.2. Now the volume of the wire is $L.A.$ So

$$\therefore \quad W/LA = \frac{1}{2} \cdot \frac{YS}{L} \cdot \frac{l^2}{L.S} = \frac{1}{2}\, \frac{YAl}{L}\, \frac{l}{L} = \tfrac{1}{2} \text{ stress} \times \text{strain} \qquad \text{(II-3.6.4)}$$

This relation gives the *strain energy stored up in unit volume* of the strained body. The result holds for all kinds of strains.

(2) The work done can be *alternatively* found thus : as the string elongates from 0 to l, the opposing stress force rises from zero to kl. So the average stress force must be $\frac{1}{2}kl$. The work done therefore for the free end to be displaced by l will be $\frac{1}{2}kl.l = \frac{1}{2}kl^2$.

$\therefore$ Elastic P. F. $= \frac{1}{2}$ Applied force $\times$ displacement

$=$ Average force $\times$ displacement (II-3.6.5)

(3) The work done on or energy stored in a stretched wire can also be found graphically by plotting F against e. For example let e_1 represent the extension for the load F_1 (fig II-3:8) and e_2 the extension with the load F_2. If F be any intermediate force and Δx the small extension for that, then

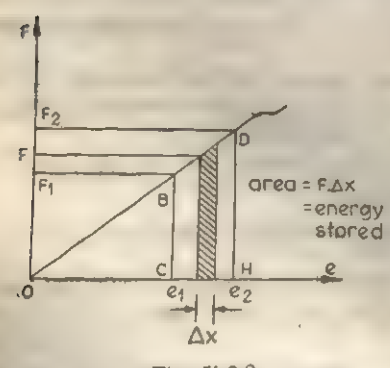


Fig. II-3.8

Work done $=$ Energy stored

$= F.\, (e_2 - e_1)$

Now you see that $F.\Delta x$ is represented by the small shaded area. So the total work done between e_1 and e_2 is given by the area $BCHD$ which is a trapezeum.

Area of trapezium $BCHD = \frac{1}{2}(BC+DH)\times CH$
$= \frac{1}{2}(F_1+F_2)(e_2-e_1)$

$\therefore$ Energy stored = Average force × increase in length

D. Rise of temperature on snapping of a loaded wire : When a loaded wire suddenly snaps it is found to rise in temperature. It so happenes because the potential energy stored in it is released in the form of heat leading to a rise in temperature which may be computed as follows :

Work done in elongating the wire is

W = Average force × elongation = $\frac{1}{2}$ Applied force × elongation

Again, elongation δl = stress × length/Young's modulus

$$\therefore \quad W=\tfrac{1}{2}F\times\delta l=\tfrac{1}{2}F\times(FL/YA)=\tfrac{1}{2}F^2L/YA \qquad \text{(II-3.6.6)}$$

Now conversion of this work into heat is $H=W/J$ where J is the mechanical equivalent of heat. If ρ be the density of the material of the wire, then its mass is $LA.\rho$. If s be the specific heat capacity of the material and its rise in temperature θ, would be given by

$$ms\theta = LA\rho s\theta$$

$$\therefore \quad H=LA\rho s\theta=W/J=\tfrac{1}{2}\frac{F^2L}{YA.J}$$

$$\therefore \quad \theta=\tfrac{1}{2}\frac{F^2}{YA^2\rho sJ} \qquad \text{(II-3.6.7)}$$

Ex. II 3.17. *A steel wire 2m long and of 1 mm² cross section is elongated through 0.1 mm. Find the work done and energy stored in unit volume.*

Solution : $W=\frac{1}{2}(YA/L)l^2=\frac{1}{2}\times2\times10^{12}\times10^{-2}\times0.0001/200=5\times10^{2}$ ergs

$$\text{(Energy/volume)}=\tfrac{1}{2}\text{ stress}\times\text{strain}=\tfrac{1}{2}\frac{Yl}{L}\times\frac{l}{L}=\tfrac{1}{2}\times2\times10^{12}\times\left(\frac{0.01}{200}\right)^2 \text{ ergs/cc}$$

$$=10^{12}\times\tfrac{1}{4}\times10^{-8}=2.5\times10^{3}\text{ ergs/cc.}$$

$$\text{Alternatively, }\frac{\text{Energy}}{\text{Volume}}=\frac{W}{LA}=\frac{5000}{200\times10^{-2}}=2.5\times10^{3}\text{ ergs/cc}$$

Problem : As the load on a wire is increased *slowly* (why ?) from 4 kgf to 6 kgf the elongation increases from 0.65 to 1.13 mm. How much work per unit volume is required during the extension ? (*Ans.* 0.0205J)

[*Hint :* Find the work done separately for the two loads and take the difference.]

Ex. II-3.18. *A 20 kg wt is suspended from a copper wire and 1 mm in radius. What will be the change in temp. if the wire suddenly breaks ? For copper Y = 12×10¹¹ dyn/cm², ρ = 9 g/cc and sp. heat = 0·1, J = 4.2 J/cal.* [Cambridge]

Solution : Work done in elongating the wire

$W=\frac{1}{2}$ Applied Force × increase in length $=\frac{1}{2}Fl$.

$$\text{Again} \quad l = \frac{F}{A} \cdot \frac{L}{Y} = \frac{20 \times 1000 \times 980 \times L}{\pi \times (0.1)^2 \times 12 \times 10^{11}} = \frac{2 \times 98L}{12\pi} \times 10^{-4} \text{ cm}$$

$$= \frac{49L \times 10^{-4}}{3\pi} \text{ cm}$$

$$\therefore \quad W = \tfrac{1}{2}\, 20 \times 10^3 \times 980 \times \frac{49L}{3\pi} \times 10^{-4} = \frac{980 \times 49L}{3\pi} \text{ ergs}$$

Now on sudden snapping, this strain energy gets converted into heat

$$\therefore \quad \frac{W}{J} = H = ms\theta = V\rho s\theta = L \times \pi r^2 \times s \times \theta$$

$$\therefore \quad L \times \pi \times (0.1)^2 \times 9 \times \theta = \frac{980 \times 49L}{3\pi \times 4.2 \times 10^7} \times \frac{\text{ergs}}{\text{ergs/cal}}$$

$$\text{or} \quad \theta = \frac{980 \times 49}{3\pi^2 \times 4.2 \times 10^7 \times 9 \times (0.1)^2} = 4.3 \times 10^{-4} \ {}^\circ\text{C}$$

Confirm by directly applying 11-3.6.7.

11-3.7. Poisson's ratio.

When a body elongates freely in the direction of a tensile force, it contracts laterally, i.e. in a direction perpendicular to the force. When a compressive force is applied, it expands laterally. When there is no bar to lateral contraction or expansion, *the ratio which the lateral change in length per unit length bears to the longitudinal change in length per unit length is called Poisson's ratio.* It is a *pure number*, being the ratio of two strains. *It is not strictly a modulus, only a fraction.*

$$\text{Poisson's ratio} = \frac{\text{lateral strain}}{\text{longitudinal strain}} \qquad (11\text{-}3.7.1)$$

It is clearly a property of solids mainly in the form of a wire or regular bar or rod. When a wire of length l is *elongated* by δl, its radius r *shrinks* by δr. On compressing a rod lengthwise its radius increases. The longitudinal strain is then $\delta l/l$ and the lateral strain $\delta r/r$. The changes are opposite and if one of them (say tensile strain) is +ve the other (lateral contraction) is −ve and vice versa. Hence for tensile strain

$$\text{Poisson's ratio } (\sigma)_T = \frac{(-\delta r)/r}{\delta l/l} \qquad \text{(A)} \qquad (11\text{-}3.7.2)$$

$$\text{and for compression } \sigma)_C = \frac{\delta r/r}{(-\delta l)/l} \qquad \text{(B)}$$

Its theoretical value from the last relation of eqn. (11-3.5.5) would lie between $+\frac{1}{2}(n = 0)$ and $-1\ (k = 0)$. For most materials it lies however between 0.2 and 0.4.

Ex. II-3.19. *Change in volume of a rubber cord due to change in linear dimensions is negligible. Show that $\sigma = 0.5$.*

Solution : For the cord $V = \pi r^2 l$. To find changes in V, r, l we have

$$dV = \pi(l\, 2r\, dr + r^2\, dl)$$

Since by condition $dV = 0$, we have $l\, 2r\, dr = -r^2 . dl$

$$\text{or} \quad \frac{-dr/r}{dl/l} = \tfrac{1}{2}$$

[Actually σ for rubber is 0.48. Metals generally increase in volume when stretched due to slipping of constituent crystals.]

Ex. II-3.20. *An aluminium rod is 2 metres long and 2 cm. in diameter. A load of 70 kg elongates it by 30 parts in a million. If Poisson's ratio for aluminium is 0.33, find the contraction in diameter due to the load.*

Solution : Longitudinal strain $= 30 \times 10^{-6}$, Lateral strain = longitudinal strain × Poisson's ratio $= 30 \times 10^{-6} \times 0.33 = 10^{-5}$.

$\therefore$ Contraction in diameter = diameter × lateral strain $= 2 \times 10^{-5}$ cm.

Ex. II-3.21. *Find the change in volume on stretching a wire 1 m long and cross section 0.1 cm² by a load of 10 kg if σ for its material is 0.33. Find also Y for the material.* [J. E. E. '76]

Solution $V = \pi r^2 l$. So change in volume $dV = \pi(l\, 2r\, dr + r^2\, dl)$

$$= \pi r^2\, dl\left(\frac{l . 2r dr}{r^2 dl} + 1\right) = \pi r^2 dl\left(1 + 2\frac{l . dr}{r\, dl}\right)$$

$$\text{Now } \sigma = -\frac{dr/r}{dl/l} \quad \text{or} \quad \sigma\, dl\, r = -l\, dr$$

$$\therefore \quad dV = \pi r^2\, dl(1 - 2\sigma) = A\, dl(1 - 2\sigma)$$

Putting these values $dV = 0.1 \times 0.1(1 - 0.66) = 0.01 \times 0.33 = 0.0033$ cc

$$Y = \frac{10 \times 1000 \times 980 \times 0.1}{0.1/100} = 9.8 \times 10^{10} \text{ dyn/cm}^2$$

Ex. II-3.22. *σ and Y for the material of a vertically suspended light rod of length 3 m and radius 3 cm are respectively 0.3 and 2×10^{12} dyn cm². Find the lateral strain when it is loaded by 1200 kg.* [H. S. '80]

Solution : We know that

lateral strain $= \sigma$ (longitudinal strain)

$$= \sigma\frac{F/a}{Y} = 0.3 \times \frac{1200 \times 980 \times 1000/\pi \times 9}{2 \times 10^{12}}$$

$$= 0.3 \times \frac{6 \times 98 \times 10^{-7}}{\pi \times 9} = 0.6 \times 10^{-5}$$

II-3.8 A. Shearing stress and strain Modulus of rigidity

A *stress* applied to a body in the plane of one of its faces is called a *shear*. It changes shape of the body, not its volume.

To understand what a shear is, we may think of a pack of cards or a thick book lying on a table. Laying the palm flat upon the pack of cards or the book we may apply a horizontal force (F so that the rectangular pile is changed into a parallelepiped, each card or the page sliding over the one beneath. The shape thus changes without change of volume. When an elastic body is changed in this way its *shape being changed without change of volume*, it is said to undergo a *shear*. A shear requires the application of a tangential force fig. II 3.9 illustrates a shear. The base DC of the rectangular block $ABCD$ is kept fixed and a tangential force F is applied to the top AB. As a result the body is deformed from the shape $ABCD$ to the shape $A'B'C'D$. The angle $ADA' = \alpha$ is the angle *of shear.*

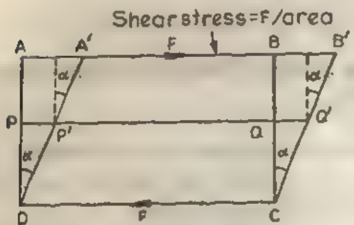


Fig. II-3.9 (a)

When a body is sheared, the ratio of the shearing stress to the shearing strain is called the *modulus of rigidity* (*or shear modulus*). If a tangential force F applied over an area α produces the shear, the shearing stress is F/α. If due to the shear a straight line perpendicular to the plane of shear becomes inclined to its original direction by an angle α, the shearing strain is taken as $\tan\alpha[=AA'/OA'](=\alpha$ when α is small, as it generally is). Thus the modulus of rigidity may be written as

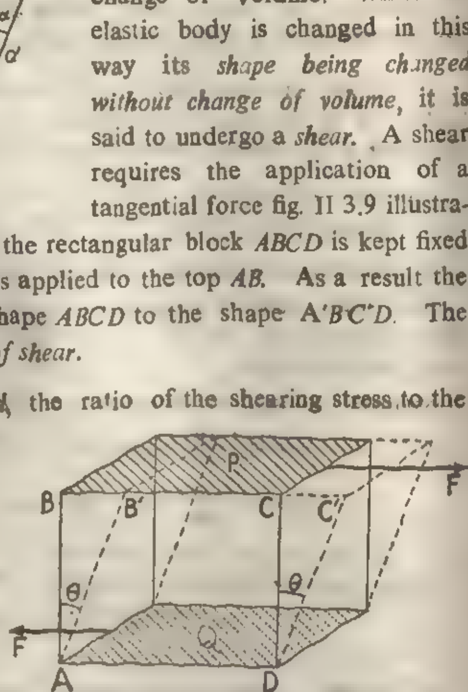


Fig. II-3.9 (b)

$$n=\frac{F/A}{\alpha}=\frac{F}{A\alpha} \qquad \text{(II-3.8.1)}$$

In cgs units it is measured in *dynes per sq. cm* (dyn/cm^2) like the other two elastic moduli. (G however is the internationally recommended symbol for modulus of rigidity).

Fig. II-3 9 (b) illustrates what we said above about shearing a pack of cards. Here the angle of shear is θ.

When two plates held by a rivet (fig. II-3.10) are pulled away from each other parallel to the common surface the rivet sustains a shear, its lower part tending to slide off the upper. The name shear is derived from the fact that when a piece of paper is severed by a pair of shears (scissors) precisely this kind of stress is applied.

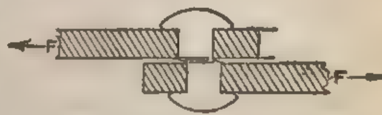


Fig. II-3.10

The twist in a wire or rod is proportional to the couple producing the twist, provided the latter does not exceed a certain value. The behaviour of a wire or rod under the action of a twisting couple is determined by its modulus of rigidity. We discuss it below.

Liquids and gases do not have any modulus of rigidity as the smallest shearing stress goes on increasing the shearing strain in them. They cannot be in equilibrium under a shearing stress. So **fluids** are defined as **those materials which cannot oppose a shearing stress.** Rigidity is a property of the solids only. The unit and dimensions of rigidity modulus are, as apparent, the same as those of Young's modulus.

Ex. II-3.23. *A square plate of metal 4 ft on a side and ½ in thick is subjected to a shearing stress which tends to twist the square surface into a rhombus. To apply this stress one edge is securely fixed, and a bar, fastened to the other edge, is pulled with a force of 180 tons. As a result the bar is observed to advance a distance of 0.069 in. in the direction of pull. Find the shearing strain, the shearing stress, and the coefficient of rigidity of the plate.*

Solution :

$$\text{Shearing strain } \theta = \frac{\text{displacement of the bar}}{\text{side of the square}} = \frac{0.069 \text{ in}}{4 \times 12 \text{ in}} = 0.00144$$

$$\text{Shearing stress} = \frac{\text{Applied force}}{\text{Area of the face parallel to it}}$$

$$= \frac{180 \times 2240 \text{ lb}}{\frac{1}{2} \text{ in} \times (4 \times 12) \text{ in}} = 16800 \text{ lb per sq in}$$

$$\text{Coefficient of rigidity} = \frac{\text{stress}}{\text{strain}} = \frac{16800}{0.00144} \text{ lb/in}^2 = 1.17 \times 10^7 \text{ lb/in}^2.$$

Ex. II-3.24. *An aluminium cube measuring 2 in. along each edge is subjected to a pair of parallel shearing forces applied to its opposite faces. How large must each of these forces be in order to shear the block through an angle of 0.01° ? ($n = 4.2 \times 10^6$ lb per sq in.)*

Solution : $\theta=0.01°=0.01\times\frac{3.14}{180}$ rad$=0.000174$ rad.

Now $n=\frac{F}{S}\cdot\frac{1}{\theta}$ or $F=n.S.\theta$.

$=4.2\times10^{6}\times2\times2\times0.000174$ lb wt$=2.9\times10^{4}$ lb wt.

B. Torsional deformation : Think of a thick rubber cord hanging from a hook. You can deform it by twisting the free end You can imagine of doing the same to a wire or cylinder. In these cases you need to apply a couple or torque at the free end instead of just a deforming force. An untwisting couple will be developed within the body and ultimately balance it. All suspended coil or suspended magnet instruments work on this principle.

That such a twist is equivalent to a shear can be understood from Fig. II-3.11. Through the cylinder imagine a rectangle *ODBX*. On twisting it becomes ODB_1X; it has suffered a shear. α radians is the *angular displacement* suffered by every point at the free end of the cylinder, θ is the *angle of twist* and *represents* the deformation or *strain*, again a pure number Under equilibrium, the stress couple is given by Hooke's law as

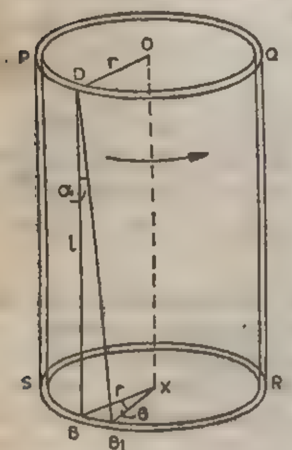


Fig. II-3.11

$$L=c\alpha$$

where c is said to be *torsion constant* the twist required for unit torsion. Angle of shear (α) between vertical lines is not equal to θ and is a constant while θ is not so for different sections between different radii.

If n be the rigidity modulus of the material of the cylinder of length and radius r then it can be shown that

$$n=\frac{2l\,c.}{\pi r^{4}} \qquad \text{(II-3.8 2)}$$

Ex. II-3.25. *A circular bar 1 m long and 8 mm diameter is rigidly clamped in a vertical position. A couple of magnitude $2{\cdot}5\times10^{7}$ dyne-cm. applied to the free end deflects a spot of light from a mirror on it through 15 cm on a scale 1 m away. Find n of its material.* [P. U.]

Solution : Lamp and scale arrangements are often utilised to measure angular deflections. The wire or bar that is to be twisted carries a very small

light mirror which reflects a very narrow beam of light on a meter scale. When the wire twists the mirror moves and the spot of light on the scale also shifts. This linear displacement divided by the lamp to-scale distance gives the angular deflection suffered by the reflected beam. Half this angle gives the rotation of the mirror and hence that of the wire, as you shall learn in the chapter on Reflection at plane mirrors. Hence from the problem the angular deflection

$$\theta=\frac{15}{200} \text{ rad. and torsion constant } c=\frac{L}{\theta}=\frac{2.5\times10^7}{15/200}$$

$$\therefore \quad n=\frac{2lc}{\pi r^4}=\frac{2\times100\times2.5\times10^7}{\pi(0.4)^4\times15/200}=8.3\times10^{11} \text{ dyn/cm}^2.$$

Work done in Shear : Refer to fig. II-3.9(b). If F be the tangential force acting over an area A, θ the angle of shear and $l\,(=AB)$ the distance between the extreme planes perpendicular to the direction of shear, then the displacement BB', $(=l\theta)$ is established by the average (from 0 along AD to F along BC) force of $\frac{1}{2}F$. So the work done is $\frac{1}{2}Fl\theta$ and energy of shear per unit volume or

$$\text{Energy density}=\tfrac{1}{2}Fl\theta/lA=\tfrac{1}{2}\frac{F}{A}\theta=\tfrac{1}{2}\text{ stress}\times\text{strain} \qquad \text{(II-3.8.3)}$$

II-3 9 Volume stress : Bulk modulus. The volume of a body whether solid, liquid or gaseous, can be reduced by applying a uniform pressure. If an *increase of pressure* Δp *reduces the volume* from V to $V-\Delta V$, the *volume strain* is $\Delta V/V$. The *volume stress* is Δp. The ratio of volume stress to volume strain is called *bulk modulus*, provided the shape of the body does not change.

$$\text{Bulk modulus }(K)=\frac{\text{volume stress}}{\text{volume strarn}}=\frac{\Delta p}{-\Delta V/V} \qquad \text{(III-9.1)}$$

Δp is measured by the force applied per unit area ; $\Delta V/V$ is a pure number. So K is expressed in *force per unit area.*

The reciprocal of bulk modulus is called the *compressibility* of the substance. Gases are highly compressible and have small values of K. For a gas the bulk modulus we shall show to be equal to its pressure. Both solids and liquids have high values of K, of the order of 10^5 kg-wt per cm^2. As obvious from above discussions and previously stated, bulk elasticity is the only elasticity

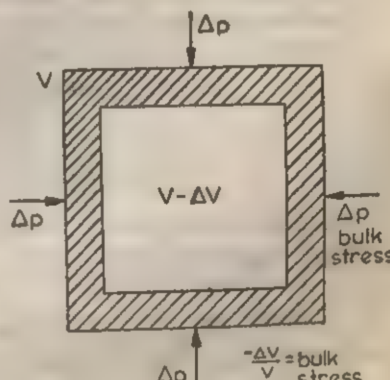


Fig. II-3.12

shown by fluids and it is the *compressibility that differentiates between the two classes of fluids.* viz liquids and gases. So far as this modulus is concerned, there is little difference between solids and liquids. The table below compares the different moduli of elasticity of several materials.

Table : Moduli of Elasticity

Substance	Young's modulus		Bulk modulus		Modulus of rigidity	
	$\frac{dyn}{cm^2}$	$\frac{lb\text{-}wt}{in^2}$	$\frac{dyn}{cm^2}$	$\frac{lb\text{-}wt}{in^2}$	$\frac{dyn}{cm^2}$	$\frac{lb\text{-}wt}{in^2}$
Aluminium	7×10^{11}	10×10^{6}	7×10^{11}	10×10^{6}	$2{\cdot}5\times10^{11}$	3.6×10^{6}
Copper	10 ,,	14 ,,	12 ,,	17 ,,	4·2 ,,	6·1 ,,
Iron (drawn)	20 ,,	29 ,,	9·6 ,,	14 ,,	5·1 ,,	7·4 ,,
Iron (cast)	11·5 ,,	16·8 ,,	—	—	—	—
Steel (mild)	20 ,,	32 ,,	16 ,,	23 ,,	8 ,,	11·6 ,,
Water	—	—	0·2 ,,	0·3 ,,	—	—
Mercury	—	—	2.6 ,,	3·7 ,,	—	—

Ex. II-3.26. *Calculate the bulk modulus of glycerine, if a litre of this liquid contracts by 0 21 cm when subjected to a pressure of 10,000 gf/cm².*

Solution : Volume strain $=\frac{\text{change in volume}}{\text{original volume}}=\frac{0.21\text{ cm}^3}{1000\text{ cm}^3}=0.21\times10^{-3}$

Stress $=10{,}000$ gf/cm².

$\therefore$ Bulk modulus $=\frac{\text{stress}}{\text{strain}}=\frac{10{,}000}{0.21\times10^{-3}}$ g/cm² $=4.76\times10^{7}$ g/cm²

Ex. II-3 27. *The Poisson's ratio of a material is σ. If e represents tensile strain show that volume strain is $e(1-2\sigma)$.*

Solution : Let V be the volume of a solid bar of length l and radius r. Now

$$V=\pi r^2 l \quad\text{or}\quad dV=\pi(l\,2r\,dr+r^2\,dl)=\pi r^2 l\left(\frac{-2r.dr}{r^2}+\frac{dl}{l}\right)$$

[When l increases r diminishes]

$$\therefore \frac{dV}{\pi r^2 l}=\frac{dl}{l}\left(1-\frac{2\,dr/r}{dl/l}\right) \quad\text{or}\quad \text{volume strain}=e(1-2\sigma)$$

Problem. Show that if $\delta\rho$ be the change in density very small compared to original density ρ then $P=K.\ \delta V/V$ may be replaced by $P=K\ \delta\rho/\rho$ for a liquid where P is the applied force and K the bulk modulus.

Volume elasticity of a gas. If the volume v of a substance changes by dv when the pressure on the substance is increased by dp, the bulk modulus is given by $K=\text{stress}\div\text{strain}=dp\div(-dv/v)=-v.dp/dv$. The negative sign is due to the fact that the volume diminishes when the pressure increases.

In the case of a gas, the relation between pressure and volume at constant temperature is given by Boyle's law, which states that at constant temperature the product of the pressure and the volume of a given mass of gas is constant. In symbols,

$$pv = \text{constant}.$$

Differentiating, we get

$$pdv + vdp = 0$$

$$\text{or} \quad p = -v.dp/dv = K. \qquad \text{(II-3.9.2)}$$

Hence, in the case of a gas, its *isothermal elasticity* is equal to its pressure.

Rapid compression of a gas heats it up, while rapid expansion cools it. If the volume stresses applied to a gas are so rapid then its temperature no longer remains constant, its bulk modulus will have a different value In the extreme case when no heat is allowed to enter or leave the gas, the relation between its pressure and volume is given by

$$pv^{\gamma} = \text{constant}.$$

It is called the *adiabatic gas equation* γ is the ratio of the specific heat of the gas at constant pressure to that at constant volume. Differentiating as before, we get

$$p\gamma v^{\gamma-1} dv + dp \quad v^{\gamma} = 0$$

$$\text{or} \quad \gamma pdv = -vdp$$

$$\therefore \quad \gamma p = -vdp/dv = K. \qquad \text{(II-3.9.3)}$$

Hence the *adiabatic elasticity* of a gas is γ times its pressure.

Remember these results well for they are basic to finding velocity of sound through a gas. Newton developed an expression for this velocity which was later corrected by Laplace on these equations.

Work done in volume strain : Let a constant presure p act uniformly on a volume V gradually reducing it by v. Then the average change in volume is $\frac{1}{2}v$ and the work done is $p \times \frac{1}{2}v$. Hence the energy stored in unit volume will be

$$\frac{p.\frac{1}{2}v}{V} = \frac{1}{2}p \times \frac{v}{V} = \frac{1}{2} \text{ stress} \times \text{strain} \qquad \text{(II-3.9.4)}$$

II-3 10 Summary of Elastic Moduli.

	Young's Modulus (Y, $\overset{*}{E}$)	Modulus of Rigidity (n, $\overset{*}{G}$)	Bulk modulus (K, $\overset{*}{B}$)
Def.	$\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$	$\frac{\text{Shear stress}}{\text{Shear strain}}$	$\frac{\text{Pressure change}}{-(\text{Volume strain})}$
Expression	Y or $E=\frac{F/s}{\delta l/l}$	G or $n=\frac{F/s}{\tan\theta}$	B or $K=\frac{\delta p}{-\delta v/V}$
Fact	(Shape or Volumeconst) Change in length due to tensile or compressive forces	Volume remaining constant Change in shape due to tangential forces or couples	Shape remaining unchanged change due to uniform change of normal pressure
Valid for	Solids only	Solids only	Both solids and fluids
Use	Stretching wires, bending beams, linear change due to thermal changes	Torsion of wires helical springs	Velocity of sound formula in all materials.

In seismography (the science of earthquakes) a fourth modulus, the axial or elongational, has been introduced to accomodate deformations produced in solids of infinite extent by longituninal waves.

II-3 11. Genaralised Stress-Strain Relation for solids

A. Metals.

Refer to fig II-3 3. The load extension graph is a straight line upto the elastic limit for by Hooke's law we have

$$\text{Load} = \text{const.} \times \text{strain}$$

*Modern Internationally accepted symbols.

i.e $y = mx$—the equation of a st. line through the origin. In the stress-strain relation [fig II-3.13 a) and (b)] we shall look further into the matter. In (a) the straight line portion OA is followed by a curve rising slowly at first and then very sharply. A then is the *proportional limit*. Along OA and upto L just beyond, the wire is found to regain its original length on removing the load. L then represents the *elastic limit*. Along OL the wire is said to undergo *elastic deformation*. Beyond L, the wire retains a small permanent set OP on removing the load. The proportional limit is distinct from elastic limit.

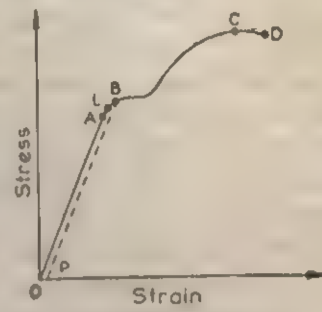


Fig. II-3,13(a)

Experiments reveal that for mild steel and iron, molecules in the wire start 'sliding' across each other soon after the elastic limit is exceeded and the material is said to have become *plastic*. In (a) the slight kink at B beyond L registers this. B is called the *yield point*. The change from elastic to the plastic stage shows itself by sudden increase in elongation as if the material has started to flow.

As the load is raised further, the extension grows rapidly along the curve BC and the wire grows a local constriction and ultimately snaps at D. The *breaking stress* of the material is the load per unit area of cross-section at the maximum point C. It is also the *ultimate tensile stress* (UTS). This multiplied by the cross-section gives the *breaking load* for the particular wire. The falling portion CD shows the unexpected fact that with diminishing load extension grows on.

Fig. II-3.13 b)

Substances that behave as above which elongate considerably, suffer plastic deformation until they break, are said to be *ductile.* Most pure metals including lead, copper, iron are such. Those that break just after exceeding the elastic limit are said to be brittle ; glass and high carbon steels are such materials. For *plastic* materials the curve runs parallel to the strain axis i e. they go on extending beyond the yield point with little increase in load, the material appearing to flow. This behaviour under varying loads are shown in fig (II-3.13b). The strength of a metal, its ductility and plasticity depend on the defects in their crystal lattice. Alloy metals like bronze or brass show no yield point, they go on increasing beyond the elastic limit without a plastic stage.

Briefly we may explain these behaviours. At *low* tensile stresses atoms get ***slightly*** displaced and return to their normal positions when the stress is released. If the metal is stretched beyond the elastic limit, the atoms cannot regain their undisturbed positions and get a permanent set or displaced position. It has then suffered ***plastic deformation***. This is caused by movement of crystal planes called ***slip*** or [illegible]nating in dislocations of crystals.

Work-hardening, also called 'cold' hardening developes by repeatedly deforming a metal wire when it becomes harder and brittle, i.e. more resistant to plastic deformation. This is because on repeated deformations the lines of dislocation move through the solid and get pinned and tangled at impurities and defects. More the deformations, greater the entanglements and hence harder or stronger the metal. You may have observed electricians breaking thick copper wires by repeatedly twisting and turning them.

Heat treatment can also increase the strength of a metal by temp. hardening If a metal is strongly heated and then rapidly quenched in cold water it becomes hard and brittle. If cooled slowly it becomes harder but more tensile.

Introducing impurities may make an alloy harder or more ductile depending on the nature of impurities.

Elastic Fatigue : Just as you fatigue yourself by repeatedly bending and then straightening yourself quickly, a metal wire loses its ductility by repeated twisting and untwisting and suddenly breaks. This happenes due to what is called *elastic fatigue*. Due to it, sometimes fan or propeller blades may break off *without any notice.* It arises because of 'cold hardening' discussed above.

B. Non-metals : Glass and Rubber :

Glass is very stiff at room temperatures ; its stiffness is then greater than that of steel. It has only a small elastic region and is

brittle. So it has no plastic region and fractures easily. Remember that glass actually is a supercooled liquid and hence melts and flows easily at high temperatures,

Rubber can easily be stretched to many times its length and hence is much less stiff than a metal, say steel. A question otfen asked runs as to which is more elastic, rubber or steel? Strictly speaking steel is so, for its modulus of elasticity is high, requiring more force to bring about equal deformation. So steel is more elastic than rubber

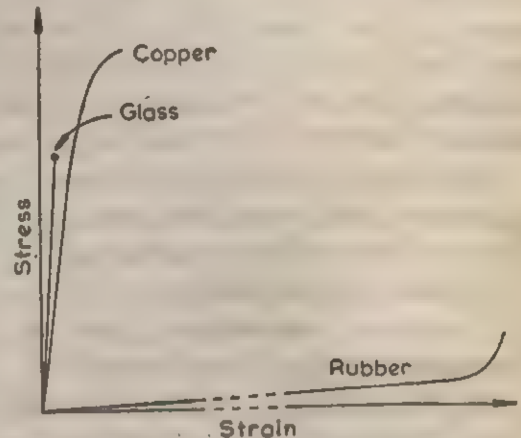


Fig. II-3.14

Popularly, the term elasticity carries just the opposite sense— easier it is to stretch a material, more elastic it is said to be. It then should have a high elastic limit and a small modulus. It is in this popular sense that we call rubber more elastic.

But at high strains such as 700%, rubber does become elastic in the strict sense whereas copper is elastic at relatively small strains such as 0·1% It is the molecular structure which is responsible for such behaviour of rubber. Unstretched rubber has coiled molecules which unwind and become straight when stretched and it is then, that it becomes really elastic i.e more difficult to stretch. There is no plastic deformation when rubber is stretched, as there is when a metal is stretched. Comparative behaviours of these are shown in fig. II-3 14. (The curves however are not to scale).

II-4

HYDROSTATICS

II-4 1. What it is : This branch seeks to understand the properties of fluids at rest. *Fluids*, as we have already learnt in § II-3.8A, *cannot resist the action of any shear or tangential forces* however small; they flow under the action of such a force (that applied over and along any free surface) and hence are called *fluids* This is their basic difference from solids, for the latter can and do oppose such forces

Hydrostatics is thus the study of *fluids at rest under gravity.* **Two fundamental laws** govern their properties which follow from their property of their inability to oppose shear. The laws are

(*i*) *Force exerted by a fluid at rest on any surface with which it is in contact is perpendicular to that surface.* The well-known **Archemedes' Principle** follows from it ; and

(*ii*) **Pascal's Law :** *Any pressure exerted on a confined mass of fluid is transmitted undiminished throughout it in every direction.*

The concept of *pressure at a point* is central to hydrostatics. We now try to clarify the idea.

II-4 2. Pressure : (1) Place vertically a pencil with its flat end standing on your palm and place a rather heavy book on it. The weight of the book exerts a force here called the **thrust** on your palm. Now invert the pencil with its sharp tip on the palm and place the book on the flat end. You feel an acute if not painful, sensation on the palm though the force remains the same.

Why is it so, though the force pressing on the palm remains unchanged in the two cases ? The difference lies in the area over which the force was applied. With the flat end on your palm the weight of the book gets spread over an area larger than when the sharp end pressed your palm.

(2) Wrap a fine string around a heavy parcel and lift it from the ground ; you feel pain. Suspend the same string from a wide handle and raise the parcel by it, the task becomes easier. With only the

string the force exerted on a unit area of your finger by the load was much greater.

(3) Your feet sink in mud and you find it difficult to cross a muddy street. But if you walk on a plank it becomes much easier to do so. In the first case, your weight is distributed over the area of your two feet while in the second the same on the much larger area of the plank makes force per unit area become much less.

(4) Children know, it is a punishment to stand on one leg. You avoid walking bare footed on broken sharp-edged pieces of rubble on a dug-up road. The sensation is painful as the areas of contact being small, force exerted per unit area is large.

Thrust and Pressure : The force exerted on the ground by your weight is said to be the thrust while that force divided by the area of your pair of soles is the pressure you exert on it.

Definition : Total force (F) exerted perpendicularly over an area A is the thrust exerted Their ratio i.e. force exerted over unit area is pressure. Hence pressure (average) is

$$p = \text{Force}/\text{Area} = F/A \qquad \text{(II-4.2.1)}$$

Dimensions of Pressure : Clearly $F/A = MLT^{-2}/L^2 = ML^{-1}T^{-2}$; this is the dimension of pressure. It is identical with stress in elastic deformation.

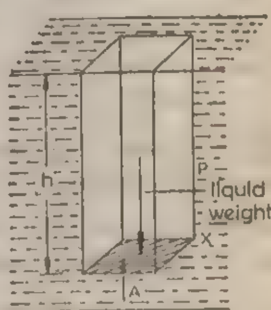


Fig. II-4.1

Units : Pressure is thus not a force and the units must be different. Force is expressed in such units as the newton, dyne or poundal. Pressure has the unit of force in the numerator and that of area in the denominator. So in *absolute units* pressure must be expressed in N/m^2, dyn/cm^2, $poundal/ft^2$ and in *gravitational* units in kgf/m^2, gf/cm^2 or lb/ft^2 etc. It may be expressed in other units as well.

Example II-4.1. *Express 1 metric ton (or tonne) per square meter in dynes per square centimeter. 1 tonne = 10^3 Kg)*

Solution : 1 tonne $= 10^3 \times 10^3$ gf $= 10^6 \times 980$ dyn. 1 $m^2 = (100\ cm)^2 = 10^4\ cm^2$

Hence the required presure $= \dfrac{10^7 \times 98\ dyn}{10^4\ cm^2} = 9.8 \times 10^4\ dyn/cm^2$

Problems. (1) Show that a pressure of 1 $ton/ft^2 = 10.72 \times 10^4\ N/m^2$

(2) Find the ratio between dyn/cm^2 and N/m^2. (*Ans.* $1/10$)

Pressure at a point : When discussing liquids we use this term. As a point has no area the expression needs an explanation. In equation II-4.2.1 if the area A is made so small (δA) that the force (δF) over it may be taken as uniform then the pressure at a point is given by

$$p = \lim_{\delta A \to 0} \frac{\delta F}{\delta A} = \frac{dF}{dA} \qquad \text{(II-4.2·2)}$$

II-4 3. A liquid at rest exerts a normal thrust on any surface in contact with it. It can be proved *theoretically.*

In Fig. II-4 2(a) let ACB be a surface in contact with a liquid *at rest under gravity.* Let a force F be exerted by the liquid at C, along if possible DC The reaction force exerted by the solid surface on the liquid must, by Newton's 3rd law act along CD. This must be equivalent to a horizontal or tangential component T along CB and a vertical or normal component N perpendicular thereto.

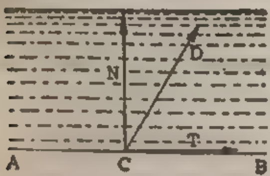


Fig. II-4.2(a)

But T provides a shear which must slide the liquid layer along CB for it would not oppose that force. But the liquid is at rest which means that T is not there So the force exerted must be entirely normal to the surface in contact.

Again, we consider two points A and A' in (fig. II-4.2(b) on the vertical sides of a bottle. Consider two inclined thrusts along PA and $P'A'$ on the wall. Their reactions along AP and $A'P'$ are equivalent to a pair of horizontal and vertical components (AB, AC and $A'B'$, $A'C'$). Now A and A' being at the same level the horizontal (here the normal) components are equal and opposite and must cancel out. But the vertical (here tangential) components being similarly directed should put the liquid in upward motion. But the liquid is at rest. Hence tangential upward

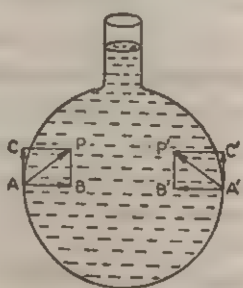


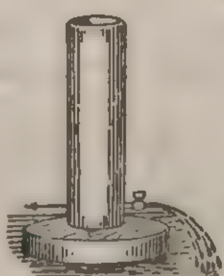
Fig. II-4.2(b)

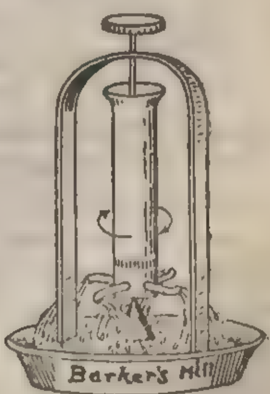
components are absent; only the normal components i.e. *normal thrusts exist* as we have proved above.

Hence *if an imaginary suface be drawn anywhere in a liquid, the force exerted by one portion on the other must be perpendicular to that surface.*

The conclusion can be *experimentally* verified. You must have observed water spurting out *normally* through small holes in an old hose pipe through which water is being driven under pressure. Walls being absent at the holes water comes out, we find normally. Push an old metal can with small punctures into a water vessel ; note that water rushes in at all holes normally. Push in the plunger into a stout multi-holed syringe full of water (as you do during holi) and water rushes out normally [fig. II-4.16(a)].

Lateral thrust exerted by a liquid is exemplified in above cases. An elegant experiment is as follows. Fix a tall metal cylinder (fig. II-4.3a) with a straight side-tap in a large piece of cork, fill it with water and float it in a large vessel of water. Open the tap and notice the cylinder being pushed back as water rushes out.





(a) Fig. II-4.3 (b)

The *Barker's mill* (fig. II-4.3b) introduced in connection with Newton's 3rd law provides a very vivid example of lateral thrust. It consists of a cylinder vessel capable of rotating freely about a vertical axis and provided with four outlet tubes near its bottom. These tubes are all bent in the same direction as shown in the

fig. II-4.3(b). Closing the outlet tubes the cylindrical vessel is completely filled with water. The outlet tubes are then opened. It will be found that as water flows out, the vessel starts rotating in a direction opposite to that of the issuing water. When the outlet tubes were closed, water exerted equal and opposite lateral pressure on the two opposite sides of the outlet tubes which balanced each other. As soon as water escapes, one of these lateral pressures at each outlet tube is removed and an unbalanced force acts at each bend of the outlet tube. These unbalanced forces produce a couple which cause the cylindrical vessel to rotate in the direction as shown.

Normal upthrust is exemplified by a floating body. A very simple experiment to prove it requires a hollow pipe with plane flanges (Fig. II-4.4) a very light aluminium disc suspended by a long light string and a deep vessel of water. Close the lower end of the pipe with the disc by pulling the string tight and lower it deep into water. Let go the string but the disc will not fall off. It is hence held there by the upward thrust exerted on the lower side of the disc. Now carefully pour water inside the pipe; when the water level inside reaches that of outside, the disc will fall off.

Fig. II-4.4.

The experiment establishes that (*i*) a liquid exerts an *upthrust* on a surface in contact (*ii*) it does so *downward* as well and (*iii*) *at the same depth inside a liquid the upthrust equals the downthrust on the same surface.*

Fig. II-4.5 shows an elaborate set-up. It is interesting to find that for pipes other than cylindrical [e. g. (b)] the same happens. This means that the thrusts are independent of the mass of water provided the depth of water is the same in the vessels—the *hydrostatic paradox.*

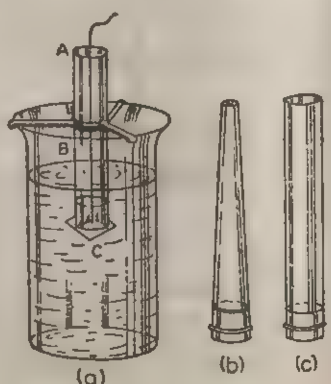


Fig. II-4.5.

II-4 4. Fluid pressure at a point acts equally in all directions. Note carefully that the fluid pressure at a point within it acts equally in all directions. Consider a very small area A surroundig the point. The pressure at the point is $p = F/A$ where F is the thrust on the surface. In whatever direction the surface is turned about the point, the ratio F/A will remain the same. But this pressure will be exerted by one portion of the liquid on the other portion across A and perpendicular to it. Thus fluid pressure at a point acts equally in all the directions but on different planes of the liquid. Had it not been so the liquid would not have been at rest, and moved from higher to lower pressure.

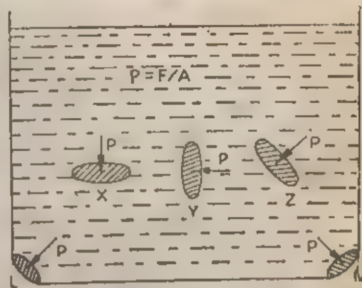


Fig. II-4.6.

In fig. II-4 6, X, Y, Z are equal *small* areas (A) at *same depth* but oriented differently. The same average pressure P acts on each of them and normally. Similarly pressure P' acts normally to the curved surfaces L and M and they are again equal.

Experimental verification (Fig. II-4.7) : Close the mouth of a

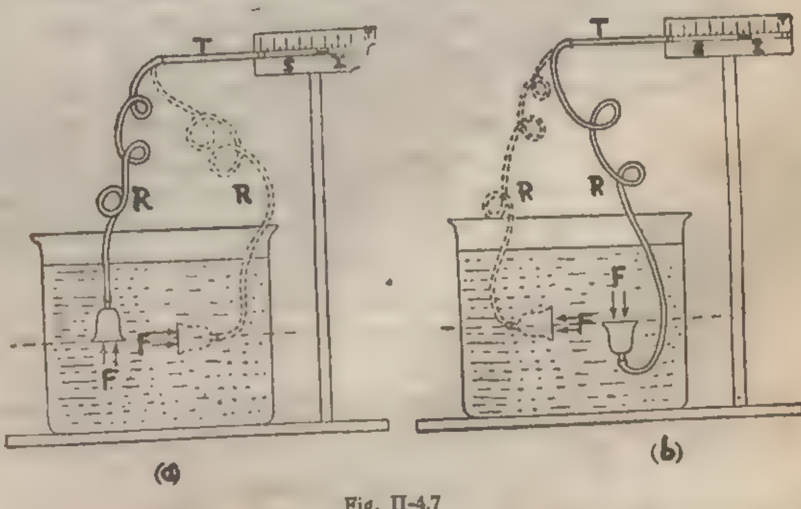


Fig. II-4.7

long-stemmed thistle funnel with a very thin rubber membrane to

make it water-tight. Connect its other end by a piece of rubber tubing to a horizontal glass tube TT mounted on a framework as shown, beside a scale S. The tube carries a drop of coloured liquid I playing the role of an index.

Insert the funnel downwards into the water vessel ; more you push down, more the index moves away indicating a rise in pressure, exerted upwards on the membrane by the liquid. This pressure compresses the air inside the funnel and the tubings pushing I away. This shows that (1) *liquid pressure increases with depth of immersion.*

Now keeping the center of the membrane if you turn the funnel into various orientations as shown, you shall find I no longer moving, i.e. (2) *at a given depth a liquid exerts equal pressures in all directions.*

Again if you use successively, liquids of *increasing densities* and immerse the funnel to the *same depth* in each, you will find I moving more and more away to the right. This shows that at a given depth (3) *pressure exerted by a liquid increases with its density.*

II-4.5. Pressure at a depth inside a liquid. Having proved that a liquid exerts a normal thrust on any surface in contact and that *the surface may even be an imaginary one in the liquid,* we proceed to find the magnitude of the pressure at a point on such a surface.

To determine the pressure at a point A, distance h below the free surface of a liquid at rest under gravity, imagine a small horizontal area a surrounding the point A. Consider the liquid contained in the vertical cylinder which has a as its base (Fig. II-4.8.). The vertical forces on the cylinder are (*i*) the upthrust on the base and (*ii*) the weight of the liquid in the cylinder. Since the cylinder is in equlibrium, these two forces are equal.

Fig. II-4.8.

The weight of the liquid in the cylinder = mass of the liquid cylinder $\times$ acceleration g due to gravity

$= \text{volume} \times \text{density} \times g = ha\rho g,$

where ρ = density of the liquid (assumed constant).

This is the total thrust on the area a.

$\therefore$ The pressure P at any point of the area

$$=\frac{\text{Normal thrust on the area}}{\text{area of the base}}$$

or $$P=\frac{ha\rho g}{a}=h\rho g \tag{II-4.5.}$$

i.e, **pressure = depth × density × acceleration due to gravity.**

If the quantities h, ρ, g are expressed in absolute cgs units, P comes out in dyn/cm². If in fps units, P will be in poundals/ft². In mks units, P will be in newtons/m².

If we divide P by g in the corresponding unit, we get pressure (P') = depth × density = $h\rho$. So written, the pressure is in *gravitational units of force* per unit area. This gives rise to an **indirect way of stating pressure** by mentioning *only the liquid and the depth.* The expression 'a pressure of 76 cm of mercury' means the hydrostatic pressure due to a column of mercury 76 cm high, i.e., the pressure at a depth of 76 cm inside mercury. Mercury has a density 13.6 g/cm³. The pressure is therefore 76 cm × 13.6 g/cm³ × 980 cm/s² = 1.013×10^6 dyn/cm². A *bar* is a pressure of 10^6 dyn/cm².

A pressure of 100 m of water will be = 10^4 g-wt/cm². In statements of pressure of this kind we often write kg or g for kg-wt or g-wt.

Experimental confirmation : The fact that pressure increases with depth has already been verified. In a simpler alternative we take a tall wide jar with holes in the side along a vertical line but closed with wax (fig. II-4.9). It is placed on a table and quick perforations are made in the wax stoppers with a needle. Water jets gush out and lower the hole further goes the jet, showing increasing pressure with depth. The jets describe parabolas under vertical gravity and horizontal liquid thrust.

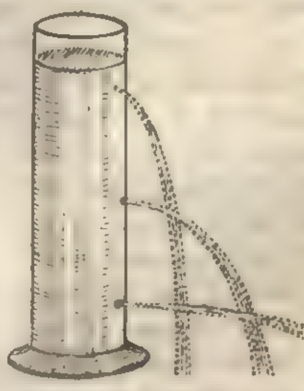


Fig. II-4.9

Where the jet strikes the ground is however not solely dependent on its height but also on its time of flight. If we imagine a base plane on table we shall see that the *jet from the middle hole* ($h/2$ below the

free surface) *has the maximum horizontal range.* This can be proved mathematically also. If the jets fall on a lower plane, say on the floor, then however the lowermost jet goes the farthest, as expected. In reaching the base plane the central jet moves longer and hence has a larger displacement.

II-4 6 Pressure of a fluid at rest is the same at all points in the same horizontal plane. The plane of the surface of a liquid in a relatively wide vessel when the liquid is *in equilibrium under gravity*, is called the **horizontal plane** at the place.

Obviously, the pressure at all points at the same depth below the free surface, will be the same, since pressure depends on depth alone. But even if the liquid surface at the top is curved (as in a liquid drop), the pressure at all points on the same horizontal plane within the liquid will be the same.

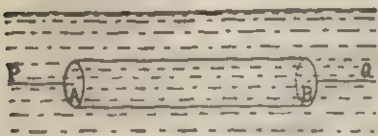


Fig. II-4.10

To prove it, consider a small uniform horizontal cylinder (Fig. II-4.10) with AB as the axis, and the ends at A and B perpendicular to the axis. Let P and Q be the pressure at A and B respectively. They act perpendicular to the end-sections each of which has an area α. The forces on the curved walls due to fluid pressure are perpendicular to the walls, i.e., the line AB Hence these forces have no component along AB. Therefore the only forces parallel to AB are $P\alpha$ and $Q\alpha$ acting in opposite directions. Since the cylinder is in equilibrium

$P\alpha - Q\alpha = 0$ or $P = Q$ (II-4.6)

Thus we find that *in a fluid at rest in equilibrium the pressure is the same at all points in any horizontal plane.* This result is put in other ways. *The pressure at two points in the same horizontal plane connected together by a liquid at rest is the same.*

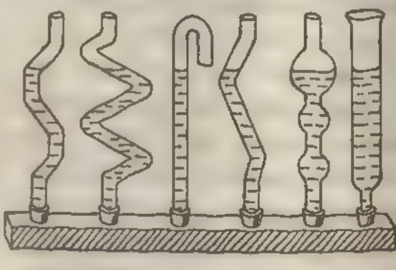


Fig. II-4.11(a)

Or, *In open vessels connected together by a liquid at rest the*

liquid stands at the same height in all. Thus in a kettle, the level of the liquid is the same in the nozzle as in the pot. In connected vessels a liquid finds its own level (fig. II-4.11a).

The fact that in connected vessels a *liquid finds its own level* has been utilised for water-supply in a town (fig. II-4. 11b). [Yet you

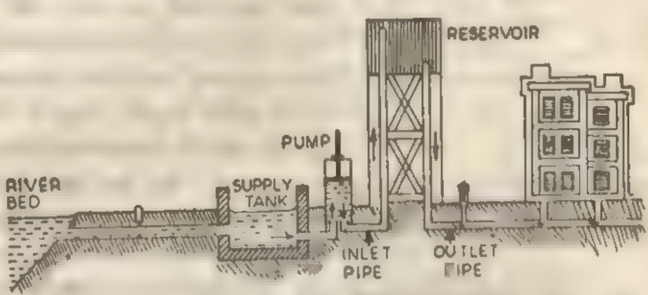


Fig. II-4.11(b)

know, even near the Tala Tank water does not rise to 3rd or 4th storey. Why not ?]. The fact is also responsible for functioning of **Artesian Wells.** In saucer-like depressions underlined by impervious rock strata, rain-water seeping through permeable top-soil gets collected. If a well or tube is sunk at the appropriate location in such cases, water gushes up by itself like a geyser.

The spirit-level is used to test whether a surface is horizontal or not. It is a slightly bent glass tube T (fig. II-4.12 *a*) with convex side uppermost filled with alcohol enclosing a small air bubble and mounted on a metal tube with a horizontal base (II-4 12*b*). The air bubble A always occupies the highest part of the tube. When placed on a horizontal surface the bubble A lies between two marks at the highest part. When the surface is not horizontal, A moves towards the highest side of the plane.

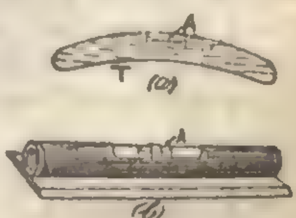


Fig. II-4.12

Free surface of a liquid at Rest is horizontal : That can be established as follows. Let AB represent a *very thin* horizontal tube imagined inside a wide, large vessel in which a liquid is at rest under gravity. Let us imagine that the free surface is not horizontal but

undulatory such that the center of the surface B (Fig. [illegible]) is at a depth h_1 less than h_2 the depth of the center of the face A. If ρ be the density of the liquid considered incompressible, the force on A is $h_2 \rho g a$ and that on B is $h_1 \rho g a$, where a is the area of cross-section of the imaginary horizontal liquid tube. Since $h_2 > h_1$ the force on the tube will be unbalanced acting towards B and hence AB [illegible] the liquid [illegible] at rest, for there is no unbalanced force and hence h_1 must be equal to h_2.

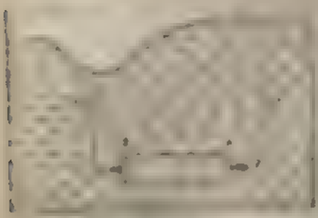
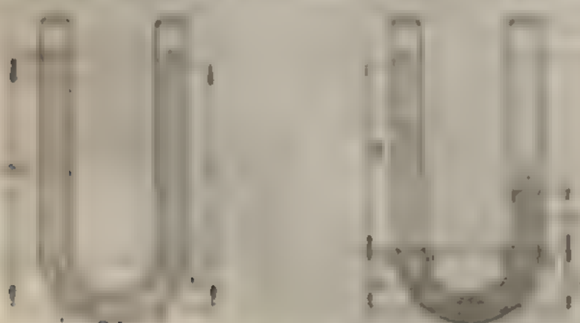


Fig. [illegible]

Hence [illegible] two corollaries [illegible] free surface of a liquid at rest must be horizontal [illegible] pressure [illegible]

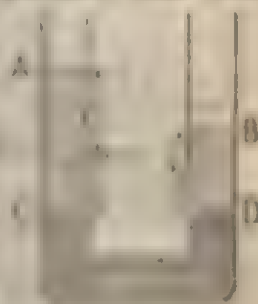
[illegible] Liquid in a U tube. [illegible] In Fig. [illegible](a) [illegible]



(a) Fig. [illegible] (b)

[illegible]

[illegible]


A
B
C
D

screwed base until its weight just exceeds the load on the pan. Then the disc is forced open, the water leaking out. A *pointer* moving along an upright is made to mark the level of water, when this occurs.

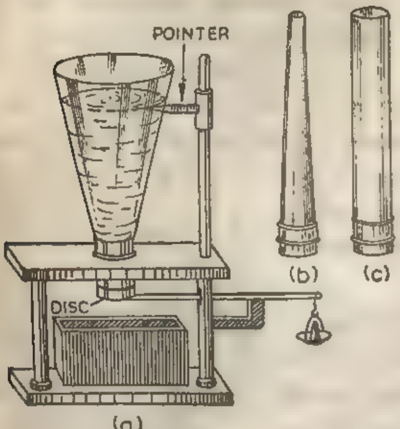


Fig. II-4.15

The vase shown, is now replaced by the one marked (*b*) which has a much smaller volume and the experiment re-performed. Precisely at the same height as for (*a*) water starts leaking. Vase (*c*) holds more water than (*b*) but less than (*a*). But water starts leaking precisely at the same height as the other two. So the surprising fact emerges that different amounts of water may exert the same pressure.

B. Explanation : There is however nothing paradoxical or surprising in it for pressure exerted by a given liquid on a surface depends only on depth and not on its weight.

In the experiment above, we find the thrust on the bases are all equal, for in each case base area is the same and so is the liquid depth and thrust is the product of these two. In fig. II-4.16 three vessels of same base area but of different shapes are shown where bases

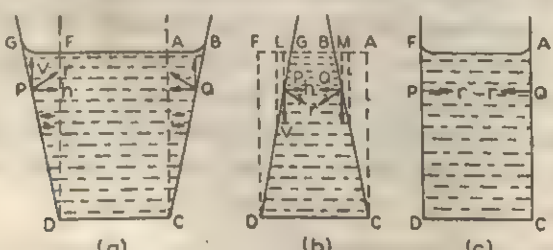


Fig. II-4.16

DC, of each are equal. In the flaring vessel (*a*) the weight of the liquid in ABC and GEF are nullified by the *upward* vertical components of reactions V to liquid pressure exerted at P and Q so that the

water inside the cylinder ACDF only exerts the down thrust. In (*b*) the *downward* vertical components of reaction of pressures at P and Q add to the weight of water in the tapering vessel GDCB which is equivalent to the weight of water in the imaginary prisms FDG and ABC so that the total thrust on CD adds up to that in cylinder of base CD as shown finally in (*c*).

II-4 9 Average Pressure and Total Thrust on a Surface inside a Liquid : The immersed surface may be of any shape and placed (*i*) horizontally (*ii*) inclined at any angle or (*iii*) vertically.

(*i*) The total thrust on any *horizontal* surface inside a liquid is

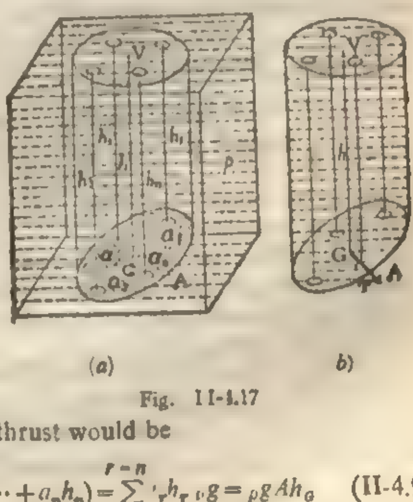
$$\text{Thrust} = \text{Pressure} \times \text{Area} = h\rho g.A$$

since pressure at all points of the surface is the same.

(*ii*) If the surface immersed be *inclined* then pressures at its different points differ, for their depths below the surface differ from point to point. It may be shown then that the **resultant thrust on any immersed surface is equal to the weight of a column of liquid with the surface area as the base and the depth of the C. G** (or rather the centroid) **of the surface as its length**. Case (*i*) is a particular case of this theorem. If then h_{og} be the depth of the C.G. of the immersed surface below the free surface of the liquid we have

$$\text{Total thrust} = \text{Average Pressure} \times \text{Area} = h_o \rho g A \qquad \text{(II-4 9.1)}$$

Thus the total thrust is (*i*) *independent of inclination and dependent only on* (*ii*) *surface area and* (*iii*) *depth of C.G. of the immersed surface.*



(*a*) (*b*)

Fig. II-4.17

Fig. II-4.17 (*a*) shows an inclined surface A of which very small bits a_1, a_2, a_3...etc are at depths h_1, h_2, h_3...etc. below the corresponding free surface areas. Clearly then the total thrust would be

$$F = \rho g(a_1h_1 + a_2h_2 + a_3h_3 + \cdots + a_nh_n) = \sum_{r=1}^{r=n} a_rh_r\rho g = \rho g A h_o \qquad \text{(II-4.9.1)}$$

where h_G is the depth of the C.G (in b) below the free surface and A the surface area. The point at which the resultant thrust acts on the surface is called the **center of pressure** of the surface

Fig. II-4.17(c) shows a vertical rectangular surface area $l \times b$, immersed with its top surface h cm below the free surface of the liquid. Clearly the *average* pressure on it acting laterally would be

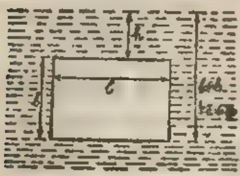


Fig. II-4.17(c)

$$\bar{P} = \tfrac{1}{2}\rho g[h + (h + l)]$$

and total lateral thrust

$$\bar{F} = \tfrac{1}{2}\rho g(2h + l) \times lb = (h\rho g + \tfrac{1}{2}\rho bg)lb \qquad \text{(II-4.9.2)}$$

Ex. II.4.2. *A vessel 20 cm long, 10 cm broad and 15 cm in depth is completely filled with water. Find in gm. wt the total thrust on one of its longer side walls.*

Solution : Area of the wall $= 20$ cm $\times 15$ cm $= 300$ sq. cm. The c.g. of the wall lies 15/2 cm below the surface.

$\therefore$ Average pressure $=$ pressure at a depth of 7.5 cm of water

$= 7.5 \times 1 \times 980$ dynes/cm^2

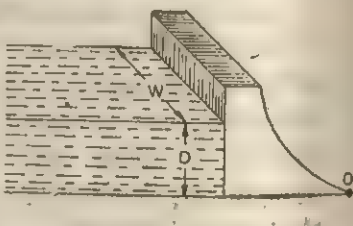
Hence, total thrust $= 300 \times \frac{15}{2} \times 980$ dynes $= 2250$ gm wt. $= 2.25$ kgs

Ex. II-4.3. *A canal 40 ft wide has a dam across it. Find the pressure on the dam at depths of 5 ft, 10 ft and 15 ft below the water level. Find in tons the total thrust on the dam if water is 20 ft. deep.*

Solution : Pressure at 5 ft

$$= 5 \text{ ft} \times 62.5 \frac{\text{lb}}{\text{ft}^3} \times g \frac{\text{ft}}{\text{sec}^2}$$

$$= 312.5g \frac{\text{poundals}}{\text{ft}^2} = 312.5 \frac{\text{lb. wt}}{\text{ft}^2}$$



The other values are double and three times this value.

[Note : *Since the pressure increases with depth, the dam must be thicker towards the base to withstand the higher pressures. See the appended fig.*]

The c.g of the submerged surface of the dam is 10 ft below the water level.

$\therefore$ Thrust $=$ average pressure $\times$ area

$=$ Pressure at a depth $\times$ of 10 ft of water $\times$ area

$$= 10 \text{ ft} \times 62.5 \frac{\text{lb}}{\text{ft}^3} \times g \frac{\text{sec}^2}{\text{ft}} \times (40 \text{ ft} \times 20 \text{ ft})$$

$$= 10 \times 62.5 \frac{\text{lb. wt}}{\text{ft}^2} \times 800 \text{ ft}^2 = \frac{10 \times 62.5 \times 800}{2240} \text{ ton wt.}$$

$= 223.2$ tons

Ex. II-4.4. *How high must a liquid be poured in a cylinder such that it exerts equal thrusts on the base as well as the walls?* (I. I. T. '79)

Solution : Let the radius of the cylinder be r and the required liquid height inside it be h and liquid density be ρ. Then

The thrust on the base $= h\rho g \times \pi r^2$ and on walls $= \frac{1}{2}h\rho g \times 2\pi rh$. By question they are equal. This happenes when $h = r$.

II-4.10. Pascal's Law of Transmission of Fluid Pressure : *Pressure applied anywhere to a mass of confined liquid is transmitted undiminished throughout the mass of the liquid and to the walls of the container.*

A. Deduction : This was arrived at by Pascal from the fact that pressure within a mass of a liquid at rest is exerted equally in all directions and depends only on density of the liquid and depth of the point. Let us consider a confined *fluid* at rest. If at any point pressure is increased, for the fluid to be at rest it must rise equally in all directions; or else the fluid will be moving and not be at rest.

The law applies equally to both liquid and gases. As it has nothing to do with weight, Pascal's law remains valid in *weightless* conditions e.g in a freely falling lift or an orbiting satellite.

B. Demonstration : (1) Make a hole in a rubber ball and fill it up entirely with water. Now close it with a finger, make a number of punctures on the ball with pin-pricks and press the ball hard with two fingers. Water will spurt out through all the holes *equally* and *normally*. Press harder, force of spurting increases, water spreading out further but again equally and normally.

2) A syringe with a long barrel and plunger ending in a perforated spherical bulb is filled with water Fig II-4 18(a) As the plunger is pushed in water shoots out equally from all holes normally. This shows that the pressure applied to the piston has been transmitted uniformly throughout the water.

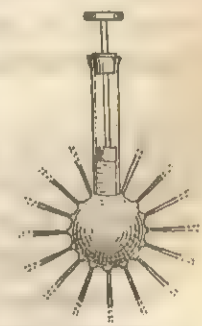


Fig. II-4.18(a)

C. Proof : Let us have a globular vessel as shown in Fig. II-4.18(b)

with a number of radial tubular outlets A, B, C, D of which the cross-sections are α, 2α, 3α and 4α. The vessel is full of water and each outlet closed with water-tight pistons. Let the piston in A be pushed downwards with a force F downwards, when all the other pistons move out. If these others are fitted with pressure gauges and prevented from moving the gauges will read 2F, 3F and 4F respectively.

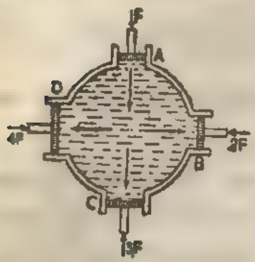


Fig. II-4 18(b)

D. **Principle of transmissibility of pressure : 1) Hydrostatic bellows.** A simple apparatus designed by Pascal, illustrates the *principle of transmissibility of pressure in a liquid.* A stout bladder or leather bellows filled with a liquid (water) has a vertical tube attached to it (Fig. II-4.19). The liquid stands at the same height in both. When a load is placed on the platform attached to the bladder (or leather bellows), the liquid in the tube rises some distance and balances the load. The condition for balance is hydrostatic pressure in the tube = pressure in the bladder (or bellows). If α is the cross-sectional area of the bellows, W the weight on it, and h the height of the liquid in the tube above the liquid-level in the bellows, then

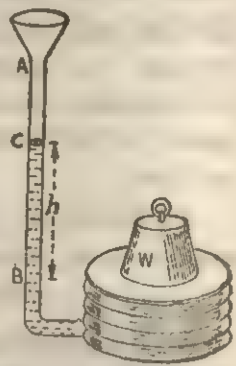


Fig. II-4.19

$$h\rho g = W/\alpha.$$

If α is big enough, a small height of liquid in the tube will be enough to support a large weight. Suppose the platform has a diameter of 30 cm. A heavy man, weighing 70 kg, can be balanced by a height of about only 1m of water in the tube.

(2) **Bursting a Cask.** You can burst open a large wooden cask or barrel strongly strapped by metal or leather thongs. Make a hole at its top, fill it with water, attach a very long tube to the hole and then just go on pouring water in it. The cask does not burst with the

water it contains which is large, but a much smaller amount poured in the pipe does it. It appears paradoxical.

In fact pressure $h\rho g$ at the top hole is transmitted everywhere and multiplied by the wall-area, developes a thrust mighty enough to break open the cask. *Stouter* the cask, *longer* the pipe you require.

II-4 12 Multiplication of Thrust : The above two appliances show you that

(*i*) Liquid pressure is transmitted undiminished and also

(*ii*) Acting on large areas develope large thrusts.

This is further exemplified in a very useful appliance the Hydraulic Press also called Brahma's press after the inventor.

A Principle of application of Pascal's law to the Hydraulic press. With the help of Pascal's law *a small force can be transformed into a large one.* Consider the arrangement illustrated in fig. II-4.20(a). It is a *confined* body of liquid connecting two cylinders fitted with pistons of areas a and A respectively. If a force f is applied to the smaller piston, the increase of pressure on the liquid will be f/a. This pressure, transmitted undiminished by the liquid, will act on the larger piston and give rise to a force $F = A \times f/a$. The force f is thus multiplied by a factor A/a. If $a = 1$ cm^2 and $A = 1000$ cm^2 a force of 1 kg applied to the small piston will cause a force of 1000 kg to be applied to the large piston.

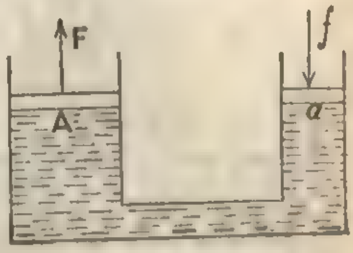


Fig. II-4.20(a)

Magnification of the force does not violate the principle of conservation of energy. The liquid may be treated as incompressible. If f is lowered through a distance s_1, the work done on a is fs_1. The volume which the piston a sweeps out is as_1. This volume of water transferred to the wider tube. If F rises through s_2 thereby, it does Fs_2 amount of work, and sweeps out a volume As_2. Since $as_1 = As_2$, for water is incompressible and $f/a = F/A$, we get, by multiplication, $(as_1)(f/a) = (As_2)(F/A)$ or $fs_1 = Fs_2$. Hence the work done on the liquid is equal to that done by i.e. recovered from the liquid.

B. Hydraulic Press. The above principle is employed in the hydraulic press a machine used for many purposes, such as compressing bales of cotton, jute etc., punching holes through metal plates, pressing metal sheets into shape, testing strength of iron beams, extracting oil from seeds etc.

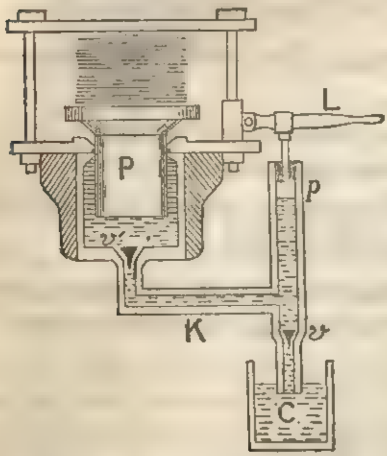


Fig. II-4.20(b)

[Hydraulic press. P=pressure piston or ram; p=plunger; v, v′=valves; C=oil tank. A tube (not shown) with a drain cock (or release valve) connects the large reservoir with the oil tank. When a compression is over, the drain cock is opened and drains the oil from the reservoir.]

Fig. II-4.20 (b) represents diagrammatically a hydraulic press in section. As the small piston p is raised, oil from the cistern C enters the piston chamber through the valve v. As soon as the down-stroke begins the valve v closes and the valve v' opens. The pressure applied on p is transmitted through the tube K to the larger reservoir. There it acts on the larger cylinder P. If the areas of cross section of p and P are a and A respectively, and the force on p is f, then the force F acting on P is $f \times A/a$.

The lever L also helps in multiplying the force. For if f' is the force applied to the lever, and m is the ratio of its longer arm to the shorter arm, then $f = mf'$. So, by exerting a force f we get a force $F = mf' \times A/a$. The ratio $F/f' = mA/a$ is called the *mechanical advantage* of the machine.

Hydraulic brakes used in automobiles, the dentist's chair, the hydraulic jack which raises heavy loads or the garage-lift which lifts automobiles, provide further examples of application of Pascal's law.

Problem. (1) The cylinders of a hydraulic press have radii 1 cm and 10 cm. respectively. The piston is attached to a handle 1 m long at a point 10 cm from the pivot, which is at one end. What force must be applied to the end of the handle for the press to exert a force of 1000 kg? (*Ans.* 1 kg-wt.)

(2) The diameters of the pistons of a hydraulic press are 1″ and 1′. A force of 56 lb is applied to the smaller one. What force is developed on the larger? (*Ans* 8064 lb f.) [H. S. '71 Comp.]

ARCHEMEDES PRINCIPLE AND FLOATATION

II-5.1. Archemedes Principle : You all know the famous story of a king's gold crown, its maker a cheat, found out by Archemedes who found *experimentally* that *(a)* a solid immersed in water displaces its own volume of the liquid and *(b)* the solid feels lighter under water and *deduced* that a force equal to the weight of the displaced water pushes the body upward. This is the *Principle of Archemedes*

Statement : *A solid immersed wholly or partly in a liquid at rest appears to lose a part of its weight which is equal to that of the displaced liquid.* The principle holds also for gases for they are also fluids and get displaced to accomodate any solid. Hence a more general statement of Archemedes Principle will be—

When a solid is submerged in a fluid, an upward force equal to the weight of the displaced fluid acts upon it.

The ascent of balloons or what we call a 'fanoosh' or soap bubbles is governed by this principle Since weights of solids and fluids are involved, *Archemedes Principle is invalid in weightless conditions* like freely falling lifts or orbiting satellites *unlike Pascal's Law.*

Let us return to a solid immersed in a liquid.

If the weight of the body in air $= W$ gf

and its weight in liquid $= w$ gf

then the apparent loss in weight $= (W-w)$ gf

By Archemedes Principle

Wt of the liquid displaced by the solid $= (W-w)$ gf

This weight of the displaced liquid acting upwards i.e. exerting an *upthrust* is called the *force of buoyancy.* It is because of this that the solid appears to lose a part of its weight when submerged in a fluid.

The term **Buoyancy** means both this apparent loss in weight and the force of buoyancy as well.

A. Demonstration : (1) Suspend a large piece of solid (preferably glass or aluminium i.e. of lower density and so larger volume) from the hook of a spring balance and note its weight (fig. II-5 1).

Take an overflow can fill it up with water and collect the overflow in an weighed empty beaker and throw the water away. Now replace

the empty beaker below the spout and carefully lower the solid into the water till completely immersed.

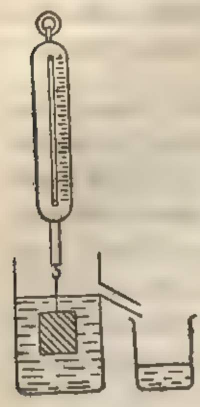


Fig. II-5.1.

Record the apparent weight as indicated by the spring balance and weigh the water collected in the beaker. The latter will be found to be equal to the difference of the weights of the solid in air and that of the water collected.

Instead of the weighed empty beaker if you collect the overflow in a *measuring cylinder*, you get the volume of water displaced. The solid here shown, is of a regular geometric shape and you can find its volume. It will be the same as that of the overflow.

In fact, the volume of any irregular solid is measured thus, by an overflow can and a measuring cylinder—a bonus from Archemedes Principle.

(2) **Bucket and cylinder experimet** (fig. II-5.2) requires a solid metal cylinder (B) filling exactly in an open one, the bucket A. A carries a hook from which B can be suspended below and another by which it itself can be suspended from that of the spring balance, above. Thus the inner volume of A is equal to the outer volume of B. B can be accomodated within A or else withdrawn and suspended below it.

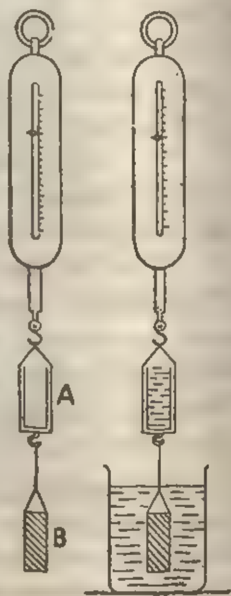


Fig. II-5.2

Suspend the cylinders from the spring balance and record the weight B when attached to the bottom of A as in fig. II-5.2. Lower B next into a beaker of water so as to be fully surrounded by water. Record the apparent weight as given by the spring balance.

Carefully pour water into A. When it is full, note that the initial reading of the balance is restored. This shows that the apparent loss in weight on immersion is equal to the weight of the displaced liquid.

B. Derivation : Let a rectangular parallelepiped of dimensions $l \times b \times d$ be immersed in a liquid at rest with its top and bottom surfaces horizontal. Let the upper surface be at a depth h below the free surface of the liquid which puts the lower surface at a depth of $h+d$ below the same (fig. II-5.3).

The liquid exerts **normal** thrusts on all the six surfaces exposed to it and so on the opposite faces they act in opposite directions. Obviously the forces acting on the *vertical* faces point by point cancel out but not so those on the horizontal faces.

If the liquid be of density ρ then the pressure on the top surface is $h\rho g$ and the force $h\rho g.lb$ (pressure $\times$ area). Similarly pressure at the plane of its bottom surface must be $(h+d)\rho g$. Hence the upward force on it $(h+d\ \rho g.lb$

$\therefore$ Net upward force on the parrallelepied will be

$$lb(h+d)\rho g \uparrow - lbh\rho g \downarrow$$
$$= lbd.\rho g = V\rho g = m'g$$

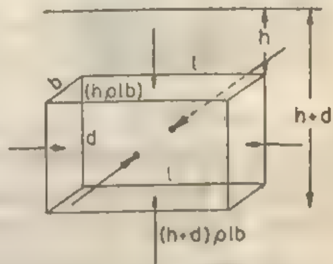


Fig. II-5 3

since lbd is the volume of the immersed body and hence also of the displaced liquid and $m'g$ the weight of it. Thus the total upthrust, the **force of buoyancy** exerted by the liquid on the solid equals the weight of the liquid displaced. This force on the body acting upwards diminishes its weight and makes it appear lighter by this amount. **Thus the apparent loss in weight of the body equals the weight of the displaced liquid.**

Center of Buoyancy. This is the the point where the force of buoyancy acts. This must be the center of gravity of the displaced liquid. For, let us replace the body in the figure above by the liquid itself. Then the weight of the displaced liquid is the same as that of the liquid parallelopiped. As the liquid is at rest the two forces must balance each other. So the **buoyancy acts upwards at the C. G. of the displaced liquid.**

C. Loss in weight is apparent but not real. For, the weight is the pull exerted on a body by the earth. It cannot change inspite of immersion of the body, as gravity acts independently of medium,

Immersion only *introduces a new force* that of buyancy acting upwards. Thus the resultant vertical force on it changes, not the true weight.

D. Reaction to the Force of Buoyancy : This force is exerted by the liquid on the body, so by Newton's third law the body exerts an equal and opposite force on the liquid. This can be easily demonstrated. Place a beakerful of water on one pan of a balance (Fig II-5.4) and counterpoise it. Hold a stick in your hand and push one end of it in water without touching any side. Note that the equilibrum is disturbed and the pan carrying the beaker gets depressed. While the water buoys up the stick, it applies an equal and opposite force on the liquid which is transmitted to the bottom of the beaker and to the balance pan We next consider two particular cases.

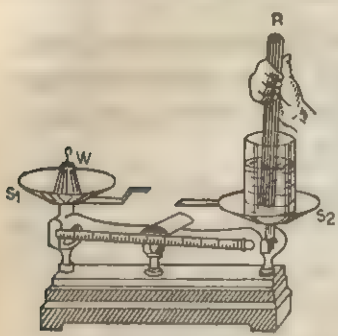


Fig II-5.4

(1) On one of the balance-pan place a beakerful of water and an *insoluble piece of solid.* Counterpoise them. Now pick up the solid and suspend it from the hook of the balance so as to immerse in the water of the beaker but touch no sides. We know that the solid would become lighter yet the balance is not found to be disturbed. That is because the solid reacts with its lost weight on the beaker base and thereby preserves the balance.

Problem : A boy walks with a bucketful of water in one hand and a fish in the other. If he drops the fish in the bucket will he feel lighter ? (J. E. E.)

No he will not. The reason is as set forth above.

(2) Let a beakerful of water on a balance pan be counterpoised. Now suspend in it a large piece of *water-soluble solid.* As reasoned above, the balance will be disturbed and the pan with the beaker will be depressed (the reaction of buoyancy). But with time the depression will diminish somewhat but not totally so. As more and more of the solid dissolves, its volume diminishes and so does the buoyancy and its reaction.

Example II-5.1 : *A weighing machine reads 700 gf when a beaker of water rests on it. A piece of glass ($\rho = 2.5$ g cc) hang from a spring balance which reads 50 gf. Find the readings on both the instruments when the piece is lowered into water.*

Solution : Volume of glass piece $= 50/2.5$ cc $= 20$ cc. = Vol. of water displaced. Then the loss in weight should be 20 gf with the glass piece in water.

$\therefore$ Spring balance should read $(50 - 20) = 30$ gf.

On the machine the downthrust as a reaction to this buoyancy would be 30 gf. Then

The weighing machine should read $(700 + 30) = 730$ gf.

Ex II-5.2 *A buoy 1000 litres in volume weighs 950 kg. It is held immersed under sea-water of sq. gr 1.02 by a light chain. Find the tension on the chain.* [H. S. Comp '68]

Solution : Tension = Wt of displaced sea water − wt of the buoy
$= 1000 \text{ litres} \times 1.02 \text{ kg/litre} - 950$ kg-wt
$= 70$ kgf

Ex. II-5 3 *A cork-piece ($\rho = 0.8$ g/cc) is taken under water. Show that it would accelerate upward with g/4 when released.*
[H. S. Comp '65]

Solution : Upward resultant force = Force of buoyancy − wt of cork
$= V\rho' g - V\rho g = g(V \times 1 - V \times 0.8)$

$\therefore$ Upward accelaration $= \dfrac{\text{Force}}{\text{mass}} = \dfrac{gV \times 0.2}{V \times 0.8} = g/4$

Ex. II-5 4 *A balance beam remains horizontal with two bodies suspended from its two ends and fully immersed in water. One has a mass of 32 g and density 8 g/cc. Find the mass of the other if that has a density of 9 g/cc.* [C U.]

Solution : Their apparent wts are equal. The volume of the first is $32/8 = 4$ cc. So its apparent weight is $32 - 4 = 28$ g That of the other one must be the same. If m be the required mass then the volume must be $m/5$ cc.

$$\therefore \quad m - \frac{m}{5} = 28 \text{ g} \quad \text{or} \quad m = 35 \text{ g}$$

Ex. II 5 5 *A stone of density 2.5 g/cc just under surface of water is allowed to sink from rest. Find how far will it sink into sea in 2s. Neglect friction of water. $g = 980$ cm/s². Density of sea water 1.025 g/cc.* [I. I. T. '69]

Solution : Let V be the volume of the stone. Then its weight is $V \times 2.5 \times g$ dynes and the weight of displaced sea water is $V \times 1.025 \times g$ dynes. So its apparent weight is

$$Vg(2.5 - 1.025) \text{ dynes}$$

Hence the acceleration with which it descends is

$$f=\frac{F}{m}=\frac{Vg(2.5-1.025)}{V\times 2.5}=g\left(1-\frac{1.025}{2.5}\right)$$

Hence the depth to which it sinks from rest is

$$d=\tfrac{1}{2}ft^2=\tfrac{1}{2}.(2)^2\times 980(1-0.41)$$
$$=2\times 980\times 0.59 \text{ cm}=11.56 \text{ m}.$$

Problem : A body of density d is gently dropped on a liquid of depth H and density ρ. Show that is reaches the bottom after a time interval given by

$$t=\sqrt{2Hd/g(d-\rho)}$$ [Pat. U.]

Ex. II-5 6 *An oil-drop rises through water with acceleration $\alpha.g$ where α is a const. Find the sp. gr of oil neglecting the friction of water.* [J. E. E. '76]

Solution : Let the volume of the oil drop be V and its weight $V\rho g$. Wt of displaced water is Vg. Then the upward acceleration will be

$$\frac{F}{m}=\frac{V\times 1\times g-V\rho g}{V\rho}=g\left(\frac{1}{\rho}-1\right)=\alpha g \text{ (given)}$$

$$\therefore\quad \frac{1}{\rho}=1+\alpha \quad \text{or} \quad \rho=\frac{1}{1+\alpha}$$

Ex. II-5.7 *Three substances P, Q, R of which one is an alloy of the other two, weigh 16, 20 and 22 g respectively in air and 14, 18 and 20 g respectively in water. Which one is the alloy and what are the weights of the other two metals in it ?* [J. E. E. '81]

Solution : Now $\rho_P=\dfrac{16}{16-14}=8,\quad \rho_Q=\dfrac{20}{20-18}=10$

$$\text{and}\quad \rho_R=\frac{22}{22-20}=11$$

ρ here representing the different sp. gr. As we find ρ_Q to be intermediate between the two it must be the alloy.

Now let there be x g of P in the alloy when that of R must be $(20-x)$g. So the volume of P in the alloy must be $(x/8)$ cc. and that of R must be $(20-x)/11$ cc.

Again the volume of Q is $(20-18)=2$ cc. Thus

$$\frac{x}{8}+\frac{20-x}{11}=2 \quad \text{or} \quad \frac{11x+160-8x}{8\times 11}=2$$

$$\therefore\quad x=5.33 \text{ g}=\text{Quantity of } P$$

and $\quad (20-x)=14.67 \text{ g}=\text{Quantity of } R$

Problem : Three ingots of gold, silver, and their alloy are of equal weights. If the gold loses 14 g in water, silver ($\rho=10.5$ g/cc) loses 26 g and the alloy 18 g, how much gold in proportion by weight is in the alloy and what is its density ? (Ans. 2/3 ; 15.17 g/cc)

II-5.2.A. Density. By *density* of a substance is meant its *mass per unit volume.* In the mks system, the unit of density is 1 kg/m^3 ; in the cgs system it is 1 g/cm^3 and in the fps system it is 1 lb/ft^3. Measurement of density of a substance involves finding the mass and the volume of a body made of the substance. These can be done in various ways.

When densities of substances are expressed in different units, we cannot immediately say which of them is denser (*i.e.* contains more mass per unit volume) without converting the values to the same system of units. But if we know the ratio of a density to the density of some standard substance, we can immediately understand from two such ratios which of the two substances is denser without any reference to any system of units. Remember that, *a ratio of two quantities of the same kind is independent of units and is a pure number.* Such a ratio is called the *specific gravity* of the substance. Generally, water at its maximum density (about 4°C) is taken as the standard substance ($\rho = 1000$ g/litre) with which to compare the densities of solids and and liquids. For gases the standard substance is hydrogen at N. T. P. ($\rho = 0.09$ g/litre). This is why though sp. gr of nitrogen is 14 and that of iron 7.8 the latter is not lighter and does not float in the former.

Note : *In modern scientific writing the term 'specific gravity' is no longer preferred. The term used in its place is relative density. The word 'specific' in modern scientific writting is used to mean 'per unit mass'. Examples are specific volume, 'specific latent heat', 'specific heat capacity', etc. However, as the syllabus mentions 'specific gravity' we shall be using it.*]

B. Specific gravity. *Specific gravity of a substance is defined as the ratio of the mass (or weight) of a volume of the substance to the mass (or weight) of an equal volume of water at its maximum density.** This is the same as the ratio of the density ρ_s of the substance to the density ρ_w of water at the temperature at which it is densest (*i.e.* about 4°C). For, let V be the volume of the substance. Then the specific gravity S of the substance is given by

* Relative density is a ratio and ratio of weights of same volumes ($mg/m'g$) of two substances is equal to the ratio of two masses.

$$S=\frac{\text{Mass of volume } V \text{ of the substance}}{\text{Mass of volume } V \text{ of water (at 4°C)}}=\frac{V\rho_s}{V\rho_w}=\frac{\rho_s}{\rho_w} \qquad \text{(II-5.2.1)}$$

In the mks system $\rho_w=1000\ \text{kg/m}^3$.

In the cgs system $\rho_w=1\ \text{g/cm}^3$.

In the fps system $\rho_w=62{\cdot}4\ \text{lb/ft}^3$.

It appears from these relations that in the

$$\left.\begin{array}{ll}\text{mks system,} & \rho_s=S\times 1000\ \text{kg/m}^3\ ;\\ \text{cgs system,} & \rho_s=S\times 1\ \text{g/cm}^3\ ;\\ \text{fps system,} & \rho_s=S\times 62{\cdot}4\ \text{lb/ft}^3.\end{array}\right\} \qquad \text{(II-5.2.2)}$$

Only in the cgs system, the numerical values of ρ_s and *S are the same.* ρ_s for gold is 19.6 g/cc ; convert this value in mks and fps systems.

From the definitions of density and specific gravity you must have understood the difference between the two quantities. Densities are expressed in mass units per unit volume, while specific gravity is a pure number.

When we say then that density of brass is 8.4 g/cc we mean that one cc of brass weighs 8.4 g, one cubic meter of it would weigh 8400 kg and one cu ft of it as much as 514.16 lbs. If we say instead, that Specific gravity or Relative density of brass is 8.4 we understand that it weighs 8.4 times as much as water of same volume.

Relative density or specific gravity of nitrogen is 14 or that of Oxygen 16, does not indicate that they are denser than brass but they are that much heavier than hydrogen, the lightest of substances known, of same volume but under the same conditions of pressure and temperature.

II-5.3. Applications of Archemedes Principle : In the laboratory, we use this principle for determining volumes of bodies and specific gravities of substances, as also the proportion of two pure metals in an alloy.

The ascent of balloons is an application of this principle.

All cases of floatation are determined by this principle, the most spectacular being that of submarines.

Buoyancy forces are very important to many marine and fresh-water animals. Many sea animals have gas-filled chambers and gas-filled spongy bones which make their density the same as that of

sea-water. This makes quick movement very easy for them. Some have a gas-filled bag, called a *swim bladder*, whose volume they can vary. Squeezing the bag, the fish makes itself heavier and sinks. Some have a control valve to adjust the amount of air in the bladder. Ease of adjustment to various depths in water is a great boon to marine life. Cartesian diver, a toy (§II-5.8F) illustrates this sort of working. The principle on which the adjustment is made is the Archemedes' principle. Submarine is a practical application of the Cartesian Diver.

A. **Application of Archemedes principle to determination of volume.** Archemedes' principle states that when a body is weighed (i) in air and (ii) when *fully immersed* in water, the difference between the observed weights is equal to the weight of the water displaced by the body.

If the weight of the body in air $= W$ g,*

its apparent weight when fully immersed in water $= w$ g,

then $(W-w)$ g $=$ weight of the water displaced.

Now, the *weight of* 1 cm³ *of water is equal to the weight of a one gram mass.*

$\therefore$ Volume of water displaced $= (W-w)$ cm³.

But this is also the volume of the solid.

$\therefore$ Volume of solid $= (W-w)$ cm³.

Ex. II 5.8. *A body weighs 20.52 g in air and 12.48 g in water. Find its volume, density and specific gravity.*

Solution : Weight of displaced water $= 20.52 - 12.48 = 8.04$ g. The volume of this water is 8.04 cm³. This is also the volume of the solid.

Hence density of solid $= 20.52$ g $\div 8.04$ cm³ $= 2.55$ g/cm³.

Specific gravity $= 2.55$.

Ex. II-5.9. *What will the above solid weigh in kerosene, of which the sp. gr. is 0.8 ?*

Solution : Volume of solid $= 8.04$ cm³. Weight of displaced kerosene $= 8.04 \times 0.8 = 6.43$ g. Hence loss of weight in kerosene is 6.43 g, and apparent weight in kerosene is $20.52 - 6.43 = 14.09$ g.

---

*Mas and weight are of course different quantities. But in expressing a weight we often use the unit of mass *when the sense is clear.* Thus it is generally accepted that we may write g (gram) to mean gf (gram force) or g-wt (gram weight) in clear cases. This practice of course violates use of proper unit symbols. But it is in use to a fair extent.

B. Specific gravity of a substance relative to any liquid and to water. From Archemedes' Principle

the weight of a body in air $= W$ g,
its weight in water $= w$ g and
its weight in a liquid $= w'$ g.

Let the density of the liquid be ρ' g/cm³. Then ρ' is also the specific gravity of the liquid relative to water.

The sp. gr. of the body relative to water $= s = \dfrac{W}{W-w}$. (II-5.3.1)

The sp. gr. of the body relative to liquid $= s' = \dfrac{W}{W-w'}$. (II-5.3.2)

$(W-w)$ g is the mass of the displaced water and
$(W-w')$ g is the mass of the displaced liquid.
Both masses have the same volume, the volume of the body.

Then $(W-w')/(W-w) = \rho'$

$$\therefore \quad s = \frac{W}{W-w} = \frac{W}{W-w'} \times \frac{W-w'}{W-w} = s' \times \rho' \qquad \text{(II-5.3.3)}$$

or specific gravity relative to water (s) = sp. gr. relative to a liquid $(s')\times$ sp. gr. of the liquid (ρ') itself.

Ex. II-5 10. *A body weighs 100 g in air and 60 g in water. What will be its weight in kerosene of sp. gr. 0.8 ? What is the sp. gr. of the body relative to kerosene ?*

Solution : Volume of displaced water $= 100-60 = 40$ cm³. This is also the volume of displaced kerosene. Weight of this volume of kerosene is $40 \times 0.8 = 32$ g. Hence weight of the body in kerosene is $100-32 = 68$ g. and sp. gr. relative to kerosene is $100/32 = 3.125$

Ex. II-5.11. *A body weighs 300 g in air and 270 g in a liquid of sp. gr. 0.9. How much will the body weigh in water ? What are its volume and sp. gr. ?*

Solution : Volume of the body in cm³ = Volume of the displaced liquid $= (300-270)$ g/0.9 g/cm³ $= 33.3$ cm³. In water it will weigh 33.3 g less than in air. Its sp. gr. = its mass in air/mass of displaced water $= 300/33.3 = 9$.

Ex. II-5.12. *When equal volumes of two substances are mixed, the resultant sp. gr. is 4.84 and when equal weights of them are mixed the sp. gr. becomes 2.08. Find the individual specific gravities.*

Solution : (*a*) Let the volume of each be V, their weights W and W', densities ρ_1 and ρ_2.

Then $V=\dfrac{W}{\rho_1}=\dfrac{W'}{\rho_2}=\dfrac{W+W'}{\rho_1+\rho_2}$. $\therefore$ sp. gr $=\dfrac{\text{Total weight}}{\text{Total volume}}=\dfrac{W+W'}{2V}$

$$=\frac{W+W'}{2(W+W')/(\rho_1+\rho_2)}=\tfrac{1}{2}(\rho_1+\rho_2)=4.84 \text{ or } \rho_1+\rho_2=9.68 \quad \text{...(i)}$$

(*b*) Here the total weight $=2W$

$$\therefore \text{ sp. gr.}=\frac{2W}{V_1+V_2}=\frac{2W}{(W/\rho_1)+(W/\rho_2)}=\frac{2\rho_1\rho_2}{\rho_1+\rho_2}$$

$$\therefore \quad 2\rho_1\rho_2=2.28(\rho_1+\rho_2)=2.28\times 9.68$$

Now, $(\rho_1-\rho_2)^2=(\rho_1+\rho_2)^2-4\rho_1\rho_2=(2.68)^2-4.56\times 9.68$

$$\text{or} \quad \rho_1-\rho_2=7.16 \quad \text{(ii)}$$

$\therefore \quad \rho_1=8{\cdot}42 \;\; \rho_2=1.26.$

C. To determine the proportion of two pure metals in alloys.

Let m_1 grams of a metal of density ρ_1 form an alloy with m_2 grams of another metal of density ρ_2. Assuming that the volume remains unchanged on alloy formation, the volume V of the alloy is $V=(m_1/\rho_1+m_2/\rho_2)$. Its density is

$$\rho=(m_1+m_2)/V=(m_1+m_2)/(m_1/\rho_1+m_2/\rho_2)$$

The proportion n of the first metal to the second in the alloy is $n=m_1/m_2$. Dividing both numerator and denominator of the last equation by m_2, we get

$$\rho=\frac{[(m_1/m_2)+1]}{(m_1/m_2)/(\rho_1+1/\rho_2)}=\frac{n+1}{n(\rho_1+1/\rho_2)}.$$

If ρ_1 and ρ_2 are known and we determine ρ of the alloy using Archemedes' principle, we can find n from this equation. You should recognise that this was the original problem passed to Archemedes by his king Hiero.

Ex. II·5.13. *A crown of gold with silver as impurity weighs 200 g. When dipped in water it weighs 185 g. Find the amount of gold and silver in the crown if sp. gr. of gold and silver are respectively 19.3 and 10.3.* **[Visva. U.]**

Solution : Let the amount of gold be x g. That of silver would be $(200-x)$g. Their respective volumes would be $x/19.3$ and $(200-x)/10.3$ and so the volume of the crown would be their sum.

Since the crown when weighed in water loses 15 gf, the displaced water must have a volume of (15 g/1 g/cc) 15 cc.

$$\therefore \quad \frac{x}{19.3}+\frac{200-x}{10.3}=15$$

$\therefore$ $x = 97.57$ g = the wt of gold. Then that of silver
$= 200 - x = 102.43$ g

Problem : The crown of Hiero weighed 20 lbs. Archemedes found it to lose 1.25 lbs in water. Find the amounts of gold and silver if their sp. gr. are 19 3 and 10.5. 1 cu ft of water weighs 62.5 lbs. *Ans* 15·08 lbs 4.92 lbs. [Dac. U.]

Ex II-5 14. *A silver ($\rho = 10$ 5 g/cc) ornament suspected to be hollow, weighs 288.75 g and displaces 30 cc. of water. Find the volume of cavity.* [P.U]

Solution : Volume of the material of the ornament = 288.75/10.5 = 27.5 cc. But its outer volume is 30 cc. So the volume of cavity = 30 − 27.5 = 2.5 cc.

Ex II-5.15. *The densities of three liquids are in the ratio of 1 : 2 : 3. Find the relative densities by combining (a) equal volumes and (b) equal weights of them.* [Gau. U. : C.U.]

Solution : Let the densities be ρ, 2ρ and 3ρ.

(a) If the volume V of each are mixed, provided no change in volume occur, the total volume will be $3V$. If ρ_v be the resultant density then the mass of the resultant mixture is $3V\rho_v$

The individual masses of liquids are $V\rho$, $2V\rho$ and $3V\rho$

So $3V\rho_v = V\rho + 2V\rho + 3V\rho$ or $\rho_v = 2\rho$

(b) If W be the wt. of each liquid then the total weight is $3W$ and the total volume $3V/\rho_w$

$$\therefore \quad \frac{3V}{\rho_w} = \frac{m}{\rho} + \frac{m}{2\rho} + \frac{m}{3\rho} \quad \text{whence } \rho_w = 1.64 \text{ (nearly)}$$

II-5 4 Principles of methods of determining specific gravity.

Specific gravity (s) is *relative density*, that is,

$$s = \frac{\text{density of the substance}}{\text{density of water at } 4°\text{C}} = \frac{\text{weight of a volume } V \text{ of the substance}}{\text{weight of the same volume of water at } 4°\text{C}}$$

So, to determine s, we have to weigh the same volume of the substance and water. In the cgs system the numerical value of the volume of water in cm^3 is equal to that of the mass in grams.

(*i*) We can use the balance and apply Archemedes' principle for measuring the masses of the same volume of the substance and water.

(*ii*) For *liquids*, we can use a *density bottle* (also called specific gravity bottle) to determine the masses of equal volumes of a liquid and water.

The density bottle can also be used to find the specific gravity of a solid powder by partially filling the bottle with powder.

(*iii*) For liquids, there are other methods.

(*a*) *Principle of balancing columns.* One depends on the hydrostatic pressure $h\rho g$ exerted by a liquid. If two liquid columns balance each other, then $h_1\rho_1 g = h_2\rho_2 g$, or $\rho_1/\rho_2 = h_2/h_1$. If ρ_1 is the density of the experimental liquid and ρ_2 that of water, then $s = \rho_1/\rho_2 = h_2/h_2$.

(*b*) We can also apply the *principle of floatation.* All hydrometers depend on this principle. Two types are in general use—*variable immersion* and *constant immersion* type exemplified respectively in Common and Nicholson's hydrometers.

The different methods for determining *s* are thus found to depend on the following principles :—

(1) Archemedes' principle, using the hydrostatic balance.
(2) Direct weighing by using density bottles.
(3) Principle of balancing columns.
(4) Principle of floatation.
(*a*) Variable immersion. (*b*) Constant immersion.

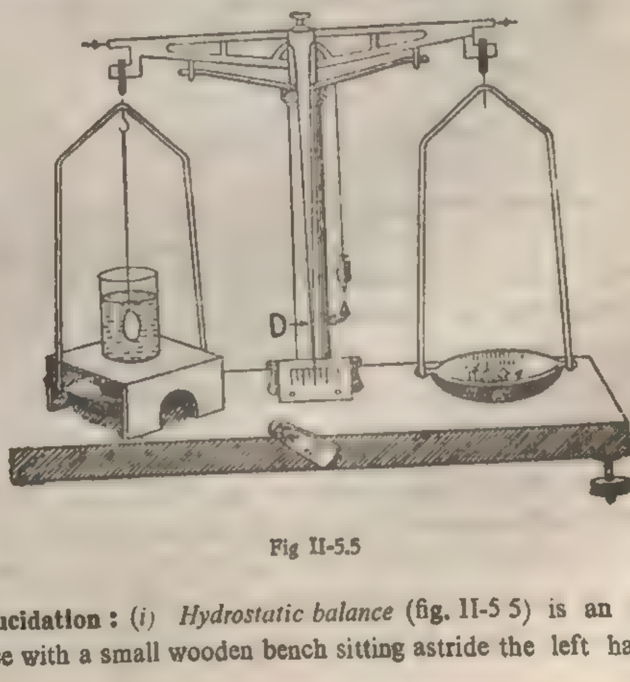


Fig II-5.5

Elucidation : (*i*) *Hydrostatic balance* (fig. II-5 5) is an ordinary balance with a small wooden bench sitting astride the left hand pan.

The solid is suspended from the hook and counterbalanced. A beaker is next introduced on the bench such that the solid hangs freely inside. Water is now carefully poured in the beaker till the solid is well immersed and then counterpoised. Sp gr. is found by applying eqn II-5.3.1. To find the sp. gr of a liquid, the same solid is weighed in water and the liquid and eqn II-5.3.3. applied. To find the sp. gr of a soluble solid its sp. gr. w.r.t. a liquid is found by eqn II-5.3.2 and then multiplied by sp. gr. of liquid w.r.t water. To find sp. gr of a solid lighter than water, a sinker is used.

Fig II-5.6

(ii) *Density bottle* (fig II-5.6,) is a stoppered bottle with a definite volume (25 cc. or 50 cc.). It is weighed empty, then when filled with water and finally with the liquid. From these the masses of same volumes of liquid and water are found and sp. gr of liquid determined.

To find sp. gr of a powdered solid four weighings are required ; empty bottle (W_1) partially filled with solid (W_2) the rest filled with water (W_3) and finally filled only with water (W_4). Then

$$\text{sp. gr} = \frac{W_2 - W_1}{W_4 - W_3 + W_2 - W_1}$$

(iii) **Balancing Columns** are of two types namely (a) *U-tube* and (b) *Hare's apparatus.* The working principle $h_1\rho_1 = h_2\rho_2$ is the same (II-4.7.1) in both ; but in the U-tube air pressure is not of any significance but the latter cannot work without that pressure. Again, in the former, liquids must be immiscible with each other as you can readily realise but in the latter they need not be so, for they are not in contact.

But in a modification shown in fig. II-4.12 (c) the U-tube can also be used to compare the densities of miscible liquids. The U-tube is mounted vertically and mercury fills up its curved part. The two liquids are poured till mercury stands at the same level in both arms. Then $h_1\rho = h_2\rho_2$

(b) **Hare's Apparatus** (fig II-5.7) is an *inverted U-tube* with a side tube at the middle of the bend and connected to a short piece of rubber tubing and a pinch cock. The open ends of the limbs dip into two beakers with the liquids. Air is partially sucked out through the side tube when the air pressure on the liquid surfaces in the two beakers pushes up the liquids into the two arms to different heights inversely as to their densities.

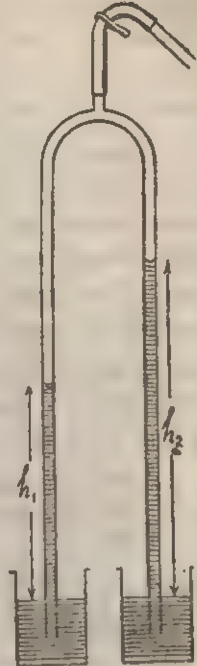
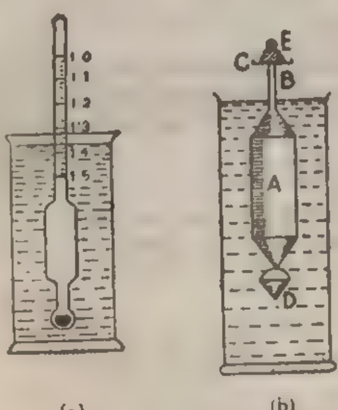


Fig. II-5.7

(iv) **Floatation and Hydrometers: A. The common hydrometer.**

It is a hollow glass chamber weighted at the bottom and has a graduated stem of uniform sectional area at the top (II-5.8a). When floated in a liquid it sinks until it displaces its *own weight* of the liquid. The lighter the liquid, the deeper it sinks before coming to rest. The scale attached to the stem is so calibrated that the specific gravity of the liquid can be read off directly at the point where the stem just projects through the liquid surface.



(a) (b)

Fig. II-5.8

The commercial importance of many liquids such as sugar solutions, sulphuric acid, alcohol etc. depends directly on their specific gravity. Hydrometers

are extensively used to find their specific gravities. Some hydrometers are designed and calibrated for specific purposes. An *alcoholometer* will give the concentration of spirit ; a *lactometer* will judge the purity of milk. A common use of a hydrometer is in testing the acid solution of the lead storage battery in cars.

B Nicholson's Hydrometer : The instrument has a hollow cylindrical body (A ; II-5.8b) to which a thin stem B is attached. The stem carries a tray C on which weights (E) can be placed. There is a small conical basket D at the bottom carrying lead shots fixed with wax. This keeps the hydrometer floating vertically. In using the instrument, weights are placed on C so as to immerse the hydrometer up to a fixed mark on B. It is therefore, known as a *constant immersion* hydrometer, and is always made to *displace the same volume* of water.

Ex. II-5.16. *5 cc of water is mixed with 7 cc of a liquid of sp. gr. 1.85 when the mixture attains a sp. gr. of 1.615. Find the amount of contraction.* [C. U.]

Solution : 7 cc of the liquid weigh $7 \times 1.85 = 12.95$ g.

Mass of the mixture $= 12.95 + 5 = 17.95$ g

Volume of the mixture $= 17.95/1.615 = 11.11$ cc

$\therefore$ Contraction $= (7+5) - 11.11 = 0.89$ cc

Ex II-5.17. *A test tube loaded with shots weighing 17.1 g in all, floats in alcohol immersed upto a certain mark. It is then placed in water and 3.2 g more of shots have to be added to sink it upto the same mark. Find the sp. gr. of alcohol.* [Pat. U.]

Solution : This is a constant volume immersion problem like the Nicholson hydrometer. Wt of displaced alcohol is 17.1 g and that of displaced water is $17.1 + 3.2 = 20.3$ g. The volumes displaced are the same.

$\therefore$ sp. gr. of alcohol $= 17.1/20.3 = 0.84$

Ex. II-5.18. A *tube 1 m long and 1 cm in internel diameter of mass 100 g weighs 150 g when filled with a liquid. How much would it weigh when full of water and what is the sp. gr. of the liquid ?* [Pat. U.]

Solution : The problem is that of a density i e sp. gr. bottle. The internal volume of the tube is $100 \times \pi \times (0.5)^2 = 78.57$ cc.

So water in it would weigh 78.57 g

$\therefore$ Sp. gr. of liquid $= (150 - 100)/78.57 = 0.636$.

II-5.5. Floating bodies. According to Archemedes principle a body, even if *partly* immersed in any liquid, will experience an upward

buoyant force equal to the weight of the liquid displaced. The *downward force* on the body *is its weight*. When the two forces are equal, the body floats. We say that *a body floats when it displaces its own weight of the liquid.* When a floating body is placed in a liquid it will sink till it displaces its own weight of the liquid. If pushed further into the liquid, it will rise when released. When a body is unable to displace its own weight of liquid, it sinks.

Floatation and density. Though we have noticed above, that the differential weights of the body and that of the displaced liquid as determining between floatation and sinking, the *essential factor is their differential densities.*

(A) Let a homogeneous body of density ρ be gently dropped in a liquid of higher density ρ' i.e. $\rho < \rho'$. The body starts sinking and displacing more and more of the liquid. Thus the weight of the displaced liquid (W') and hence the upward thrust of buoyancy grows till it equals that wt. of the body (W) and the body sinks no further ; part of it remains exposed above the liquid surface. Left to itself the body will come to rest and be in equlibrium, for the two forces $W\downarrow$ and $W'\uparrow$ act along the same line equally and oppositely and no couple would act on it.

If the body is further depressed the volume of the displaced liquid W' exceeds W. An *additional* downward force ($W' - W$) is required to hold the body in that position. If released, this force pushes the body upwards*, and is proportional to the additional immersion So *the body ascends, executes S.H.M. for sometime* (fig. III-1.5) before coming to rest ; when at rest it maintains the condition of $W = W'$. A piece of cork or wood in water behaves thus.

(B) Let the density of the homogeneous solid be equal to that of the liquid i.e. $\rho = \rho'$. Then it will displace its own volume of the liquid before the condition, $W = W'$ reached. Thus it must be fully submerged for $V\rho g = (W)$ equals $V\rho' g = (W')$. However it will be at rest anywhere within the liquid. Its *apparent weight is zero* anywhere within the liquid. A drop of olive oil in a suitable mixture of water and alcohol behaves thus.

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* See example II-5.3 under Buoyancy.

(C) The body does not float but continues sinking when $W > W'$ i.e. $\rho > \rho'$ for then $V\rho g > V\rho' g$ [See Ex. 11.5.5 and the associated problem]. *Remember this is no case for condition of floatation but the foregoing two are.* A piece of iron or most of the metals sinks in water but floats in mercury. Li, Na, K, etc. are however lighter than water though metals, and so float in it.

Equilibrium of Floating bodies. Resultant force *and* couple on a body must be zero for a body to be in equilibrium. Hence, not only the wt. of the body and the displaced liquid must be equal but they must be collinear and act oppositely. If they are not collinear, a couple would result. So we have :-

Conditions of floation : For a solid to float on a liquid

(i) its wt must equal that of the liquid displaced (no resultant force).

(ii) the C. G. of the solid and that of the displaced liquid must be in the same vertical line (no resultant couple).

For stability of the floating body, C. G. of the solid must be below the centre of buoyancy i.e. the C. G. of the displaced liquid.

Stability of a floating body : For a floating body to be in stable equilibrium the forces acting on it must provide a restoring torque when it is disturbed, same as for bodies resting on a plane surface. In floatation the torque results from W, the wt of the solid and F the force of buoyancy. In Fig. 11 [illegible] these two form a couple W tending to rotate the body back to its undisturbed position.

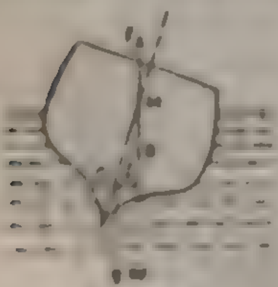


Fig. [illegible]

When a floating body is *slightly* displaced about a horizontal axis without changing the volume of the displaced liquid, the point where the vertical line cuts the straight line joining the C. G. of the body to the *original* C. G. of the liquid, is called the **meta center or centre of buoyancy**, M in fig. [illegible].

For equilibrium the metacenter or centre of buoyancy i.e. the C. G. of the displaced liquid and the C. G. of the immersed solid must lie in the same vertical line.

For stable equilibrium C. G. of the body (G) must lie below the centre of buoyancy (B) as shown in fig. 11.5.1(a) depicting a floating ship.

If the ship leans over (fig. 11.5.1(b)), B shifts position and l, the lever arm increases in length. So long as B is above G and l does not

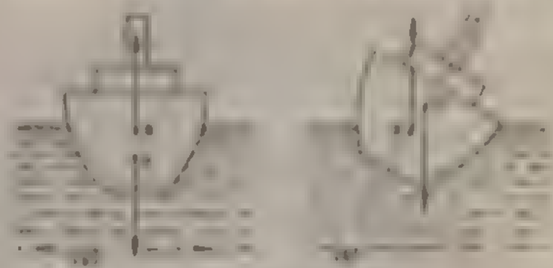


Fig. 11.5.1

change the ship rights itself. But if she is loaded heavily with cargo on the deck, G may rise too high and as shown figure (b), the couple would capsize the ship. This state of affairs had happened for [illegible]. Hence for stable equilibrium the metacentre must always stay above the C. G. of the floating body.

11-5.6 Some Relations in connection with Floatation

A. If for a body of volume V floating in a liquid of density ρ, the immersed volume is v then the liquid exerts an upthrust $v\rho g$. By the law of floatation, the wt. of the body = wt. of displaced liquid

or $$V\sigma g = v\rho g \quad \text{or} \quad \sigma V = \rho v$$

$$\therefore \frac{\text{immersed volume of the body}}{\text{total volume of the body}} = \frac{\text{density of the body}}{\text{density of the liquid}} \qquad (11.5.6.1)$$

B. Let V be the volume of a floating body of density σ or specific gravity s. Let v be the volume of its volume immersed in water when the body floats. Then its weight is Vsg and the weight of the displaced water is $v \times 1 \times g$ (in c.g.s. units).

$$Vsg = v\,1\,g \quad \text{or} \quad s = \frac{v}{V} \qquad (11.5.6.2)$$

If the liquid in which it floats has a density ρ, we shall have

$$Vsg = v\rho g \quad \text{or} \quad s = \frac{\rho v}{V} \qquad (11.5.6.3)$$

This result may be expressed in words as follows:

If a solid floats in a liquid with a fraction n of its volume immersed, then the density (or the specific gravity) of the solid is n times that of the liquid. [Iron has a lower density than mercury. So iron floats on mercury with a fraction of its volume immersed.]

Ex. II-5.19 *Ice has a specific gravity of 0.917. What fraction of its volume will emerge out of water ?*

Solution : If V is the volume and a fraction n is submerged, then $V \times 0.917 = nV$ or $n = 0.917$. The emergent fraction is $(1 - 0.917) = 0.083$ (nearly one-twelfth).

Ex. II-5.20 *If the emergent volume of an iceberg (sp. gr. = 0.917) is 1000m³, how much of it is below sea-water (sp. gr = 1.028) ?*

Solution : If V m³ is the volume below the surface, then $V \times 1.028 = (V + 1000) \times 0.917$, whence $V = 8261$ m³.

Problems. (1) A piece of wax of volume 22 cm³ floats in water with 2 cm³ above the surface. Find the weight and specific gravity of the wax. [C. U.] (*Ans.* 20 g, 10/11)

(2) A block of ice weighing 1000 kg is thrown into the sea. Determine the volume of ice submerged. The density of ice is 0 917 g/cm³ and density of sea water is 1.03 g/cm³. [*Ans.* $9{\cdot}71 \times 10^5$ cc]

C. Let l_1 and l_2 be the lengths to which a cylinder of cross-section A sinks in two liquids while floating in them. Let ρ_1 and ρ_2 be the respective densities ot the liquids. By Archemedes' principle the weight of the cylinder would be

$$W = Al_1\rho_1 = Al_2\rho_2 \quad \text{or,} \quad \frac{\rho_2}{\rho_1} = \frac{l_1}{l_2}. \qquad \text{(II-5.6.4)}$$

Now, if ρ_1 and l_1 refer to values for water, ρ_2/ρ_1 is the specific gravity of the other liquid. Hence we may say that

The specific gravity of a liquid is equal to the ratio of the depth l_1, to which the cylinder sinks in water, to the depth l_2, to which it sinks in the liquid.

Ex. II-5.21 *A wooden cylinder of uniform cross-section is 10 cm long. It floats in water wtih 2 cm above the surface. In a salt solution it floats with 3 cm above the liquid surface Find the density of the salt solution.*

Solution : Let A cm² be the area of cross-section of the cylinder. and ρ_1 the density of the solution in g/cm³. Then the weight of the cylinder = weight of displaced water = weight of displaced liquid.

Now, weight of displaced water $= 8A$ g-wt.

Weight of displaced liquid $= 7A\rho$ g-wt.

$\therefore \quad 7A\rho = 8A \quad$ or $\quad \rho = 8/7$.

Ex. II-5 22. *In the above problem find how much of the cylinder will be above the liquid surface when floating in a liquid of specific gravity 1.25.*

Solution : If x cm is the length sought, then $(10-x)$ cm will be below the liquid surface. Hence $8A = (10-x)A \times 1.25$ or $x = 3.6$ cm.

Problems (1) Sea water is 1.03 times as dense as fresh water. How many cubic metres of sea water will be displaced by a ship of total weight 5,000 tonne ? (1 tonne = 100 kg). [Ans. 4874 m^3]

(2) A large block of ice (density 0.9 g/cm^3), 5 metres thick, has a vertical hole drilled through and is floating at the middle of a lake. What is the minimum length of a rope required to scoop up a bucket-ful of water through the hole ? [I. I T. '83] (Ans. 50 cm)

The average density of the human body is very slightly less than that of water. When we breathe out air our body becomes smaller and so denser than water and we sink. But when air is inhaled the body becomes lighter than water. The art of swimming consists in keeping the head (which is heavier) out of water. Animals need not learn swimming, for their heads are lighter.

The water of the Dead Sea contains so much salt in solution and is so dense that one does not sink in it. It is easier to swim in salt water than fresh water.

Problems. (1) A cubical block of wood of sp. gr. 0·7 floats in water, just completely immersed, when a body of unknown weight is placed in it. Find the weight of this body, if the volume of the block of wood is 100 cm^3. [Ans. 30 g.]

(2) A piece of iron is placed on a piece of cork and the two together float on water in a tumbler. If now the piece of iron is taken off the cork and dropped into the water, will the level of water in the tumbler rise or fall ?

[**Hint :** While on cork, the iron displaces its own weight of water. While in water, it displaces its own volume of water. The former is larger than the latter. Smaller displacement causes smaller rise in water level.]

Ex. II-5.23. *Show that a hollow sphere of radius r and sp. gr. s will float on water only if the thickness of its wall is less than $3r/s$* [Nag. U.]

Solution. Let the wall thickness be x for which the sphere would *just* float. Water then displaced has a volume of $\frac{4}{3}\pi r^3$.

Volume and mass of the *material* of the sphere are $\frac{4}{3}\pi[r^3-(r-x)^3]$ and $\frac{4}{3}\pi[r^3-((r-x)^3]s$ respectively. For it to just float then

$$\tfrac{4}{3}\pi[r^3-(r-x)^3]s=\tfrac{4}{3}\pi r^3\times 1$$

$$\therefore \quad r^3\left[1-\left(\frac{r-x}{r}\right)^3\right]s=r^3 \quad \text{or} \quad \left(\frac{r-x}{r}\right)^{\frac{1}{3}}=1-\frac{1}{s}$$

$$\text{or} \quad \left(1-\frac{x}{r}\right)^{\frac{1}{3}}\simeq 1-\tfrac{1}{3}.\frac{x}{r}=1-\frac{1}{s} \quad \text{or} \quad x/r=3/s$$

$\therefore \quad x=\frac{1}{3}.r/s$ Thus if $x<3r/s$ the sphere would float.

Ex. II-5.24. *A body floats in water with $\frac{1}{4}$th its volume above the liquid surface. It is released from a depth of x under the liquid surface. Show that it reaches the surface after a time interval of $\sqrt{6x/g}$.*

Solution ; If the density of a floating solid be ρ with its nth fraction under a liquid of density ρ' then $\rho=n\rho'$. Here $\rho=\frac{3}{4}\rho'$

Now the weight of the body is $V\rho g$ and that of the displaced liquid is $V\rho' g=V\frac{4}{3}\rho g$.

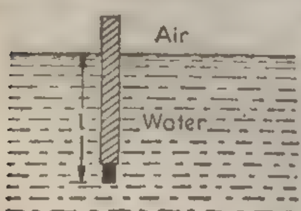
$\therefore$ Upward thrust $=V\rho' g-V\rho g=\frac{1}{3}\rho g V$

and upward acceleration $f=\dfrac{F}{m}=\dfrac{\frac{1}{3}V\rho g}{V\rho}=g/3$

Now $x=\frac{1}{2}ft^2$ or $t=\sqrt{\dfrac{2x}{f}}=\sqrt{\dfrac{6x}{g}}$

Problem : Why a uniform wooden stick floats horizontally but not vertically without being loaded ? [J. E. E. '72]

Ans. Since wood has a density lower than that of water it would float. The cylinder would float horizontally because it then exposes a large surface area to water and can displace its own volume of water by small immersion.



The same would happen (i.e. large surface exposure) when it is properly loaded and drags vertically a large portion of the cylinder down. It will not float vertically without load as then the C.G of the rod being far above the C.B of the liquid, chance of unstable equilibrium is very much greater. For horizontal floating the two are very close and in the same vertical line.

Note : If the vertically floating cylinder is depressed a little (say by x) and then let go it will be acted upon by a net upward

force $Ax\rho g$ (where A is cross sectional area) and execute an up-and-down S.H.M. (Chap. III-1)

Ex II-5.25. *A hollow cube of side 25 cm is floating half submerged in water. Find the volume of lead attached to its lower end so as to sink it a further 1 6 cm in water. ρ of lead = 11.5 g/cc.*

[J E. E. '67]

Solution : On depressing the cube by 1.6 cm further, the volume of water displaced will be $25 \times 25 \times 1.6 = 1000$ cc and the upward thrust would be 1000 gf. To counterbalance it, the mass of lead required must be 1000 g and of volume 1000 g/11.5 g/cc $\simeq$ 87 cc.

II-5.8. **Some special examples of floatation.** (i) When **ice floats** in a tumbler, full to the brim with water, will the melting of ice cause water to overflow ? When the ice floats it displaces its own weight of water. Let V be the volume of water displaced. On melting the ice will form exactly this volume V of water and just fill up the space which the ice displaced. So when ice melts there will be no change in the level of water in the tumbler.

(ii) A **Floating ship**—A ship is made of steel and yet it floats. For it is given such a concave shape with the hollow upwards that, as it sinks into water, it displaces a large volume and soon the weight of that water equals its own weight. *The factor which makes it possible for a ship to float, is the shape.* Tbe density of the material of which it is built is greater than that of sea-water. Buoys marking the channel in a river are hollow iron spheres. They float because each one can displace a weight of water greater than its own weight.

B When a *ship sinks* it reaches the bottom of the sea. The density of water at great depths differs but little from the value at the snrface. Even at a depth of 5 miles the density does not exceed the value at the surface by more than 5%. An object of density very slightly greater than that of sea-water, but not exceeding the latter by more than the above value (*i.e.* lying within the approximate range of densities from 1.03 to 1.08 gm/cm³), may be supported at a suitable level in sea-water. But a ship of a material of higher density sinks to the bottom. *Compressibility of water, a liquid, is very small.*

The density of fresh river water is less than that of sea-water. So greater volume of fresh water must be displaced to match the weight

of the ship than of sea-water, That is why a ship just afloat in the Bay of Bengal may sink in the river Hooghly on her way to Calcutta. That is why a ship gets depressed more in rivers and rises slightly when sailing out to sea.

Ex. II-5.26. *A cargo ship sinks by a cm going into a river from the sea. On discharging the cargo she rises by b cm. When the empty ship sails out in the sea she further rises by c cm. Taking the ship-sides as vertcial to the sea-water show that the sp. gr. of sea water is given by $b/(c-a+b)$.* [Pat. U.]

Solution: *The ship*, note, *behaves like a giant constant weight hydrometer.* Let its sides be immersed by x cm into sea-water before coming into the river. Then

$x+a=$ Immersion depth in river-water with cargo

$x+a-b=$ " " " " without cargo

$x+a-b-c=$ " " " sea " " "

Let ρ be the density of river water and ρ' that of sea-water. Then

$$\text{Wt of ship with cargo} = \rho' x = \rho(x+a) \quad \ldots \quad (1)$$

$$\text{Wt of ship without cargo} = \rho'(x+a-b-c) = \rho(x+a-b) \quad \ldots \quad (2)$$

Subtracting (2) from (1) we get $\rho'(c+b-a)=\rho b$

$$\therefore \frac{\rho'}{\rho}=\frac{b}{c+b-a}$$

Carrying capacity of a ship is the difference in weights displaced by the fully loaded ship and the empty ship. When we say a giant Oil tanker is of 100,000 tonnage we mean that when fully loaded it will displace 100,000 tons of sea-water when floating.

Plimsoll lines are white lines painted on sides of a ship indicating the *maximum depth* upto which the loaded ship can be immersed. *FW* is the level of maximum sinking in *fresh water* and *W* the minimum sinking, *IS* stands for Indian Ocean in summer ; *S* and *W* are for summer and winter in other seas. The variations are due to different salinities in different oceans and different seasons. Obviously *FW* is the maximum permissible immersion in sea. Immersion is always less. They are named after Plimsoll, a British M.P. who initiated an Act of Parliament (1776) to stop overloading and consequent hazards of sinking, by greedy tradesmen. A ship of displacement 100,000 tons imply that the loaded ship must weigh that much to sink her upto the FW Plimson line. A ship drawing 30 ft of water implies that its keel-to-water surface distance is that much.

C. **Floating dock** is a giant recangular trough into which Ocean-liners can move in, when it is full of water. It has large chambers at the base from which water can be pumped out and air pumped in so that the dock becomes lighter and floats up along with the liner. Its base than becomes dry to carry out necessary repairs. One such, has been found at Lothal, Gujerat amongst Indus Civilization ruins.

D. **Life belts** used in ships and boats are large belts inflatable with air, like tyres so that with a small immersion it can displace large volumes of water, and float and support ship-wrecked persons or novice swimmers.

E. **Submarines** (Fig. II-5.11) The submarine is so built that it can float like an ordinary ship. It has two shells, one inside the

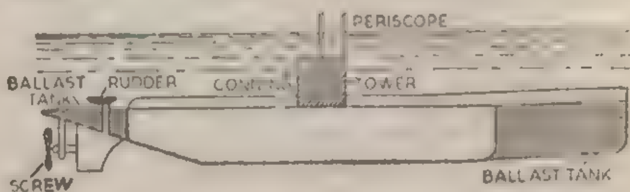


Fig. II-5.11

other. The inner shell is much stronger than the outer. The space between the two shells is divided into chambers. When they are full of air, the submarine floats. When the chambers are filled with water, the submarine sinks. When it is required to rise to the surface, water is expelled from the chambers by pumps driven with compressed air. This lightens the sub, and it floats up.

Fig. II-5.12

F. **Cartesian Diver** (Fig II-5.12) is a toy doll invented by Descartes which long anticipated the workings of a submarine. It is a comic hollow small glass doll with a hole in its tail. It can be simulated by a small hollow glass bulb with a taper opening downwards. The doll contains some air and water inside, in such proportion that the combination floats totally submerged, in water in a tall glass jar. The top of the

jar is closed with a rubber membrane enclosing below it some air above the water surface. On pressing the membrane the air gets compressed and the pressure is transmitted (Pascal's law) through the water to the air inside the doll, contracting it. More water then enters the doll making it heavier and it sinks. On releasing the pressure on the top, air inside expands and the doll comes up. By adjusting your finger pressure, you can make the doll stop anywhere within the water. At different depths the volume of trapped air being different the weight of the doll differs (why ?).

II-6

PNEUMATICS AND ATMOSPHERIC PRESSURE

II-6.1. Pneumatics.

The subject-matter refers to the properties of gases at rest and belongs to that broader branch, Hydrostatics. In common with the other class of fluid the liquids, gases are found to

(i) exert normal thrust on any surface they are in contact with

(ii) obey Pascal's law in transmitting pressure applied to any part, undiminished, to the containing walls in all directions and

(iii) obey Archemedes' Principle in exerting a buoyant force on any solid submerged in it.

Note that Pascal's law would be obeyed in artificial satellites but not Archemedes Principle which is concerned with weights (non-existent in satellites) whereas the former deals with pressure of the gases which must be there present.

Gases however differ from liquids in being highly compressible. This is because of their structural difference we have discussed before, in Chapter II-2

II-6 2. Gases have Weight and exert Pressure :

A. A hollow 1-litre glass globe full of air (fig. II-6.1) is weighed and re-weighed after pumping out air from it. In the second case the globe will weigh *nearly* 1.3 g less. The weight of air however would change from this value at significantly different temperatures and pressures, for their changes induce that in density (mass of unit volume), volume of a gas in the container remaining unchanged.

Fig. II-6.1

Voltaire is said to have concluded that air has no weight from the following experiment. He weighed a toy balloon in a sensitive balance and then inflated it and re-weighed it. No change in weight was noticed. The fallacy lay in the fact that when inflated, the balloon was acted upon by

the upward buoyant force of the air it displaced, which had the same weight as that of the enclosed air, their volumes being equal.

Q.S.S. A soft plastic bag weighs the same when empty as when filled with air at atmospheric pressure. Why? Would the weights be the same when measured in vacuum?

Ans. In air, the apparent weight = true weight − buoyant force due to air on it. When the bag is full of air, the buoyant force due to air increases by the weight of the additional air displaced. Hence whether empty or full we get only the weight of the bag (less the weight of the little air it displaces). But in vacuum, there is no force of buoyancy. Hence the bag containing air will weigh more.

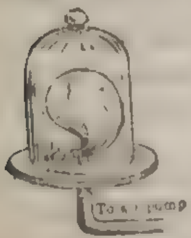


Fig. II-6.2

B. That a gas does exert pressure on its containing walls can be very easily proved by small toy rubber balloons. The mouth of one such containing a little air or any gas, is sealed and the balloon placed on the receiver of a vacuum pump under a large bell-jar (fig. II-6.2). As the bell-jar is slowly evacuated the small shrunken balloon swells up to almost a sphere. The same will happen if the shrunken balloon is slowly heated up. The swelling is obviously due to the pressure of the gas inside, which in both cases, rises above the outside atmospheric pressure.

II-6.3 Pascal's Law applied to Gases and Simple Manometers.

If pressure be applied from outside on an enclosed volume of gas thereby diminishing its volume, as by squeezing an inflated balloon, or pressure inside, it increased by pumping in air, pressure of the trapped gas on the containing walls *rises and equally so at all points.* This can be verified by attaching small *manometers* at different points of the walls. Manometers measure gas pressures.

That Pascal's law si applicable to gases renders possible application of compressed air for diverse purposes. Air is compressed by electric motors when demand of electricity is low and stored in large steel tanks. It is used to drive turbines that run electric generators when the demand goes up. This arrangement allows generators to be run steadily at lower loads all the time, preventing shut-down

during lower and straining them during peak demands. Besides, compressed air drives pneumatic devices to cut or drill or chip stones and hard metals. These tools are a must for mining in hilly regions. It is the transmissibility of gas pressure within our bodies or our rooms that keeps the pressure within and without equal. Had it been not so we would have been squeezed tight by atmospheric pressure of about 14.7 lb-wt on each square inch of our body. An ordinary person has a body surface area of about 16 sq. ft thereby subjected to a total of about 15 ton-wt.

To measure gas pressure inside a closed vessel when it is not much different from atmospheric pressure a **U-tube manometer** is used. It is a very simple device as shown in fig. 11-6.3. It is just a U-tube open at both ends containing some oil for low pressure differences, mercury for higher. The end A is open to atmosphere and the end B is connected to the closed vessel where the gas pressure is required. If the gas pressure there exceeds the atmospheric pressure it pushes up the liquid in the open tube and the gas pressure in gt is the difference in heights of the liquid columns (CD) in the two arms multiplied by the density. When the required pressure is lower the liquid stands at a lower height in the open arm. The gas pressure from the closed vessel in transmitted undiminished through the gas in the connecting tube through the liquid to D where it is balanced by the downward pressure of the atmosphere + the liquid pressure of the column CD.

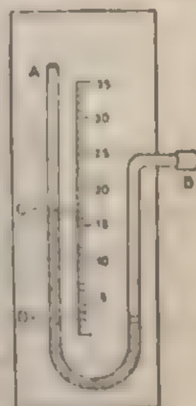


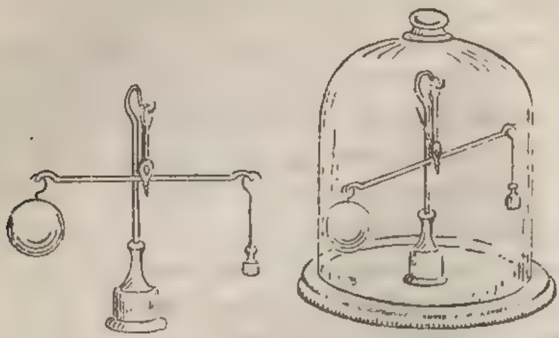
Fig. 11-6.3

11-6.4. Archemedes Principle in Gases

Gases being fluids and having weight on earth, exert upthrust on solids immersed in them. Question of partial submerging however does not occur here.

Buoyancy of air can be demonstrated easily by the following arrangement (fig. 11-6.4). A light but large hollow sealed glass sphere is suspended from one arm of a sensitive common balance placed on the receiver of a vacuum pump and counterpoised (fig. i).

The balance is totally enclosed by a bell jar, its periphery on the receiver being made thoroughly air-tight. As the pump is run,



(*i*) (*ii*)

Fig. II-6.4

gradually the globe is found to descend (fig. ii). On readmitting air fully the balance is restored. If a spring balance is used the same is found to happen.

The explanation is simple. Air exerts upthrust both on the globe and on the balancing weights. But because of the much larger volume of the globe, much larger is the upthrust of air on it, and so is its apparent loss of weight. In absence of air therefore, the globe is found to be heavier than the weights.

Because of this buoyancy, *a kg of cotton would weigh much more than a kg. of lead or iron in vacuum*, though they may weigh equal in air. Lead is far denser than cotton ; a kg of it hence occupies a much smaller volume and so suffers much less upthrust, much less apparent loss in weight than an **equal mass** of cotton.

A. Buoyancy Correction for Weighing in Air. From the above example we find that a body to be weighed and the standard counter-poising weights have different volumes and hence suffer different upthrusts of air and so different losses in weight. So a correction is needed, to get which we proceed as follows :

Let M=*true mass* of the body, m its *apparent mass* in air which is also the apparent mass of the standard weights used, D=density of the material of the body, σ that the material of the standard weights and ρ that of air at the time of experiment.

Then the volume of the body is M/D and the upthrust on it due to air is $(M/D)\rho g$. The upthrust on the counterpoising weights is similarly $(m/\sigma)\rho g$. Since the *apparent weights are equal* we shall have

$$Mg-(M/D)\rho g=mg-(m/\sigma)\rho g$$

whence $M=m\dfrac{1-\rho/\sigma}{1-\rho/D}\simeq m\left(1-\dfrac{\rho}{\sigma}\right)\left(1+\dfrac{\rho}{D}\right)\simeq m\left[1-\rho\left(\dfrac{1}{\sigma}-\dfrac{1}{D}\right)\right]$ (II-6.4.1)

expanding by the binomial theorem and neglecting higher terms.

Thus with the body and the weights of the same material $(D=\sigma)$, no correction arises. But with $D>\sigma$ (the body being of greater density) the *true wt is less than the apparent* i. e. recorded weight. With $D<\sigma$, the true wt is greater.

Example II-6 1. *A body weighs 40 g in air, the density of its material being 0.76 g/cc that of the counterpoising weights 8.4 g/cc and that of air 1.293 g/litre. Find the true mass.* [I.I.T. '69]

Solution. Using the formula deduced above we have

$$M=m\,\frac{1-\rho/\sigma}{1-\rho/D}=40\frac{1-0.001293/0.76}{1-0.001293/8.4}=40.062 \text{ g.}$$

Ex. II-6 2. *Calculate the percentage error arising from neglecting the buoyancy of air in weighing an object of density 12 g/cc with brass weights of density 8 g/cc. Density of air $=1.2\times10^{-3}$ g/cc.* [J. E. E. '81]

Solution : From eqn. II-6.4.1 we have

$$\frac{M}{m}-1=-\rho\left(\frac{1}{\sigma}-\frac{1}{D}\right)=\rho\left(\frac{1}{D}-\frac{1}{\sigma}\right)$$

$\therefore$ the percentage error $=\dfrac{M-m}{m}\times100=1.2\times10^{-3}\left(\dfrac{1}{12}-\dfrac{1}{8}\right)\times100$

$$=0.005\%$$

B. Lifting Power of Balloons. You must have observed that children's balloons filled with hydrogen tends to rise up. Scientists now-a-days use balloons filled with helium which carry up self-recording instruments to study the conditions of atmosphere at various heights and those of weather. Von Hess sent up balloons with electroscopes to study Cosmic Rays in the upper atmosphere (1911). Man ascended in a balloon to 13.7 miles in 1935. Airships or Zeppelins prior to 1940 used for passenger transport from Europe to America were specially adapted balloons utilising hydrogen.

They rise because of Archemedes' Principle. The gases hydrogen and helium are much lighter than air. When inflated with these

gases balloons displace an equal volume of air which has a much larger weight. The *balance of upward force between buoyancy of air and the weight of the balloon* including those of the envelope and the enclosed light gas, is its **lifting power**. It may raise that amount of weight. If V be the internal volume of the inflated balloon and ρ, ρ' densities of air and the enclosed gas under same temperature and pressure then

the total lift of balloon $= V\rho g - V\rho' g = Vg(\rho - \rho')$

and the available lift = total lift – wt. of balloon and its contents

$$= Vg(\rho - \rho') - W \qquad \text{(II-6.4.2)}$$

Ex. II-6.8 *The envelope of a balloon has a 500 cu. m. capacity and is filled with hydrogen of density 0·089 g/litre. If the total weight of the balloon and its load is 200 kg and density of air 1 293 g/litre find the available lift.*

Solution : Total lift $= Vg(\rho - \rho')$N

$= 500 \times (1.293 - 0.089)$ kgf $= 602$ kgf

$\therefore$ Available lift $= 602 - 200 = 402$ kg. wt.

It must however be rememberd that as the balloon rises the density of air surrounding it, diminishes and so does the external pressure. So the buoyancy falls as the balloon ascends and (though it inflates tending to raise buoyancy) a height is reached where buoyancy and the weight just counterbalance and the balloon stops rising (may be said to just float). Hence for a rising balloon a **ceiling ascent** exists depending on its volume, load, enclosed gas and density of air at that height. For a liquid however no ceiling of depth exists for its density does not alter much with depth and the buoyancy on a sinking ship changes but little.

II-6.5. Work done by an Expanding Gas and Boyle's Law :

We have seen above that compressed gases drive machinary so it can do mechanical work. The work it can do *against a constant pressure* can be calculated. We imagine a gas contained in a vertical cylinder under a gas-tight piston. The enclosed gas exerts a constant upward pressure on the underside of the piston because of molecular bombardments. If this pressure is P and the piston cross-section A then the thrust exerted by the gas on the underside is PA. Let us press down the piston *slowly* (why ?) and uniformly through a

distance l. Then the work done against the gas pressure is $PA.l$. See that Al is the extent by which the gas volume has diminished. So the work done is $\quad W = PA.l = P(V_1 - V_2)$

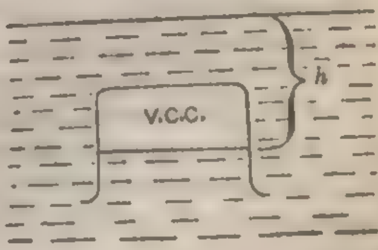
e.g. work done by a gas against atmospheric pressure in expanding by one litre is 1 *litre-atmosphere* $= (1.013 \times 10^6) \times 10^3$ ergs $= 101.3$ joules.

But work done in compressing a gas leads to rise in pressure on the underside of the piston for molecular bombardments become quicker as they have less distances to cover to the bottom of the cylinder and back and forth. This fact leads to the **Boyle's Law** well known to you. It states that

For a given mass of gas at constant temperature, pressure varies inversely as the volume. If in the above example the initial pressure and volume of the enclosed gas be P and V and their final values P' and V', then we have $PV = P'V'$. The law will be discussed in full in HEAT (Chap. IV-5).

Ex III6-4. *An empty beaker floats in water bottom upwards. It is gradually pushed down under water in that condition. Show that after reaching a certain depth it loses all its buoyancy and sinks of itself.*

Solution : Let the empty beaker contain V cc of air at pressure P. As it is forced down mouth downwards, water enters it compressing the air. With depth the volume of enclosed air diminishes and pressure on it increases. Let h cm be the depth of immersion when the required condition arises. Then at that depth, the wt. of the beaker W = that of water displaced by *Archemedes Principle*.



$\therefore \quad W = (v + W/\rho) \times 1$ where v is the volume of air and ρ the density of the beaker material, 1 the density of water.

$$\therefore \quad v = W - W/\rho$$

If H is the height of *water barometer* exerting pressure P, then by Pascal's law pressure on the enclosed air is $(H+h)$ cm. So by Boyle's law $\quad VH$ (at surface) $= v(H+h)$ at immersion h

$$= (W - W/\rho)(H+h)$$

$$\text{or } H+h = \frac{\rho VH}{\rho W - W} \quad \text{or } h = \frac{H(\rho V - 1)}{W(\rho - 1)}.$$

At this depth the weight of the beaker is just balanced by the buoyancy. A little more immersion, the enclosed air is compressed to a volume less than the critical and the beaker sinks.

Problems (1) : How far should a long wide glass tube closed at one end be dipped in water open mouth downwards such that $\frac{2}{3}$rds of it is filled up with water ? Atmos Press = 76 cms of Hg.

[*Ans.* 20.7m]

(2) Force a small inflated balloon some distance under water. It will then sink. Why ? [I.I.T. '72]

II-6.6. Atmosphere : We and everything else on the surface of the earth is submerged in an ocean of air. The gaseous envelope of the earth is called the *atmosphere*. This gas—our air—without which we would not survive beyond a few minutes, is really a mixture of several gases, about 78% of N_2, 21% of O_2 and 1% of A. Besides, there are CO_2(0.03%), Ne(18×10^{-4}%), He(5.3×10^{-4}%), Kr(1×10^{-4}%) H_2(0.5×10^{-4}%), Xe(0.08×10^{-4}%), O_3(0.01×10^{-4}%) of which the amount is variable and increases with height and a variable quantity of water vapour. The composition stated, is by volume. Atmosphere is the only example of a gaseous mass *held in equilibrium without a container.*

It is not possible to state precisely the height of the atmosphere. Air is densest at sea-level, thins out with height gradually and finally fades away in the interstellar space. There, where vacuum is considered to be perfect contains matter, one or two molecules per cc. The atmosphere is broadly divided into two belts the *troposphere*, a region of turbulence upto about 8 miles and beyond, the *stratosphere*, a region of serenity and tranquility. Molecules of air move about very fast. Only the earth's gravity prevents them from flying off in space. Gravity acts towards the earth's center and as you have seen, grows stronger as its surface is approached. Hence the air molecules tend to crowd together near the surface of the earth. A schematic diagram of the atmosphere upto about 250 miles is shown in fig. II-6.5. 59% of the total weight of atmosphere is due to air within 3½ miles and 99% within about 20 miles. From 30 miles to 250 miles above extends the *Ionosphere* from which short radio waves are reflected ; matter is present there very sparingly in the

form of ions. Crowding of molecules close to the earth's surface explains why air there is the densest and thins away with height. Complete crowding is prevented only because of thermal motion of molecules.

All our winds, storms, transport of heat energy and moisture occur in the turbulent troposphere, which is barely 1/1000th of earth's diameter. Had there been no air circulation most of the earth would have been either uninhabitably hot or similarly cold. The main driving force of circulation is the heating of air by land and water in the equatorial region. There, air continuosly rises and spreads out

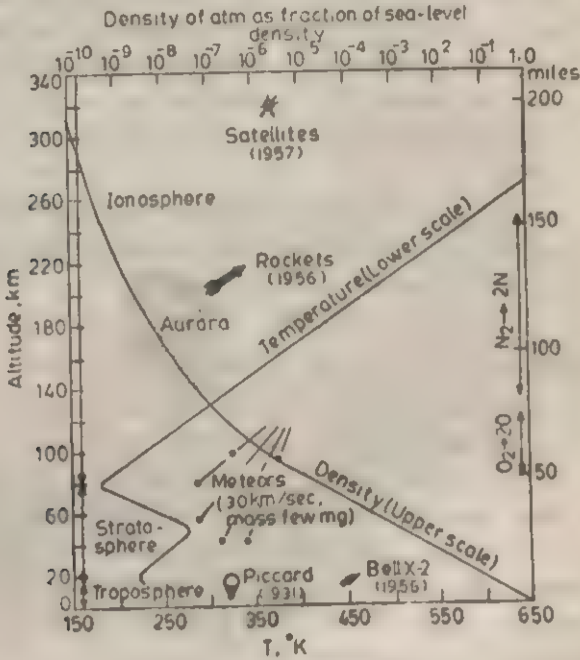


Fig. II-6.5

north and south in the upper troposphere. The air then cools and descends near the middle latitudes with considerable fluctuation and variation. A second circulation occurs beyond, in the higher latitudes and yet another at the poles. The rotation of the earth and variation of temperature due to myriad causes complicate these circulations very much. Intense investigations were carried out in 1979 on the

Indian Ocean to ascertain the origin and nature of the mysterious monsoons, the preserver and life-giver of our agricultur. These circulating winds carry energies beyond comprehension. At any moment the K.E. of all air currents moving over the earth is estimated to be equivalent to 7 million of Hiroshima nuclear bombs ($\simeq 2\times10^4$ tons of TNT each) !

II-6.6. Atmospheric pressure. The *atmosphere* is the only known example of a gas at rest in equilibrium under gravity. In considering the atmosphere, we shall here ignore the causes which make air masses move and take it to be at rest. Like a liquid at rest under gravity, the atmosphere exerts a hydrostatic pressure on all bodies immersed in it. This pressure is called the **atmospheric pressure**. *It is equal to the weight of a column of air contained in an imaginary vertical cylinder of unit cross-section extending up to the top of the atmosphere.*

If at any place, H is the height of such a cylinder, ρ the *average* density of air in the cylinder and g the acceleration due to gravity, then the atmospheric pressure P at the place is given by

$$P = H\rho g \qquad \text{(II-6.6.1)}$$

P diminishes with height above the surface of the earth.

Diverse experiments have been devised to demonstrate the existence of atmospheric pressure. Like liquid pressure, it acts in all directions normal to any surface with which it is in contact. If air exists on both the sides of a surface, there would be no *resultant* thrust on it they being equal and opposite on the two sides. So to demonstrate the existence of atmospheric pressure on a surface, *air from its other surface* has to be fully or partially removed. This can be easily done by a *vacuum pump.*

A very simple experiment shows *upward pressure* of atmosphere. Fill a sharp, even-edged glass tumbler to the brim with water and cover it with a cardboard so that no air exists above water. Now invert the tumbler and remove your palm from the cardboard. It will *not* fall off as atmospheric pressure acting upward will balance out the weight of water inside. To show the downward pressure cover a wide cylinder with a very thin tin plate and place it over

the receiver of a vacuum pump. As air is sucked out the tin plate buckles downwards. A glass plate there would be shattered.

That sideways pressure of atmosphere also exists can be shown (fig. II-6.7(a)) by taking a little water in a very thin tin gallon can with a stopper and boiling the water vigorously till steam drives out air from inside; now it is stoppered tightly and allowed to cool; steam condenses reducing the pressure within when the side walls of the can are found to cave in and crumple into a grotesque shape. The famous Magdeburg hemisphere experiment (1654) carried out by Guericke, the inventor of vacuum pump, demonstrated dramatically the force of atmospheric pressure. Before a royal and large gathering he put together a pair of smoothly fitting copper hemispheres about 2 feet in diameter (fig. II-6.7(b)) in an air-tight joint. Two teams of eight horses each pulling from opposite sides failed to part them when air had been pumped out from inside. The force cementing the hemispheres amounted to nearly three tons.

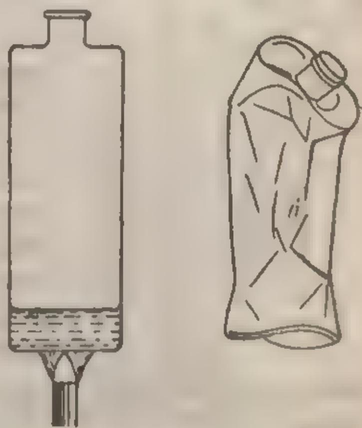


Fig. II-6.7(a)

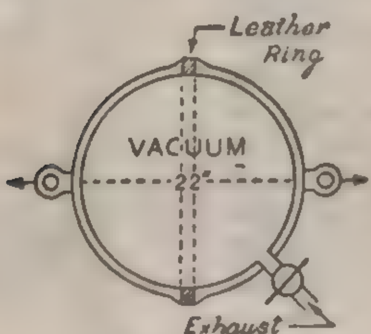


Fig. II-6.7(b)

Sipping up a drink through a straw or *the action of a self-filling fountain pen* are two of many interesting effects of atmospheric pressure. In the first case we diminish air pressure inside the straw and our mouth cavity by sucking when higher air pressure outside on the drink in the tumbler (fig. II-6 8) or bottle forces the liquid up. In the second case, by raising the lever we squeeze out air from

within the rubber bag inside the pen. With lowering of the lever the bag regains its original volume but not the pressure. Atmospheric pressure on the ink in the ink-bottle forces some ink in the partially evacuated bag.

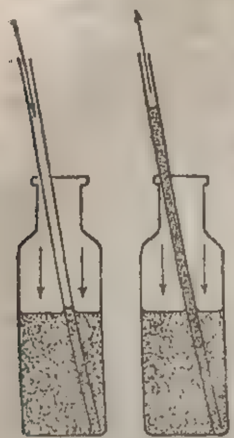


Fig. II-6.8

B. **Nature abhors vacuum** had been taught by Aristotle and the above examples seemed to bear that out. But again Galileo cast a doubt on it when he found no suction pump could raise water from a deep well, water rising to about 28 feet and no more. He opined that apparently there is a limit to abhorrence of vacuum by Nature. Torricelli, a pupil of and successor to Galileo gave the correct explanation that the event was due to atmospheric pressure which can support a water column of at most 34 ft. high. Fig II-6.9 shows an arrangement to prove it. Take a glass tube about a metre long and dip one end of it vertically in a large bowl of mercury (fig II 6.9). Connect the other end of the tube to an air pump through a piece of pressure tubing (i e., thick walked rubber tube). As the pump sucks air out of the tube atmospheric pressure, which is pressing on the surface of the mercury in the bowl, forces mercury up into the tube. If it is a good pump and you are not doing the experiment at a hill station you will find that the mercury rises to a height of about 30 inches and no more. Water under the same condition will rise to a height of about (30"×13.6/12 or) 34 feet. The column of liquid is supported by atmospheric pressure.

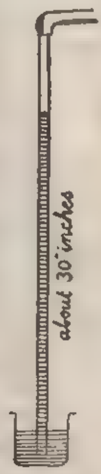


Fig. II-6.9

C. **Torricelli's experiment.** Torricelli an Italian, was the first to conclude (1642) that it was atmospheric pressure which supported a column of mercury in an evacuated tube. He completely filled

a tube about a metre long with mercury and inverted it in a vessel of mercury (fig II-6.10). The mercury in the tube came down to a height about 76 cm above the mercury in the vessel. It fell no further because atmospheric pressure, pressing on the free surface of mercy in the wider vessel, was able to support the weight of the column of mercury. The condition for the balance is

Pressure exerted by the atmosphere = Hydrostatic pressure exerted by the column of liquid, both of them on the mercury in the bowl.

To test Torricelli's theory that the mercury in the tube was supported by atmospheric pressure, Pascal had the experiment performed at a height of about 1700 m above sea level If

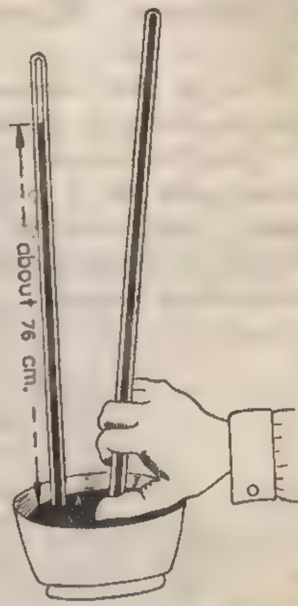


Fig. II-6.10

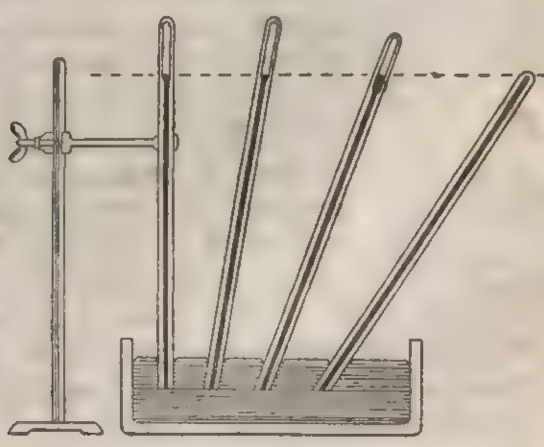


Fig. II-6.11

atmospheric pressure at a place is due to the weight of the air

above the place, the pressure should decrease as we go higher. The height of mercury supported in the tube should therefore be less the higher we go. Pascal found this to be true. The baromater read about 7.5 cm less.

The space above the mercury column (in Torricelli's experiment) is a vacuum since air has been excluded from the tube.* It is known as *Torricellian vacuum.* If the tube is gradually inclined, more mercury goes into it, but mercury in the tube remains at the same vertical height above the mercury outside (fig. II-6.11). If the tube is tilted enough to make the mercury strike the end of the tube, it does so with a sharp metallic click, showing that there is no air inside it.

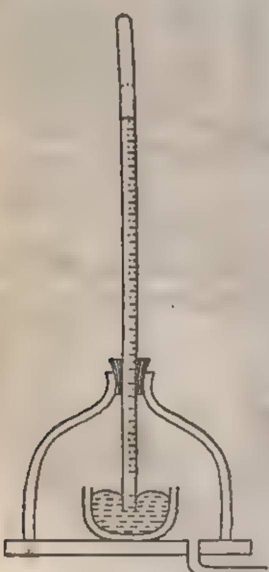


Fig. II-6.12

If Torricelli's experiment is done in an enclosure from which air is slowly taken out (fig II-6.12) the mercury column would slowly fall and on readmitting air will as slowly rise. This confirms that air pressure supports the weight of the mercury column.

The height to which the mercury rises is independent of the diameter of the tube provided it is not very narrow. In very narrow tubes another effect, due to what is called *surface tension*, the height is slightly reduced.

II-6.8. A. Measure of atmospheric pressure. Atmospheric pressure is measured by the height of the mercury column it can support. It varies with the height of a place above the sea-level. Even at a given place it varies slightly according to the weather conditions. For convenience of reference a standard value of atmospheric pressure has been defined as follows :

A pressure of one **standard atmosphere** *is the hydrostatic pressure*

* There will, however, be a little mercury vapour in the space.

*exerted by a column of mercury 76 cm high at 0°C, 45° latitude and mean sea-level.*** (Also see the portion marked 'An indirect way of stating pressure' under §II-4.5).

A pressure of one atmosphere may be expressed in other units as follows :

(1) 1 atmospheric pressure = hydrostatic pressure exerted by 76 cm of mercury $= 76 \times 13.6 = 1034$ g-wt per cm²

$= 1034 \times 980 = 1.013 \times 10^6$ dyn/cm² [II-4.5]

(2) If we take the atmospheric pressure to equal 30 inches of mercury, which is the value we accept while using the British system of units, we shall have

Pressure of 1 atmosphere = hydrostatic pressure due to 30 inches of mercury $= 30 \text{ in} \times \dfrac{13.6 \times 62.4 \text{ lb-wt.}}{12 \times 12 \times 12 \text{ in}^3}$

$= 14.7$ lb-wt per sq inch.

The definition of the standard atmosphere has since been changed. Now 1 standard atmosphere = 1013250 dyn/cm². This value closely approximates the above definition, but is more precise.

Problem. Express *in atmospheres* the pressure at a depth of 300 feet of water, given that the density of water is 62.4 lb per cubic foot, and that a pressure of one atmosphere equals 14.7 lb per square inch.

[*Ans.* 8.8]

B The millibar. Meteorlogists usually express atmospheric pressure in *millibars.* As one millibar is equal to a pressure of 1000 dynes per sq. cm, a pressure of one atmosphere is equal to 1013 millibars (*i.e.* 1.013 **bars**). Study the Weather maps daily published by the Statesman and you will see the *isobars* (same pressure lines) marked out in millibars. *Baros* in Greek means *heavy.*

The mm Hg or **torr.** When pressure is expressed in millimetres of mercury we write mmHg or *torr* as the unit of pressure. One

** Mean sea-level is defined by an Act of the British Parliament as the half-way level between the average high and low tides at Newlyn in Cornwall.

atmosphere is a pressure of 760 mmHg. The word *torr* has been framed to honour Torricelli.

C Height of the water barometer. The hydrostatic pressure due to 30 inches of mercury is equal to the pressure of 30×13.6 inches i.e. about 34 feet of water. Atmospheric pressure can therefore support a column of water 34 ft high. This is known as the height of the *water barometer.* In metres, the height of a water barometer is $0.76 \times 13.6 = 10.34$. But if you really construct a water barometer whose top is closed, you cannot get the water rise to 10.34 m. In the closed space above the water, there will be water vapour which will press down on the water with a pressure dependent on temperature (about 3.2 cm of mercury around 30°C).

In the Kensington Museum London, there is a **glycerine barometer** As density of glycerine is 1.26 g/cc, the height of this barometer would come out to be $76 \times 13.6/1.25$ or nearly 8.2 m or 26.9 ft. Pascal used a barometer with *red wine* in a glass tube about 46 ft high. Thus any liquid may be used in a barometer.

D. Requisites of a suitable Barometric liquid : (1) *The height should be convenient to handle.* Obviously mercury is most suitable for the height is 0.76 m whereas water and glycerine soars to beyond 10 m and 8 m respectively. This is brought about by the *high density* of mercury.

(2) The liquid *should have very low vapour density :* i.e. it should be *non-volatile with high boiling point.* Mercury fulfills this condition the best. The respective boiling points of the three liquids are 357°C. 100°C and 290°C.

(3) The liquid *should be easily visible.* Opaque but shining mercury is better than the other two colourless liquids.

(4) The liquid *should be pure* so that changes in density and other physical conditions should be accurately known. Mercury can be obtained very pure but not so the others.

(5) The liquid height should change sufficiently for a small change in pressure so as to be more sensitive. Mercury being the

heaviest is at a disadvantage here but use of suitable vernier scales has removed this shortcoming.

E. Height of the homogeneous atmosphere The height of a column of air at S. T. P. which would exert the same pressure as the standard atmosphere is known as the height of the homogeneous atmosphere. If h cm is this height, the weight of a column of air at S. T. P. and of height h and cross section unity should be equal to the weight of a 76 cm tall column of mercury of unit cross-section. Since density of air at S.T.P. $= 0.001293$ g/cm^3 we shall have

$h \times 0.001293 = 76 \times 13.6$ whence $h = 8$ kilometres (i.e. 5 miles) less than the height of Mt. Everest (8.8 km or $5\frac{1}{2}$ miles). The atmosphere would extend to this height if the density of air remained as at S. T. P. throughout this height ; but it does not for air thins out as we go up, as pressure falls off very rapidly, so does the temperature.

Fig. II-6.13(a)

II-6.9. The Barometer. A barometer is a device for measuring atmospheric pressure. Since atmospheric pressure can support a column of liquid the measurement of atmospheric pressure consists in measuring the height of a liquid column it supports. The liquid chosen is *mercury*. The advantages of doing so have been detailed above.

A. Fortin's barometer. The most reliable barometer is the one devised by Fortin. It acts on the principle of Torricelli's experiment. In fig. II-6.13(a), C is a mercury cistern in communication with the atmosphere, over which a glass tube A is inverted. The two are enclosed in a metal tube B on which a scale S is etched.

There is also a vernier *V* which can slide in a slot made in the metal tube. The tube is so slotted as to make only the top of the mercury column visible. The zero of the scale starts from the tip of an ivory pointer *F* which projects downwards from the ceiling of the cistern. The lower part of the instrument is shown magnified in fig. II-6.13.(b). The surface of the mercury in the cistern can be adjusted by the screw *E* so that the tip of the ivory pointer *F* just touches it The bottom of the cistern is made of leather. through the pores of which air presses the mercury upwards in *C*.

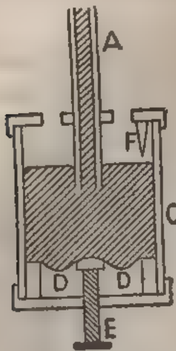


Fig. II-6.13(b)

The scale reading corresponding to the top of the mercury column gives the value of atmospheric pressure at the moment in terms of the height of the column.

B. Siphon barometer. It is very simple in construction and is portable, but it is not so reliable as Fortin's. It is (fig. II-6.14) a U-tube with one arm about 90 cm long with the upper end closed. The other arm is short and open to the atmosphere. The tube is set up on a wooden board fitted with a scale. The difference of the mercury levels in the two arms gives the barometric height, which can be read off from the scale. Unless very carefully constructed a siphon barometer may have traces of air and water vapour in its Torricellian space. They lower the reading.

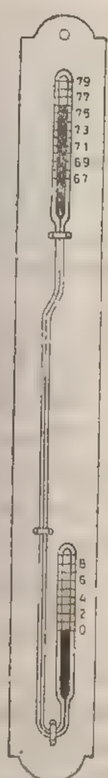


Fig. II-6.14

Weather glass or household barometer is a siphon barometer carrying a dial graduated for pressure in inches and marked as "stormy", "rainy", "variable" and "fair".

C. Aneroid barometer. This type does not require any liquid for its working ('aneroid' means without liquid) and has the great advantage of ruggedness and portability. It consists of a partially evacuated metal box (B: fig. II-6.15) with a flexible corrugated top. A box of this type will collapse under pressure of air. This is prevented by a stiff steel spring S which pulls the top upwards.

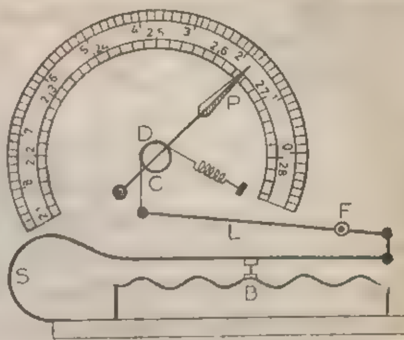


Fig. II-6.15

The free end of the spring S is connected to a lever L which has its fulcrum at F. The other end of L is connected to a small chain C which is wrapped round a spindle D carrying a pointer P which moves over a dial. The movement of the corrugated top due to changes in the atmospheric pressure is magnified by the lever L and transmitted to the pointer P. The dial of an aneroid barometer is calibrated by comparison with a standard mercury barometer.

Barograph. It is a modified aneroid barometer which is self-recording It carries a pen attached to the end of a long lever that takes the place of the pointer. A continuous record is drawn on a piece of squared paper wound on a uniformly rotating cylinder driven by clockwork. From this we can obtain a continuous record of changing pressure throughout a day.

II-6 10 Corrections to Barometer Readings. To convert the readings taken by a *faultless* Fortin's barometer, corrections have to be applied for the temperature and the locale. They are as follows—

(*a*) *Temperature correction:* Rise in temperature expands the metal scale and lowers the density of mercury. The observed height appears less due to the former and more due to the latter. They will be discussed in Expansion of Liquids under HEAT.

(*b*) *Correction for Height:* As pressure depends on g which diminishes with height and Standard Pressure refers to Sea-Level, the barometer reading is to be corrected by reducing the value to the sea-level.

(*c*) *Latitude-correction* : Since again g varies with latitude and pressure with g and Standard Pressure involves 45° latitude, correction for latitude is necessary.

The mercury height (H reduced to sea-level at 45° latitude is given by

$$H = h(1 - 0\ 000257 \cos 2\lambda - 1.96l \times 10^{-9})$$

where h is the observed height reduced to 0°C, λ is the latitude of the place and l the height of the place in cm above sea-level. This height represents the length of the mercury column which would be supported by the existing atmospheric pressure at 0°C and at sea-level and 45° latitude. As you may realise that except for needs of very accurate reading, the *above corrections may be totally ignored.*

Faulty barometer : It gives a reading less than it should. That happenes when the Torricellian vacuum contains air or moisture. With change in temperature the intruder gas changes volume and pressure introducing variability in the barometer readings. To test whether there is air above mercury the barometer tube should be tilted ; if air or moisture happens to be there mercury will *never* reach the top whatever be the tilt. If no air be there mercury will hit the top on sufficient tilting with a metallic click.

The faulty reading can be corrected by utilising Boyle's law for the air trapped in the Torricellian vacuum.

Ex. II 6 5 *A faulty barometer reads 28" and 30" when a true barometer reads 28.5 and 31 inches respectivelp. Find the true reading when the faulty bcrometer reads 29".* [H. S. 1981]

Solution : Let the Torricellian space measure l'' when the faulty barometer reads 28". If s be the area of cross-section of the tube then the air occupies a volume of ls. The pressure exerted by the trapped air must be $(28.5'' - 28'')$ i.e. 0.5" of Hg

Again when the faulty barometer reads 30" the air column is $(l-2)''$ long and exerts a pressure of $(31'' - 30'')$ or 1". Hence by Boye's law we have

$$0.5 \times ls = 1 \times (l-2)s \qquad \therefore\ l = 4''.$$

Finally when the faulty barometer reads 29", the air column would be $(l-1)'' = (4'' - 1'') = 3''$. If the required pressure is H inches of Hg we have

$$(H - 29) \times 3 \times s = 0\ 5 \times l \times s$$

$$\text{or} \quad (H - 29) \times 3 = 0.5 \times 4$$

$$\text{or} \quad H = 29\tfrac{2}{3}''.$$

II-6 11 Change of pressure with height. At sea-level the atmospheric pressure changes *approximately by* 1 mm of mercury for every 11 metre increase of height or 0.1 inch per 90 ft of ascent. The difference of pressure between the two levels is equal to the weight of a column of air of unit cross-section and height equal to the difference in the levels. Thus with a barometer you may *measure the height of a hill.*

Let two stations be at an altitude difference of H cms from each other where the barometer readings are h_1 and h_2 cm of mercury. Then H cms of air column exerts a pressure equal to (h_2-h_1) cm of Hg. If ρ and ρ be the *average* densities of air and mercury between the two stations then

$$H\rho g=(h_2-h_1)\,\rho' g \quad \text{or} \quad H=(h_2-h_1)\,\rho'/\rho \text{ cm} \qquad \text{(II-6.11.1)}$$

With this formula the heights of a building or a hill can be easily determined But the value is *not* very accurate. (Why not ?)

Altimeters Since the atmospheric pressure diminishes as we go up to higher altitudes, the barometer may be conveniently used to determine elevation as indicated above. Altimeters are simply aneroid barometers with an *altitude scale attached* They are very handy and may be small enough to be carried in a pocket. Others may be sensitive enough to indicate a change in elevation of only 1m.

Ex II-6.6 *The atmospheric pressure at the ground floor af a building is 76·85 cm of mercury and that on the top of the same building, 75 63 cm If the average density of air outside is 0·00125 g/cm³, how tall is the building ?*

Solution : The difference of pressure (0·22 cm. of mercury) is due to the weight of a column of air of height (h) equal to that of the building and of cross-section 1 cm².

$$0{\cdot}22\times 13{\cdot}6=h\times 0{\cdot}00125 \text{ whence } h=23{\cdot}9 \text{ metres.}$$

Ex II 6.7 *The atmospheric pressure at the top of a mountain is 4.5 inches of mercury less than the value at the base. If the mean density of air is 0 075 lb/ft³, find its height.*

Solution : Let h be the required height in feet. Then the pressure due to a column of air h ft in height and of density 0·075 lb/ft³ is equal to 4 5 in of mercury.

$\therefore\ h\times{\cdot}075\times g=(4{\cdot}5/12)\times 13{\cdot}6\times 62{\cdot}5\times g$, since the specific gravity of mercury is 13·6 and 1 cu. ft of water weighs 62·5 lb.

$$\therefore\quad h=\frac{4{\cdot}5\times 13{\cdot}6\times 62{\cdot}5}{12\times 0{\cdot}075}=4249 \text{ ft.}$$

Ex. II 6.8. *Barometer readings at the roof and basement of a high-rise building are 76 and 75 cm of Hg. Use Avogadro hypothesis to find the average density of air and the height of the building. Take the temp. of air as 0°C and density of mercury as 13.7 g/cc.*

[J. E E '78]

Solution : We take a *mole* of air. Air is 4 parts of N_2 and 1 part of O_2 and their gm. molecular weights are 28 g and 32 g respectively. So a mole of air would contain 22 4 g of N_2 $(28 \times \frac{4}{5})$ and 6·4 g of O_2 so that 1 mole of air weighs 28 8 g Now the average pressure of air along the building would be 75·5 cm. of Hg. Let the volumes of a mole of air at pressures P_0 and P be V_0 and V. Since temp. remains at 0°C we have by Boyle's law $PV = P_0V_0$ or $V = P_0V_0/P$. Now P_0 is 76 cm of Hg and by Avogadro hypothesis $V_0 = 22{\cdot}4$ litres. Hence

$V = (76 \times 22{\cdot}4 \times 10^3/75.5)$cc. and the average density of air at 0°C is $\rho_0 = \dfrac{M}{V} = \dfrac{28{\cdot}8 \times 75{\cdot}5}{76 \times 22{\cdot}4 \times 10^3} = 0{\cdot}001277$ g/cc.

If the height of the building be H then

Pressure at basement = Pressure at top + Pressure due to H cm. of air

$$\therefore \quad 76 \times 13{\cdot}7 \times g = 75 \times 13{\cdot}7 \times g + H\rho g$$

$$\therefore \quad H = \frac{1 \times 13{\cdot}7}{1{\cdot}277 \times 10^{-3}} \text{ cm.} = 106{\cdot}5 \text{ m.}$$

Check up the result with the rule of thumb indicated above that for each 11 m rise pressure decreases by 1 mm.

II·6.12. *Pressure law of Atmospheres :

The following table shows how air pressure diminishes with height.

Height in miles above Sea Level	0	3·7	5	10	15	20
Air Press. in mm of Hg (torr)	760	380	270	76	24	7

Note how fast the pressure falls at a height beyond 5 miles. This fall can be deduced from the so-called *pressure* law of atmospheres which can be established if we *assume* Boyle's law to hold i.e. temp throughout remains constant and so does g. [As we know, none do]. From Boyle's law we find that pressure of a gas varies directly with density. Now since

$$p = h\rho g \quad \text{we shall have} \quad \frac{dp}{dh} = -\rho g$$

* For more inquisitive students.

$-$ve sign indicating fall of pressure with increase in height. From Boyle's law $(\rho/\rho_o)=(p/p_o)$ where p_o and ρ_o are air pressure and density at the sea-level.

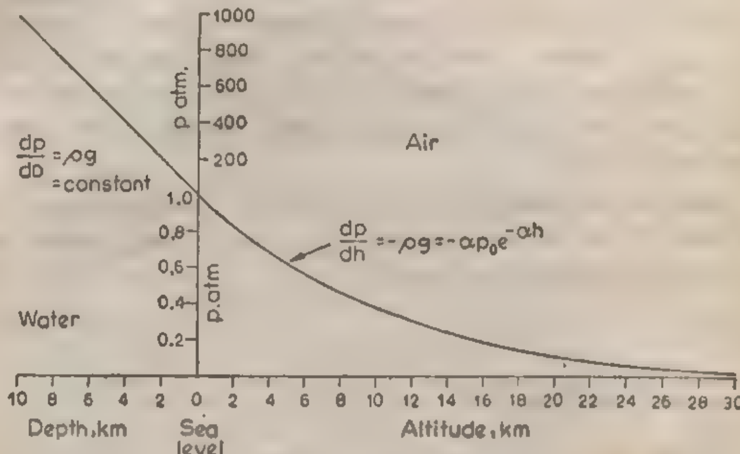


Fig. II-6.16

$$\therefore \quad dp/dh = -g\rho_0(p/p_0) \quad \text{or} \quad (dp/p) = -(g\rho_0/p_0)dh$$

$$\text{or} \quad \int_{p_0}^{p}\frac{dp}{p} = -\frac{\rho_0}{p_0}\int_0^h dh \quad \text{or} \quad \ln p/p_0 = -\frac{g\rho_0}{p_0}h = -\alpha h$$

where $\alpha(=g\rho_0/p_0)$ is a constant $=0.116$/km

$$\therefore \quad p = p_0 e^{-\alpha h} \qquad \text{(II-6.12.1)}$$

The adjoining graph (fig. II-6.16) shows this variation of air pressure with height to the right; and to the left is shown the variation of pressure with depth under water where ρ does not change. Hence the former is an *exponential decay* curve, while the latter is a rising straight line curve.

II-6 12 Weather Forcasting.

Mean atmospheric pressure at a station depends on the altitude above sea-level, latitude and the prevailing temperature there. Again moisture-content lowers the density of air and hence the atmospheric pressure. Rise in temperature similarly lowers the density and hence the pressure. Thus the weather conditions at a place at a given time governs the atmospheric pressure.

So it is possible to make a rough and short-range forecast by observing barometric changes at a given place. If a barometric height falls rapidly a storm and possibly rain can be expected for air would rush in to fill up the partial vacuum. A steady rise of mercury in the barometer indicates replacement of water vapour by dry air and hence a clear dry weather. Rain is possible when the barometer slowly falls. Remember, this forecast is only tentative, for there are many unknown long-term and short-term variables in weather conditions.

Uses of a Barometer : They are three fold—measurement of air pressure at a given place, finding its altitude above sea-level and roughly forecasting the weather there.

II-7

SOME HYDROSTATIC AND PNEUMATIC APPLIANCES

II-7.0. Material

We shall here discuss the action of some simple machines that depend upon the properties of liquids or gases *at rest*. The hydrostatic appliances to be discussed are

(i) siphon and some of its adaptations, and

(ii) some varieties of water pumps

while the pneumatic appliances that will be considered are

(i) simple air pumps and

(ii) the diving bell including caissons

II-7.1. Siphon A siphon is a simple device for transferring a liquid from one vessel to another *at a lower level* withont tilting the vessel. When it is not possible or convenient to lift a vessel of liquid, such as a petrol tank, to empty it, we often use a siphon. We also use it to draw off the upper layers of a liquid without disturbing the lower layers.

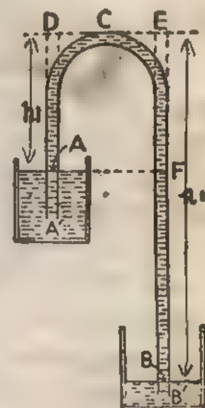


Fig. II-7.1

A. Description. In its simplest form a siphon (fig. II-7.1) consists of an inverted U-tube of unequal limbs, ***completely filled with liquid.*** The shorter limb dips into the liquid to be transferred; the longer limb leads outside and extends below the level of the liquid to be drained out.

B. Action of the siphon may be understood as follows: since the siphon tube is full of liquid, the liquid must flow through the tube if the force pushing on the liquid at one end of the tube is greater than that at the other end. In fig II-7.1 the upward pressure P_A at A, a point inside the tube at the same level as the liquid outside, is equal to the atmospheric pressure P_0 *minus* the downward pressure due to the column of liquid DA. If $DA = h_1$ and density of the liquid in the

siphon is ρ, then we have $P_A = P_0 - h_1\rho g$. The upward pressure P_B at a similar point B inside the tube at the lower end is $P_B = P_0 - h_2\rho g$, when $h_2 = BE$. Hence the pressure at A is greater than the pressure at B by on amount equal to

$$P_A - P_B = (P_0 - h_1\rho g) - (P_0 - h_2\rho g) = (h_2 - h_1)\rho g.$$

This is the pressure due to the liquid column FB = EB − DA. which moves the liquid from the shorter to the longer tube.

Note that the rate of flow of liquid depends on the pressure difference between D and E which is

$$(P_0 - h_1\rho g) - (P_0 - h_2\rho g) = (h_2 - h_1)\rho g$$

So the rate of flow is independent of the air pressure.

C. **Conditions of working.** The siphon will cease to act

(i) when the liquid is at the same level in the two vessels, for then $FB = 0$ and the forces acting at the two ends of the siphon are equal and opposite.

(ii) when EB = AD or EB < AD for there will not be pressure enough to push the liquid out.

(iii) If the liquid is water it will also cease to act if the bend C is more than 34 ft above the surface of water in the higher vessel, for then the atmospheric pressure will be unable to lift the water up to the bend C. For any liquid whatsoever it will not work if **AD > the height of that liquid barometea.**

(iv) A siphon will not work in vaccum, for it is the atmospheric pressure which pushes the liquid through.

What happenes if a hole is made on any arm of a siphon? If it is at A or anywhere on the shorter arm the siphoning ceases; for pressure inside equals that outside. But it does not stop once started, if the longer arm springs a leak.

D. **A discussion:** Experiments have shown however that (i) siphons do work in vacuum and *in certain cases* (ii) it can work with the shorter arm longer than the corresponding liquid barometer height i.e. *atmospheric pressure is not a must* for a siphon to work.

Modern explanation maintains that after the siphon has started working the greater weight of the liquid column in EB pulls up the liquid in AB. This pull acts because of the cohesive force of liquid molecules. It is like this that, if a longer part of a heavy chain

passing over a pulley is given a gentle tug and then released, the weight of this longer part pulls up the shorter part dangling on the other side of the pulley.

Remember however, the liquid in the siphon *needs* to be pure for its working in vacuum. Impurities or dissolved gases lower the cohension between molecules and bubbles form. Then atmospheric pressure becomes essential to compress the bubbles and prevent disruption of the liquid column.

Example II-7.1. *A siphon tube has an internal radius of $3/\sqrt{\pi}$ inches and its two arms are 14" and 20" long. The shorter arm dips 6" inside a liquid. Find the amount of liquid discharged per sec. ($g=32 ft/s^2$)* [J. E. E '72]

Solution : The flow depends on (h_2-h_1) the difference in height. As the liquid flows continuously and starts coming down from D it has no initial downward velocity. Hence the velocity i.e. time rate of flow will be

$$v=\sqrt{2g(h_2-h_1)}=\sqrt{2\times 32[20''-(14''-6'')/12]}$$

$$=\sqrt{64\times(12/12)}=8\text{ft/s}$$

$\therefore$ Volume of liquid discharged per sec is

$=$ Rate of flow $\times$ area of cross-section

$=8\text{ft/s}\times\pi\times(3/\pi)^2\times(\frac{1}{12})^2$ sq ft $=8\times\frac{9}{144}$ cu ft/s $=0.5$ cu ft/s

Ex. II-7.2 *Kerosene of sp. gr 0.8 is to be transferred from a tank 30 m deep by siphoning in to another vessel. The smaller arm reaches the bottom of the tank. After some time siphoning stops though kerosene still remains. Why ? Find the level of kerosene left in the tank. [The barometric height is 11 m of water column].*

Solution : Height of kerosene barometer $h=\frac{1100\times 1}{0.8}=1375$ cm.

$=13.75$ m. So kerosene can be raised by atmospheric pressure thus far and no further. Hence the flow of kerosene stops when the amount to a depth of 13.75 m has been discharged.

The depth of kerosene left is therefore $30-13.75=16.25$ m.

Ex II-7.8. *A cylinder 1 m long and area of cross-section 30 sq. cm has mercury up to 85 cm of it. What volume of mercury can be siphoned out ? [$P_0=750$ mm of Hg]*

Solution : The mercury level in the tube must fall 75 cm below the edge of the container for the siphoning to stop, i.e. it must fall through 60 cm. so that from the top of the vessel to the surface of mercury it may be 75 cm. So the volume siphoned out $=60$ cm $\times$ 30 sq. cm. $=1.8$ litres.

Problem : You are to siphon gasolene ($P = 0{\cdot}7$ g/cc) over an obstacle. Find the maximum height of the obstacle over which siphoning would be possible if barometric height is 76 cm of H_g.

[Ans 14·77 m]

E. Applications of Siphon : Another use of the siphon is to convert a *continuous* stream of water into an *intermittent* stream. The principle may be illustrated by a toy called the *Tantalus' cup* (fig. II-7.2a) where a hidden siphon prevents the level of water in a vessel from rising beyond a certain height, the lip of the figure. When water is poured *slowly* into the vessel (fig. II-7.2b) the water

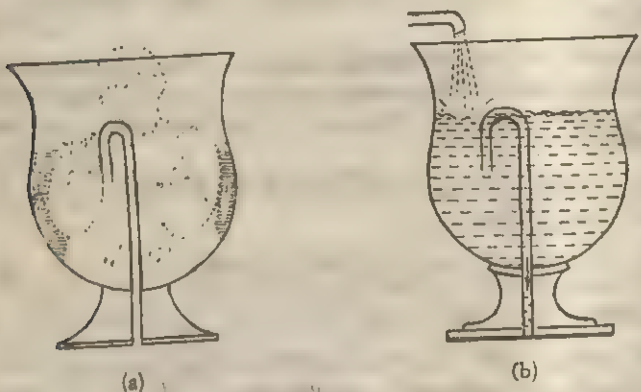


Fig. II-7.2

level in it slowly rises until it reaches the top bend of the siphon when it starts functioning. Water flushes out quickly and if the rate of water supply is less, the water level falls below the shorter arm. The vessel then refills and starts the siphon action again.

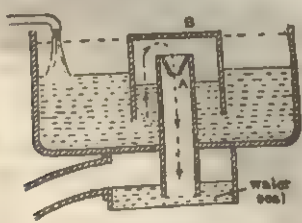


Fig. II-7.3

This is utilised for (i) washing troughs in use for photographic plates and **automatic flushing systems** used in public lavatories (fig. II-7.3). A float (not shown in the figure) rises with the water level and causes the cap B to turn round the fulcrum and close the outlet. The water is discharged, the float goes down re-opening the outlet and the whole cycle is repeated.

II-7.2 Water pumps. Water pumps are devices for rising water from a lower to a higher level. Since liquids have a natural tendency to flow from a higher to a lower level, the lifting is possible only at the expense of energy supplied from outside.

Similarity of the pumps. Though there are different types of pumps, they have some similarities in construction and action :

(a) Each type has valves which allow flow of water *in one direction only.* The difference between the types lies mainly in the position of the valves.

(b) In the first stage of action, each pump sucks up water by the vacuum it creates. Atmospheric pressure forces the water up into this vacuum. Hence none of these pumps can work if the level of water below the pump is greater than the height of the water barometer (34 ft). In practice, however, due to leakage and to vapour pressure such pumps would not work if the height is greater than 30 ft or so.

A. **Common or suction pump.** The construction and action of the pump will be clear from Fig II-7.4. A piston *B*, with a valve *C* in its head, fits airtight into the cylinder *A*. The valve C opens upward and so allows flow only in the upward direction through the piston. From the bottom of the cylinder a pipe goes down into the reservoir from which water is to be pumped up. At the junction of the pipe and the cylinder there is another value *D* which also opens upwards.

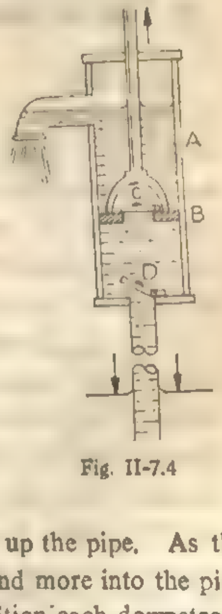


Fig. II-7.4

Suppose the piston occupies the lowermost position at the start. During the upstroke, a partial vacuum is created in *A* between the piston-head and the valve *D*. The higher pressure from below lifts the valve *D* and air from inside the pipe enters the cylinder. As a result a partial vacuum is created in the pipe also.

Atmospheric pressure now forces some water up the pipe. As the pump is gradually worked water rises more and more into the pipe and ultimately enters the cylinder. In this condition each downstroke will cause some water to pass through *C* and collect above the piston

head. During the next upstroke this water will flow out through the spout, while more water will flow in from the pipe into the cylinder. This *intermittent out-flow of water at each upstroke* will continue so long as the pump is worked. This pump is widely used in tubewells

Priming. The valve C is generally not air-tight, particularly when the pump has been in use for some time. As a result the degree of vacuum within the cylinder is not sufficient to make the valve D open. To avoid this difficulty a part of the cylinder above C is filled with water from outside to make the valve C airtight. Suction is now effected easily. This operation is called 'priming'.

Ex. II 7 4 : *A pump has a diameter of 1 foot, stroke of 2 feet and worked at 20 strokes a minute. How much water it discharges per minute ?*

Solution : Vol of the barrel is $= \pi(\frac{1}{2})^2 \times 2 = 1{\cdot}57$ cu-ft.

In a single acting pump, only the upstroke being effective, here we have only 10 such. So volume of discharge $= 15{\cdot}7$ cu-ft

and mass of discharge $= 15{\cdot}7$ cu-ft $\times\, 62{\cdot}5\ \dfrac{\text{lb.}}{\text{cu-ft}} = 981{\cdot}3$ lb.

B The lift* pump. A suction pump, as described above, cannot raise water to more than 30 ft or so for reasons stated earlier. To lift water to greater heights a lift pump may be used. The mechanism will be clear from fig II-7.5. In place of the spout, a delivery tube F is attached to the cylinder. At the junction there is a valve E which opens in the direction of flow. The other valves C and D are in the same position as in a suction pump. The principle of action is as in the suction pump. When the cylinder is filled with water, every upstroke forces some water into F through E. The water gradually rises into F and is delivered at the desired height. Note that here also atmospheric pressure forces the water from the reservoir

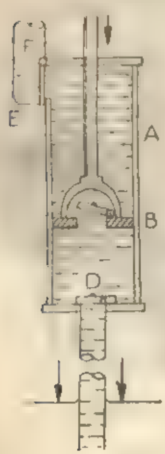


Fig. II-7.5

* There is no uniformity in the use of the word 'lift' pump. Some authors use it to mean the 'common pump described above. Some use it to mean 'pumps' which can lift water to heights above the height of the water barometer.

into the cylinder of the pump. Hence the level of water in the reservoir below the level of the pump should not exceed about 30 ft. Extra energy is needed at each upstroke to push the water into the pipe *F* against the hydrostatic pressure of the liquid column growing gradually in *F*.

The valve *E* may also be placed near the lower end of the cylinder. In that case, water is forced up *F* during the down stroke of the piston.

C. **The force pump** The force pump (fig. II-7.6) can also raise water to heights greater than 30 ft. But here the piston is a solid rod and the delivery pipe *H* is connected at the bottom of the cylinder through a valve *G* which opens only in the direction of flow. The valve *D* remains unaltered in position. Its action is also the same as in the suction pump. During the upstroke the valve *D* openes to let in water into the cylinder, the valve *G* remaining closed. It is the downstroke which forces water into *H* through *G*. It is subject to the same limitations as the force and the suction pumps.

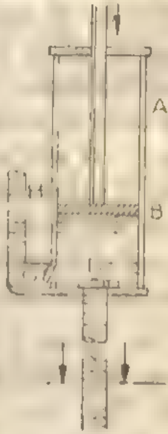


Fig. II-7.6

The continuous action force pump. By adding an air-chamber (*J*, Fig. II-7.7) to the force pump, the outflow of water may be made continuous. The action of the pump keeps the air in *J* compressed. The compressed air forces the water up the delivery tube *K* and maintains a continuous outflow.

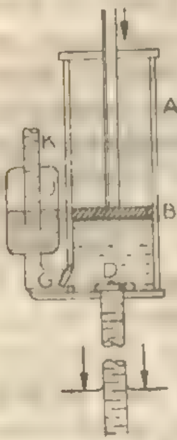


Fig. II-7.7

Fire engine The fire engine has two force pumps connected to a common air chamber. The handles of the pumps are so arranged that while one descends the other ascends. This makes the delivery of water still more regular. Such pumps are used for spraying water on fires and are miscalled fire-engines.

II-7.8 Air pumps. Air pumps are used to pump air *out* of a vessel or *into* a vessel. When used to pump air out of a vessel, an air pump is generally called a *vacuum pump.* A pump that forces air into a vessel is generally called a *compression pump* or *air compressor.*

Vacuum pumps. Though there are various forms of vacuum pumps, we shall consider first the *piston pump*, commonly used in beginners' laboratories.

A The piston pump A simple piston pump is illustrated in fig. II-7.8. It consists of a metal cylinder *A* fitted with an airtight piston *B* which has a *light* valve *C* on it opening outwards. *A* is connected through the tube *E* to the vessel *G* to be evacuated. Another *light* valve *D*, which opens into the cylinder, closes the mouth of the tube *E*. It will be clear that the arrangement of valves is the same as in a suction water pump. The valves *C* and *D* are however *much lighter* than in the suction pump, and open or close with a very small difference of pressure between the two sides. To make good seals there are layers of oil on *C* and *D*.

The action is the same as that of a suction pump. When the piston rises, the volume of the air below *B* increases and its pressure falls. The valve *C* closes due to the pressure above it being greater than that below. As the pressure in *A* is less than atmospheric, the air in *G* forces the valve *D* open and some air from *G* enters *A*. When the piston descends, air in *A* is compressed. This increased pressure in *A* closes *D*. When pressure in *A* rises above atmospheric, the valve *C* opens and air from *A* passes out. Thus at each upstroke some air from *G* enters *A*, and during each downstroke it leaves *A*. The vessel *G* is thus exhausted of air.

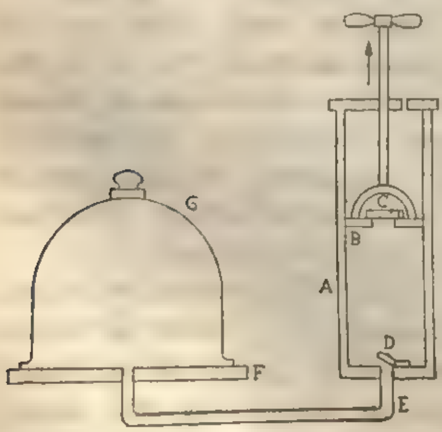


Fig. II-7.8

The action of the pump ceases when the pressure in G fails to lift the valve D. The valve is generally made of oiled silk. The pressure in G may thus be reduced to about 1 or 2 *torr* by this pump.

Pressure and Density of Air in G. To find these after a given number of strokes in a single barrel air pump, let

V = volume of the vessel to be exhausted

v = " " " cylinder between extreme positions of the piston

P_0 = initial air pressure in G.

On the *first* upward stroke, the air volume of air rises from V to $V+v$ while the pressure falls from P_0 to P_1 which can be found from Boyle's law. Thus

$$P_1(V+v)=P_0V \quad \text{or} \quad P_1=P_0V/(V+v)$$

On the *second* upward stroke air volume V at pressure P_1 increases to $V+v$ again and the pressure falls to P_2 where

$$P_2(V+v)=P_1V$$

$$\text{or} \quad P_2=\frac{V}{V+v}\cdot P_1=\left(\frac{V}{V+v}\right)^2 P_0$$

Arguing similarly, after n strokes upwards, the pressure P_n is

$$P_n=\left(\frac{V}{V+v}\right)^n P_0 \qquad \text{(II-7.3.1)}$$

Again if ρ_0 be the initial density of air in G and ρ_n the final one,

$$\frac{\rho_n}{\rho_0}=\frac{P_n}{P_0}\text{ (By Boyle's Law)}=\left(\frac{V}{V+v}\right)^n \qquad \text{(II-7.3.2)}$$

Ex II-7.5 *An exhaust pump has a volume of 100 cc and the volume of the receiver is 500 cc. If the initial air pressure in the receiver is 76 cm of Hg what it will be after 10 strokes?* [J.E.E.'78]

Solution: $P_{10}=P_0\left(\frac{V}{V+v}\right)^{10}=76\cdot\left(\frac{500}{600}\right)^{10}=12.27$ cm of Hg

Note: The receiver can never be completely evacuated however high the value of n i. e. the number of strokes be, for $[V/(V+v)]^n$ can never be zero. Also density cannot be zero i. e. there must be some air molecules always left inside the barrel, fewer after every stroke however.

Problem. If the pressure in a pump were reduced to 1/3rd of the atmospheric pressure in 4 strokes to what would be reduced in 6 strokes? (*Ans.* $1/3\sqrt{3}$) [Pat. U.]

B **Rotary pump.** This pump is a modern device, very rapid in action, and produces vacua as high as 10^{-8} *torr* or even better. It is largely used for laboratory and industrial work.

Construction Fig. II-7.9 represents diagrammatically a rotary pump. *A* is a hollow steel cylinder inside which another cylinder *B* (called the *rotor*) can rotate eccentrically touching the casing *A* (called the *stator*) as it rotates. *C* and *D* are two openings in *A* through which air enters and leaves it. The vessel to be exhausted is connected with the pump through *C* by a thick rubber tube. Air from this vessel can enter *A* through *C*. The outlet opening *D* can be closed by a valve *E* operated by a spring. If the pressure in *A* exceeds a certain value, *E* opens and the air escapes through *D*. Steel vanes F_1, F_2, separated by a spring press against the casing *A*. They lie in slots cut at the opposite ends of a diameter of the cylinder *B*. The place of contact between *A* and *B* completely cuts off connection between the parts *C* and *D*.

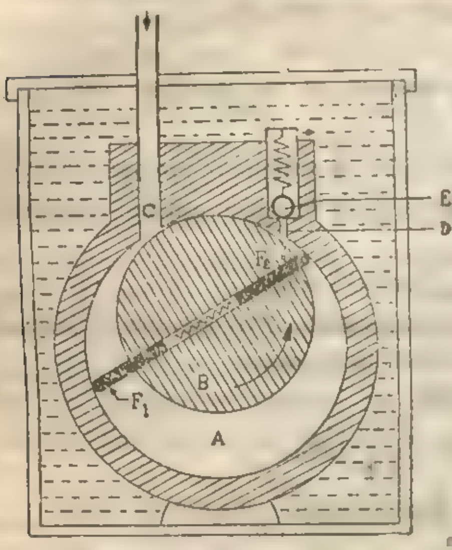


Fig. II-7.9

Action : The eccentric cylinder *B* is rotated by an electric motor coupled to its shaft. Let it turn anticlockwise. As F_1 crosses *C*, the region behind it increases in volume and air from the vessel to be exhausted enters through *C* into this region. At the same time air in the region in front of F_2 is compressed until the valve *E* opens. This air then passes out through *D*. When F_2 passes *C*, a new volume of air is drawn in behind it. The air in front of F_1 is compressed and expelled.

The entire system is kept immersed in oil. This prevents the

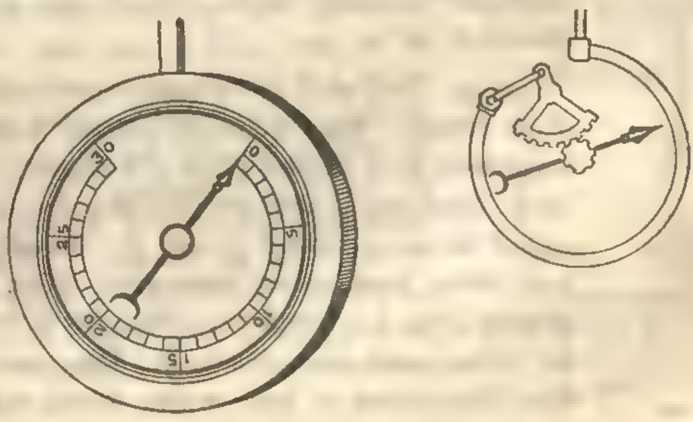
leakage of air into *A*. When the pump is shut down it should be disconnected from the vessel to be evacuated. Otherwise, atmospheric pressure will slowly force the oil into *A* through the minute clearances between the casing and *B* as well as the vanes. This oil may even be forced into the evacuated chamber. In some designs, a reservoir is placed within the pump between *C* and the vessel. It accomodates the oil so pushed in.

Well-constructed rotary pumps can produce vacua of the order of 0·001 *torr* or mm Hg (1 *torr* is the unit of pressure used in vacuum practice). 1 *torr* = 1333 dyn/cm^2.

High vacuum pumps called *diffusion pumps* can ordinarily attain vacua of the order of 10^{-5} to 10^{-6} torr. With special care they can reach 10^{-8} torr. They are much in demand for producing vacua in filament bulbs, X-ray, electronic and discharge tubes and many others.

II-7 4. Pressure gauges. While the barometer is used for measuring the pressure of the atmosphere, *pressure gauges* (or *manometers*) are used for measuring the pressure of gases confined in vessels. We have alrady considered the open-tube manometer, and will discuss the Bourdon gauge for measuring pressure.

Bourdon gauge (fig. II-7.10). It can be adapted to measure pressures above or below atmospheric pressure, and is suitable for



(a) (b)

Fig. II-7.10

many purposes such as measuring the steam pressure inside the boiler of a steam engine or the pressure of compressed air in a cylinder.

The gauge consists of a bronze tube of elliptical cross-section, bent into a nearly complete ring and closed at one end (fig. II-7.10). The open end of the tube is connected with the chamber in which the gas pressure is to be measured. The closed end is connected by levers and a curved rack and pinion to a pointer [fig.(b)] which moves over a graduated scale. As the pressure in the flat tube increases, it tends to straighten out. This moves the pointer over the scale which is generally graduated in pounds/in^2, and reads zero at one atmosphere pressure. A pressure of 10 lb. *by the gauge* is, therefore, a pressure of 10 lb. per sq. in. *above* one atmosphere.

In Bourdon gauges meant for measuring pressures less than one atmosphere, the scale is marked from 0 to 30 (fig. II-7.10a) The graduations are in inches of mercury. A reading of 30 would mean a nearly perfect vacuum. A reading of 10 would mean that 10/30 i.e. $\frac{1}{3}$rd of the air has been removed.

II-7.5. Compression pump. A simple form of compression pump is represented in fig, II-7.11. The piston *B* moving in a cylinder has a valve *C* which opens *inward*. The vessel *G* into which air has to be pumped is connected by a tube with a nozzle attached to the cylinder. Between the nozzle and the cylinder lies the valve *D* which opens towards *G*.

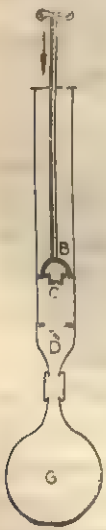


Fig II-7.11

Action Suppose *G* and *A* contain air at atmospheric pressure. When the piston is drawn upward the pressure of air between *C* and *D* is reduced. The valve *C* opens and atmospheric pressure pushes some air into the space between *C* and *D*. During the down stroke, known as the *compression stroke*, air between *C* and *D* is compressed. The valve *D* opens and allows some air to enter *G*. Thus some air enters the cylinder at each suction stroke, and is then forced into the vessel *G* at each compression stroke. The valve *D* remains closed during each suction stroke, since the pressure on the lower side of *D* is greater than that on the upper during suction.

The rotary pump may also serve as an air compressor. The end C (fig. II-7.9) is kept open to the atmosphere while a delivery tube is fitted to the outlet port D. As the pump runs, it forces air through D.

Pressure in G. With symbols as in exhaust pumps, we have initial mass of air in $G = \rho V_0$. During each stroke a mass $v\rho$ is forced into G. Hence after n strokes the total mass of air in $G = V\rho + nv\rho$. If the density of this air is ρ_n then

$$V\rho_n = V\rho + nv\rho$$

$$\therefore \quad \rho_n = (1 + nv/V)\rho \qquad \text{(II-7.5.1)}$$

If P_n be the pressure of this compressed air then by Boyle's law

$$\frac{P_n}{P} = \frac{\rho_n}{\rho} = 1 + \frac{nv}{V}$$

$$\therefore \quad P_n = P(1 + nv/V) \qquad \text{(II-7.5.2)}$$

Ex. II-7.6 *The barrel and receiver of a condensing pump contain 0·075 and 1 litre of air respectively, at one atmoshere pressure. After h w many strokes the pressure would rise to 4 atmospheres ?*

Solution : From II-7.5.2, $4 = (1 + n \; 0{\cdot}075/1) \times 1$
or $3n = 120$ or $n = 40$

II·6 6. The Diving Bell With this appliance one can descend to considerable depths under water. It consists of a more or less cylindrical, large vessel (fig. II·7.12) closed at the top but open at the bottom and weighs more than the water it can contain. As it is lowered into water with the open end downwards, it sinks. As the air in it is gradually compressed, water rises in the bell, but never fills it. The internal pressure may be found by applying Boyle's law. Compressed air may be pumped into it to keep out water as desired.

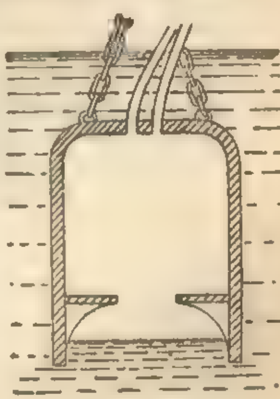


Fig. II-7.12

Sounding lead : A miniature cylindrical bell of the above type, coated internally with a soluble pigment, serves as a useful device for measuring the depth of water. When lowered into water, the pigment

dissolves up to the distance to which the water level rose within the cylinder. From this distance the depth of immersion can be computed. See the problem on page 141.

Caissons. They are used to lay the foundations of a bridge under water and are essentially diving bells. Powerful air compressors, operated on land, pump compressed air into caissons to keep water out of them.

Problem: A cylindrical diving bell 1.50m tall is lowereed to the bottem of a tank, when 0.5 m of water rises inside the bell. Find its depth. [I.I.T. '67]

Take temp to remain const and atmopheric press to be 10 m of water. Refer to Ex. II-5.4. (Ans. 5.5 m)

EXERCISES

[illegible]

[illegible]

Exercise—II.1 (Gravitation and Gravity)

[A] **Essay type questions**

1. State the law of universal gravitation and explain it in symbols. What is meant by the universal gravitation constant ? What is its value in cgs units ? Why is it called a universal constant ?

2. Two spheres have masses M_1 and M_2 and radii r_1 and r_2. What will be the gravitational attraction between them if their centres are at a distance d apart ? Will the formula apply if the bodies, instead of being spherical, has some other sphape ? Explain your answer.

3. What role does gravitation play in the motion of planets ? Assuming that the orbit of a planet is circular, derive a relation connecting the periodic time of the planet and the radius of its orbit.

4. What is meant by acceleration due to gravity ? How is it related to the mass of the earth ? Is it a constant ?

5. Discuss why g changes with latitude. At a given place why does g have the greatest value on the surface of the earth than above or below it ?

6. Distinguish between mass and weight. Under what circumstances will a body have no weight ? Will it have no mass at the same time ? Explain ?

A spring balance can detect differences of weight at different places on the earth ; but a common balance cannot. Why ?

7. Why does a body appear weightless in an artificial satellite ? Will it require a force to move a body inside it ? Explain.

8. State the laws of falling bodies. How can you deduce from them that all freely falling bodies move with the same constant acceleration ?

9. What is meant by escape velocity ? What is its value on the earth's surface ? Is it the same on all planets ? If a body is to leave a planet not to return, in what direction should it be given the necessary velocity ?

10. What are meant by the term sgravitational field, potential and intensity ? How do the gravitational force differ from electric and magnetic forces ?

11. What is a simple pendulum ? State the laws of a simple pendulum. How can you determine acceleration due to gravity with its help ? What preceautions woald you taxe ?

12. State Kepler's laws of planetary motion. How can you establish Kepler's third law from the Newton's law of gravitation ? Ared vice verea ?

13. How can you compare the masses of different planets from Kepler's third law ?

Show that the smallest period of revolution of an artificial satellite in a circular orbit around a planet is determined only by the average density of matter of the planet. [J. E. E.]

14. Obtain expressions for the orbiting velocity and period of revolution of a planet moving round the sun in a circular orbit. Hence derive an expression for the mass of the sun.

Prove that the length of the year is shorter for a planet nearer to the sun.

15. What is meant by orbiting velocety of a satillite ? Calculate the orbiting velocity and period of revolution of an artificial satellite. Find a relation between orbiting velocity and escape velocity.

16. What is a Geo-synchronous satellite ? Find its height.

[B] Short answer type questions

17. If the force of gravity acts on bodies proportional to their masses, why does not a heavy body fall faster than a light body ?

18. Explain clearly why there is no atmosphere on the surface of the moon but on the surface of the earth.

19. Where will a body weigh more—at the pole or at the equator ? If the body a somehow taken to the centre of the earth, then what will be its weight ? Why so ?

20. The gravitational force exerted by the sun on the moon is greater than the gravitational force exerted by the earth on the moon. Why then does not the moon escape from the earth ?

21. The sun attracts all bodies on the earth. At midnight, when the sun is directly below, it pulls an object in the same direction as does the earth ; at noon, when the sun is directly above, it pulls the object in a direction opposite to the pull of the earth. Hence, all objects should be heavier at midnight than they are at noon. Explain why this is not.

22. The gravitational attractions of the sun and the moon on earth produce tides The solar tidal effect is about half as great as that of the lunar. The direct pull of the sun on the earth, however, is about 175 times that of the moon. Why is it then that the moon causes higher tides ?

23. Neglecting air friction and technical difficulties, can a satellite be put into an orbit by being fired from a huge cannon on the earth's surface ? Explain.

24. An artificial satellite is in a circular orbit about the earth. How will its orbit change it one of its rockets is momentasity fired (a) towards the earth, (b) away from the earth, (c) in a forward direction, (d) in a backward direction, (e) at right angles to the plane of the orbit.

25. If a planet of given density were made larger, its force of attraction for an object on its surface would increase because of the planet's greater mass but would decrease because of the greater distance of the object from the centre of the planet. Which effect predominates ?

26. For communication purposes it is desirable to have a satellite which stays vertically above one point on the earth's surface. Explain why the orbit of such a satellite must be circular and must lie in the plane of the equator. What is it called ?

27. A body is falling freely towards the earth. At some instant when it is falling the gravity ceases to act. What happens to its motion ?

28. (i) Elucidate why a space vehicle in orbit is a freely falling body with an acceleration towards the earth equal to the value of g at that point.

(ii) Hence explain weighlessness.

29. Weight is not an essential property of a body—Explain.

30. State and explain whether the time period of the pendulum will change in the following cases :—

(i) If a hollow bob is taken instead of a solid bob.

(ii) If the hollow bob is partly filled with water.

(iii) If the pendulum is taken to the top of a mountain.

(iv) If the pendulum is taken to the bottom of a mine.

(v) If the hollow bob is fully filled with water.

(vi) If the pendulum is kept in a lift moving with (1) uniform velocity. (2) uniform upward or dounward acceleration ($f<g$)

(vii) If the pendulum is inside a satellite.

31. A hollow sphere is filled with water and is hung by a long thread. A small hole is made at the bottom of the sphere and water trickles out slowly through the hole. It is observed that the period of oscillation of the sphere first increases, then decreases and then restored to original value—Explain.

32. It is observed that the period of a simple pendulum is much larger and it continues to oscillate much longer on the surface of the moon than on the earth. Why ?

33. The bob of a pendulum is made of iron. A powerful magnet is placed below the bob. It is observed that the time period of the pendulum decreases—Explain.

34. A body is released from an orbiting satellite. What will happen to the body ?

35. From the top of a tower, a ball is dropped while another is thrown horizontally simultaneously. Which one strike the ground first ? Explain the answer. [I. I. T.]

36. When a ball is thrown up the magnitude of its momentum decreases and then increases. Does it violate the conservation of momentum principle ? [I. I. T.]

37. The point of suspension of a simple pendulum moves with a uniform acceleration in the horizontal direction. How is the period of the pendulum affected ? [H. S. '83]

38. How can you explain rarity of certain gases in the earth's atmosphere ?

39. A pendulum is suspended from the ceiling of a lift. The pendulum is brought out of equilibrium through an angle and released. At the moment when the pendulum passed through its lowermost position, the lift began to fall freely. How will the pendulum move with respect to the lift.

40. How the length of the year changes if the distance between the sun and the earth changes ?

[C] Numerical problems

41. Two iron spheres of radii 10 cm and 1 cm are placed with their centres 10 cm apart. Calculate the force of attraction between them. Given, density of lead $= 11{\cdot}5$ gm cm^{-3} and $G = 6{\cdot}67 \times 10^{-8}$ cm^3 gm^{-1} sec^{-2} [$15{\cdot}45 \times 10^{-4}$ dyne.]

42. Find the force of attraction between two protons at a seperation distance of 10^{-10} m. The mass of a proton is $1{\cdot}67 \times 10^{-27}$ kg. [$1{\cdot}86 \times 10^{-44}$ N.]

43. A space rocket flies to the moon. At what point of the straight line connecting the centres of the moon and the earth will the rocket be attracted by the earth and the moon equally. The mass of the earth is 81 times that of the moon and the distance between them is $3{\cdot}84 \times 10^5$ km. [$3{\cdot}46 \times 10^5$ km.]

44. At what altitude above the earth's surface would the numerical value of g be half of that at the surface ? Radius of the earth is 6400 km. [2550 km.]

45. If the earth's mass $= 5{\cdot}96 \times 10^{27}$ g, its radius $= 6{\cdot}367 \times 10^3$ cm and gravitational constant $= 6{\cdot}67 \times 10^{-8}$ cgs unit, what will be the value of g on the surface of the earth ? Explain the difference form the known value. [$101{\cdot}02$ cms^{-2}]

46. Calculate the value of g the moon's surface, given that its radius is 0·27 times the earth radius and its mass is 1/81 of the earth's mass.

How heavy will a boy weighing 60 kg on the earth feel on the moon ? [about 1/6 the earth's gravity ; about 10 kg on the earth.]

47. In a free fall from an aeroplane a terminal velocity of about 192 kmh^{-1} is reached when the force due to air resistance equals the gravitational force. When the parachute opens, the velocity is reduced to about 6 ms^{-1} in approximately one second. Find the deceleration caused by the opening of the parachute.

If the parachutist weighs 100 kg what is the decelerating force ? Express the deceleration as a multiple of g ($=9·8$ ms^{-2}) and the force in terms of the weight of the man. [4·8 g ; $4·8 \times 100$ kgf.]

48. The mass of the earth is 80 times that of the moon. Its diameter is 4 times that of the moon. If a body is weighed on the moon in what ratio will it decrease ? (1 : 5) (H. S. '79]

49. Determine the mass of the sun if the mean radius of the earth's orbit is 149×10^6 km. [2×10^{33} g]

50. Calculate the change of apparent gravity at the equator of a planet if it were suddenly to contract and change its radius by $\frac{1}{10}$th of its former value. [Increase by $\frac{1}{8}$th of the present value.]

51. A satellite of earth appears to be stationary with respect to the earth. Find its distance from the centre of the earth and also the direction of its motion. Given, the radius of the earth = 6400 km, mass of the earth = $6·03 \times 10^{27}$ g and G = $6·67 \times 10^{-8}$ cgs unit. [42460 km ; from west to east.]

52. The ratio of the radius of the earth to that of the moon is 4, whereas the ratio of the accelerations due to gravity on the earth and on the moon is 6. Find the ratio of the escape velocity from the earth and the moon. [5]

53. Calculate the escape velocity from the following data : Radius of the earth = 6400 km ; $g = 980$ cms^{-2}. [11·2 km/s]

54. The ratio of the acceleration due to gravity in a very deep mine and on the surface on the earth is 790/800. Assuming the earth to be of uniform density throughout and its radius 6400 km, calculate the depth of the mine. [80 km]

55. At what altitude and at what depth will the acceleration due to gravity be 25 percent of that at earth's surface ? [R ; 0·75 R]

56. At what distance from the earth's surface is the acceleration due to gravity equal to 1 ms^{-2} ? Given that radius of the earth = 6400 km. [31632 km]

57. How much faster than its present rate should the earth rotate about its axis in order that the weight of a body on the equator

may be zero and how long would it take to make one revolution? What would happen if (i) the rotation becomes faster and (ii) if the rotation were stopped altogether? The value of $g = 9.8$ [illegible].

[17 times, 1.42 hr, objects would start leaving the earth's surface; g increases by [illegible]]

58. If the earth were to stop rotating about its axis, what will be the change in the value of g at a place of latitude 45°, assuming the earth to be a sphere of radius 6.38×10^8 cm? [1.687 $cm\,s^{-2}$]

59. A uniform sphere has a radius of 2 cm. Find the percentage increase in weight when a spherical sphere of radius 20 cm and density 12 gm cm^{-3} is brought underneath it and nearly touching it.

[5.65×10^{-6} %]

60. If an object at the equator is weightless, then what would be the length of the day? The radius of the earth = 6400 km and $g = 980\ cm\,s^{-2}$. (C.U.) [1.41 hr]

61. A ring is made of a thin wire of radius r. Find the force with which the ring will attract a material particle of mass m on the axis of the ring at a distance L from the centre. The radius of the ring is R and the density of the material of the wire is ρ. Hence find where the attraction will be maximum.

$$\left[F = \frac{2\pi^2 Gm\rho r^2 RL}{(R^2 + L^2)^{3/2}},\ L = R/\sqrt{2} \right]$$

62. If the earth were a solid sphere of iron of radius 6.37×10^8 cm and of density 7.86 gm cm^{-3}, what would be the value of acceleration due to gravity at its surface, taking $G = 6.66 \times 10^{-8}$ cgs unit? [1400 $cm\,s^{-2}$]

63. A body is suspended on a spring balance in a ship sailing along the equator with a speed v. Show that the scale reading will be very close to $W(1 \pm 2\omega v/g)$, where ω is the angular speed of the earth and W is the scale reading when the ship does not move.

64. Two artificial satellites A and B of same mass are circling the earth at distances R and 3R respectively from the surface of the earth, R being the radius of the earth. Assuming the orbits of the satellites circular, compare total kinetic and potential energies.

[2 : 1 in both cases]

65. A satellite Titan of Saturn completes one revolution in 16 days, the radius of the orbit being 1.2×10^6 km. The moon completes an orbit round the earth once in 27 days, the distance of the moon from the earth being 3.8×10^5 km. Compare the masses of Earth and Saturn. [96.8]

66. The planet Pluto is 40 times as far from the sun as the earth is. What is Pluto's orbital period? [253 yrs]

[illegible] What is the potential energy of the earth assuming it to be a uniform sphere of radius [illegible] m and mass [illegible] kg ?

(Bombay [illegible])

[illegible] Two particles having masses m and M respectively attract each other according to the law of gravitation. [illegible]

[illegible] A pendulum consists of a long thread carrying a small lead sphere at its lower end. [illegible] Find the original length of the thread and the value of the acceleration due to gravity.

[illegible]

[illegible] Calculate the acceleration due to gravity at a place where a simple pendulum [illegible]

[illegible] If the length of [illegible] pendulum [illegible] find the length of a pendulum [illegible]

[illegible] What is the ratio between the length [illegible] of a simple pendulum [illegible]

[illegible] A [illegible] pendulum [illegible] building [illegible] top of the building? [illegible]

[illegible] The mass and [illegible] of a planet are each [illegible] of the earth. What will be the period of oscillation of a [illegible] on the planet [illegible] pendulum?

[illegible] A simple pendulum is [illegible] a lift. [illegible] when the lift is [illegible] moving with uniform velocity, [illegible] with uniform acceleration [illegible] with uniform [illegible]

acceleration [illegible]

[illegible] The length of a [illegible] pendulum is increased [illegible] How many [illegible]

[illegible] Two pendulums [illegible] period [illegible] and [illegible] are started [illegible]

[illegible]

[illegible] A [illegible] pendulum is [illegible] at a particular place, its true period being T. A certain mass M is brought under it and caused

along such that the centre to centre distance remains always equal to d. Find the new time period in terms of T. $\left[T\Big/\left(1+\frac{GM}{gd^2}\right)^{\frac{1}{2}} \right]$

79. The period of a simple pendulum is increased by 1/100 s when the length is increased by 1 cm. Find the original length of the pendulum. (Poona) [100 cm.]

80. A small ball of mass 100g suspended from a fixed ponit by a light inextensible string 200 cm. long describes a horizonal circle of radius 100 cm, the string sweeping out a conical surface. Find the frequency of revolution of the ball and the tension in the string.

[0·38 s^{-1}, 115·59 *g*-wt]

81. Suppose, a man is measuring force by the gravitatinal attraction exerted by two unit masses separatcd by unit distance as the unit. What will be the value of G, according to this new unit ?

(J. E. E. '76) [$1{\cdot}5 \times 10^7$]

82. A clock-pendulum made of Aluminium has a length of 100 cm, when the temperature is 20°C. How many seconds will this clock lose in a day if the temperature is maintained at 30°C ? Co. efficient of linear expansion of Al = $25 \times 10^{-6}\,°C^{-1}$. [10·8 s]

Exercise—II.3 (Elasticity)

[A] Essay type questions

1. Explain the terms elasticity, strain and stress. What do you understand by elastic limit and Hooke's law ? What is meant by modulus of elasticity ?

2. Explain the terms tensile stress and strain, compressive stress and strain and shearing stress aad strain. Define Young's modulus, bulk modulus and modulus of rigidity. Distinguish between solids, liquids and gases on the basis of these three moduli.

3. What is Poisson's ratio ? In what unit is it enpressed ? What are the egs units for the three elastic moduli ? What cgs units do you use for stress and strain ?

4. Describe a laboratory method for the determination of Youug's modulus of elasticity of a material in the form of a wire.

5. Find the work done in stretching a wire. What happens to the energy when the wire is allowed to return to its original length ?

6. With the help of a stress-strain curve, explain the terms perfect elasticity, yield point, elastic lemit, breaking point, breaking stress and breaking load.

[B] Short answer type questions

7. Steel is more elastic than rubber—Explain.

8. What is force the constant of a spring ? What property of the spring does it express ? 'A spring balance measures the weight of a body—Explain.

9. When a wire is stretcted potential energy is stored in it. Why ? When the stretching force is removed and the wire regains its original length then what happen to this potential energy ?

10. A stretched wire is suddenly snapped. Would there be any change in its temperature ?

11. In an experiment to measure the Young's modulus for steel a wire is suspended vertically and loaded at the free end. In such an experiment, (i) Why is the wire long and thin, (ii) Why is a second steel wire suspended adjacent to the first ?

12. State one advantage and one disadvantage in using a long wire rather than a short stout bar when measuring the Young's modulus by direct stretching.

13. A system of balanced forces produces deformation but a system of unbalanced forces produces acceleration—Explain.

14. Which is more rigid—steel or diamond ? Explain with reasons.

15. Show that the value of Poisson's ratio lies between 0·5 and −1.

16. Two springs have their force constants K_1 and K_2 ($K_1 > K_2$). On which spring is more work done (i) when their lengths are increased by the same amount, (ii) when they are stretched by the same force. [I. I. T. '76]

[C] Numerical Problems.

17. Express the force constant in terms of Y, length and the area of cross-section of the spring wire. [$K = YA/L$]

18. A spring of force constant K is cut into three equal pieces. Find the force constant of each part. [3K]

19. Two vertical weightless springs of equal length hang from a ceiling and the lower ends of the springs are attached to a bar. It the force constants of the springs are K_1 and K_2, find the force constant of the spring system. [$K_1 + K_2$]

20. A spring of force constant K_1 and another of force constant K_2 are joined together and hung from a ceiling. Find the force constant of the spring system. $\left[\dfrac{K_1K_2}{K_1+K_2}\right]$

21. A railway engine of mass 20 tons struck a wall and each of its two buffers was found to be compressed by 10 cm. If the spring of each buffer is compressed by 1 cm under the action of a force of 1 ton force find the velocity with which the engine struck the wall. (1 ton = 1000 kg.) [3 56 km/hr.]

22. A 2 kg load elongates a spring by 2·5 cm What will be the elongation for a load of 1 5 kg ? What load will be required for an elongation of 1 75 cm ? [1·875 cm, 1·4 kg]

23. A piece of copper wire is 2m long and 0 5 mm in diameter. A load of 10 kg increases its length by 2·38 mm. What is the Young's modulus for copper ? [$4{\cdot}28\times10^{6}$ kgf. cm.$^{-2}$]

24. A 10 kg. weight suspended by a 5m long wire causes a strain of 0 1%. If the cross-section of the wire is 1 mm^2, what are its elongation, strees and Young's modulus ?

[5mm, $9\ 8\times10^{8}$ dyn cm^{-2}, $9{\cdot}8\times10^{11}$ dyn cm^{-2}]

25. A piece of wire, 600 5 cm long and 1mm^2 in cross-section has a load of 20 kg suspended from one end If the load is removed the length diminishes by 0 5 cm. What is the value of Young's modulus ? [$2{\cdot}35\times10^{12}$ dyn cm^{-2}]

26. A horizontal plank supported at two ends is depressed in the middle by 2 cm. when a load of 5 kg. in placed there. How much will a load of 7 5 kg depress it in the middle ? What load will be required for a depression of 3·75 cm ? [3 cm, 8·75 kg.]

27. A rod of length 250 cm and radius 2·5 cm has Young's modulus $= 2\times10^{12}$ dyn cm^{-2} and Poisson's ratio $=0\ 3$. What strain in the diameter will be caused by a load of 1000 kg ? [$1{\cdot}47\times10^{-4}$]

28. A metal mass of volume 500 cm^3 hangs from the end of a wire whose upper end is rigidly fixed. The diameter of the wire is uniform and equal to 0·4 mm and its Young's modulus is 7×10^{11} dyn cm.$^{-2}$. When the metal mass is completely immersed in water the length of the wire changes by 1 mm. Find the length of the wire. [179·4 cm.]

20. If the coefficient of linear expansion of steel is 12×10^{-6} per °C, Young's modulus is 2×10^{9} gm wt per cm^2, calculate the increase in tension in a steel wire of cross-section 0·005 cm^2 tightly fixed between two rigid supports when the temperature falls by 10°C. [1200 gm wt.]

26. A piece of rubber pressure tubing 30 0 cm long extends by 0 60 cm when stretched by a load of 300 gm. If the internal

diameter of the tube is 4·0 mm and the thickness of wall also 4·0 mm, Calculate Young's modulus for the rubber. [$1{\cdot}46 \times 10^7$ dy cm^{-2}]

27. A steel cylinder is heated through 50°C and is then clamped in rigid supports at each end. Calculate the stress in the cylinder when it has cooled down to its original temperature. (Y for steel = 2×10^{12} dyn cm^{-2}, co-eff. of linear exp. for steel = $1{\cdot}2 \times 10^{-5}$ per °C).
[12×10^8 dyn cm^{-2}]

28. A steel wire of diameter 1·0 mm is stretched between two rigid supports fixed at right angles to a thick brass base, the wire and the base being parallel. The tension of the wire is 1·5 kg wt. What will the tension become if the temperature of the apparatus rises by 10°C ? The co-efficient of linear expansion of steel and brass are $1{\cdot}09 \times 10^{-5}$ deg C^{-1} and $1{\cdot}89 \times 10^{-5}$ deg C^{-1} respectively. Young's modulus for steel is $2{\cdot}0 \times 10^{12}$ dyn cm^{-2}. [2·78 kg wt]

29. A wire of length 1m and cross-sectional area $0{\cdot}5 \times 10^{-2}$ cm^2 is stretched horizontally between two pillars. A mass of 100 g is suspended from the midpoint P of the wire. Calculate the depression of P. ($Y = 2 \times 10^{12}$ dyn cm^{-2} ; $g = 980$ cms^{-2}) [1·07 cm]

30. A vertical steel wire and a parallel brass wire, each 400 cm long and 0·0040 cm^2 cross-section, hang from a ceiling and are 30 cm apart. The lower end of the wires are attached to a light horizontal bar. Find the mass of the load which must be hung from the bar to cause each wire to extend by 0·10 cm. At what distance from the brass wire must the mass be suspended ? (Y for brass = $1{\cdot}0 \times 10^{12}$ dyn cm^{-2} and for steel, twice as much.) [3·06 kg , 20 cm]

31. A steel wire 1m long and a brass wire 70 cm long each 0·01 cm^2 in cross-section are fastened together end to end and are then subjected to a tension of 10 kg wt. Calculate the elongation of each wire. Given Y for steel = 2×10^{12} dyn cm^{-2} and that for brass = 1×10^{12} dyn cm^{-2}. [0·049 cm, 0·0686 cm]

32. Elastic limit is exceeded when the strain of a wire becomes just greater than 1/2000. If the cross-section is 0·02 cm^2 and Young's modulus for its matrial is 14×10^{11} dyn cm^{-2}, find the value of maximum load that can be hanged from the wire without causing permanent deformation. [14·3 kg wt]

33. A light rod of length 200 cm is suspended from the ceiling horizontally by two vertical wires of equal length tied to its two ends. One of the wires is made of steel and is of cross section 0·1 cm^2 and the other is of brass of cross-section 0·2 cm^2. Find the point on the rod at which a weight may be hung to produce (i) equal stresses in both wires, (ii) equal strains in both wires. Y for brass = 10×10^{11} dyn cm^{-2} and that for steel = 20×10^{11} dyn cm^{-2})
[66·67 cm from brass wire ; At the middle of the rod.] (I. I. T , '74)

34. A wire of length 1m when stretched by a load of 10 kg increases in length by 0·1 cm. The cross-section of the wire is 0·1 cm^2. Calculate the Young's modulus of the material. If the Poission's ratio be $\frac{1}{3}$, calculate also the change in volume of the wire.
(J. E. E. '82.) [$9 \cdot 8 \times 10^{10}$ dy cm^{-2}, 0 0033 cm^3.]

35 A vertical s eel columm 10 m high supports a load of 80 metric tons. Taking Young's modulus for steel to be 2×10^6 kg wt cm^{-2}, area of cross section to be 100 cm^2 and Poission's ratio to be 0·2, find the decrease in length and decrease in volume of the column when the load is applied. (J. E E. '79) [0·4 cm ; 56 cm^3]

36. A steel wire of cross-sectional area 0·5 mm^2 is held between two fixed supports. If the tension in the wire is negligible and it is just taut at a temperature of 20°C, find the tension when the temperature falls to 0°C. (α for steel $= 12 \times 10^{-6}$ per °C and $Y = 2 \cdot 1 \times 10^{12}$ dyn cm^{-2}) (I. I. T. '73) [$2 \cdot 52 \times 10^6$ dyn]

37. A copper wire and a steel wire each of length 1·5 m and diameter 2 mm are joined at one end to form a composite wire 3 m long. The wire is loaded till its length becomes 3·003 m. Calculate the strain in copper and steel wires and the force applied to the wire. Y for copper $= 1 \cdot 2 \times 10^{12}$ Nm^{-2} and that for steel $= 20 \times 10^{12}$ N m^{-2}. [$1 \cdot 25 \times 10^{-3}$, 75×10^{-4}, 471 N]

38. A uniform glass tube containing water is suspended vertically from a support. When a weight attached to its lower end, 1 m length of the tube is found to be stretched by 0·05 cm, while the same length of the water column in the tube is lengthened by 0·03 cm. Find the Possion's ratio for glass. [0·299]

39. Find the value of the Poission's ratio for gold given that a force of 330 gf elongates a gold wire of diameter 0·32 mm by 1 mm and that equal and opposite torques of 145 dyn-cm applied at its ends twist it through 1 radian. (Bombay). [0·429]

40. Find the work done in stretching a wire of 1 mm^2 cross-section and 2 m long through 0·1 mm. Young's modulus for the material of the wire is 2×10^{12} dyn cm^{-2}.
(Punjub Univ.) [5000 erg.]

41. A copper wire 2 m long is stretched by 1 mm. If the energy stored in the stretched wire is converted into heat, calcultate the rise in temperature of the wire (Y for copper $= 12 \times 10^{11}$ dyn cm^{-2}, density of copper $= 8$, sp. heat of copper $= 0 \cdot 1$ and $J = 4 \cdot 2 \times 10^7$ erg/cal) [0·004°C]

42. A copper wire has twice the radius of a steel wire. One end of the copper wire is joined to one end of the steel wire so that both can be subjected to same longitudinal force. By what fraction of its length will the steel wire have stretched when the length of

copper wire has increased by 1% Y for steel is twice that for copper. [2%]

43. Calculate the minimum tension with which a platinum wire of diameter 0·1 mm must be mounted between two points in a stout invar frame if the wire is to remain taut when the temperature rises to 100 K. Platinum has a coefficient of linear expansion of $9 \times 10^{-6}\ K^{-1}$, $Y = 17 \times 10^{10}\ Nm^{-2}$. The thermal expansion of inver may be neglected. [1·20 N]

44 The rubber cord of a catapult has a cross sectional area of 1·0 mm^2 and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a missile of mass 0·005 kg From energy considerations or otherwise, calculate the velocity of projection of the missile. Take Y for rubber as $5{\cdot}0 \times 10^8\ N\ m^{-2}$. State the assumptions made in your calculations. [20 m s^{-1}]

45. Two rods of different metals having the same area of cross-section A are placed end to end between two massive walls. The first rod has a length l_1 coefficient of expansim α_1 and Young's modulus Y_1. The corresponding quantities for the second rod are l_2, α_2 and Y_2. The temperature of both the rods is raised by T degrees Find the force with which the rods act on each other in terms of the given quantities. (I.I.T.) [$ATY_1Y_2(l_1\alpha_1 + l_2\alpha_2)/(l_2Y_1 + l_1Y_2)$]

46. A copper ring of radius 100 cm and cross-sectional area 4mm^2 is fitted on to a steel rod of radius 100·125 cm. With what force will the ring be expanded if the modulus of elasticity of copper is 12×10^3 kgf/mm^2. [60 kgf.]

47. An iron wire 5m long is suspended vertically. By how much will the volume of the wire change it a load of 10 kgf is attached to it ? Poission's ratio = 0·3. [0·001 cm^3.]

48 A sphere of radius 10 cm and mass 25 kg is attached to the lower end of a steel wire which is suspended from the ceiling of a room. The point of support is 521 cm above the floor. When the sphere is set swinging as a simple pendulum, it lowest point just grazes the floor. Calculate the velocity of the ball at the lowest position. The original length of the wire is 500 cm, radius of the wire is 0·05 cm and Young's modulus for its material $= 2 \times 10^{12}$ dyn cm^{-2}. (I.I.T.) [3·756 ms^{-1}]

49. A sphere of mass 50 g is attached to one end of a steel wire 0 315 mm is diameter and 1 m long. In order to form a conical pendulum, the other end is attached to a vertical shaft which is set rotating about its axis. Calculate the number of revolutions per second necessary to break it. The wire will break when the force exceeds $4{\cdot}5 \times 10^6$ dyn cm^{-2}, g = 980 cms^{-2}. [151 cps.]

50. A steel ball of radius 10 cm and mass 1000 g is suspended from a point 312 cm above the floor by a steel wire of unstretched length 290 cm. The radius of the wire is 0·2 mm. If the ball is

set swinging like a pendulum so that its centre passes through the lowest point at 6 ms^{-1}, by how much does it clear the floor ? Y for steel $= 2 \times 10^{12}$ dyn cm^{-2}. [1·748 cm]

51. 200 cm^3 of air is at a pressure of 760 mm of mercury. If the pressure is increased by 1 mm of mercury and the temperature remains constant, the volume of air increases by 0·263 cm^3. What is the bulk modulus of air ? [760 mm of Hg]

52. If a torque of 1000 dyn-cm is applied at the lower end of a suspended wire it is twisted by 90°. What torque will be required to produce a torque of one radian ? [$2000/\pi$ dyn cm.]

53. Obtain a value for the density of sea water at a depth of 4 mi below the surface, assuming that the density at the surface is 1·025 gm ml^{-1} and that the compressibility of sea water is $4{\cdot}3 \times 10^{-5}$ per bar. (1 bar $= 10^6$ dyn cm^{-2}) [1 053 gm ml^{-1}]

54. Find the work done in twisting a steel wire of radius 1 mm and length 25 cm through 45°, the modulus of rigidity of steel being 8×10^{11} dyn cm^{-2}. (Lucknow). [$1\ 55 \times 10^6$ erg.]

55. The Young's modulus and the modulus of rigidity for a substance are $11{\cdot}25 \times 10^{11}$ and $4{\cdot}55 \times 10^{11}$ dyn cm^{-2} respectively. What will be the change in volume of 1000 cm^3 of the substance when subjected to a pressure of 10 atmosphere ?
(1 atm. $= 10^6$ dyn cm^{-2}). (Gujerat). [0·014 cc.].

56. A disc of mass M is placed on a table. A striff spring is attached to it and is vertical. To the other end of the spring is attached a disc of negligible mass. What minium force should be applied to the upper disc to press the spring such that the lower disc is lifted off the table when the external force is suddenly removed ?
(I. I.T. '76). [Mg]

57. When a rubber cord is stretched, the change in volume due to change in linear dimensions is negligible. Obtain value of Poission's ratio for rubber. (Bihar) [0·5]

58 Calculate the bulk modulus for a specimen of steel, given that the Young's modulus and the rigidily modulus for the specimen are 21×10^{11} and 8×10^{11} dyn cm^{-2}. [$18{\cdot}7 \times 10^{11}$ dyn cm^{-2}]

59. Poission's ratio of a material is 0 379, its rigidity modulus is $2{\cdot}87 \times 10^{11}$ dyn cm^{-2}, find (Baroda) [$7{\cdot}916 \times 10^{11}$ dy cm^{-2}.]

60. A wire of diameter 0·6 mm and length 300 cm hangs vertically. A load of 4 kgf is applied at its middle point and also a further load of 4 kgf at its free end. Find the total elongation, assuming Young's modulus for the material of the wire $= 2 \times 10^{12}$ dyn cm^{-2}. [0·312 cm.]

61. A copper wire 8 m long hangs vertically. Find the elongation due to its own weight. Y for copper $= 1{\cdot}0 \times 10^{12}$ dyn cm^{-2} and density of copper $= 8\ 9$ gm cm^{-3}. [0·00215.]

II.4 (Hydrostatics)

[A] **Essay type questions :**

1. Distinguish pressure and thrust. It pressure involves an area, what is meant by pressure at a point ?

2. What is the characteristic property of a fluid ? How can you prove that a liquid exerts a normal pressure on any surface with which it is in contact ? Explain if this will apply also to an imaginary surface within the liquid.

3. Derive an expression for the pressure at a depth h in a liquid of density ρ.
What do we understand when we say that fluid pressure at a point acts equally in all directions ?

4. How can you prove that the pressure of a fluid at rest is the same at all points in the same horizontal plane ?
What do you mean 'horizontal' and 'vertical' ?

5. If there be several open vessels connected together by a liquid, why should the liquid levels be the same in all ?
What is the meaning of the s atement 'a liquid finds its own level' or 'a liquid stands at the same height in communicating vessels'.

6. State Pascal's law of transmission of liquid-pressure.
Explain how the law can be applied to magnify a force. Show how the principle of conservation of energy is not violated in such magnification.

7. Briefly discuss the construction of a hydraulic press and explain the principal on which its acts. Derive an expression for mechanical advantage of the machine.

[B] **Short answer type questions :**

8. Things can be cut by the sharp edge of a knife easily but cannot be cut by the blunt edge. Explain the reason. (H. S. '82)

9. Why it is necessary to make the dam of a water reservoir thicker at the bottom than at the top ? (H. S.'82)

10. Why does a pointed nail penetrate into the wall more than a thick nail when struck ?

11. Explain how it can be that pressure is a scalar quantity when forces, which are vectors, can be produced by the action of pressures.

12. Will a hydraulic press work if its cylinder be filled with air instead of liquid ?

13. Water is poured to the same level in each of the three vessels of different shape but of the same base area. If the pressure is the same at the bottom of each Nessel the force experienced by the base

of each versel is the same. Why then do the three vessels have different weights when put on a balance ?

14. Can a hydraulic press multiply energy ?

15. Why should we get a stronger flow of water from a tap lowers down than another, but having the same diameter of the mouth ?

[C] **Numerical problems :**

16. A 20kg table stands evenly on four legs 5 centimeltre square. What are the values of thrust on each leg and the pressure at a point of the leg ? [5 kg, 0·2 kg cm^{-2}]

17. The water supply tank in a household is 30 m above ground. When a tap is opened 10m of water head is lost for overcoming friction. What is the pressure of water in a tap 8 m above the ground ?

If the tap has a diameter of 1 2 cm, what force should be applied to its mouth so as to stop flow of water ? [1200 gf cm^{-2}, 1560 gf.]

18. How much length of mercury will exert pressure equal to that of 34 ft of water ? Given density of mercury = 13·6g cm^{-3}. [30 inch]

19. A bottle has bottom area equal to 30 cm^2. It is filled with water and corked. The area of the cork is 1 cm^2. If a force of 40 gf be applied on the cork, find the thrust on the bottom. [1200 gf]

20. The pressure at the bottom of a well is 4 times than at a depth of 2 ft. If the pressure of atmosphere is equal to a column of water 30 ft high, find the depth of the well. [98 ft.]

21. A cubical tank having each side 2m is filled with water. Calculate the force on the bottom and the force on a vertical side. [8000 kgf, 4000 kgf.]

22. What is the pressure at the bottom of a clear take of water 10m deep ? Atmospheric pressure = 76 cm. Hg, density of mercury = 13·6 g cm^{-3}. (H. S. '82) [2033·6 gf cm^{-2}]

23. A cubical tank with side 40 cm is closed at the top. At 25 cm from the base, a vertical pipe is joined to one side of the tank. The pipe contains water upto a length of 55 cm. If the cross-section of the pipe is 100 cm^2, calculate the thrust on each side of the tank including the top. [1254 5N on bottom, 627·2N on top, 940 8N on each vertical side without pipe, 886 9N on each vertical side with pipe.]

24. A lockgate separates water on two sides, the height of water on one side is 4 ft, on the other side 20 ft. If the gate is 20 ft wide, find the resultant thrust on the lockgate. [24×10^4 lbf.]

25. A cylindrical vessel is filled with equal amounts by weight of mercury and water. The overall height of the two layers is 29·2 cm.

Determine the pressure of the liquid at the bottom of the vessel. The density of mercury is 13 6g cm^{-3}. [4 cm Hg.]

26. A cubical box is full of water, show that the thrust on any vertical wall is half the weight of the water contained in the box.

27. If the pressure of water at the ground floor is 34 lbf/inch^2 and on roof 18 lbf/inch^2 of a certain house, calculate the height of the house. [36.86 ft.]

28 A vertical U-tube of uniform inner cross-section contains mercury in both its arms. A glycerine (density 1.3 g cm^{-3}) column of length 10 cm is introduced into one of the arms. Oil of density 0 8 g cm^{-3} is poured into the other arm until the upper surfaces of oil and glycerine are in the same horizontal level. Find the length of the oil column. Density of mercury is 13.6 g cm^{-3}. (I. I. T. '72) [9.61 cm]

29. A U-tube, whose ends are open, whose section is one square inch, and whose vertical, branches rise to a height of 33 inches, contains mercury in both branches upto a height of 6.8 inches. Find the greatest amount of water that can be poured into one of the branches, assuming density of mercury to be 13.6 g cm^{-3}. (London Univ.) [27.2 inch^3]

30. A U-tube is partly filled with mercury of density 13·6 g cm^{-3}. Salt solution of density 1.10 is poured into one limb so as to make the difference of levels of mercury in the two limbs equal to 1 cm. What is the height of the column of solution ? [12.4 cm]

31. The two arms of a U-tube have cross-section of 1 cm^2 and 0.1 cm^2 respectively. Some water is poured into the tube which enters into both the limbs of the U-tube. What volume of a liquid of density 0·85 g cm^{-3} should now be poured into the wider limb so that water level in the narrower tube may rise 15 cm ? (J. E. E. '76) [17·91 cm^3]

32. A U-tube partly filled with water. Another liquid which does not mix with water, is poured into one limb until it stands a distance h above the water level on the other limb, which has meanwhile risen a distance d. Find the density of the liquid. [$2d/(2d+h)$]

33. The smaller piston of a hydraulic press has a diameter 1 inch, and the larger one has a diameter 1ft If a force of 50 lb be applied on the smaller piston, how much force will the larger one develop ? (P. U. '62) [8064 lb]

34. The diameter of two pistons of a hydraulic press are respectively 3 inches and 30 inches. The smaller piston is attached at distance of 2 ft from the fulcurm of a lever of length 12 ft. What force must be applied at the free end of the lever in order to produce a force of 5000 lb at the bigger piston ? (Tripura H. S '81) [8·3 lbf.]

35. A hollow right circular cone of height h and semi-vertical

angle α rests with its base on a horizontal tabe. If the cone is filled with a liquid of density ρ, the weight of the empty cone becomes equal to the weight of the liquied it contains. Find the thrust on the base of the cone and the pressure exerted on the table when the cone is filled with the liquid. [$\pi\rho gh^3 \tan^2\alpha$, $\frac{2}{3}h\rho g$]

II-5. (Archemedes' Principle and flotation)

[A] Essay type questions :

1. State and explain Archemedes' principle. Describe an experiment in support of the principle.

What is meant by up thrust on an immersed body ?

2. How can you find the volume of a body with the help of Archemedes' principle ?

Distinguish between Density and Specific gravity. In what sense can we say specific gravity is relative density ?

3. What is meant by 'force of buoyancy' ? Where does it act ?

Show theoretically with the help of a simple example that the upthrust on an immersed body is equal to the weight of the displaced liquid.

What relation has the apparent loss of weight of an immersed body with the fact that a liquid exerts a normal pressure on any surface with which it is in contact ?

4. Find the condition that a solid will float or sink in a liquid. Why does a ballon filled with hydrogen or helium ascend ? Though steel is heavier thrn water why does a ship made of steel float ?

[B] Short answer type questions :

5. Explain in which of these cases the greater part of your body will remain under water—(i) breathfully inhaled, (ii) breathfully exhaled.

6. When a ship sinks in an ocean does it go down to the bottom or remain suspended somewhere ? Give the reason for your answer.

7. The top of the tower of a submarine was just submerged when it was in a river. Explain how the position will change when the submarine reaches the sea.

8. Show that 'specific gravity' and 'relative density' are equivalent.

9. The specific gravity of mercury is 13·6. What is its density in c.g.s., f.p.s. and m.u.s. systems ?

10. Why in defining specific gravity, water is taken at 4°C ?

11. If a body is taken to a greater depths inside a liquid, then does the force of buoyancy acting on the body changes ? Explain your answer.

12. 'The ascent of a balloon in air illustrates the action of buoyancy'—Explain.

13. Is 'centre of buoyancy' coincides with 'centre of gravity' of the displaced liquid ?

14. A piece of cork is embedded inside an ice block which floats in water. What will happen to the level of water when all the ice melts ? (I I T. '76)

15. Does Archemedes' principle hold inside a freely falling lift ? Explain.

16. Does Archemedes' principle hold in a satellite moving in a circular orbit ? Explain.

17. Why does an ordinary balloon rise to the ceiling when filled with hydrogen, but drops on the floor when filled with carbon-di-oxide ?

18. Explain how the position of 'meta centre' determine the stability of floating bodies.

19. Explain why goods are loaded in the lower deck of a ship.

20. A ship which just floats in the sea sinks when it enters the river – Explain.

21. Sometimes a submarine which sinks down to a sea-bed of sand or clay cannot raise itself again—Explain.

22. When a piece of iron immersed in mercury and then released, it floats up. Will this happen in outer space ? Give reasons for your answer.

23. A solid wooden cylinder is placed in a container in contact with the base in such a manner that when water is poured into the container, no water can go beneath the solid. Will the cylinder float up ? Give reason for your answer. (H. S. '80)

24. 'A floating body has no weight'—Explain the statement.

25. A boy is carrying a bucket of water in one hand and a fish in the other. Is the weight carried by him less when he transfer the fish to the bucket without spilling any water during the transfer ? (I. I. T. '70)

26. A flat disc, a cube and a sphere, all made of iron and having the same mass, are immersed completely in water. Which one of them will experience the (i) mininum buoyancy, (ii) maximum buoyancy ?

27. A ballon filled with air is weighted so that it barely floats in water. Explain, why it sinks to the bottom when it is submerged a short distance in water. (I. I. T. '73)

28. Explain why a uniform wooden stick which will float horizontally if it is not loaded, will float vertically if enough weight is added to one end. (J. E. E. '72)

29. A boat floating in water tank carrying a number of large stone. If the stones are unloaded into water, what will happen to the water-level ? (I. I. T. '79)

30. A beaker containing water is placed on the pan of a balance which shows a reading of Mg. A lump of sugar of mass m g and volume V cm^3 is now suspended by thread in such a way that it is completely immersed in water without touching the beaker and without any overflow of water. What will be the reading of the balance just when the lump of sugar is immersed ? How will the reading change as time passes on ? (I. I. T. '78)

31. A block of ice is floating in a liquid of specific gravity 1·2 contained in a beaker. When the ice melts completely, will the liquid level in the beaker change ? (I. I. T. '74)

32. A piece of ice with a stone frozen in it floats on water kept in a beaker. Will the level of water increase, decrease or remain the same when the ice completely melts ? (I. I. T. '73)

33. Discuss the conditions under which it becomes safer for a vessel floating on water to take in more goods and load them to get rid of loads. (J E.E. '74)

34. A man, sitting on a boat floating in a pool, drinks some water from the pool, so that water level of the pool gets down a little. Is it true ? Explain with reasons. (I.I·T. '80)

35. A beaker filled with water is placed on left scale-pan of a balance and a piece of iron is suspended from the hook of the scale pan outside the beaker. This is balanced by placing weights on the right hand scale-pan. The piece is now immersed into water and weighed again. Will there be any change ?

36. Two bodies of equal weight and volume and having the same shape, except that one has an opening at the bottom and the other is sealed, are immersed to the same depth in water. Is less work required to immersed one than the other ?

37. A piece of ice, containing an air bubble, floats in a glass filled with water. How will the level of water in the glass change when ice melts ?

38. Which one is greater—'real weight' or 'apparent weight'—Why.

39. Two identical sphere—one solid and the other hollow are immersed completely in water Which one of them will experience greater upward thrust ?

40. Why is it easier to move a heavy piece of stone in water ?

41. Why is it easier to swim in sea-water than in fresh water ? (H S. '80)

42. How does a ship, made of iron, float in water while a lamp of iron sinks ? (H. S. '80)

43. An egg sinks in pure water but floats in saline water—Why ?

44. Why are plimsoll lines different for sea water and fresh water ?

45. Two vessels are of identical volume. Both are filled with water upto the brim, but in one of them, a block of wood is floating. Which vessel has greater weight ?

[C] Numerical Problems :

46. A body weighs 50 g in air and 40 g in water. Find its volume, density and sp. gravity. [10 cm^3, 5 g cm^{-3}, 5]

47. The volume of a body is 36 cm^3 and it can float on water with 3/4th of its volume immersed. What are the mass and density of the body ? [27 g, 0.75 g cm^{-3}]

48. A piece of wood is 5 cm long, 4 cm broad and 3 cm high. If it floats on water with 2·5 cm of its hight immersed, what will be the weight and density of the piece ? (H. S '83) [50 g, 0.83 g cm^{-3}]

49. The mass of a piece of alloy, made of iron and copper, is 30 g and its volume is 30 cm^3. Find the volume of copper and iron. Sp. gr. of copper = 8·89 and that of iron = 11 37. (Tripura H S. '80) [8·5 cm^3, 21·5 cm^3]

50. A steamer weighs 10 metric tonne. When it enters into a sweet water lake from the sea, if displaces 50 litres more water. What is the density of sea-water ? 1 litre of water weighs 1 kg. (H. S. '82) [1 004 g cm^{-3}]

51 A silver ingot weighing 2·1 kg is held by a string so as to be completely immersed in a liquied of density 0·8 g cm^{-3}. Find the tension in the string. (Density of silver is 10 5 g cm^{-3}.) [1 94 kgf]

52. A specific gravity bottle completely filled with water, mercury and with copper sulphate solution weighs respectively 45 g, 297 g and 49 g. Calculate the density of the solution, that of mercury being 13.6 g cm^{-3}. (H. S. '60) [1.2 g cm^{-3}]

53. A piece of cork weighing 19 g is attached to a bar of silver whose weight is 63 g and the combination is found to just float in water. Find the sp. gr. of cork, assuming that of silver to be 10.5. Calculate also the weight of the combination in a liquid of sp. gr. 0.8. (H. S '67) [0.25, 16.4 g]

54. A cubical block of wood 10 cm along each side floats at the interface between an oil and water with its lower surface 2 cm below

the interface. The heights of the oil and water column are 10 cm each. The density of oil is 0.8 g cm^{-3}. (i) what is the mass of the block ? (ii) what is the pressure at the lower surface of the block ? (I. I. T. '77) [840 g, 1043.6 cm of water]

55. A dilute H_2SO_4 acid has specific gravity 1.28 and contains 40% H_2SO_4 by weight. Find the amount in gramme of H_2SO_4 in the acid. [51.2 g]

56. A glass capillary tube weighs 4.576 g. A thread of mercury is drawn into the tube which then weighs 4.925 g. The length of mercury thread as measured by travelling microscope is 3.435 cm. Assuming the capillary bore to be unifrom, find the diameter of the bore Given sp. gr. of mercury = 13 6. [0.9757 mm.]

57. 10 lb of a liquid of sp. gr. 1.25 is mixed with 6 lb of liquid of sp. gr. 1.15. What is the sp. gr. of the mixture ? [1 214]

58. The density of ice is 0.918 g cm^{-3} and that of sea water is 1·03 g cm^{-3}. What is the total volume of an ice burg which floats with 700 cm^3 exposed ? [6437.5 cm^3]

59. When equal volumes of alcohol and water are mixed, the volume of the mixture is 96% of the origoial volume. Find the sp. gr. of the mixture, if the sp. gr. of alcohol is 0.76. [0.92]

60. A hollow sphere has an internal and external diameters of 10 cm and 12 cm respectively. If it floats in a liquid of density 1 2 g cm^{-3} just fully immersed, find the density of the material of the sphere. [2.84 g cm^{-3}]

61. A thin cylindrical vessel of diameter 10 cm and height 15 cm contains 200 cm^3 of water. The weight of the vessel alone is 114 g. The cylindrical vessel with its contents are placed in a large trough. Water is now poured gradually into the trough. What is the maximum height to which water can be poured in the trough before the vessel begins to rise ? (I. I. T. '70) [4 cm]

62. A piece of cork of sp. gr. 0 25 and a metallic piece of sp. gr. 8.0 are bound together. If the combination neither floats not sinks in alcohol of sp. gr. 0.8, calculate the ratio of the masses of cork and metal. (J. E. E '81) [9 : 22]

63. A sphere of radius 1 cm sinks in water but floats just immersed if a layer of wax 2mm thick be given on it. If the density of wax be 0.8 g cm^{-3}, what is the density of the material of the sphere ? [1·20 g cm^{-3}]

64. A man and a stone are floating on a raft in a swimming pool 10 m long and 5 m wide. The stone weighs 40 kg and has a specific gravity 3·0. If the man drops the stone off from raft into the pool, by how much will the water level on the side of the pool rise or fall ? (J. E. B. '73) [0·53 cm fall]

65. An ornament is suspected to be hollow. It weighs 288.75 g.

in air and 258.75 g in water. If the sp gr. of the material of the ornament be 10.5, calculate the volume of the cavity of the ornament. (H. S. '69) [2.5 cm^3]

66. A gold ring fitted with a stone weighs 5 g in air and 4.25 g in water. The sp. gravities of gold and stone are 19.3 and 2.5 respectively. Calculate the amount of stone in the ring. [1.41 g]

67. When equal volumes of two substances are mixed together, the sp. gr. of the mixture is 4.84. But when equal weights of the same substances are mixed together, the sp. gr. of the mixture is 2.28. Find the sp. gravities of the two substances. [8.36, 1.32]

68. Two identical cylindrical versels with their bases at the same level each contain a liquid of density ρ. The height of the liquid in one vessel is h_1 and that in the other is h_2. The area of either base is A. What is the work done by gravity in equilising the levels when two vessels are connected ? (I. I. T. '81)
[$A\rho g(h_1 - h_2)^2/4$]

69. A wooden plank of length 1 m and unifrom cross-section is hinged at one end to the bottom of a tank. The tank is filled with water upto a height 0.5 m. The sp. gr. of the plank is 0·5. Find the angle θ that the plank makes with the vertical in the eqilibrium position (exclude the case $\theta = 0°$). (I. I. T. '84) [45°]

70. A rubber ball with a mass m and a radius R is submerged into water to a depth h and released. What height will the ball jump up above the surface of water ? Neglect resistance of air and water. Density of water is ρ. [$(\frac{4}{3}\pi R^3\rho - m)h/m$]

71. A solid cubical block of side a, made of unifrom material of sp. gr S (>1), is held just above the water surface in a large vessel, the lower horizontal face of the cube just touching the water surface. Prove that when the cube sinks down so that a depth x is immersed, the potential energy of water and the block increases by $W(x^2 - 2x\,as)/2\,as$, where W is the weight of the block. Show that the equilibrium position of the block when it is floating freely corresponds to a minimum value of this potential energy.

72. Two cylinders of same cross-section and length l but made of two meterials of densities ρ_1 and ρ_2 are connected togather to form a cylinder 2l. If the conbination floats in water with a length l/2 above the surface of water, show that if $\rho_1 > \rho_2$ and the density of water be unity, then $\rho_1 > 3/4$. Show also that the equilibrium is unstable if the heavier part of the cylinder is at the top. (J. E. E. '82)

73. A solid floats on the boundary betweən two non-miscible liquids. The density of the upper liquid is ρ_1 that of the lower is ρ_2. The density of the material of the solid is ρ $(\rho_1 < \rho < \rho_2)$ What fractions of the volume of the solid will be above and below the boundary of the liquids ? $\left[\frac{\rho_2 - \rho}{\rho_2 - \rho_1}, \frac{\rho - \rho_1}{\rho_2 - \rho_1}\right]$

74. A rod of length 6 m has a mass of 12 kg. It is hinged at one end at a distance of 3 m below of water serface. (i) What weight must be attached to the other end of the rod so that 5 m of the rod are submerged ? (ii) Find the magnitude and dircetion of the force exerted by the hinge on the rod. Sp gr. of the material of the rod = 0·5. (I. I. T. '76) [2·33 kg, 5·67 kgf]

75. A flat bottomed thin walled glass tube has a diameter of 4 cm and it weighs 30 g. The centre of gravity of the empty tube is 10 cm above the bottom. Find the amount of water which must be poured in the tube so that when it is floating vertically in a tank of water, the centre of gravity of the tube and its contents is at the mid-point of the immersed length of the tube. (I.I.T. '68) [110·6 g].

76. When a body is immersed in three liquids of sp. gravities S_1, S_2 and S_3 respectively, its apparent weights are w_1, w_2 and w_3. Show that $S_1(w_2-w_3)+S_2(w_3-w_1)+S_3(w_1-w_2)=0$.

77. A body floats in a liquid of sp. gr. S_1 with a certain fraction f of its volume immersed. In another liquid of sp. gr. S_2 it floats with a fraction $(1-f)$ of its volume immersed. Show that the sp. gr. of the solid is $S_1S_2/(S_1+S_2)$.

II-6 (Pneumatics and atmospheric pressure)

[A] Essay type questions :

1. 'Gases have weight and exert pressure'—explain the statement with suitable experiments.

2. What is meant by atmospheric pressure ? How do we express it ? What is meant by standard atmospheric pressure ?

3. How did Torricelli prove that it is atmospheric pressure which supports a column of mercury in an evacuated tube ? What is Torricellian vacuum ?

4. What is a barometer ? Describe a Fortin's barometer and explain how atmospheric pressure can be measured with it.

5. What are the requisites of a suitable barometric liquid ? How can height be measured by noting change of pressure ?

[B] Short answer type questions :

6. What do you mean by standard atmospheric pressure ?

7. What do you mean by the height of homogeneous atmosphere ? What is its magnitude ?

8. How can weather forecasting be made from barometric observation ?

9. What are the advantages of using mercury as a barometric liquid ?

10. What are the disadvantages of using water as a barometric liquid ?

11. What is Torricellian vacuum ? Is it perfectly vacuum ?

12. "The atmospheric pressure at a place is 760 mm of mercury" —What do you understand by this statement ? (H. S. '78)

13. A glass tube 1 m long and closed at one end, is filled with mercury and then inverted over a trough of mercury with the open end dipping into the mercury in the trough, in a vertical position—Explain with reason what will happen. Is there be any difference if a wider tube is taken.

Further if the glass tube is gradually filted what will happen to the mercury column ?

14. Is Archemedes' principle applicable to gases ? Illustrate with suitable experiment.

15. A large glass bulb is balanced by a small brass weight in a sensitive beam balance. State and explain what will happen when the balance is covered by a bell jar which is then evacuated (I. I. T. '72)

16. A ball floats on the surface of water in a vessel exposed to atmosphere. Will the ball remain immersed at its former depth or will it sink or rise if the vessel is covered and the air is (i) removed, (ii) compressed ?

17. A hallow glass sphere, containing atmospheric air, is weighed in air by a balance. The sphere is then broken and the broken pieces are carefully collected (none being lost) and weighed again in air. Will there be any difference in the two weights ? Give reasons for your answer. (J. E. E. '74)

18. A thin-walled balloon weighs equaly when deflated and inflated with air. Why it is so ? If the weighings are done in vaccum, will they be equal ?

19. Why it is necessary to mention the temperature, latitude and height relative to sea level in defining atmospheric pressure.

20. How does the reading of a barometer change if it is within a vessel from which air is sucked out by an exhaust pump ?

21. A gas ballon rises upwards gaining both kinetic and potential energy. Does this imply a violation of principle of conservation of energy ? Explain your answer with reasons.

22. How do you account for the facts that one kg of cotton appears to be lighter than one kg of iron ?

23. What would you conclude if you observe that (i) the height of the mercury column of a barometer is steadily increasing, (ii) the height of the mercury column decreased abruptly.

24. A man carries a tyre tube and decides to make it lighter. He inflats the tube thus increasing the volume. Will his aim be achieved ?

[C] Numerical Problems :

25. If the height of a mercury barometer is 75 cm, what will be the height of a water barometer ? Density of mercury $= 13.6 \text{ g cm}^{-3}$.
[10.34 m]

26. Find the magnitude of the atmospheric pressure in the c. g. s. system. 'g' at the place $= 980 \text{ cm s}^{-2}$ and the density of mercury $= 13.6 \text{ g cm}^{-3}$. (H. S. '61) [1.013×10^{6} dyn cm^{-2}]

27. If the height of water barometer be 32 ft, what will be the height of glycerine barometer ? sp. gr. of glycerine $= 1.25$.
(Tripura H. S. '80) [25.6 ft]

28. A toy balloon is to be filled with hydrogen (density $= 0.09 \text{ gl}^{-1}$). When empty, the balloon weighs 5.8 g. What volume in litres of hydrogen at normal pressure is required to make the balloon just rise when the density of air is 1.25 gl^{-1} ? [5 litre]

29. The materical of a balloon weighs 100 g. It is filled with hydrogen at atmospheric pressure. If the volume of the balloon is 100 litre, calculate the tension in the cord with which the balloon is attached to the ground. Given density of air at atmospheric pressure $= 1.29 \text{ gl}^{-1}$, density of hydrogen at atmospheric pressure $= 0.09 \text{ gl}^{-1}$.
[20 gf.]

30. A barometer is used to determine the height of a mountain. It was observed that the barometer reads 760 mm at sea level and 667 mm at the top of the mountain. Assuming the average density of air to be 1.24 gl^{-1} and density of mercury to be 13.6 g cm^{-3}, calculate the height of the mountain. [1020 m]

31. A barometer reads 760 mm at the ground floor of a tall building 102 m high. What will it read at the top of the building ? Density of mercury $= 13.6 \text{ gcm}^{-3}$ and that of air (average) $= 1.24 \text{ gl}^{-1}$.
[750.7 mm]

32. A balloon is descending with a uniform acceleration a ($<$g). Weight of the balloon with the bucket and its contents is W. What weight of the ballast should be released so that the balloon will begin to rise with the same unifrom acceleration a ? Neglect air resistance.
$\left[\frac{2Wa}{g+a} \right]$

33. A balloon filled with hydrogen has a volume of 1000 litre and its mass is 1 kg. What would be the volume of a block of a very light material which it can just lift ? One litre of this material has a mass of 91·3 g. (Density of air $= 1.3 \text{ gl}^{-1}$) (I. I. T. '75) [3.33 litre]

34. The height of a barometer is 30 inch and the Torricellian vacuum above mercury column is 1 inch long. A quantity of air which occupies 1 inch of the barometer tube under atmospheric pressure is introduced into the barometer. What will be the height of mercury column in the barometer now ? [25 inch]

35. When a faulty barometer reads 28.5 inch and 29.5 inch, a true barometer reads 29.5 inch and 30.7 inch respectively. What is the true barometric height when the faulty barometer reads 29.9 inch ? [31.2 inch]

36. A bottle with its mouth down-words in sunk into a pond. Initially the bottle contained 320 cm^3 of air and the weight of the empty bottle is 400 g and the density of glass is 2.5 g cm^{-3}. When the bottle is taken to certain depth and released, it sinks and does not float up. If the atmospheric pressure be 10.5 m of water, show that the depth of the pond is greater than 3.5 m. (J. E. E. '69)

37. Find the depth of the lake such that an air bubble diminishes to half its volume at the surface, the atmospheric pressure is 76 cm and density of Hg = 13·6 g cm^{-3}. [1033·6 cm]

38. A vertical cylinder of total length 100 cm is closed at the lower end and is fitted with a movable, frictionless, gas-tight disc at the other end. An ideal gas is trapped under the disc. Initially the height of the gas column is 90 cm, when the disc is in equilibrium between the gas and the atmosphere. Mercury is then slowly poured on top of the disc and it just starts overflowing when the disc has descended through 30 cm. Find atmospheric pressure. (I.I.T. '71) [76.115 cm Hg]

39. A bottle whose volume is 800 cm^3 is sunk mouth downwards below the surface of a pond. How far must it be sunk for 300 cm^3 of water to run into the bottle ? Given barometric height = 76 cm Hg and density of mercury = 13·6 g cm^{-3}. [6.2 m]

40. An ideal gas is trapped between a mercury column and the closed lower end of a narrow tube of uniform bore. The upper end of the tube is open to the atmosphere. The length of the mercury and the trapped gas columns are 20 cm and 43 cm respectively. What will be the length of the gas column when the tube is tilted slowly is a vertical plane through an angle of 60° ? [48 cm]

II-7 (Some hydrostatic and pneumetic appliances)

[A] Essay type questions :

1. Explain the action of a Siphon. For what purpose is it used ? Under what conditions does a siphon fail to act ?

2. Describe the action of a piston pump that can draw water from a well. Why is there a limit to the depth of the water level up to which the pump will act ?

How would you alter the construction of the pump so that it can lift water to heights greater than the above limit ?

3. Describe the actions of a vacuum air pump and an air compression pump. Why should the valves of air pump be light ?

[B] **Short answer type questions :**

4. Can a liquid be raised to a vessel placed at a higher level with the help of a siphon ?

5. Can a sipnon work in vacuum ?

6. Explain why water in the bottom of a floating boat cannot be siphoned over the side.

7. A small hole exists on the longer arm of a siphon. Show that whether the siphon will function or not is determined by the location of the hole.

8. What is the highest limit of vacuum that a (i) vacuum pump (ii) rotary pump can produced ?

9. If the atmospheric presseure changes, will the rate of flow of liquid through a siphon undergo any change ? Explain your answer. (H. S. '82)

10. Can an enclousure be completely evacuated by an exhaust pump ?

11. Sometimes a suction pump does not work properly. But when some water is poured on to the top of the piston, the pump starts working. Why ?

12. What modifications will render an air exhaust pump into a bi-cycle pump ?

[C] **Numerical Problems :**

13. A liquid of sp. gr. 1.7 is contained in a tank of height 23 ft. Calculate the minimum height for which it is just possible by siphon action to bring the liquid out of the tank. Atmospheric pressure = 30 inch Hg. [3 ft.]

14. The two arms of a siphon having an internal radius of 1 inch and 20 inch in length respectively. The shorter arm is immersed in a liquid upto a depth of 6 inch. It $g = 32$ ft s^{-2}, find the velocity of of flow of the liquid and also the volume of the liquid discharging through the siphon in 1 second. [8 ft s^{-1}, 0.174 ft^3]

15. The cross-sectional area of the vessels X and Y are respectively 75 cm^2 and 150 cm^2. The vessel X is 4 cm higher than the vessel Y. 1.5 litre of water are poured in the vessel X and 0.35 litre in Y. What is the maximum amount of water that can be siphoned out from the vessel X to the vessel Y ? [0.80 litre]

16. A lift pump is used to lift oil of sp. gr. 0.8 through a maximum height. Find the height if the mercury of the barometer stands to a height of 76 cm. Density of mercury = 13.6 g cm^{-3}.

[12.92 m]

17. When the piston of air pump moves 5 times the pressure reduces to 25 inch from 30 inch. What would be the pressure after 15 stroks ? [17.36 inch]

18. A rubber bladder is in a completely deflated state When the air pressure in the bladder becomes equal to and double than the atmospheric pressure, its volume becomes 1000 cm^3 and 1200 cm^3 respectively. An air compression pump, whose barrel has an effective volume of 200 cm^3 is connected to the bladder. How many strokes of the pump will be necessary to make the air-pressure in the bladder equal to and double than the atmospheric pressure ? [5, 12]

19. After four strokes, the density of the air in the receiver of an air pump is found to bear to its original density the ratio of 256 to 625 What is the ratio of the volume of air in the barrel and receiver ? [1 : 4]

20. The volume of a bi-cycle tube is 100 $inch^3$ and that of its barrel is 10 $inch^3$. How many strokes will be necessary to make the pressure of the tube double than that of atmosphere ? It may be assumed that when inflated, the volume of the tyre increases by 1/10.

[11]

21. If the volume of the barrel of a vacuum pump is 1/3 of the volume of the vessel to be exhusted, what fraction of the initial pressure will remain in the vessel after 5 strokes ? [0·237]

22. If the volume of a receiver of an air pump is n times that of the piston, after how many strokes would the densisy of air be one-third ? [3]

23. A bicycle pump is used to pump air into the tyre of a bicycle. The volume of the tyre is 2 litre. If the cross-sectional area of the pump be 5 cm^2 and the length of each strokes of the piston be 20 cm, what will be the air pressure in the tyre after 40 strokes ? Initial pressure of air in the tyre was 75 cm Hg. [225 cm Hg]

PART III

Vibrations and waves

PART III

VIBRATIONS AND WAVES

III—1

Simple Harmonic Motion

III-1.1. Periodic motion. All motions in the physical world is either linear or periodic. In Chapters I-1.1 to I-1.4 all motions discussed had been linear. In the following two, they have been rotational. The latter motion repeats itself after definite intervals of time ; they are periodic. There are plenty of others ; in fact most of the motions we come across are such. Spinning of a top or our earth, motion of planets and satellites, electrons in an atom, your journey to your school and back or your heartbeat, all types of vibrations, whirring of your fan blades—their number is endless. The simplest of them is the S.H.M.

III-1.2. Simple harmonic motion. The relation between the particle displacement and the time required for it, may be quite complex in a periodic motion. It depends on the acting forces, that produce the displacements. The motion in which the above relation is the simplest is called simple harmonic motion or S.H.M. in brief (already introduced in § I-5.9). Besides, any periodic motion however complex, may be expressed as the sum (or superposition) of the right number of S.H.M.s with appropriate amplitudes and frequencies. (This result is known as Fourier's theorem : but the theorem cannot be intelligently discussed at this stage of our study.)

Definition of S.H.M. It can be defined in *two* ways ; *kinematic* and *kinetic* —

(i) The projection of a uniform circular motion on any diameter is called a simple harmonic motion. We have barnt that in § I-5.9.

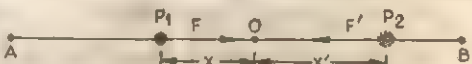


Fig. III-1·1

(ii) If a particle is (P) acted on by a force (F) proportional to the particle displacement (x) and is always directed towards the normal position of rest (O) of

the particle (fig. II-1.1) the motion that occurs is called simple harmonic motion. This is the *kinetic* definition. (A force of the above kind is called a *linear, restoring* force. By the term 'linear' we mean propotional to particle displacement. 'Restoring' means trying to bring the particle back to its normal position) Look up eqn I.5.10.10.

The two definitions look very different, but both represent the same motion, and we can go to one from the other. The first one gives the characteristics of the motion very easily, but does not say anything about the nature of the force that causes the motion. It is the *kinematic* or *geometrical* definition. The second definition deals with the nature of the force causing the motion. It is the *dynamical* or *physical* definition.

To the beginner, the first definition is more helpful. But to the student of physics who has advanced a little in the subject, the second definition is better. In older textbooks, SHM was generally introduced by presenting the characteristic features of its motion. Then it was shown that the projection of a uniform circular motion has that character. That is what we have done before. *But this approach should no longer continue.* SHM should be defined kinetically or dynamically and the characteristics determined from the definition.

Characteristics of S.H.M. From the kinetic definition, it follows that

(*i*) *the force being a restoring* one, it must always be opposite in direction to that of the displacement and

(*ii*) *the force being linear*, it must be proportional to its displacement from the equilibrium position. Eqn I-5.10.10 shows that $F=-m\omega^2 x$.

These properties lead to the facts that follow—

(*iii*) Motion must vanish at a certain displacement from the mean position which may be on either side of it. This means that the motion must be *periodic* and *rectilinear* i.e. repeating along a straight line.

(*iv*) At the farthest displacement ($x=a$) velocity is zero and the restoring force maximum. Eqn I-5.10.18. has given us

$$v=\omega\sqrt{a^2-x^2}$$

III-1.3. Differential Equation of S.H.M. and its Solution : We can straightway convert the kinetic definition into a differential equation for S.H.M.

If x is the displacement at any instant, we may write

$$F=-kx \qquad \text{(III-1.3.1)}$$

where F is the force acting at that moment, and k, the factor of proportionality between the force and the displacement. k is therefore, the spring factor or force const i.e. the *force per unit displacement.* The negative sign indicates that F and x are oppositely directed.

But force = mass × acceleration. In writing an equation of motion, the acting forces are equated to the *inertial reaction* (i.e., mass × acceleration).

$\therefore \quad F = m\dfrac{d^2x}{dt^2}$ where m is the mass of the moving body.

We, therefore, have as the equation of motion

$$m\frac{d^2x}{dt^2} = -kx \quad \text{or} \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega^2 x \text{ (writing } \omega^2 \text{ for } k/m)$$

Hence $\dfrac{d^2x}{dt^2} + \omega^2 x = 0$ * (III-1.3.2)

To solve this equation multiply both quantites by $2\,dx/dt$.

Denoting instantaneous velocity by v, we have $v = \dfrac{dx}{dt}$ and $\dfrac{d^2x}{dt^2} = \dfrac{dv}{dt}$

$$\therefore \quad 2\frac{dx}{dt}\cdot\frac{d^2x}{dt^2} = -\omega^2 . 2x\frac{dx}{dt} \quad \text{or} \quad 2v\frac{dv}{dt} = -\omega^2 . 2x\frac{dx}{dt}$$

or $\dfrac{d}{dt}(v^2) = -\omega^2\dfrac{d}{dt}(x^2)$. Integrating we get

$$\tfrac{1}{2}v^2 = -\tfrac{1}{2}\omega^2 x^2 + \tfrac{1}{2}C \quad \text{or} \quad v^2 = -\omega^2 x^2 + C$$

where C is the integration constant. To find its value let $x = a$ when $v = 0$. (From the known nature of S.H.M., we can identify a as the amplitude).

$$\therefore \quad 0 = -\omega^2 a^2 + C, \quad \text{or} \quad C = \omega^2 a^2.$$

Hence $v = \pm\sqrt{\omega^2(a^2 - x^2)}$ or $\dfrac{dx}{dt} = \pm\omega\sqrt{a^2 - x^2}$

$$\text{or} \quad \pm\frac{dx}{\sqrt{(a^2 - x^2)}} = \pm\frac{1}{a}\cdot\frac{dx}{\sqrt{1 - x^2/a^2}} = \omega\,dt$$

Taking the expression with the positive sign and integrating, we get

$$\sin^{-1}\frac{x}{a} = \omega t \pm \epsilon \text{ or } x = a\sin(\omega t \pm \epsilon) \text{ [Refer to eqn 0-2.7.7]}$$

where ϵ is an integration constant.

---

* This the well known differential equation for S.H.M. In your Maths Paper II you shall follow this deduction.

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But force = mass × acceleration. In writing an equation of motion, the acting forces are equated to the *inertial reaction* (i.e., mass × acceleration).

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$$\text{Hence} \quad \frac{d^2x}{dt^2} + \omega^2 x = 0 \; * \qquad \text{(III-1.3.2)}$$

To solve this equation multiply both quantites by $2\,dx/dt$.

Denoting instantaneous velocity by v, we have $v = \frac{dx}{dt}$ and $\frac{d^2x}{dt^2} = \frac{dv}{dt}$

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$$\text{or} \quad \frac{d}{dt}(v^2) = -\omega^2\frac{d}{dt}(x^2). \text{ Integrating we get}$$

$$\tfrac{1}{2}v^2 = -\tfrac{1}{2}\omega^2x^2 + \tfrac{1}{2}C \quad \text{or} \quad v^2 = -\omega^2x^2 + C$$

where C is the integration constant. To find its value let $x = a$ when $v = 0$. (From the known nature of S.H.M., we can identify a as the amplitude).

$$\therefore \quad 0 = -\omega^2a^2 + C, \quad \text{or} \quad C = \omega^2a^2.$$

$$\text{Hence} \quad v = \pm\sqrt{\omega^2(a^2 - x^2)} \quad \text{or} \quad \frac{dx}{dt} = \pm\omega\sqrt{a^2 - x^2}$$

$$\text{or} \quad \pm\frac{dx}{\sqrt{(a^2 - x^2)}} = \pm\frac{1}{a}\cdot\frac{dx}{\sqrt{1 - x^2/a^2}} = \omega dt$$

Taking the expression with the positive sign and integrating, we get

$$\sin^{-1}\frac{x}{a} = \omega t \pm \epsilon \text{ or } x = a\sin(\omega t \pm \epsilon) \text{ [Refer to eqn 0-2.7.7]}$$

where ϵ is an integration constant.

* This the well known differential equation for S.H.M. In your Maths Paper II you shall follow this deduct'on.

With the negative sign we are led, on integration, to the result

$$\cos^{-1}\frac{x}{a}=\omega t\pm\epsilon \quad \text{or} \quad x=a\cos(\omega t\pm\epsilon). \quad \text{[eqn 0-2.7.8]}$$

Thus the solution of the differential equation gives the **displacement**

$$x=a\sin(\omega t\pm\epsilon) \qquad \text{(III-1.3.3)}$$

$$\text{or} \quad x=a\cos(\omega t\pm\epsilon) \qquad \text{(III-1 3.4)}$$

Look up equations I-5.10.3 and 4. Clearly a is the amplitude and ϵ the phase angle at the initial moment, the two integration constants arising out of two integrations.

$\therefore$ **velocity** $v=\dfrac{dx}{dt}=\omega a\cos(\omega t\pm\epsilon)$

$$=\omega\sqrt{a^2[1-\sin^2(\omega t\pm\epsilon)]}=\omega\sqrt{a^2-x^2} \qquad \text{(III-1.3.5)}$$

and **acceleration** $f=\dfrac{dv}{dt}=-\omega^2 a\sin(\omega t\pm\epsilon)=-\omega^2 x$ (III.3.6)

See that the solutions are same as equations I-5.10-7. and I-5.10.8. In § I-5.10 we had started with expressions for displacement of S.H.M. and ended up in that for the force. Here we have just gone the reverse way. Hence the two definitions of S.H.M. we started with, are entirely equivalent. In fig III-1.2 are shown their graphical representations. If the particle starts from D and moves towards A (equivalent to D towards B) the the sine curve occurs; i.e. at $t=0, x=a$. On the other hand, starting from

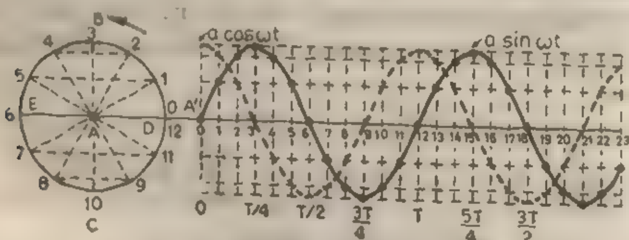


Fig. III-1.2

A and moving towards D (equivalent to moving from B to D) it is the cosine curve ; i.e. at $t=0, x=0$.

Time-Period in S.H.M. The motion analysed is periodic. The periodic time is given by

$$T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}} \qquad \text{(III-1.3.7)}$$

$$\therefore \quad T=2\pi\sqrt{\frac{\text{mass of the moving body}}{\text{force per unit displacement}}} \qquad \text{(III-1.3.8)}$$

$$=2\pi\sqrt{\frac{\text{Inertia factor}}{\text{spring factor}}} \qquad \text{(III-1.3.9)}$$

$$=2\pi\sqrt{\frac{1}{\text{acceleration at unit distance}}} \qquad \text{(III-1.3.10)}$$

To prove it let t increase by $T=2\pi/\omega$. If the values of x at the instants t and $t+T$ be represented by x_t and x_{t+T}, we have

$$x_t = a \sin(\omega t+\epsilon),$$

and
$$x_{t+T} = a \sin\{\omega(t+T)+\epsilon\}$$
$$= a \sin(\omega t+2\pi+\epsilon)$$
$$= a \sin(\omega t+\epsilon) = x_t.$$

So, the values of x repeat themselves at intervals of $T=2\pi/\omega$. In the same way it may be shown that the values of the velocity $v=dx/dt$ repeat themselves after the same interval.

Example III-1.1 A body in S.H.M. has an amplitude of 5 cm and a mass of 10 g. The restoring force acting on it at the end of its swing is 1000 dynes. Calculate the periodic time.

Solution : The restoring force per unit displacement = 1000/5 = 200 dynes.

Hence the acceleration at unit distance = 200/10 = 20 cm/s².

III-1.4 Examples of S.H.M. In many physical problems we find linear restoring forces. Remember that the motion in all such cases will be simple harmonic with periodic times given by Eq III-1.3.7. to .10. A number of such examples are discussed below.

(i) The simple pendulum. *A heavy particle suspended by a weightless, inextensible and perfectly flexible string constitutes a* **simple pendulum.** We have already discussed this motion before, in I-5.11 and deduced the time period of vibration in equations 0-1.9.1 and 1-5.11.1

(ii) Compound pendulum. In a simple pendulum conditions are idealized. The mass is considered concentrated at a point. The string is weightless, inextensible and perfectly flexible.

Such conditions cannot be realized in practice. In all actual cases where a body oscillates in the manner of a pendulum, the mass is distributed and not concentrated.

Any rigid body capable of oscillation about a horizontal axis under gravity is a *physical* or *compound* pendulum. Let the axis pass

through O (Fig. III-1.3) at a distance l *from the centre of gravity G of the body* ($OG=l$). If θ is the angular displacement of the body at any moment from its equilibrium position, and m its mass, the restoring force is $mg \sin\theta$. The restoring couple is, therefore, $mgl \sin\theta$. Let I be the moment of inertia of the body *about the axis of rotation* through O. Since, for a rotating body, *moment of inertia × angular acceleration = couple acting*, we may write

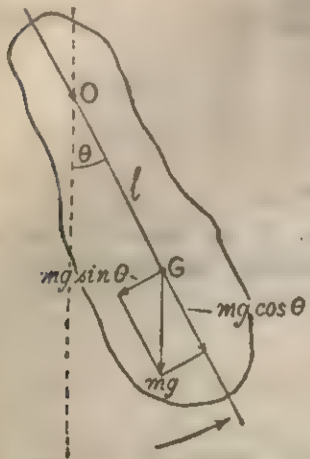


Fig III-1.3

$$I\frac{d^2\theta}{dt^2} = -mgl \sin\theta.$$

The negative sign indicates that the couple and the displacement are oppositely directed. When θ is very small, we may replace $\sin\theta$ by θ (in radians). The above equation then reduces to

$$I\frac{d^2\theta}{dt^2} = -mgl\theta \quad \text{or.} \quad I\frac{d^2\theta}{dt^2} + mgl\theta = 0 \quad \text{or,} \quad \omega^2 = \frac{mgl}{I}$$

The periodic time $= 2\pi/\omega = 2\pi/\sqrt{mgl/I}$

$$= 2\pi\sqrt{I/mgl} \qquad \text{(III-1.4.1)}$$

Ex. III-1.2 Two simple pendulums of lengths 24 in. and 25 in. respectively hang vertically, one in front of the other. If they are set in motion simultaneously, find the time taken for one to gain a complete oscillation on the other ($g=32$ ft/sec^2.)

Solution : Since $T \propto \sqrt{l}$, we shall have $T_2^2/T_1^2 = l_2/l_1 = 25/24$. If coincidence occurs after n oscillations of the slower, $T_2.n = T_1(n+1)$ or $T_2^2/T_1^2 = (1+1/n)^2$.

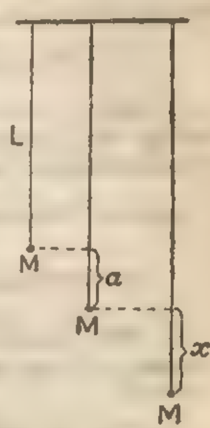
$\therefore$ $(1+1/n)^2 = l_2/l_1 = 25/24$ or $(1+1/n) = \sqrt{1+1/24} = 1+1/48$

$\therefore$ $n=48$. The faster will have executed 49 oscillations. For the faster, $T_1 = 2\pi\sqrt{2/32} = \pi/2 = 1{\cdot}57$ sec. $\therefore$ The required time $= 49 \times 1{\cdot}57 = 77$ sec nearly.

(iii) **Motion of a vertical loaded elastic string.** We have already discussed the vibrations of a spring in Chap. II-3. Consider a string of length L and negligible weight hanging vertically from a fixed support and having a load of mass M at its lower end. The string extends somewhat, and the increased tension in it supports the load, as we have seen already.

Let the load be pulled downwards through a small additional vertical distance x from this position (fig. III-1.4). The string will exert a restoring pull on the load which, for small extensions, is proportional to the extension (by Hooke's law). The additional upward pull on the load will therefore be sx, where s is the pull per unit extension of the string. If the mass M stretched the string initially by a length a, then $Mg=sa$ where s is the spring const i. e. force per unit displacement. The acceleration at unit ex tension is $s/M=g/a$. Hence from Eq. III-1.3.7 the period

$$T=2\pi/\sqrt{g/a}=2\pi\sqrt{a/g} \qquad \text{(III-1.4.2)}$$



g. III-1.4

The mass M executes an S.H.M. about its normal position of rest with this periodic time, which equals that of a simple pendulum of length a.

Ex. III-1.3 A body of mass 100 g stretches a vertical spiral spring by 0.5 cm. If it is set into vertical oscillations of small amplitude, what is the periodic time of the oscillation ?

Solution : Assuming that the extension of a spring is proportional to the force, the force required to produce an extension of 1 cm is 100 gm-wt/0·5 =200 gm-wt=200 g dynes. Hence the acceleration of the body at unit distance=force/mass=200g/100=2g.

$$\therefore \text{ Time period}=2\pi/\sqrt{2g}$$

Ex. III-1.4. A point mass m is suspended from a massless wire of length l and cross section A. If Y is the Young's modulus for the material of the wire find an expression for the frequency for vertical S.H.M. [I.I.T. '78]

Solution : From the foregoing analysis you establish that

$$T=2\pi\sqrt{a/g}. \quad \text{Now by definition } Y=\frac{mg/A}{a/l}$$

$$\therefore \; a/g=ml/YA \text{ or } T=2\pi\sqrt{ml/YA}$$

$$\text{or Frequency } n=1/T=\frac{1}{2\pi}\sqrt{\frac{YA}{ml}}$$

Ex. III-1.5. A mass M at the end of an elastic thread executes a vertical S.H.M. With an additional mass m, time period changes in the ratio 5/4. Find the ratio of the masses.

Solution : If a_1 and a_2 be the initial elongations of the thread due to the masses M and $(M+m)$ and the corresponding periods are T_1 and T_2 then from eqn III-1.4.2 we have

$$T_1/T_2=4/5=\sqrt{a_1/a_2} \text{ or } a_1/a_2=16/25$$

Again if A be the cross-section, l the length and Y the Young's modulus of the material of the thread then

$$\frac{a_1}{a_2}=\frac{Mgl/YA}{(M+m)gl/YA}=\frac{M}{M+m}=\frac{16}{25} \quad \text{or} \quad m/M=9/16$$

Ex. III-1.6. A motor car has a mass of 1000 kg while of a passenger the same is 60 kg. When the person gets in, the C.G. of the car is depressed by 0.3 cm. What is the frequency of the empty car ? [J.E.E. '84]

Solution : The spring const. $k=F/x=\dfrac{60\times 9.8\text{N}}{3\times 10^{-3}\text{m}}=196\times 10^{3}$ N/m

$$\text{Now frequency}=\frac{1}{T}=\frac{\sqrt{k/M}}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{196\times 10^{3}}{10^{3}}}=2.22/\text{s}$$

Ex. III-1.7 A mass attached to a spring oscillates in 2s. If the mass is increased by 2 kg. the period increases by 1s. Find the initial mass if Hooke's law holds. [I.I.T. '79]

Solution : Let the initial mass be M. If is acceleration is f then $Mf=-kx$ from Hooke's law where k is the force constant. Now then $f=-(k/M)x$ and so $T=2\pi\sqrt{M/k}$

Again $T'=2\pi\sqrt{(M+m)/k}$ or $T'/T=3/2=\sqrt{(M+2)/M}$

$$\therefore\ 1+\frac{2}{M}=\left(\frac{3}{2}\right)^2=\frac{9}{4} \quad \text{or} \quad M=1.6 \text{ kg.}$$

Problem : A helical spring elongates under a tension of 5×10^5 dyn. Find the load necessary to make it vibrate twice per sec. If the amplitude be 1 cm find also the maximum velocity. (**Ans.** 317 g. ; 12.5 cm/s.) [J.E.E. 79]

(iv) **Floating cylinder.** Consider a cylinder of mass M and area of cross-section α floating vertically in a liquid of density ρ, being immersed to a length l. If it is depressed a distance x below its position of rest, it will displace a further mass $\alpha x\rho$ of the liquid and hence experience an upthrust of $\alpha x\rho g$ dynes (fig. III-1.5)

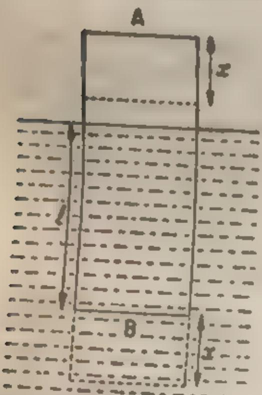


Fig III-1.5

This is a restoring force which is proportional to the displacement. The resulting motion is, therefore, simple harmonic The force per unit displacement is $\alpha\rho g$. From Eq. III-1.3.8., the periodic time is given by

$$T=2\pi\sqrt{\frac{M}{\alpha\rho g}}=2\pi\sqrt{\frac{\alpha l\rho}{\alpha\rho g}}=2\pi\sqrt{\frac{l}{g}}. \qquad \text{(III-1.4.3)}$$

Due to viscous friction with the liquid, the motion of the cylinder falls off rapidly in amplitude, and soon dies out. Time period is that of a simple pendulum of length equal to that of the immersed length.

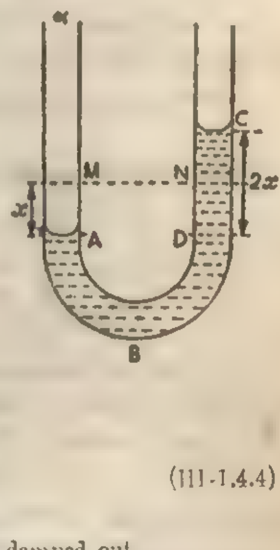
Problem : (i) A test-tube of external diameter 2 cm. and weighing 4 g is floated vertically in water by placing 12 g of mercury in the tube. Find the time of oscillation when the tube is depressed by a small amount and then released.

[**Hint.** Area of cross-section $=2\pi$ cm^2. Depth of immersion $l=16/\pi$ cm]

(**Ans.** 0.45s)

(2) A rectangular wooden block of 10 cm^2 cross-section floats with 200 cc. of it below water. Find its period of vibration. (**Ans.** 1.11s) [**J.E.E.**]

(v) **Liquid in a U-tube.** Let there be a column of liquid of total length l in a U-tube of uniform cross-section α. If ρ is the density, the mass is $\alpha l\rho$ gm. When the liquid in one limb is depressed by x cm, a difference of height of $2x$ cm is set up. The weight of this 'head' of liquid is $2x\alpha\rho g$ dynes. It acts as a restoring force and is proportional to the displacement (fig. III-1.6).



The motion of the liquid column is, therefore, simple harmonic. The mass moved is $\alpha l\rho$ gm, while the restoring force per unit displacement is $2\alpha\rho g$ dynes. Hence by Eq. III-1.3.9 the periodic time is

$$T=2\pi\sqrt{\frac{\alpha l\rho}{2\alpha\rho g}}=2\pi\sqrt{\frac{l}{2g}}. \qquad \text{(III-1.4.4)}$$

As in the previous case the motion is quickly damped out.

Problem. Mercury of density 13.6 gm/cm^3 is contained in a U-tube with its arms vertical. Neglecting damping find the time of oscillation of the mercury if the total length of the tube occupied by mercury is 30 cm. If the area of cross-section of the tube is 2 cm^2, find the energy of the motion when the amplitude is 5 cm. [**Ans** 0.78 s ; 6.7×10^5 ergs.]

(vi) **Gas enclosed in a Cylinder under a Piston.** Let an *ideal* gas be enclosed in a cylinder under a piston as shown in fig. III-1.7. Let at a certain instant the piston be in equilibrium with the gas under it. The piston is slightly depressed and then let go ; the increased pressure pushes

the piston upwards ; as it rises the volume of air increases, pressure falls. This developes into a restoring force proportional to displacement and a vertical oscillation results, very much like what would happen if you jump down on a cushion. The enclosed gas undergoes alternate condensations and rarefactions. Let the volume and mass of the gas be V and M under atmospheric pressure P.

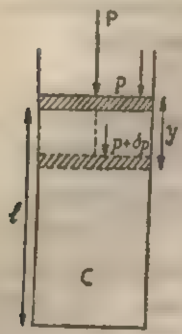


Fig III.1-7

A. Let the change be *isothermal*, with rise in pressure dP of the enclosed gas and diminution in volume $(-dV)$. By Boyle's Law

$$PV=\text{const.} \quad \text{or,} \quad P.dV+V.dP=0$$

$$\text{or,} \quad dP=-PdV/V$$

Now if α be the area of cross-section of the cylinder and y the displacement of the piston the $dV=\alpha y$

$$\therefore \quad dP=-\frac{P\alpha}{V}.y=-ky$$

Now the unbalanced force on the piston is $\alpha.dP$. $\therefore$ Acceleration

$$f=\frac{\alpha.dP}{M}=-\frac{P\alpha^2}{MV}y=-\omega^2y$$

$$\therefore \quad T_{iso}=\frac{2\pi}{\omega}=\frac{2\pi}{\alpha}\sqrt{MV/P} \qquad \text{(III-1.4.5a)}$$

B: Let the volume change be made *adiabetic* by enclosing the ideal gas in a cylinder of *perfectly insulating* material under a similar piston and of cross section A. Then the relevant pressure-volume relation becomes

$PV^\gamma=$ const. where γ is the ratio of specific heats

$$\text{Then} \quad V^\gamma dP+\gamma V^{(\gamma-1)}dV.P=0$$

$$\text{or,} \quad dP=-\gamma P.\frac{dV}{V}=-\gamma P.\frac{Ay}{V}$$

$$\text{As above } f=\frac{AdP}{M}=-\frac{\gamma PA^2}{MV}.y$$

$$\text{or,} \quad T_{Ad}=\frac{2\pi}{A}\sqrt{\frac{MV}{\gamma P}} \qquad \text{(III-1.4.5b)}$$

Problem : An ideal gas is enclosed in a vertical cylinder and supports a freely moving piston of mass M. The piston and cylinder are of same cross-section A. Atmospheric pressure is P_o and the volume of the enclosed gas V_o under equilibrium conditions. Assuming the system to be completely isolated from its

surroundings show that the piston executes S.H.M. and find the frequency of oscillation. $\left(\text{Ans. } n=\frac{1}{2\pi}\sqrt{\frac{\gamma gA}{V_0}}\right)$ [I.I.T. '81]

[**Hint :** The downward pressure exerted by the piston on the enclosed gas is Mg/A that is equal to the pressure exerted *upward* on the piston by the enclosed gas.]

(vii) Body suspended by a wire. Consider a rigid body suspended from one end of a thin vertical wire, the axis of the wire passing through its C.G. If, instead of displacing the body laterally, we twist the body a little about the axis of suspension and let it go, it will execute angular oscillations about the axis. Such a motion is known as *torsional oscillation*. See fig III-2.2(c).

When the wire is twisted by applying of a couple to the body, the wire tends to untwist itself and thereby exerts a restoring couple on it. If the twist does not exceed a limiting value, (which is pretty large for thin wires) the *restoring couple is proportional to the twist.* Then the body executes torsional oscillations that are simple harmonic in nature. Such a motion is called *angular simple harmonic motion.* It is of much practical importance.

Let θ be the angle of the twist at any instant. If c is the restoring couple for unit twist, $c\theta$ will be the restoring couple for a twist θ. If I be the moment of inertia of the body about the axis of twist, the equation of rotational motion of the body will be given by the relation

$$I\frac{d^2\theta}{dt^2}=-c\theta. \quad \text{or} \quad \frac{d^2\theta}{dt^2}=-\frac{c}{I}\theta=-\omega^2\theta.$$

$$\therefore \quad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{I}{c}} \qquad \text{(III-1.4.6)}$$

You shall recall these results when discussing movements of the coil in suspended coil galvanometers.

(viii) Horizontal bar magnet (fig. III-1.8). When displaced from its position of rest in a magnetic north-south direction by an angle θ, the magnet experiences a restoring couple $MH \sin\theta$, which for small values of θ may be written as $MH\theta$.

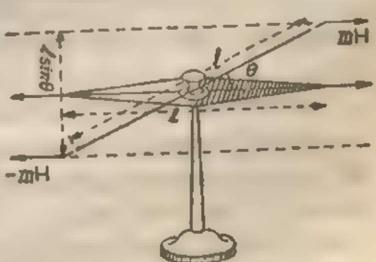


Fig. III-1.8

Its equation of motion is, therefore, given by

$$I\frac{d^2\theta}{dt^2}=-MH\theta. \quad T=2\pi\sqrt{\frac{I}{MH}}. \qquad \text{(III-1.4.7)}$$

The motion will be simple harmonic with this periodic time.

Note that M = magnetic moment of the bar magnet and H = horizontal component of the earth's magnetic field. I is the moment of the inertia of the magnet about the axis of rotation.

Problem. A suspended magnet of magnetic moment 1200 cgs units and of moment of inertia 2500 gm cm² is allowed to oscillate in the earth's horizontal field of strength 0·31 oersted. Find the periodic time. (**Ans.** : 15·5 s)

When discussing determination of H, the horizontal component of earth's magnetic field with a vibration magnetometer you shall have to refer to this result.

(ix) Time to cross the earth through a tunnel. Suppose a smooth straight tunnel is bored through the center of the earth and a body is dropped into it. Assuming the earth to be a uniform homogeneous sphere and the motion of the body to be unimpeded by any resisting force, we can show that (*i*) the body will execute S.H.M., (*ii*) find the periodic time of the motion. It is also independent of the direction of the tunnel. But we do not consider that.

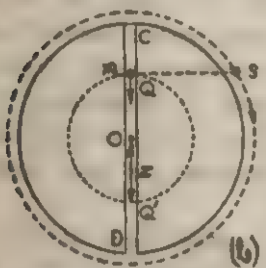


Fig. III-1.9

In the figure III-1.9 let the body be at Q at any moment. It experiences a pull towards O, the centre of the earth. This pull is the gravitational force of attraction between the body and the sphere of radius OQ. The spherical shell of thickness CQ does not contribute to this pull.

Let m = mass of the body, ρ = density of the earth considered as a homogeneous sphere, and $OQ = x$. Then the force acting on the body at Q is

$= G \times$ mass of the sphere of radius $x \times$ mass of the body$/x^2$

$= G.\frac{4}{3}\pi x^3 \rho \times m/x^2 = G\frac{4}{3}\pi\ \rho m x.$

This force is constantly directed towards O, and is proportional to the distance of the body from O. Taking O as the origin of co-ordinates, we can write the equation of motion of the body as

$$mf = -G.\tfrac{4}{3}\pi\rho m x \quad \text{or} \quad f = -G\tfrac{4}{3}\pi\ \rho x = -\omega^2 x$$

where $\omega^2 = G\frac{4}{3}\pi\rho = f/x$ The periodic time the is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{3}{4\pi G\rho}} \qquad \text{(III-1.4.8a)}$$

If g is the acceleration due to gravity at the surface of the earth $g = \frac{4}{3}G\pi R^3\rho/R^2 = \frac{4}{3}G\pi R\rho$ where R is the radius of the earth.

$$\therefore \quad \frac{R}{g} = \frac{3}{4\pi G\rho}. \quad \text{Hence} \quad T = 2\pi\sqrt{\frac{R}{g}} \qquad \text{(III-1.4.8b)}$$

Note that *the periodic time is the same as for a small satellite* (s) *going round the earth close to its surface.* The motion through the tunnel is the same as the ***projection of the circular motion of the satellites on the earth diameter*** coinciding with the tunnel. See eqn. II-1.16.3.

If numerical values of G and ρ or R and g are known, T can be calculated. Taking $R = 4000$ miles and $g = 980$ cm/sec^2, values which are often to the remembered,

$$T = 2\pi\left\{\frac{4000 \times 1760 \times 3 \times 30.48}{900}\right\}^{\frac{1}{2}}$$

$$= 5082 \text{ sec.} = 1 \text{ hr. } 24 \text{ min. } 42 \text{ sec.} \qquad \text{(III-1.48c)}$$

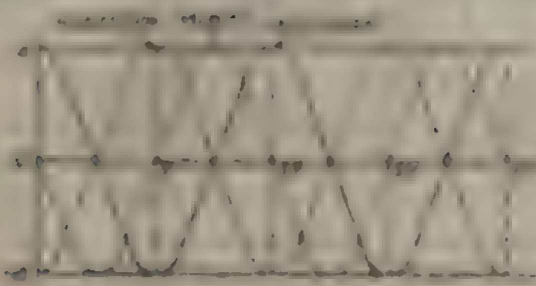
III-1.5. Phase in S.H.M. The concept of *phase* is very important in connexion with propagation and superposition of vibrations. Try to understand its meaning clearly.

In S.H.M. we find that the state of motion of the particle, that is, its displacement, velocity and acceleration are changing continuously. They come back to the same set of values after a definite interval of time. So, there is a continuous cycle of changes in the motion. The word **phase** means the state of motion of the particle in this cycle of changes. (Compare the 'phases' of the moon.) Any quantity which can give the displacement and velocity of the particle at any moment, may be taken as the measure of phase. The angle $(\omega t + \epsilon)$ is such quantity. It is called the **phase angle.** The angle ω is called the ***initial phase*** or ***epoch*** : it gives the phase at the initial moment.

Two S.H.M.s of the same frequency may be in different phases at the same moment. The difference between their phase angles is called their **phase difference.**

Note the following relations of S.H.M. :

$$\left.\begin{array}{lll} \text{(i)} & \text{Particle displacement} & x = a \sin \omega t \\ \text{(ii)} & \text{Particle velocity} & v = a\omega \cos \omega t = a\omega \sin (\omega t + 90^\circ) \\ \text{(iii)} & \text{Particle acceleration} & f = -\omega^2 x = a\omega^2 \sin \omega t \\ & & \quad = a\omega^2 \sin (\omega t + 180^\circ) \end{array}\right\} \text{(III-1.5.1)}$$



on the particle) is kxx'. The force increases uniformly from 0 to kx as the displacement increases from 0 to x. We may therefore assume that the displacement x has occurred under an *average* force $\frac{1}{2}kx$. Hence the work done on the particle for displacement x is $\frac{1}{2}\ kx.x=\frac{1}{2}kx^2$. *This* is its potential energy V at x. Eq. III-1.3.2 tells us that $k=m\omega^2$.

$$\therefore \quad V=\tfrac{1}{2}kx^2=\tfrac{1}{2}m\omega^2x^2=\tfrac{1}{2}ma^2\omega^2\sin^2\omega t \qquad \text{(III-1.6.2)}$$

$$\text{The total energy}=K+V=\tfrac{1}{2}ma^2\omega^2(\cos^2\omega t+\sin^2\omega t)$$
$$=\tfrac{1}{2}ma^2\omega^2. \qquad \text{(III-1.6.3)}$$

The total energy is obviously constant, since m, a and ω are constants in a given case [fig. III-1.11(a).] [Eq. III-1.6.3.] also shows that when kinetic

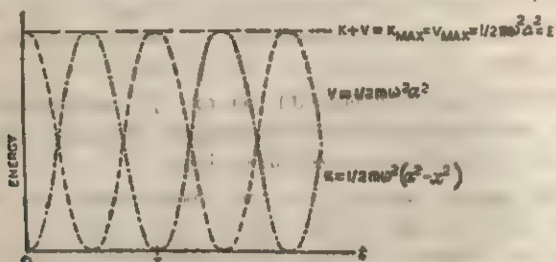


Fig III-1.11 (a)

energy is maximum, potential energy is minimum. The potential energy

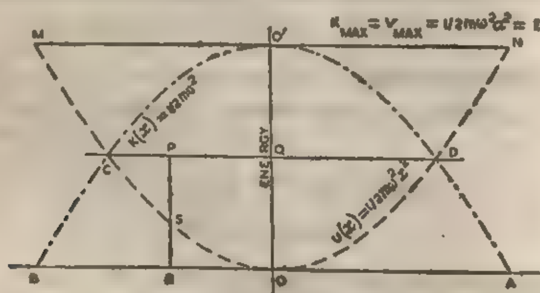


Fig III-1.11 (b)

is zero when $x=0$, and maximum at the end of a swing ($x=a$). **S.H.M. is an example of the conservation of mechanical energy.**

Note that since k is the restoring force for unit displacement, k/m is the acceleration at unit distance. By eq. (III-1.3.7) we then have

$$T=2\pi/\omega=2\pi/\sqrt{k/m}=2\pi\sqrt{m/k}$$

Conservation of mechanical energy in S.H.M. is graphically explained in fig III-1.11.(b). In it displacement is along the x-axis and energy along y-axis. Note that with increase in x, the P.E. increases and K.E. decreases. Both the representative curves are parabolic as $E \propto x^2$ (compare $x^2 = 4ay$ for a parabola). From the shape of the P. E. curve it is said to be a *potential well*, a very important concept in higher physics.

Expression for P.E. can be *alternately* derived. At a displacement of x from the equilibrium position the restoring force is kx ; the work done in moving the particle against this force through a very small distance dx is $kx.dx$. So the total work done on the particle through a displacement of x is the potential energy.

$$\therefore \text{ P. E.} = \int_0^x kx.dx = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 \qquad \text{(III-1.6.4)}$$

We may arrive at the **differential equation of S.H.M.** from the expression of total energy for S.H.M. We have

$$\text{K.E+P.E} = \tfrac{1}{2}\, mv^2 + \tfrac{1}{2}m\omega^2x^2 = \text{const.}$$

Differentiating $v.\,dv + \omega^2x.dx = 0$ or, $v.\dfrac{dv}{dt} + \omega^2 x.\dfrac{dx}{dt} = 0$

$$\text{or, } \frac{dv}{dt} + \omega^2x = 0 \quad \left[\because \quad v = \frac{dx}{dt} \neq 0\right] \quad \text{or, } \frac{\mathbf{d^2x}}{\mathbf{dt^2}} + \boldsymbol{\omega^2}\mathbf{x} = \mathbf{0}$$

III 1-7. Superposition of two simple harmonic motions in the same direction : Let a particle have two S.H.M.'s at the same time. Its displacement at any instant will be the *vector* sum of the displacements due to each S.H.M. at the moment considered. If both S.H.M's are in the same direction vector addition becomes simple *algebraic* addition. We shall consider two such cases. The superposed S.H.M.'s will be in the *same direction* and have the *same frequency*. In one case (fig. III-1.12) they will be in the *same phase*. In the other case (fig. III-1.13) they will be in *opposite phases*. In both figures, the continuous curves represent the S.H.M.'s to be added. Mathematically, they are represented by $x_1 = a_1 \sin \omega t$ and $x_2 = a_2 \sin \omega t$.

In fig. III-1.12, the two motions are *in phase*. Hence they always produce displacements in the same direction. Their resultant is, therefore, the algebraic sum of the two. This means that the resultant

displacement at any time t is $x=x_1+x_2=a_1 \sin \omega t+a_2 \sin \omega t$ $=(a_1+a_2) \sin \omega t$. It represents an S.H.M. of the same angular

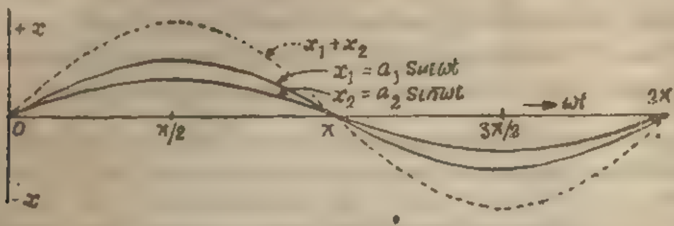


Fig III-1.12

frequency (ω) as the other two, and has an amplitude (a_1+a_2) equal to the sum of the amplitudes of each. In the figure, it is given by the broken curve.

In fig. III-1.13, the same S.H.M's are added but *in opposite phases*. If the phases are opposite, the phase angles will differ by 180°. Suppose

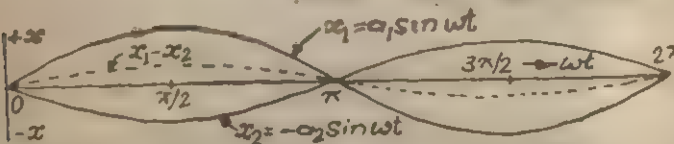


Fig III-1.13

$x_1=a_1 \sin \omega t$ has the larger amplitude. The other motion will be $x_2=a_2 \sin (\omega t \pm 180°)=-a_2 \sin \omega t$. This means that the resultant motion will be given by $x=x_1-x_2=a_1 \sin \omega t-a_2 \sin \omega t=(a_1-a_2) \sin \omega t$. This is an S.H.M. in phase with the S.H.M. of larger amplitude but an amplitude equal to the difference of the two amplitudes. If $a_1=a_2$, the superposition of the two motions (two S.H.M.'s of the same amplitude and frequency but in oppsite phases) will *cancel* each other. No motion will be produced by their combined effect. This will appear in stationary vibrations.

In both figures, broken curves represent the resultant motion.

III-2

VIBRATIONS

III-2.1. Vibrations. A particle or a body is said to *vibrate* if it describes a to-and-fro motion along a path. Examples are, the pendulum of a clock, children's swing, motion of the needle of a sewing machine, the to-and-fro motion of the piston of a railway engine, motion of any point of a string under tension, etc. Some of these are *longitudinal* and some *transverse*. Vibrations along the length of the body are longitudinal; those at right angles to that are transverse. We also have *torsional* oscillations.

If the vibrating particle reaches the same point of its path after equal intervals of time, its motion is a type of **periodic motion.** When the motion is in a straight line, as in the case of the nedle of a sewing machine or the piston of an engine it is *rectilinear* vibration.

The words 'vibration' and 'oscillation' are used almost synonymously. There is no hard and fast difference in their meanings. Generally, when the motion is fast we call it a vibration. A relatively slow vibration is often called an oscillation as in examples of S.H.M. discussed before. An *oscillation* also means a complete period of vibratory or periodic motion i.e. the whole succession of states that takes place before the motion begins to repeat itself. Vibration is *sometimes* taken to be half an oscillation.

Certain terms are very useful in describing a periodic motion, namely

(i) Periodic time (T) or **Period.** It is the time required by the vibrating particle (or body) to execute one complete oicillation, say, in going from one end of its path to the other end and back to the first end. It is also called the *time of oscillation.*

(ii) Frequency (n). The number of oscillations completed in one second is called the frequency. From the definitions, it follows that

$$nT=1 \quad \text{or} \quad n=1/T \qquad \text{(III-2.1.1)}$$

(iii) Amplitude. The maximum distance through which the particle moves from its normal position of rest is called the amplitude of the vibration.

(iv) Particle displacement. The distance, at any moment, of the vibrating particle from its normal position of rest is the *particle displacement.*

Fig III-2.1

We shall illustrate the meanings of these terms by reference to the motion of a pendulum (fig. III-2.1). The pendulum is suspended from O. A is the normal position of rest of the bob. It oscillates between the extremes B and C. $AB = AC$ is the amplitude of the motion. The *angular* amplitude is $\angle AOB = \angle AOC$. The periodic time (or *period*) is the time required by the bob in going from B to C and coming back to B (or from A to B, B to C and back from C to A). The reciprocal of the period (T) is the frequency (n).

In this chaper we shall *remain confined to elastic vibrations* i.e. where the restoring forces are due to elastic deformations only. In the next chapter, we shall be considering elastic waves i.e. those in material media. They are caused by elastic vibrations.

III-2.2. A Free vibration. Any elastic body may be made to vibrate under suitable conditions. The vibration may be *transverse, longitudinal* or *torsional.* For example, the motion of a pendulum bob or of a metal strip clamped at one end is transverse (a); that of a stretched spring is longitudinal (b), while the oscillatory motion of a twisted wire carring a load at the free end (c) is torsional (fig III-2.2). The vibrations of a tuning fork is transverse. All of these vibrations will be simple harmonic if the restoring force or couple is proportional to the displacement. It is generally so when the displacements are small. After vibrations have been excited in an elastic body and the exciting forces removed, the body continues to vibrate with a frequency characteristic of its own. Such a vibration is called **free vibration.** The periodic time of vibration is called the *free* or *natural period.* It depends upon the mass and the elastic properties of the vibrating body. Its reciprocal is the frequency, and is called the *natural frequency.* S.H.M. *may be taken to be undamped free vibration.*

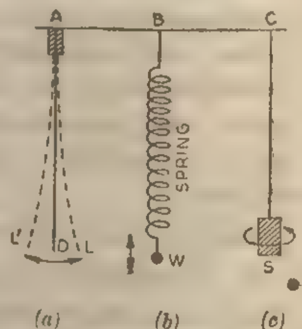


Fig III-2.2

Recording the time displacement graph for a free vibration is shown in fig. III-2.3. A heavy cylinder *W* hanging from a spring moves up and down. It carries a sharp inked stylus past which moves unformly, a paper strip unwinding from one and winding on another. The amplitudes are *almost* constant for all oscillations.

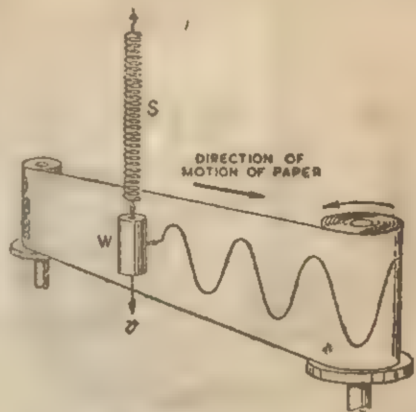


Fig. III 2.3

B. Damped vibration : In all practical cases, it is found that when left to itself the vibration of a body gradually diminishes in amplitude and finally dies away. This is so because the motion is resisted by various frictional effects, internal as well as external. When a tuning fork is vibrating its different layers are sliding, one over another. This brings into play a resisting force due to viscosity or internal friction. This

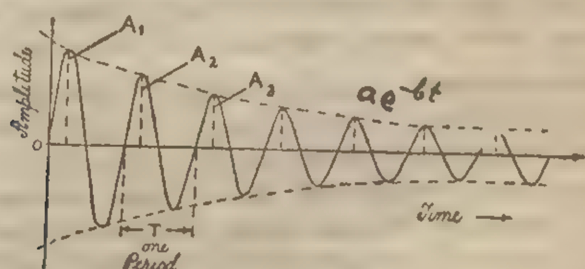


Fig III-2.4

internal friction as well as resistance to motion offered by air, gradually diminishes or *damp* the amplitude and finally brings the body to rest. A vibrating pendulm is similarly damped because of resisting forces between its suspension and support, and between the air and the bob. Vibrations thus opposed from within and without, gradually diminish in amplitude. They are called *resisted* or *damped vibrations* (fig III-2.4) Actual free vibrations are such.

In fig III-2.5 the same arrangement as above for recording damped vibrations is shown but with a modification. A piston fastened to the

bottom of the vibrating cylinder moves up and down in a glass of water so as to damp the motion. See that the record is a replica of fig III-2.4.

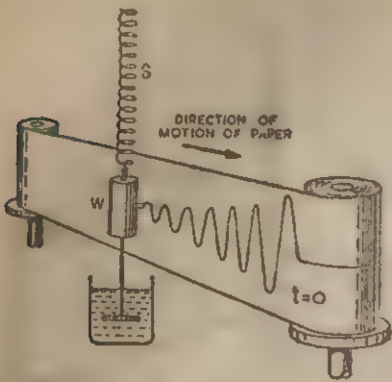


Fig III-2.5

Fig. III-2.4 represents a case of *damped simple harmonic motion.* At each vibration some energy is lost in overcoming the resisting forces. If the resisitng forces are large the rate of loss of energy is also large. The body comes quickly to rest. With small damping, the vibrations continue longer, the amplitude diminishing slowly. In figure III-2.5 replace water by kerosene and glycerine. In the first, oscillations last longer, in the latter shorter, than for water.

III-2.3. **Forced vibration and resonance.** The study of forced vibration and resonance is of special interest in wave motion. Sound waves and radio waves are detected by the forced vibration or resonance they produce in the receiver. Resonance is a special case of it.

A. Forced vibration. A vibrating body gradually loses amplitude due to opposing forces, which are always present. Energy must be supplied from without if the body is to be maintained in vibration. Let an *external periodic force* act on a vibrating body. The body tends to vibrate with its own natural frequency. But the applied force tries to impress its own frequency of vibration on the body. Initially, vibrations of both frequencies are present at the same time (fig. III-2.7.). In course of time, the natural vibration dies away due to the resisting forces. Finally, only the vibrations due to the impressed force remain. The vibration of a body with a period same as that of an impressed periodic force is called a *forced vibration.* To keep the swing of a child in motion, you apply an impulsive force periodically *i.e.*, after a definite time-interval. This is forced vibration.

B. Resonance. The amplitude of forced vibration is generally small. If the period of the applied force is the same as the natural period of the vibrating body however, the vibrations build up quickly. The amplitude may be large even with a small impressed force. As the motion

grows, the resistance to the motion increases also. A state of steady oscillation is reached when the energy supplied by the external source is fully used up in overcoming the resistance to the motion. The particular case of forced vibration where the applied force has the same period as the natural period of *unresisted* vibration of the body is known as **resonance.** A resonant vibration is also called *sympathetic vibration.* Note that the *motion is uniform, for no net force acts* on the vibrator.

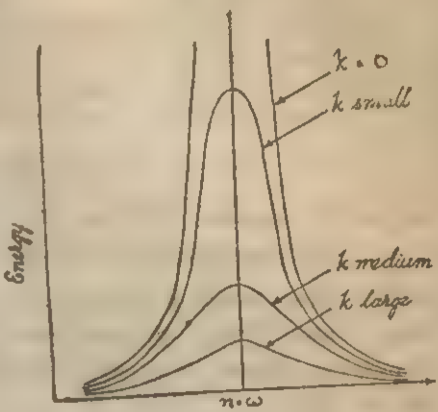


Fig. III-2.6

It can be shown mathematically that if resonance occurs for a body whose vibration is undamped—an ideal case never realised in practice—the amplitude would be infinitely large. In an actual case damping forces limit the amplitude. In fig. III-2.6. are shown the response of a vibrator to an applied force at various frequencies for different dampings (k) or (b).

There are, in fact, two cases of resonance to distinguish, namely (i) *amplitude resonance* and (ii) *velocity or energy resonance*. In the former the amplitude of the forced vibration acquires a maximum value. In the latter, the velocity is a maximum. This corresponds to maximum energy transfer between the forcing and the forced systems. The foregoing sketch is actually that for energy resonance. When damping is negligible both resonances occur at practically the same frequency, which is the natural frequency of the vibrator. When there is appreciable damping, the two resonances occur at slightly different frequencies. Velocity resonance occurs always at the frequency equal to the undamped frequency of the body in forced vibration ; this is the more important case.

C. Characteristics. Forced vibration shows the following characteristics :

(1) Initially, vibrations with the natural frequency of the vibrating body and the frequency of the impressed force are both present. If the frequencies are close enough they may form 'beats' that is a rise and fall in amplitude (Fig. III-2.7.). With passing of time the natural vibration dies out and the body vibrates with the forcing frequency.

(2) The amplitude of forced vibration is generally small except in

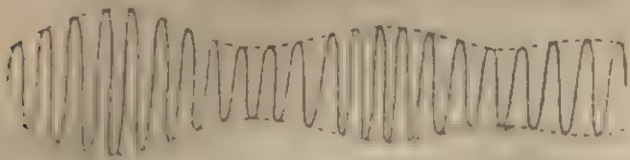


Fig. III-2.7.

the case of resonance. This amplitude may then be quite large.

D. Difference between free and forced vibration.

(i) Free vibration is executed by a body under the action of its own elastic forces without being subjected to any external force. But forced vibration is executed by a body under the action of an externally applied periodic force.

(ii) Amplitude of free vibration may have any value, large or small, depending on the amount of initial supply of energy. If damping is present the amplitude is diminished exponentially (ae^{-kt}). Amplitude of forced vibration is generally small except when resonance occurs. Under resonance conditions the amplitude of vibration is large. Near resonance, the amplitude increases rapidly as the frequency of the applied force approaches the frequency of the unexcited free vibration. See fig. III-2.7.

(iii) Frequency of free vibration of a body depends on its mass and elasticity. Frequency of forced vibration is equal to that of the externally applied force.

Fig. III-2.8

(iv) Free vibration finally ceases due to action of resisting forces. But forced vibration continues so long as the applied force acts.

Fig. III-2.8 shows an arrangement to demonstrate in a simple way the difference between forced vibration and resonance. C is one of several light pendulums of different lengths suspended from the same horizontal suspension. C is another pendulum having

the same length as C'. C and C' therefore have the same natural frequency; but the frequencies of the others are different as they differ in length. When C is set into oscillation C' gradually picks up an increasing amplitude, while others vibrate with very small amplitudes. C' resonates with C, but the others are thrown into forced vibration. The vibration of C is transmitted to the others through the suspension.

III-2.4. Some examples of forced vibration and resonance.

Forced and resonant vibrations are fairly common. A few examples are given below. You should be able to think up others.

A. Mechanical examples. (1) The various parts of a motor car, such as loose fittings, brake rods, gear lever etc., have their own natural frequency of vibration. The periodic motion of the engine applies to them a forcing frequency proportional to the speed of the car. As the speed alters, the frequency of the motion may match the natural frequency of some part so that it is thrown into resonant vibration and rattles vigorously.

(2) A child on a swing applies periodic impulses to the motion, keeping time with the swing. The swing gradually increases in amplitude. It is a case of resonance.

(3) If one leans on one side of a heavy boat and then on its other side, a considerable roll can be built up if the motions of the body have the same period as the swing of the boat. The boat may then capsize.

(4) Resonant vibration is of great practical importance to structural and mechanical engineers. If quite a small periodic force acts on some structure or machine having the same natural period, vibrations of large magnitude may develop. Vibrations produced by resonance, if of large magnitudes, the resultant stresses may exceed the elastic limit and damage the structure. Hence in designing a structure it is necessary to find out what external periodic forces may act on the structure. The structure is then built up so as to have a different natural frequency. Such are the cases of buildings beside rail roads with heavy traffic and cantilever bridges across rivers.

(5) Soldiers break step while marching over a [illegible]. Bridges have been known to vibrate dangerously and even to collapse when resonance occurs between the period of steps of the soldiers' march and the natural period of vibration of the bridge.

Jointless rails are used to prevent railway-bridges being affected by the periodic impact of wheels crossing the joints.

B. Acoustical examples. (1) A vibrating fork held in the hand produces a feeble sound. When its stem is pressed on a table-top the sound is greatly magnified. The vibration of the fork is transmitted to the table-top. The top is thrown into forced vibration with a frequency equal to that of the fork. The large vibrating surface of the table sets a large mass of air into vibration. This rouses the volume of sound.

In this as well as in all other cases of forced vibration the kinetic energy of the sounding body is transferred to the forced vibrator. Because of the large surface of the table the *loss of energy* to the surrounding air is much faster than when the fork vibrated alone. So the vibration of a fork dies away much more quickly when it is pressed on a table-top.

(2) Take a tall empty glass jar. Hold a vibrating tuning fork just above the upper end of the jar and slowly pour water into it. When the water reaches a particular level, a loud sound may be heard. Once the level is crossed the loud sound ceases. The loud sound is due to resonance between the fork and the air column in the jar above the particular level of water. You observe the same if the jar is full of water which is slowly leaked out through a tap, with the vibrating fork held over.

(3) Stretch two identical wires side by side under the same tension, between two bridges on a wooden board (a *sonometer*). Place a third wire under a different tension beside them. The first two have the same natural frequency while that of the third is different. If one of the identical wires be disturbed its vibration will be transmitted to the other two wires. In the identical wire resonant vibration will occur and grow to a large amplitude. A paper rider placed on it may be thrown off. But this will not occur on the third wire where the vibration is forced and consequently the amplitude is small.

(4) In stringed instruments like the sitar, esraj, guitar, violin, etc., strings are stretched on a thin wooden board. The vibration of a string produces forced vibration of the board and thence of air. This makes the emitted sound stronger.

Many of these instruments have several strings tuned to different frequencies. When a note is sounded on the principal wire resonant vibrations are excited in those that have the same frequency as that of the note sounded. This fact increases both the intensity and the pleasantness of the note played. In 'percussion' instruments like drums,

the intensity of sound is increased by the forced vibration of air inside them.

(5) It has at times been noticed that when some particular note is played loudly on an organ or a piano, some object inside the room, such as a large empty vase, resounds. This happens when the frequency of the note agrees with some natural frequency of the object. (An object may have several natural frequencies corresponding to the various modes in which it can vibrate.)

Loudspeakers of poor quality sometimes produce a magnified response to certain parts of the musical scale because of such resonance. The result is boomy and unpleasant distortion.

C. Electromagnetic examples. The principle of resonance is utilised in tuning a radio or TV receiver. A circuit of low resistance has a natural frequency of oscillation depending on its inductance and capacitance ($n=1/2\pi\sqrt{LC}$). To receive the radio waves from a given station the frequency of the oscillatory circuit in the receiver is adjusted to the value of the incoming waves. The latter then sets up resonant electrical oscillations in the receiver. These are amplified by valves and operate the loudspeaker. Reception of radio signals is thus brought about by resonance. This is why only one particular station (*i.e.* frequency) can be "tuned" at a time.

The phenomena of forced and resonant vibration of electrons under the impact of electromagnetic waves have been utilised to explain scattering, dispersion and absorption of light.

1.13. Sharpness of resonance. The term refers to the fall in amplitude of a body in forced vibration as the frequency of the driving force changes from the resonant frequency.

Resonance is said to be sharp when the response of the driven body falls off quickly as the impressed frequency moves away from the resonant frequency. Between two cases of resonance we call that one sharper in which the response is smaller for a given fractional (or percentage) departure of frequency from that a resonance. Sharpness of resonance depends primarily on damping ; the smaller the damping the sharper the resonance (See fig. III-2.6.), is found to be.

The concept of sharpness of resonance is of much practical importance. The following two cases may give us some idea of it.

(*a*) If we blow across the mouth of a glass tube, the air in it is thrown into vibration and a sound is produced. But the sound stops as

soon as the blowing is stopped showing that the vibrations of the air column are *heavily damped.*

Over the open end of a glass tube of suitable length hold in turn a series of vibrating tuning forks of nearly equal frequencies. The air in the tube will vibrate giving a maximum response for the fork whose frequency agrees with the natural frequency of the air in the tube. It will also respond to the other forks, though to a lesser extent. The response diminishes slowly as the two frequencies differ more and more.

The experiment shows that a *heavily damped system* responds to a *band of frequencies* around the resonance frequency, *i.e.*, its *selectivity* is poor. Sharpness of resonance is then said to be small.

(*b*) Vibrations of tuning forks persist for a long time. This shows that their motions are *lightly damped.* Take two tuning forks, *A* and *B*, of the same frequency and mount them on resonance boxes with the open ends facing each other. Excite *A*, and after a few seconds stop its vibration by touching it. *B* will then be heard sounding by resonance, although it was not struck. If *A* is loaded with a small piece of wire and the experiment is repeated, *B* will not respond.

Again refer to the experiment described in **B (3)** above. The experiment consists in placing a heavy vibrating tuning fork on the wooden plank (the *sonometer*) and adjust the position of one of the bridges till the paper rider is thrown off because of resonance. This is an experiment you have to do in your practical class. It is difficult to adjust, for if you cannot get the exact length, the rider will vibrate only a little or not at all and will not fall off, for forced vibration produces a small amplitude. The vibrating wire disturbs a very small volume of air and so is lightly damped.

The experiments suggest that *for light damping* response occurs practically at resonance frequency and falls off very rapidly when frequencies differ. Here *sharpness of resonance* or *selectivity is* said to be *high.*

Sharpness of resonance is measured by the ratio of resonant frequency to the range of frequencies for which energy of the forced system falls to half the value at resonance. These half-power frequencies occur on the two sides of the central line in fig. III-2.6. Sharpness of resonance also plays a significant role in the selectivity of reception in radio and television circuits.

PROPAGATION OF VIBRATIONS : WAVE-MOTION

III-3.1. Introduction. We shall now consider the *propagation* (i.e. transmission) *of vibrations through an elastic medium.* This is wave motion. We have considered before, the simplest kind of vibration, the S.H.M. Now we consider transmission of simple harmonic vibrations only, through an elastic medium as we have decided before.

An 'elastic' medium is a material medium in which elastic forces act between neighbouring particles. To propagate vibrations through an elastic medium, *two factors* are necessary. One is *inertia.* It is necessary for vibration, for the particle must not stop at its normal position of rest where there is no force on it. It must be carried through this position by 'inertia'. The other is *elasticity.* Elastic forces must act between neighbouring particles, to enable a vibrating particle to force the next particle also to vibrate similarly. It is through such elastic forces that vibrations are transmitted from particle to particle through the medium. So a *material medium that will propagate elastic vibrations must have inertia and elasticity distributed throughout it.*

A medium in which the properties are the same in all directions, is an *isotropic* medium. Air, water, glass are isotropic. So are many other homogeneous materials. In an isotropic medium, vibrations are transmitted with the same velocity in all directions. We shall confine ourselves to the transmission of simple harmonic vibrations in *one direction* only. The resulting motion *in space and time* is known as *plane, progressive harmonic wave motion.* We shall return to it later. But first let us add a few words in general about wave motion.

III-3.2. Wave motion. Wave motion in matter is the kind of motion that takes place *in time and space* when vibrations are being propagated through an extended material medium i.e. vibrations at one point at an instant is found some time later to have moved to another point. The commonest kind of wave motion that we see occurs, when we throw a stone on the still surface of water. A portion of the water where the stone has struck, rises and falls a few times, that is, that portion

vibrates. This rise-and-fall pattern spreads out over the surface of water in all directions as concentric circles. A floating leaf on the water surface, will simply rise and fall with the undulataing surface, but would not advance with the waves. This is a distinguishing characteristic of wave motion, *In wave motion, no portion of the medium advances with the waves; it executes a vibratory i.e. undulatory motion about its normal position of rest.*

What then advances in wave motion? *It is the state of vibration, i.e. the* **phase** *of motion, that advances.* Here, the waves advance along *radial lines* from the centre of disturbance. Any such radial line is a direction of wave propagation in this case. Particles of water along any such line are thrown into vibration one after another as the wave advances. A particle farther from the centre of disturbance gets at some later moment the same state of motion (same phase) as a particle nearer the centre. All particles lying on a circle concentric with the centre of disturbance are in the same state of motion at a given instant. They all arrive at their maximum elevation over the still water surface, or maximum depression below it, at the same instant. They are all said to be in the same phase at any given moment. But this phase (that is, the state of vibration of the given point) changes with time.

Difference between vibrations and waves. A vibration spreading out through a medium is a wave. Both require *elasticity* and *inertia.* In a vibration, such as the vibration of a load held by a spring, *inertia is localized* in the load and elasticity is localized in the spring. When such a vibrating body is placed in an extended medium (such as water or air) waves spread out into the medium. Vibration occurs at every point of the medium. So the *medium must have elasticity and inertia distributed* throughout it, so as to be able to carry the waves.

Particles along the line of advance of a wave are in different states (or phases) of vibration at a given instant. Particles away from the source aquires at a later instant the state (or phase) of vibration of a particle lying nearer to the source.

In the above case the rise-and-fall (or vibration of a particle) occurs only a few times and then stops. Besides, the amplitude of vibration dimininishes with increasing distance from the centre. Such factors complicate any elementary discussion of wave motion. We shall, therefore, for simplicity of discussions *idealize* our waves. This is like introducing idealizing conditions in other fields of physics, like the point

particle, the perfectly smooth surface, the weightless and perfectly flexible string of a simple pendulum, etc. They don't reeally exist ; but they are of great help in understanding problems and solving them.

III-3.3. The idealized wave : Plane, progressive, harmonic wave. In our idealised wave, the vibrations of all particles of the medium will be simple harmonic. That is why we call it a *harmonic* wave. We call it *plane* because all particles on a plane perpendicular to the direction of propagation are in these same phase at any given instant. It should be clear that these waves propagate (move forward) in *one direction only.* (They do not spread out in all directions along a plane as for water waves just described.) The waves are called *progressive* because they do not come across any boundary on their way which will modify the waves and move on unhindered.

The fact that you cannot have a real wave satisfying these conditions does not matter. You cannot realize a simple pendulum, you cannot realize a point particle or a ray of light or a frictionless surface. Nevertheless, these idealized concepts help you in getting useful results. The same applies to idealized waves.

Later in Sec. III-3.10. we shall discuss the properties of these idealized waves. Now we speak of 'transverse' and 'longitudinal' waves.

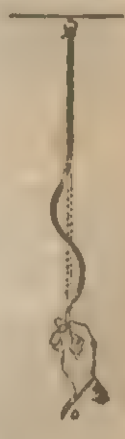


Fig. III-3.1

II-3.4. Transverse waves. Waves in which the particles of the medium vibrate *perpendicular* to the direction of propagation, are known as *tranverse waves.* You may think that the *water waves* are transverse. But strictly, they are not so. (The particles actually move in circles or ellipses.)

Visible examples of transverse waves are few. If the free end of a long suspended heavy cord is jerked perpendicular to its length, transverse vibrations move towards the upper end in the form of a wave (fig. III-3.1), A stretched string plucked or struck laterally vibrates transversely. Light, heat and radio waves are transverse in nature, but we cannot see their vibrations.

When a wave advances along a string (fig. II-3.1) you see that in one half of a wave the particles are displaced to one side and in the other half to the other side. A complete wave is made up of two such

halves in which the displacements are in opposite directions. In a transverse wave travelling *horizontally*, the part of the wave above the undisturbed part is called a **crest** and the part below is called a **trough** *A crest and a trough together make up one wave.* The terms are often used to represent two halves of any transverse progressive wave. Particles on a crest and a trough, which are at equal distance from respective equilibrium positions but move in opposite directions, are said to be in opposite phases. A succession of waves of a finite number is called a **wave-train.** Remember that ripples on water surface show crests and troughs but are not strictly transverse waves. They form a wave-train.

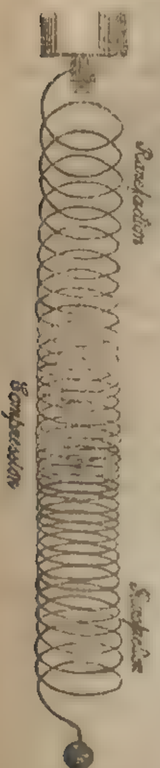


Fig. III 3 2

III-3.5 Longitudinal waves. Waves in which the direction of particle vibration is *along* the direction of propagation of the waves, are called *longitudinal waves.* To visualize such a wave, take a long, vertical closely wound spring (fig. III-3.2) and load it with a bob at the lower end. Pull it slightly downward and then release it. The turns of the spring will vibrate up and down along the length of the spring. If you mark a point on the spring with a short piece of white thread, you will find the mark moving up and down. Take the spring as an elastic medium, the marked point as a particle and the motion of the spring, the wave motion. The layers of the spring may be considered as layers of the medium, all particles of a layer being in the same phase of motion.

Since the particles in this kind of waves vibrate in the direction of wave propagation, their displacements do not produce crests and troughs In that part of the wave where the particles *move in the direction of wave propagation* they come nearer together than their normal separation in the medium and form a **condensation.** A layer of medium in this portion is in a state of compression. (Longitudinal waves are also called *compressional waves.*) If the medium is a fluid, *pressure* in the compressed layer will be higher than the normal value (that is, when there are no waves). If the medium is a solid, compression will increase the *stress* in the layer. In that part of the wave where the particles *move*

opposite to the direction of wave propagation, they are further apart than their normal separation and form a **rarefaction.** The medium in this portion is rarefied. The *pressure* or the *stress* in a rarefied layer will be lower than the normal. A complete longitudinal wave consists of one compressed and one rarefied region. *The pressure or stress therefore alternates about the normal value in a longitudinal wave.* Sound waves are longitudinal waves. Waves are not necessarily only transverse or longitudinal ; there are other types also. We would not consider them.

III-3.6. Elastic properties of a medium determine the nature of wave. Whether a wave generated by the vibration of a particle will be transverse or longitudinal depends on the nature of the medium. This is so because the direction in which a vibrating particle will displace the next, depends on the acting elastic forces.

Since a fluid cannot resist a shearing force the displacement of any of its layers cannot drag a parallel layer in its own direction of motion. Hence *a transverse wave cannot be produced in a fluid.* A fluid, however, resists a volume change. Hence a sudden compression or rarefaction applied in a given direction to a gaseous or a liqnid layer is transmitted along the line of the applied force to adjoining parallel layers. *A longitudinal wave, therefore, can be propagated in a gas or a liquid.*

A solid resists deformation of both of size and shape. So, if a particle in a solid is disturbed, it displaces others lying in directions both parallel and perpendicular to its own direction of motion. Hence *in a solid both longitudinal and transverse waves are possible.*

Earthquakes are caused by large vibrations in the interior of the earth. Both transverse and longitudinal waves are produced during an earthquake. As they reach the earth's surface with different velocities (longitudinal waves at 7·2 km/s and transverse waves at 4 km/s) their responses on the seismograph are separate. From this time interval, it is possible to estimate the distance of the *epicentre* of the earthquake.

When the *displacement of the disturbed particle is small*, the restoring force is proportional to displacement and acts towards the equilibrium position of the particle. Under such conditions the motion is simple harmonic. Such waves are called *simple harmonic waves.* In our discussions we shall assume this condition to hold.

III-3.7. Waves transmit energy. From where do the particles that vibrate in wave motion, derive their energy ? Waves spread out through a medium only when a portion of the medium is displaced. The agent that causes this displacement supplies the energy. The

energy that the agent transfers to the medium spreads out through the medium in the form of waves. *Wave motion is the most important form of energy transmission, particularly over long distances.*

Summary. *In material wave motion, the particles on a line of propagation vibrate about their normal positions of rest. A point on a line of propagation farther from the source of disturbance acquires at some later instant the same state of motion (phase) as a point nearer to the source. Wave motion means propagation of the phase of vibration. What advances in wave motion is the phase of motion, and not any portion of the medium.*

Though waves may be of various kinds, we shall consider only longitudinal and transverse waves. Longitudinal waves can pass through all kinds of media solid, liquid or gaseous. Transverse waves are possible in solids only. *In longitudinal waves, layers of the medium are alternately compressed and rarefied. Compression occurs in the layer in which particle velocities are in the direction of wave propagation. When particle velocity is opposite to the direction of wave propagation, rarefaction occurs. In longitudinal waves, pressure (in fluids) and stress (in solids) alternate about their normal values at any point. Sound waves are longitudinal. Light waves are transverse as also are radio waves. Waves carry energy received from the source on to the receiver.*

III-3.8. Propagation of vibrations. Let us consider the particles of a medium lying along a straight line (fig. III-3.3). Let one of them be displaced perpendicular to the line and forced to execute a simple harmonic motion along its line of displacement. Its neighbour will follow suit but will start a little later. Between these two vibrating particles there will be a small phase difference because of this time lag. The vibration is then transmitted from particle to particle with a small phase difference between two successive particles. *The phase difference between any two particles on the line of propagation will be proportional to the distance separating them.*

Individual particles will vibrate with a constant periodic time T. In Fig. III-3.3 the particles are shown after intervals of time $T/12$. Each particle starts to move $T/12$ after the particle preceding it. The figure indicates that the vibrations gradually pass on from particles on the left to those on the right. The direction along which the vibrations are transferred is the direction of wave propagation.

When the 'zero' particle has completed one vibration (*i.e.* when it is passing through its initial position with the initial velocity in the initial direction) the 12th particle just starts moving. These two particles therefore vibrate in phase. But since the former has completed one vibration when the latter begins to

move, the phase difference between them will also be 2π radians. The phase difference of any particle relative to the zeroeth particle will increase

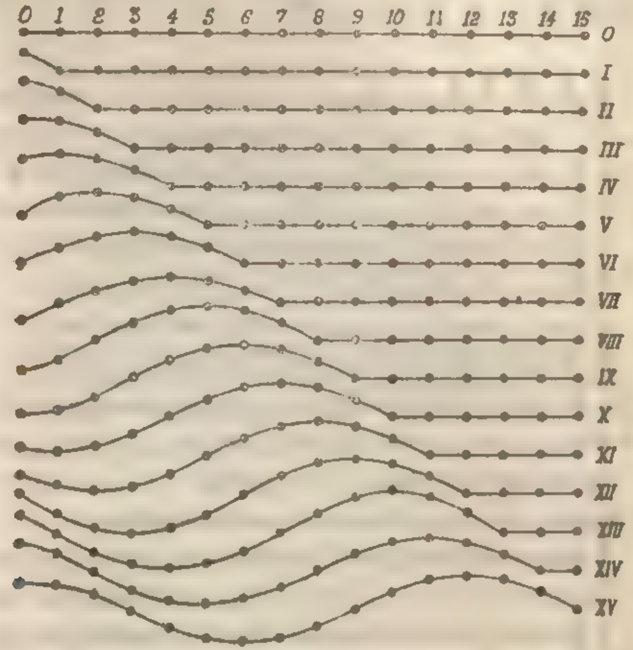


Fig III-3.3

with its distance from O, as has already been noted, till it grows to 2π radians at the 12th particle.

The above analysis indicates that the *difference in phase angles* of the vibrating particles will *increase linearly with distance.* For some particular distance it will be 2π radians. Such particles as 1 and 12 in the above figure are in the same phase of vibration ; they have the same displacements and are moving in the same direction. *The minimum distance, measured along the line of propagation of a wave, between two particles in the same phase of vibration is called the* **wavelength** (λ) *of the wave.* Particles on the line of propagation separated by a distance that is a multiple of λ, will differ in phase by the same multiple of 2π radians.

The displacements of particles in a *longitudinal* wave can be investigated exactly as above and are shown in Fig. III-3.4. The particles marked 0, 1, 2, etc. are on the line of propagation. The figures marked

I, II, III, etc. indicate the positions of particles at intervals of $T/12$ where T, as before, is the time period of vibration of a particle. A particle executes an S.H.M. about its mean position of rest in the

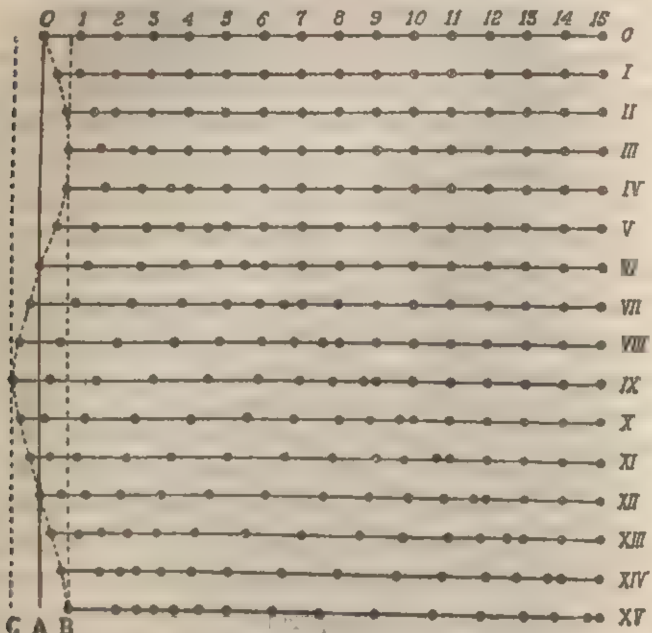



Fig III-3.4

direction of wave propagation left to right. AB and AC represent the amplitude of this vibration. Any two neighbouring particles have a small relative phase difference.

III-3.9. Some important definitions in connexion with waves.

(i) Wave velocity or phase velocity (c). It is the velocity with which the phase of vibration advances along a line of wave propagation. We shall denote it by the letter c.

(ii) Wave length (λ). It is the *shortest distance* between two particles on a line of wave propagation which are in the same phase of vibration. Particles on a line of propagation which are separated by distances equal to integral multiples of λ are all in the same phase.

It is the distance between two *successive crests* or *troughs* in a *transverse* and between *successive compressions* or *rarefactions* in a *longitudinal* wave.

(iii) Frequency of a wave (n). Frequency of a wave is the number n of wavelengths contained in a distance equal to the wave velocity c. Hence the relation between c and λ is

$$c = n\lambda. \qquad \text{(III-3.9.1.)}$$

We look at it from another angle. As a particle in a wave starts vibrating, it starts at the same time disturbing the particle ahead of it on the line of propagation. By the time the first particle has finished one complete vibration, the disturbance has gone over a certain distance. This distance is the wavelength. If there are n vibrations per sec, we have as many waves and they cover a distance equal to $n\lambda$ which must be the wave velocity, i.e. the distance crossed by the disturbance in one sec.

If T is the time period of vibration of the particle (and they are all alike in a medium), we must have $\lambda = cT$. The reciprocal of T is the frequency of the particle. So $c = \lambda/T$. But $c = n\lambda$ also. Therefore $n = 1/T$ = frequency of vibration of the particle as well. Thus

$$c = n\lambda = \lambda/T, \qquad \text{(III-3.9.2.)}$$

$$\text{and } n = 1/T. \qquad \text{(III-3.9.3.)}$$

Alternatively, T is the time for one vibration and n, the no. of vibrations in 1 sec. So $nT = 1$.

If the vibrations in a wave are simple harmonic,

$$n = 1/T = \omega/2\pi \qquad \text{(III-3.9.4.)}$$

where ω is said to be the **angular frequency** or **pulsatance** of vibration.

T may also be defined as the time a wave takes in moving through a distance λ (one wavelength). It is the **period of the wave** as well (besides being the period of vibration of a particle). *A wave thus has a periodicity both in space and time while a vibration, only in time.*

Problems. (Take c in air = 340 m/s and c in water = 1480 m/s.)

1. A tuning fork has frequency 512 Hz. What is the wavelength of the sound wave produced in air ? (**Ans.** 0.664 m)

2. The disc of a siren has 40 holes and rotates at 20 rev/s. What is the frequency of the note produced ? (**Ans.** 800 Hz)

3. A disc siren rotates at 15 rev/s. If the note produced has a frequency of 300 Hz how many holes are there in the disc ? (**Ans.** 20)

4. Find the wavelength in air in water of sounds of frequeucy 20 Hz and 20,000 Hz. [**Ans.** 17 m, 74 m ; 0.0179 m, 0·07 m.]

(iv) Amplitude of a wave. The maximum displacement that a vibrating particle undergoes in a wave is called the *displacement*

amplitude of the wave. Similarly, we shall have 'velocity amplitude', 'pressure amplitude', etc. for maximum values of the relevant quantities.

(v) **Wave front and ray.** When a wave is passing through a medium it is always possible to find at any instant a surface through any point (of the medium) on which (surface) the particles are in the same phase of vibration. **The continuous locus of points in the same phase of vibration in a medium through which a wave is travelling is called a wave front.** A wave front does not remain stationary but advances through the medium with a definite velocity called the **wave velocity** or **phase velocity.** It is different from particle velocity of the medium.

A normal to a wave-front is called a **ray.** The energy that a wave carries, moves along the rays. In a *homogenous* medium, a disturbance produced at a point inside it, spreads out in all directions with *equal* velocity. Particles equidistant from the source will vibrate in the same phase. Hence the wave front will be spherical. Such waves are called **spherical waves.** A wave in which the wave fronts are plane surfaces is called a **plane wave.** A finite portion of a wave front coming from a source very far away is practically a plane surface and may be treated as a plane wave front.

(vi) **Wave form or wave profile.** In fig. III-3.3, if O executes an S.H.M, at any instant the space-displacement curve of the particles lying between 0 and 12 will be a complete sine curve (fig. III-3.5.). The

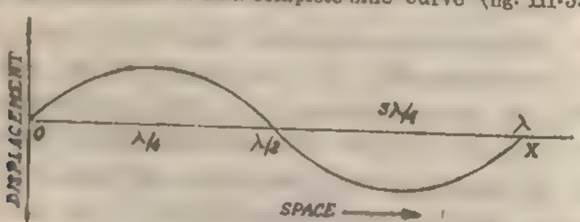


Fig. III-3.5.

space-displacement curve (for a complete wave at any moment) is known as the **wave-form** or **wave-profile.** It pictorially represents the displacements at any instant of all the particles lying within a distance of one wavelength along the line of propagation. (Compare curve XII fig. III-3.3.). The curve is what you get by taking a snapshot of a wave.

In a simple harmonic wave, if we plot the displacement curve of *any single particle* for a complete period T, then also a sine curve

result (fig. III-3.6). This means that the *time-displacement* curve of a single particle of the medium transversed by simple harmonic waves throughout a time-period is exactly similar to the wave-form (i.e., the *space-displacement* curve of all the particles at any instant), i.e. the *cinemato-*

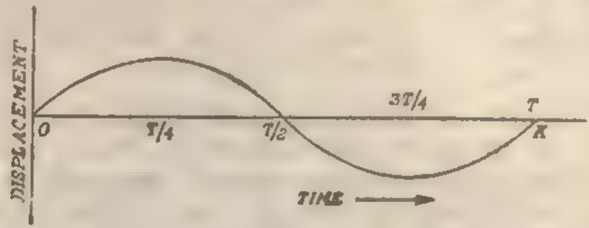


Fig. III-3.5

graphic picture of displacements (*i.e.* a record of successive positions) of a single particle of the medium in a time interval T agrees with an instantaneous or *snapshot* photograph of displacements of all the particles within a distance λ.

III-3.10. Equation of plane, progressive, harmonic waves. We have introduced before the *idealized* wave, which we called a *plane, progressive, harmonic wave.* Let us derive a mathematical expression for *particle displacement* in such a wave. For simplicity, it is a plane, harmonic wave.

Take the positive x-axis as the direction of propagation, and any convenient point on it as the origin ($x=0$). The equation of particle displacement at the origin will be $y=a \sin \omega t$. (The waves may be transverse or longitudinal.) A particle at a distance x from the origin will acquire after an interval t' the phase that the particle at the origin has at time t. Hence the displacement equation for the particle at x will be $y_x=a \sin \omega(t-t')$. If c is the wave velocity $x=ct'$, and we shall have

$$y_{x,t}=a \sin \omega(t-x/c)$$
$$=a \sin (\omega/c)(ct-x)=a \sin (2\pi/\lambda)(ct-x) \qquad \text{(III-3.10.1)}$$

Introducing the various quantities n, T, λ from above into this equation and writing $k=2\pi/\lambda$ (k is called the *wavelength constant*), we can throw eqn. III-3.10.1 into various equivalent forms. Dropping the subscripts x, t from y for convenience (since the sense is clear) we have

$$y=a \sin 2\pi n(t-x/c)=a \sin 2\pi(t/T-x/cT)$$
$$=a \sin 2\pi(tT-x/\lambda)=a \sin (\omega t-kx) \qquad \text{(III-3.10.2)}$$

The last is the most compact form of writing the equation. We shall use it more often. a is the amplitude of the wave. Remember $k=2\pi/\lambda$ is often called the *wave constant.*

Properties of the wave : (1) *The waves are plane,* because the phase angle depends on x alone. For a given x, all points on the y-z plane perpendicular to the x-axis are in the same phase. Hence the **wave front is plane.**

(2) *The waves are progressive.* If in (eqn III-3.10.1) we increase t by 1 and x by c, we get the same value for y. This means that y at $x+c$ and $t+1$ is the same as y at x and t, *i.e.* the phase of the vibration has moved over a distance c in one second. Using symbols, we find

$$y_{(x+c),\,(t+1)}=a\sin\omega\{(t+1)-(x+c)/c\}=a\sin\omega(t-x/c)=y_{x,t}.$$

The phase advances along the direction of the *positive* x-axis with **velocity c. The equation**

$$y=a\sin\omega(t+x/c)=a\sin(2\pi/\lambda)(ct+x) \qquad \text{(III-3.10.3)}$$

when analysed as above, will show that it represents a wave moving in the direction of the *negative* **x-axis with velocity c.**

Note : The waves represented by above equations have no ending. They do not indicate any stoppage of motion. They belong to an *infinite* **wave-train.**

(3) That *the vibrations in the wave are simple harmonic* is our basic assumption.

(4) *The waves have a two-fold periodicity, one in time and one in space.* The *time periodicity is T*, for as t increases by T the vibrations repeat themselves. They are also repeated at intervals of space equal to λ, the wavelength. *λ is the space period.*

Example. III-3.1. A wave has an amplitude of 0·1 mm, velocity of 350 m/s and frequency 500 c/s. What are the wavelength and period? Write down its displacement y at the point x.

Solution : **Wavelength** $=\dfrac{\text{velocity}}{\text{frequency}}=\dfrac{350\ \text{m/s}}{500/\text{s}}=70$ cm

Period $=1/\text{frequency}=1/500\ \text{s}^{-1}=0.002$ s.

Equation : $y=0.01\sin 2\pi\times 500\,(t-x/35000)$ cm.

Here y and x are in cm and t in seconds. **If y and x are in metres, then**

$$y=1.10^{-4}\sin 1000\pi(t-x/350)\ \text{m}.$$

Ex. III-3.2. Show that the equation $y=0.5\sin\dfrac{2\pi}{3.2}(64t-x)$ represents a progressive wave. Find from the equation the values of the amplitude, frequency, wavelength ane phase velocity of the wave. (The units are in c.g.s.)

Solution : Comparing the given equation with Eq. III 3-10.2 we can at once write down (in the c.g.s. system) :

Amplitude $(a) = 0.5$ cm ; wavelength $(\lambda) = 3.2$ cms.

Phase velocity $(c) = 64$ cms/s. Frequency $(n) = 20$ per sec.

Consider the displacement of a particle at co-ordinate $x+64$ at time $t+1$. From the equation, this is

$$y' = 0.5 \sin \frac{2\pi}{3.2} \{64(t+1) - (x+64)\}$$

$$= 0.5 \sin \frac{2\pi}{3.2}\left[(64t - x)\right] = y.$$

Thus we find that a particle at $x+64$ cm acquires the motion of a particle at x after one second. This means that the disturbance travels a distance of 64 cm in one second, i.e., it is a progressive wave.

Problems (1) The displacement equation in a wave is $y = 10^{-4} \sin 2\pi \left(\frac{t}{0.01} - \frac{x}{200}\right)$ metre. If t is in seconds and x in centimetres, find (a) the amplitude, (b) period, (c) frequency, (d) wavelength and (e) wave velocity.

[**Ans.** : (a) 10^{-4} m ; (b) 0·01 s ; (c) 100/s ; (d) 200 cm. ; (e) 200 m/s]

(2) If the wavelength is 1 metre, what is the phase difference in the vibration of two particles which are 10 cm apart on the line of wave propagation ?

[**Ans.** : 36°. Note that a distance λ corresponds to a phase difference of 2π].

III-3.11. Periodic waves. Vibration of particles in a wave may be periodic, but not simple harmonic (fig. III-3.7c). Such a wave is a *periodic wave*, but not a harmonic one. Musical instruments or radio transmitters may produce them. We have said before that a periodic vibration may be expressed as the sum of a suitable number of S.H.Ms of appropriate amplitudes. Similarly, any periodic wave may be expressed as the sum of a suitable number of harmonic waves of appropriate amplitudes. Curve III-3.7(c) is the combination of pure simple harmonic waves III-3.7(*a*) and (*b*).

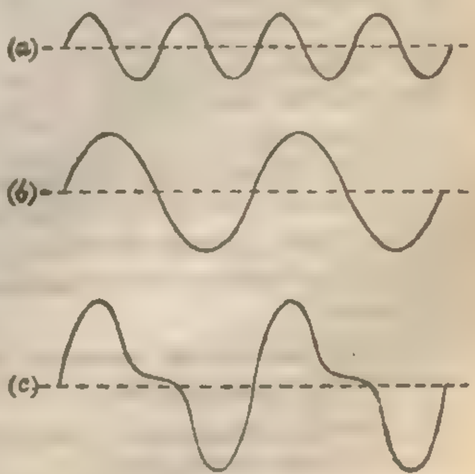


Fig. III-3.7

III-3.12. Characteristics of progressive wave motion. We may now summarize the characteristics of progressive wave motion as follows :

(*i*) Progressive waves are produced by continuous vibration of a a portion of the medium. They advance through the medium with a velocity determined by the elasticity and density of the medium.

(*ii*) Each particle of the medium executes the same vibration about its equilibrium position with identical frequency and amplitude. The vibrations may be transverse, longitudinal (or torsional) relative to the line of propagation of the waves.

(*iii*) The state of vibration, *i.e.* the phase, of one particle is transmitted to the next along the line of propagation. The phase difference between two particles along this line is proportional to their separation.

(*iv*) *Progressive wave-motion is thus a disturbance recurring both in space and time.* The space-displacement curve at any instant (of the particles on a line of wave propagation) agrees with the time-displacement curve of any single particle. Both are sinusoidal in a simple harmonic wave.

The wavelength λ gives the periodicity in space and $T=1/n$ gives the periodicity in time. In time T the wave moves a distance λ, giving a velocity of wave propagation $c=\lambda/T$, a ratio of the two periodicities.

(*v*) Progressive wave motion carries energy from one point to another along the wave normals, *i.e.*, the rays, without any bodily transfer of the medium.

(*vi*) As a compressional (*i.e.*, longitudinal) wave proceeds, every point of the medium suffers the same changes in pressure and density (in fluids) or stress and density (in solids).

There are various kinds of wave apparatus to demonstrate the above characteristics.

III-3.13. Characteristic properties of waves. Waves have the following characteristic properties :

(a) Through a homogeneous, isotropic medium waves travel with the same velocity in all directions.

(b) When a wave travelling in a medium falls on a surface (*large compared with the wavelength*) separating it from another, part of it turns back into the first medium while the rest passes into the second. The

first phenomenon is known as **reflection** and the second as **refraction**. They occur at any boundary, the elastic properties or densities on the two sides of which, are different. In both cases there is a change in direction of wave propagation. The laws governing reflection and refraction are already familiar to the students of light.

(c) When a wave meets an obstacle of *size comparable with its wavelength* or passes through an opening of similar size, it bends round the corners to some extent and encroaches into the region of geometrical shadow. This phenomenon is known as **diffraction**.

(d) If two *identical wave-trains* passing simultaneously through a medium *continue to meet* in opposite phases at some points, the particles at these points will remain permanently at rest. Two waves in this way neutralise the effects of each other at these points. The phenomenon is called destructive **interference**.

(e) If a wave meets a small body in a medium, that body is thrown into forced vibration by the wave. The body (of *size small compared with the wavelength*) absorbs energy from the wave and radiates out this energy in the form of spherical waves through the medium. The phenomenon is known as **scattering**. It weaknes the waves.

(f) Transverse waves have a special property known as **polarization** which longitudinal waves do not possess. The transverse vibrations may be confined to a plane containing the direction of propagation. With appropriate devices, these vibrations may be prevented from advancing. But this cannot be done to longitudinal waves. Light waves show polarization ; sound waves do not. Polarization distinguishes between transverse and longitudinal waves.

III-3.14. Comparison of transverse and longitudinal waves.

Transverse wave	*Longitudinal wave*
1. The vibration of particles is at right angles to the direction of wave propagation.	1. Vibration of particles is in the direction of wave propagation.
2. Can be produced only in solids but not in gases or liquids.	2. Can be produced in solids, liquids and gases.
3. A crest and a trough make up a complete wave.	3. A complete longitudinal wave consists of a compressed and a rarefied region.
4. Exhibits polarization.	4. Does not show polarization.

III-4

SOUND WAVES, VELOCITY OF SOUND

III-4.1. Sound waves are elastic waves. (1) It is well known that sound is due to vibration of a body and that it requires a *material medium* for propagation. Sound cannot pass through vacuum.

(2) As sound travels through air, the air does not advance with the sound. Even in a loud sound, there is no air current. Waves behave so. An advancing wave does not carry the medium with it.

(3) In all media (solid, liquid or gas), sound travels with a definite velocity characteristic of the medium. The velocity is determined by the *elasticity* and *density* of the medium. Other waves behave similarly. So sound must be wave-like in nature—waves in a material medium.

(4) All waves have the common properties of reflection, refraction, diffraction and interference. Sound shows all these properties. This supports the wave nature of sound.

(5) Only longitudinal (also called *compressional*) waves can move through gases and liquids. Transverse waves cannot do so. Hence sound waves must be *longitudinal* (that is, compressional) in character.

(6) Only transverse waves show the phenomenon of polarization. Longitudinal waves do not have this property. Sound does not show polarization. This supports the longitudinal character of sound waves.

(7) Sound waves have actually been photographed.

These facts convincingly prove that sound waves are elastic waves, longitudinal (compressional) in character.

III-4.2. Definition of sound. Sound has been defined by the American Standards Association as follows :

Sound is an alternation in pressure, stress, particle displacement or particle velocity, which is propagated in an elastic material or the superposition of such propagated vibration.

The definition is *objective* ; it does not depend on whether anybody can hear the sound or not. Sound is audible when the frequency of

alternation lies within the approximate limits 20 and 20,000 Hz. Dogs, bats, birds can hear higher frequencies. (1Hz = 1 cycle per sec).

As you go through different aspects of sound, try to notice how they agree with the definition. For practical purposes, the most important aspect is the change of pressure in sound waves. Most methods of detection and measurement in sound depend on the changing sound pressure.

Audible and inaudible sounds. Whether compressional waves will produce the sensation of sound in the human ear or not, depends upon the wave frequency. Ordinarily, the human ear responds to compressional frequencies in the approximate range, 20 to 20,000 Hz. The range, however, varies from person to person. In the same person, the range is reduced with age.

Compressional waves of frequency lower than 15 Hz are inaudible to most people. Still they are sound waves and are called *infrasonic waves*. Compressional waves of frequency higher than 20,000 Hz are called *ultrasonic waves*. They are also inaudible to the human air. Study of ultrasonic waves now forms a very important branch of physics. It has found useful applications in industry and other fields including medical.

III-4.3. Sources of sound. Any solid, liquid or gaseous body which can (1) vibrate between 20 and 20,000 times a second, and (2) transfer the vibrations to the medium in which it is situated, (3) producing compressional waves in the medium, is a source of sound. If the frequency is beyond this range, *the sound is inaudible, but the waves are there.*

Clearly, a classification of sources of sound does not carry much sense, unless you decide on the basis of classification. When we hear a sound we can try to identify the source of sound. Strike a solid ; you will hear a sound. The solid is the source. When rain-drops fall on the water in a pond, the sound that you hear is due to the vibrations of portions of water. These vibrating portions are the sources. In musical instruments we can identify the sources often easily. In stringed instruments, such as the sitar, esraj, piano, guitar, etc., the vibrating string is the source. In percussion instruments, such as drums etc., the vibrating membrane is the source. In flutes or clarinets, the vibrating air column inside, is the source.

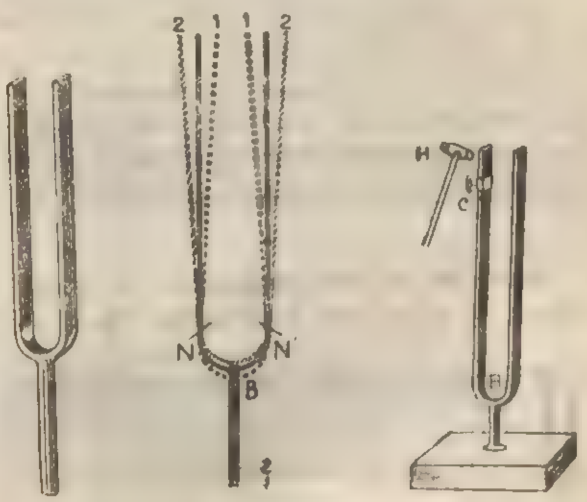
Our *voice box*, a small organ inside the throat, is the most important source of sound to us. It has two thin membranes (vocal cords) nearly

closing the gap in the box. Air from the lungs is forced through this gap, causing the membranes to vibrate. These vibrations produce the sounds we emit.

For experiments on sound we often require a source which generates simple harmonic vibrations in air. There are complicated electrical devices for the purpose. But a very simple one which you will use in the laboratory is the *tuning fork.* It is described below.

The Tuning Fork a special source of sound. In considering the action of waves it is very helpful to have a source which produces waves of a single frequency. For sound waves, the tuning fork is such a source.

A tuning fork a rectangular bar bent into the form of a U with a stem attached at a bend (fig. III-4.1a). If any of the arms of the fork (called *prongs*) is struck, both arms begin to vibrate. *The vibrations (unless very*



(a) (b) (c)

Fig. III-4.1

strong) are simple harmonic. The nature of the vibrations has been shown in fig. III-4.1b. Both prongs simultaneously move inward (as at 1, 1) or simultaneously outward (as at 2, 2). Two points (*N*, *N*) near the bend do not vibrate. They are called *nodes.* The bend (B) between the nodes rises and falls as the fork vibrates. So the stem executes an up-and-down motion.

A tuning fork has the special characteristic that it *gives out a sound of only one frequency.* A sound of single frequency is called a '*pure tone*'. Ordinarily any sound has several frequeucies in it at the same time. Because of its ability to produce a sound of a single frequency, the tuning fork is an essential equipment in a sound laboratory.

The frequency of a tuning fork can be *lowered* by loading one of its prongs. Some forks are provided with sliding weights called *collar* (c) (fig. III-4.1c). If the collar is fixed near a free end of the fork, the lowering of frequency is greater than if the weight is fixed nearer the nodes. Instead of sliding weights we can use one or more turns of thin or thick wire wound round a prong. Frequency can be slightly *raised* by filing off some material from near the end of the prongs. The frequency of a tuning fork is

$$n=(k/l)\sqrt{q/\rho} \qquad \text{(III-4.3.1.)}$$

where k is a const, l the length of the prong, q the Young's modulus and ρ the density of the material of the fork.

A standard tuning fork is mounted on a wooden box open at one end (fig III-1.4c). It is of such a size that the air mass inside, can resonate with the fork, its *frequency varying inversely as the prong length.*

If the mass of a prong is reduced, say, by filing, its frequency increases.

III-4.4. Mechanism of sound propagation. Let us now study the state of motion of air when sound is propagated through it. Consider

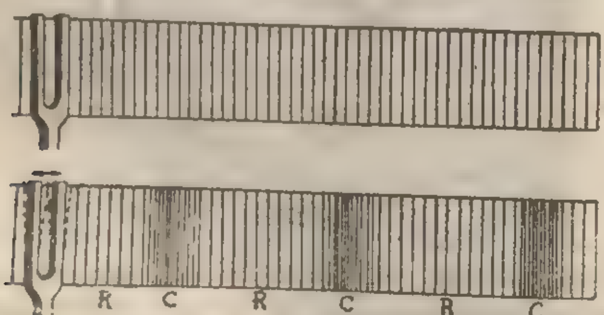


Fig. III-2(a)

a vibrating tuning fork emitting sound (fig. III-4.2a). *Imagine* the undisturbed layer in front of the prong to be divided into thin vertical

layers of equal thickness. As the arm moves outward it compresses the layer in front of it. This layer presses the next layer into a smaller volume. The second layer in its turn compresses the third layer and so on. Thus *the compression advances to the right from layer to layer with a definite velocity.* By the time the arm moves from the left end to the right end of its swing, the state of compression advances through a certain distance depending on the period of vibration of the fork and the density and elasticity of the medium.

As the prong starts swinging backwards to the left, it creates a partial vacuum behind it and the air layer in contact with it expands. The next layer being relieved of pressure also expands. The following layers follow suit one after another. Thus *a state of rarefaction is passed on from layer to layer travelling with the same velocity as that of the compression.* A compression together with the rarefaction following it, forms a complete longitudinal wave of sound. The particles vibrate simple harmonically to and fro since the vibrations of the fork are so. Actually you do not have air layers, you have air particles. Their

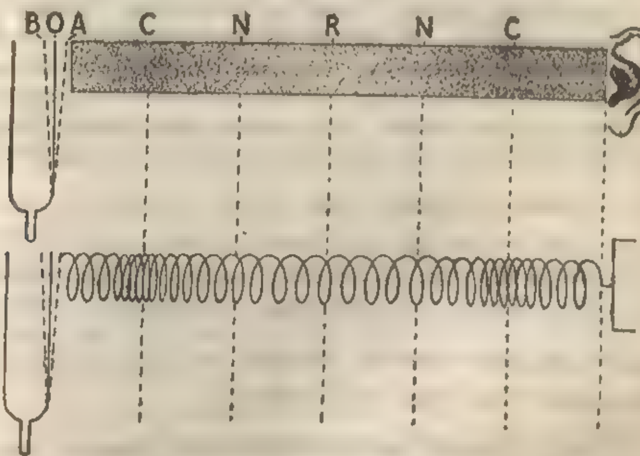


Fig. III-4.2(b)
[*C* = Compression : *R* = Rarefaction]

vibrations and a model provided by longitudinal vibrations of a spring is shown in fig. III-4.2b.

So long as the fork vibrates, alternate compressions and rarefactions occur in the surrounding medium. *The moving pattern of alternate*

compressions and rarefactions of the medium constitute sound waves. From a small source in a homogeneous medium the generated sound waves spread out spherically as shown in fig. III-4.3.

Fig. III-4.3

Particle of air. We clarify two ideas in this connexion. One is the word 'particle' when the medium is air. A particle of air does *not* mean a molecule of air, but a small volume, say 10^{-15} cm^3, of air. At N.T.P. it contains about 10^4 molecules. Air molecules in this volume may be supposed to have the same displacement from the mean position of the volume. In the displaced position, the volume will not contain exactly the molecules it had at the rest position. Many molecules will leave the volume due to their thermal motion, while new ones will enter it. Density over the volume however is supposed to remain unchanged. The same applies to liquids. In spite of their granular structure, *we treat gaseous and liquid media as continuous* so far as wave propagation is concerned.

Particle velocity and compression. The other is the relation between particle velocity and compression. As the right prong of the fork moves from its extreme left to its extreme right position, it gives the air particles a velocity in the direction of wave propagation. The particles disturbed in this half of the motion of the prong constitute a *compression.* Note that during the first half of the left-to-right motion of the right prong, the air particles in contact with it were to the left of their rest position. Hence particle displacements during this quarter of the fork's motion were opposite to the direction of propagation.

III-4.5. Velocity of sound. The velocity of sound in a medium depends on its elasticity and density. Newton proved that for longitudinal waves in a medium, the wave velocity c is given by $c = \sqrt{E/\rho}$, where E is the modulus of elasticity of the material and ρ its density.

In liquids and gases the relation reduces to $c = \sqrt{K/\rho}$ where K is the bulk modulus and ρ the density of the medium.

***A. Velocity of sound in a gaseous medium.** Consider a *very long* tube of unit cross 11 section fitted with a gas-tight but frictionless piston (P) (fig. III-4.4a)]. Let the gas be at rest under a pressure p and have density ρ. Push the piston *suddenly* with a constant velocity u along the tube from A to B. It will compress a layer of air directly in front of it and the compression will pass on to the succeeding layers with a speed equal to

*Derivation not in the syllabus.

that of the velocity of sound in a gas. Let the pressure and density in the compressed layer be p' and ρ' respectively. After a time t the piston has

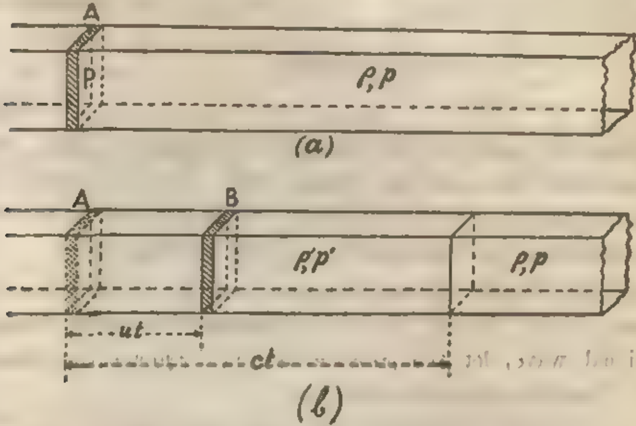


Fig. III-4.4

moved through a distance ut and the compression, through ct. So the layers that would have covered the length ct without compression, must be packed together within the length $(ct-ut)$ because of the compression (fig. III-4.4b). Since the mass of these layers must remain consant and we take their cross-sections to be unity, we have

$$(ct-ut)\,\rho' = ct\rho \quad\text{or}\quad (c-u)\,\rho' = c\rho \tag{III-4.5.1}$$

$$\text{or}\quad c(\rho'-\rho) = u\rho', \quad\text{or}\quad \rho'/(\rho'-\rho) = c/u. \tag{III-4.5.2}$$

Now the impulse given by the piston is equal to the increase of momentum of the layers. Impulse is *force* × *time* and force is provided by the *change in pressure* before and after the motion of the piston. The layers being originally at rest, we have

increase of momentum = *mass* × *velocity*

$$\therefore\quad (p'-p)\times t = [(ct-ut)\,\rho']u$$

$$\text{or}\quad p'-p = (c-u)\,\rho'.u \tag{III-4.5.3}$$

But volume elasticity K is given by

$$K = \frac{\text{increase in pressure}}{\text{reduction in volume per unit volume}}$$

$$= \frac{p'-p}{(V-V')/V} = (p'-p)/\left(\frac{m/\rho - m/\rho'}{m/\rho}\right)$$

$$= (p'-p)/\frac{p'-p}{p'} = (p'-p)\cdot\frac{p'}{p'-p}$$

$$= (c-u)\,\rho'.u.\frac{c}{u} \qquad \text{(from eq. III-4.5.3. and III-4.5.2)}$$

$$= c^2\rho \quad \text{(from eq. III-4-5.1)}$$

$$\therefore\ c = \sqrt{K/\rho}. \qquad \text{(III-4.5.4)}$$

B. Newton's equation for velocity of sound in a gas. Newton had deduced this relation on theoretical grounds. It can be easily shown that for a perfect gas $K=P$, the pressure of the gas, when the volume change is *isothermal* (i.e. takes place at a constant temperature).

Proof. Let V be the volume of a given mass of a gas in a layer when the pressure is P. As the layer is compressed due to the passage of a compressional wave, let the pressure rise to $(P+p)$ and the volume fall to $V-v$. Then from Boyle's law we have

$$(P+p)(V-v) = PV \quad \text{or} \quad PV-Pv+pV-pv = PV$$

As the product pv is very small, we neglect it and get $Pv=pV$,

$$\text{or} \quad P = \frac{p}{v/V} = \frac{\text{change of pressure in the layer}}{\text{its change of volume per unit volume}}$$

$$= \frac{\text{stress}}{\text{strain}} = K. \quad [\text{ Look up eqn. II-3.9.2 }]$$

In a perfect gas we should therefore have (*pressure changes* being assumed isothermal)

$$c_1 = \sqrt{P/\rho}. \qquad \text{(III-4.5.5)}$$

Application of Newton's formula to sound waves in open air. Newton's formula for the velocity of compressional waves in a perfect gas is applied to calculate the velocity of sound in open air. Assuming conditions of standard temperature and pressure, i.e., $T=273$K and $P=76\times13{\cdot}6\times980$ dyn/cm^2 and ρ at STP$=0{\cdot}001293$g/cm^3, we get

$$c=\sqrt{\frac{76\times13{\cdot}6\times980}{0{\cdot}001293}}\ \text{cm/s}=286\ \text{m/s}.$$

The experimental value, **331 m/s**, is much higher than this calculated value. Evidently there must have been some error in Newton's assumptions.

C. Laplace's correction to Newton's formula. Laplace argued that air is a bad conductor of heat and the volume changes in sound waves occur very quickly. Hence such volume changes cannot be isothermal.

A compressed layer must remain a bit hotter and a rarefied layer a bit cooler than normal. He considered the volume changes to be *adiabatic.* This means that heat cannot leave a compressed layer nor enter a rarefied layer so as to equalise their temperatures; this is due to (*i*) the bad conductivity of air and (*ii*) quickness of changes.

Under adiabatic conditions the pressure-volume changes of perfect gos is governed by the relation $PV^{\gamma}=\text{const}$ where γ is a constant for a given gas. Differentiating as in eqn II-3.9.3 we get

$$P.\gamma V^{\gamma-1}.dV+V^{\gamma}.dP=0$$

$$\text{or} \quad \gamma P.V^{\gamma-1}.dV=-V^{\gamma}.dP$$

$$\text{or} \quad \gamma P=-V\frac{dP}{dV}=K$$

$$\therefore \quad c=\sqrt{K/\rho}=\sqrt{\gamma P/\rho} \qquad \text{(III-4.5.6)}$$

Under adiabatic conditions the bulk modulus of a gas is thus γP where γ is a constant characteristic for a gas. For air $\gamma=1{\cdot}41$. Laplace found that when we take $c=\sqrt{1{\cdot}41\ P/\rho}$, the value agrees very well with the experimental value. So, in a gas the sound velocity $c=\sqrt{\gamma P/\rho}$.

γ of a gas is the ratio of its molar specific heats at constant pressure (C_p) to that at constant volume (C_v) i.e $\gamma=C_p/C_v$.

For monatomic gases like A or He, $\gamma=\frac{5}{3}$ or $1{\cdot}66$ while for diatomic gases it is $\sqrt{2}$ or 1.41. Air being mainly a mixture of O_2 and N_2, its γ value can be taken as $\sqrt{2}$ so that velocity of sound at $N.T.P$ becomes

$$c_a=\sqrt{2}.c_i=332 \text{ m/s}$$

which is very close to the experimental value for velocity of sound, $331{\cdot}4$ m/s. Thus Laplace's assumption stands justified.

Stokes has shown from thermodynamical considerations that the pressure change must be either very nearly isothermal or very nearly adiabatic and nothing else; otherwise, *attenuation* would be very high. The fact that sound waves travel over large distances before fading out, therefore, precludes a type of change midway between or appreciably different from adiabatic and isothermal.

III-4.6. Factors which affect the velocity of sound in air. All factors which affect the *ratio* of pressure to density of air also affect the velocity of sound in air.

(i) Effect of pressure. So long as temperature of air is constant a change of pressure alone does not change the velocity, because at constant temperature P/ρ is constant, density being proportional to pressure. *Sound velocity in a gas is thus independent of pressure alone.*

(ii) Effect of temperature. Density of air changes with temperature according to Charles' Law. If ρ_0 and ρ_t are the densities of a gas at 0°C and t°C respectively, then according to Charles' Law,

$$\rho_0=\rho_t(1+\alpha_p t)$$

where α_p is the coefficient of volume expansion of air at constant pressure per °C and is very nearly equal to 1/273 for air. If c_0 and c_t are the velocities of sound in air at 0°C and t°C respectively, then

$$c_0=\sqrt{\gamma P/\rho_0} \text{ and } c_t=\sqrt{\gamma P/\rho_t}$$

$$\therefore \quad \frac{c_t}{c_0}=\sqrt{\rho_0/\rho_t}=\sqrt{1+\alpha t}=\sqrt{\frac{273+t}{273}}=\sqrt{\frac{T}{T_0}} \qquad \text{(III-4.6.1)}$$

in which T is the absolute temperature corresponding to t°C and T_0 that to 0°C.

Thus *sound velocity in a perfect gas is proportional to the square root of its absolute temperature.*

When the change of temperature is small we have, from eq. III-4.6.1

$$c_t/c_0 = \sqrt{1+\alpha_p t} - 1+\tfrac{1}{2}\alpha_p t \text{ or } c_t=c_0(1+\tfrac{1}{2}\alpha_p t) \qquad \text{(III-4.6.2)}$$

Now. $\alpha_p=1/273$ per °C nearly. Taking $c_0=331$ m/s, we find that the increase in the velocity of sound per °C rise in temperature around 0°C is $\frac{1}{2}\ c_0\times\frac{1}{273}\times 331$ m/s $=61$ cm/s. *The velocity of sound in air increases by about 0·61 m/s for every 1°C rise in temperature around 0°C.*

(iii) Effect of simultaneous changes of pressure and temperature. Let P_1, t_1, ρ_1 be respectively the pressure, celsius temperature and density of air initially. Let the final values be P_2, t_2 and ρ_2.

Then t_1 °C $=(T_0+t_1)$ K $=T_1$K and t_2°C $=(T_0+t_2)$ K $=T_2$K.

From the perfect gas, laws, we have

$$\frac{P_1}{\rho_1 T_1}=\frac{P_2}{\rho_2 T_2} \text{ or } \frac{c_1^2}{T_1}=\frac{c_2^2}{T_2} \text{ or } \frac{c_1}{c_2}=\sqrt{\frac{T_1}{T_2}} \qquad \text{(III-4.6.3)}$$

Therefore, *the velocity of sound in a perfect gas will be propotional to the square root of absolute temperature when both pressure and temperature change simultaneously.*

(iv) Effect of humidity. Water vapour is lighter than air and has a density of 5/8 relative to air. Hence the density of air diminishes as its humidity, i.e. *moisture content*, increases. Sound, therefore moves faster in humid air than in dry air.

(v) **Effect of wind.** The velocity of sound in air means the velocity with which sound waves move *relative to air.* If the air itself is in motion then to a stationary observer, the velocity of sound will be the resultant of the two velocity vectors. *Air in motion is wind.* Hence if a wind blows in the same direction as the sound, the velocity of sound will be the sum of the two velocities. Wind velocity does not affect the velocity of sound relative to air. *It is the velocity relative to the earth that changes.*

Over a very wide range, the velocity of sound in air is not affected by frequency or intensity of its waves.

(vi) **Velocity of sound in other gases.** For simplicity we can treat other gases like hydrogen, oxygen, nitrogen, etc. as perfect. They will therefore, be similarly discussed as for air above. Where γ *is the same for two gases*, the velocities in them under the same conditions of pressure and temperature are inversely proportional to the square roots of their densities. Since the density of a gas is proportional to its molecular weight, we may also say that the *velocities in two gases of the same γ and under identical conditions, are inversely proportional to the square roots of their molecular weights.*

An Alternative Method for Considering Factors affecting Velocity of Sound. We know that the equation of state for an ideal gas of which a *mole* is considered, is

$PV=R_0T$ and $V=M/\rho$ where M is the molecular wt.

Then we have $PM/\rho=R_0T$ and $\gamma P/\rho=\gamma R_0T/M$

$$\therefore \quad \text{Velocity of sound } c=\sqrt{\gamma P/\rho}=\sqrt{\gamma R_0T/M} \qquad \text{(III-4.6.4)}$$

This relation tells us that the velocity of sound through a gas is

(i) independent of pressure, for P is absent from the expression
(ii) varies directly as the square root of absolute temp. T
(iii) varies inversely as the square root of molecular wt. M i.e. ρ.
(iv) varies as the square root of γ which depends on the number of atoms in the gas molecule.

Example. III-4.1. *An observer sets his watch by the sound of gunfire from a distant tower. His watch is found to run 2 s slow. If the temp of air be 15°C and velocity of sound at 0°C is 332 m/s find the distance of the tower.* [**Pat. U. Gau. U.**]

Solution : Velocity of sound increases by 61 cm/sec for 1°C rise in temperature. Hence the velocity of sound in air at 15°C is $c=332+0.61\times15$ $=341.55$ metres/sec.

Since the observer finds his watch too slow by 2 seconds, sound took 2 sec. in travelling from the tower to the observer and covered in that time a distance equal to $2 \times 341.15 = 682.3$ metres.

Ex. III-4-2. *Find the wavelength in air at 30°C corresponding to 512 c.p.s when the velocity of sound at 0°C is 332 metres/sec and coefficient of volume expansion of air is 0.00366.* **[And. U.]**

Solution : Velocity of sound in air at 30°C is given by

$$c = c_0(1+\tfrac{1}{2}\alpha t) = 332(1+\tfrac{1}{2}\times 0.00366\times 30)$$
$$= 332\times 1.0549 \text{ metres/sec.}$$

$$\therefore \text{ Wavelength } \lambda = c/n = 332\times 1.0549/512$$
$$= 68.4 \text{ cm.}$$

Ex. III-4.3. *Find the barometric pressure when the velocity of sound in air is 340 m/s and density of air is 1.22×10^{-3} g/c.c. given that $\gamma = 1.41$.* **[U. P. B.]**

Solution : We know that $c = \sqrt{\gamma P/\rho}$. $\therefore$ $P = c^2\rho/\gamma = H\rho' g$

where H is the barometric height and ρ' the density of mercury.

$$\therefore H = c^2\rho/\gamma\rho' g$$
$$= (340\times 100)^2 \times 1.12\times 10^{-3}/(1.41\times 13.6\times 981) = 74.99 \text{ cm.}$$

Ex. III-4.4 *Calculate the velocity of sound in air at 100°C if the density of air is 0.001293 gm/c.c. and density of mercury 13.6 gm/cc. both at 0°C, specific heats at constant pressure and constant volume for air are 0.2417 and 0.1715 respectively.* **[Lond. H. S. C.]**

Solution : The required velocity at 100°C is

$$c_{100} = c_0\sqrt{\frac{T'}{T}} = \sqrt{\frac{\gamma P}{\rho}}\cdot\sqrt{\frac{T'}{T}}$$

where P is the *normal atmospheric pressure*, ρ the density of air at 0°C and γ the ratio of its specific heats. Now

$$P = h\rho g = 76\times 13.6\times 981 \text{ and } \gamma = 0.2417/0.1715.$$

$$\therefore c_{100} = \sqrt{\frac{0.2417}{0.1715}\cdot\frac{76\times 13.6\times 981}{0.001293}}\times\sqrt{\frac{373}{273}}$$
$$= 383.6 \text{ metres/sec.}$$

Ex. III-4.5. *The planet Jupiter has an atmosphere composed mainly of methane (CH_4) at a temperature of −130°C. Find the velocity of sound on the planet assuming γ for the gas to be 1.3. ($R = 8.3$. joules/°C/gm. molecule).* **[Oxf.. U.]**

Solution : Eq. III-4-6.4 shows that velocity of sound is given by $c = \sqrt{\gamma RT/M}$

Now from our data $\gamma = 1.3$. $R = 8.3\times 10^7$ ergs/°C/gm.-molecule, M for $CH_4 = 12+4 = 16$ and $T = -130+273 = 143$K

$$\therefore c = \sqrt{\frac{1.3\times 8.3\times 10^7\times 143}{16}} = 310.6 \text{ metres/sec.}$$

Problems : (1) If the pressure in a vessel containing hydrogen be equal to that due to 76 cm of mercury, find what will be the velocity of sound through hydrogen, given that its density under the given condition is 0.09 g/litre. Take $\gamma = 1{\cdot}41$ and $g = 981$ cm/s^2. (*Ans.* $1{\cdot}26\times 10^5$ cm/s)

(2) At 30°C the velocity of sound in air is 350 m/s. What is the value at 0°C? (*Ans.* 331·8 m/s)

(3) If the velocity of sound in open air at STP be 330 m/s, what will be its value at 50°C and 70 cm/Hg of pressure?

(*Ans.* 360·2 applying Eq III-4.6.2. It is better to use the previous Eq.)

III-4.7. Velocity of sound in liquids. When sound travels through a liquid the waves are still longitudinal. Hence $c=\sqrt{K/\rho}$ for velocity of sound still holds. The compression of the liquid is so small that it is immaterial whether the changes in the medium are taken as isothermal or adiabatic. Though strictly speaking, adiabatic volume elasticity should be considered, experiments yield its isothermal value. For water at least, the two are almost equal. Thus velocity of longitudinal waves in water is given by the square root of the ratio of isothermal elasticity to density i.e. $\sqrt{K/\rho}$.

An accurate knowledge of the speed of sound in sea-water is required in the calculation of its depth by the *echo depth-sounding* method.

Velocity of sound through sea-water can be determined easily by the *radio-acoustic* method where two ships take part. From one, a radio signal is sent out and simultaneously an under-water depth charge detonated. The two signals are received by the other ship at a known distance away. The reception of wireless signal marks the start of the sound waves and the method is exactly analogous to the signal method of finding the velocity of sound on land.

Underwater signalling experiments have given the following results for sea-water :—

(1) The velocity near the surface at temperature t°C is $c=1445.5+3.92t-0.024t^2$ m/s for salinity 3.5% by weight. For 1% increase in salinity the velocity increases by about 13 m/s.

(2) Loss of intensity with distance, i.e. *attenuation*, is small and is due mainly to non-homogeneity in water caused by changes of temperature and salinity. That is why sounds under water can be heard at very great distances. For this reason lightships and buoys are provided with massive underwater bronze bells, whose sounds are picked up by the ships' hydrophones far far away.

(3) The range of sound is much greater in winter than in summer.

For solids in the form of rods and bars the velocity of sound is

$c=\sqrt{Y/\rho}$ where Y is the Young's modulus of the material

and $c=\sqrt{(K+\frac{4}{3}n)/\rho}$ for extended solids where K and n are bulk and rigididity moduli of the material respectively.

Velocity of sound in other media. Velocity of sound in solids is in general higher than that in liquids, and that in liquids higher than that in gases. Some values are given in the table below.

Velocity of sound in different media

Gases at S. T. P.		*Solid rods and water*	
	metres/sec.		metres/sec.
Air	331.5	Copper	3790
Hydrogen	1286	Steel	5150
Oxygen	314.8	Glass (crown)	4710—5300
Sulphur dioxide	211	Brass	3130—3450
Carbon dioxide	260.3	Water (25°C)	1497
Ammonia	415	Sea-water (18°C)	1516

Ex. III-4-6. *An explosive on a rail is detonated by the passage of a train passing over it. A listener one kilometre away with one ear on the rail hears two reports. Explain the phenomenon and calculate the time interval between the two reports. Given, Young's modulus E for steel* $= 2 \times 10^{12}$ *dyn/cm*^{2}, ρ *of steel* $= 7.8$ *g/cm*^{3}, ρ *of air* $= 0.0013$ *g/cm*^{3}, γ *for air* $= 1.4$ *and atmospheric pressure* $= 10^6$ *dyn/cm*^{2}. **[Oxf., Gen. U]**

Solution: With one ear on the rail, the listener receives the sound of the signal through the rail and with the other ear in air he hears the sound coming through the air. Velocity in the two media being different two reports are heard.

Velocity of sound through the rail,

$$c' = \sqrt{E/\rho} = \sqrt{2 \times 10^{12}/7.8} = 5064 \text{ m/s}.$$

$\therefore$ Time required for this sound to travel through 1 km of steel is $t' = 10^3/5064 = 0.2$ s.

Velocity of sound through air,

$$c = \sqrt{\gamma P/\rho} = \sqrt{1.4 \times 10^6/0.0013} = 328.4 \text{ m/s}.$$

$\therefore$ Time required for sound to travel through 1 km of air is $t = 10^3/328.4 = 3.0$ s. Hence the time-interval $= (t - t') = 2.8$ s.

III.4.8. Doppler effect. It is an effect associated with waves when the source or the receiver is moving. When any one or both are in motion, the *apparent* frequency *heard* is different from that emitted by the source. The effects upon the apparent frequency of a wave is produced (i) by the motion of the source towards or away from an observer at rest, and (ii) by the motion of the observer towards or away from a stationary source, are known as *Doppler Effect*.

Broadly speaking, we may say that the ***Doppler effect*** *is the apparent change in frequency as perceived by an observer when there is relative*

motion between the observer and a source emitting periodic waves, the motion occurring along the line joining the source and the observer.

The whistle of a railway engine or the sound emitted by a moving aeroplane appears to have a higher frequency as it approaches an observer at rest. As the source moves away from the observer, the frequency appears to be reduced. When a moving observer approaches a source of waves, the apparent frequency increases. When the observer moves away from the source, the apparent frequency falls. These are all cases of Doppler effect. The effect is common to all wave motions including sound and light. For the effect to be appreciable the *velocity of the source or observer must be an appreciable fraction of the wave velocity.* In calculations let c stand for wave velocity.

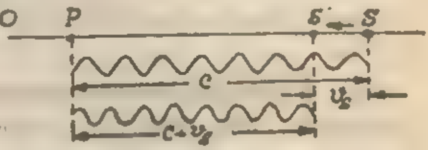


Fig. III-4.5 (a)

A. (i) Source moving : Observer at rest. Let the source S be moving *towards* the observer with velocity v_s. The n waves emitted by it in one second will be contained in a distance $c-v_s$ (fig. III-4.5a) and will have a wavelength $\lambda' = (c-v_s)/n$. The observer will take them as waves of frequency $n' = c/\lambda' = nc/(c-v_s)$. The change in frequency is

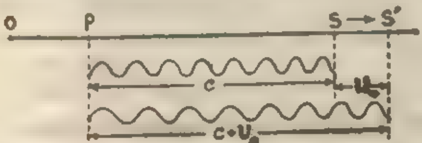


Fig. III-4.5(b)

$$n'-n = nv_s/(c-v_s). \qquad \text{(III-4.8.1)}$$

If the source is moving *away* from the observer, the sign of v_s will be negative. In that case we shall have [fig. III-4.5(b)]

$$n' = nc/(c+v_s) \text{ and } n-n' = nv_s/(c+v_s). \qquad \text{(III-4.8.2.)}$$

(ii) Observer moving ; source at rest. When the observer moves towards the source with a velocity v_o (fig. III-4.6) he receives in one

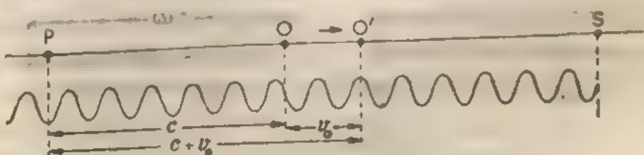


Fig. III-4.6

second all waves contained in a distance $c+v_o$. This number is

$n'=(c+v_o)/\lambda=(c+v_o)/(c/n)$ as n waves are contained in a distance c. Hence the apparent frequency is $n'=n(c+v)/c$. (III-4.8.3.)

It is greater than n.

If the observer is moving away from the source with velocity v_o, the sign of v_o in the above relation will be negative. Thus

$$n''=n(c-v_o)/c. \qquad \text{(III-4.8.4.)}$$

It is less than n.

Remember that in all these cases n' is the frequency as it appears to the observer.

Problem. For the same relative velocity of approach between a source of sound and an observer, show that the rise in frequency is greater when the source is moving.

(iii) Source and observer both moving. Let the source *approach* the observer with velocity v_s, while the observer at the same time approaches the source with velocity v_o. Then waves of wavelength $(c-v_s)/n$ approach the observer. Then the waves received by the observer in one second are contained in a distance $c+v_o$. Since these waves have the wavelength $(c-v_s)/n$, the number contained in the distance $c+v_o$ is given by

$$n'=\frac{c+v_o}{(c-v_s)/n}=n.\frac{c+v_o}{c-v_s} \qquad \text{(III-4.8.5.)}$$

This is the apparent frequency perceived by the observer. If either the source or the observer move *away* from the other, the sign of its velocity in the above relation will have to be accordingly reversed.

(iv) Effect of wind. If there is a wind of velocity w in the direction of c, c is effectively increased to $c+w$. This applies to all of the last three equations. Thus from eq. III-4-8.5., we shall then have

$$n'=n\frac{c+w-v_o}{c+w-v_s} \qquad \text{(III-4.8.6.)}$$

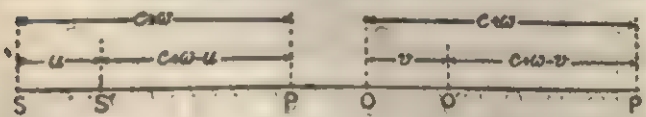


Fig. III-4.7

The equation may be taken as the most general one in Doppler effect when *all motions are in the same line.* With proper considerations

of signs of v_s and v_o and with $w=0$ (medium not moving), it reduces to the appropriate relations stated earlier. You must note carefully the sign of v_s and v_o.

Ex. III-47. *A motor car approaches and passes a stationary observer with a speed of 20 mph. If the horn emits a frequency of 300 Hz, what will be the (i) absolute change in frequency, (ii) percentage change in frequency as the car passes the observer? (Speed of sound = 720 mph.)*

Will the percentage change alter if the horn had some other frequency?

Solution: The apparent frequency n' at approach is $n'=300\times(720/700)=308{\cdot}7$. That at recession is given by $n''=300\times(720/740)=291{\cdot}9$. Absolute change in frequency $=308.7-291{\cdot}9=16{\cdot}8$Hz. Percentage change $=16{\cdot}8\div300)\times100=5{\cdot}6$. It will not alter so long as the speed remains the same.

B. Doppler Effect in Light. This is common to all types of wave motion. But for demonstration, the relative motion between the source and the observer should be a sizeable fraction of the wave velocity. So Doppler effect is easily demonstrable in sound. Since light has a very high speed, its Doppler effect is much more difficult to detect, because the relative motion between the light source and the observer is very small. Stellar bodies moving towards or away from us do show Doppler effect. The wavelength of light from receding bodies, when analysed with a spectroscope shows a shift towards the red end of the spectrum.

When the spectra of light from the eastern and western edges of the sun are separately examined, it is found that Fraunhofer lines are displaced towards the red end in one case and towards the violet in the other. *A shift towards the red* means an increase in wavelength and hence a lowering of frequency. Lowering of frequency is explained if we assume that the source is moving away (receding) from us. *Violet shift* means that the source is moving towards the observer. The red and violet shifts of spectral lines from the two edges of the sun indicate that the sun is rotating about a north-south axis. (Its period of rotation has been determined in this way.)

Spectral lines of light from very distant galaxies show a *red shift*. From the amount of shift one can calculate the speed with which such a galaxy is moving away from us. Accurate measurements gave the very important and astonishing result that *distant galaxies are all moving away from us with a speed proportional to their distances from us*. This observation gave rise to the theory of the *expanding universe* in cosmology. (Cosmology is the science of the nature, origin and history of the *universe*.)

C. General Case. When the motion between the source and the observer is not along the line joining the two, but inclined to it, the Doppler effect will be due to the component of the motion along the line. The perpendicular component does not contribute to the Doppler effect. Only the axial component is effective.

In such a case the frequency changes continuously. Let the source be moving along AB (fig-III-4.8) with a velocity u while the observer remains stationary at O. At S_1, the component velocity along S_1O is $u \cos\theta$ and is towards O. The observer therefore hears a note of higher pitch whose frequency is given by

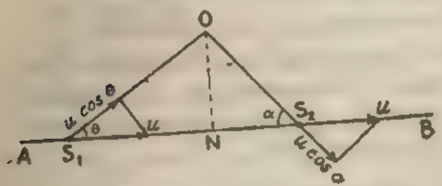


Fig. III-4.8

$$n' = n \frac{c}{c - u\cos\theta} \qquad \text{(III-4.8.7a)}$$

where c is the velocity of sound and n the frequency of the source. When the source is at S_2, the component of u acts along OS_2 and away from O, with a magnitude $u \cos\alpha$. So now the frequency becomes lower and is given by

$$n'' = n \cdot \frac{c}{c + u\cos\alpha} \qquad \text{(III-4.8.7b)}$$

At N, both θ and α are 90° ; so the components vanish and the observer hears a note of the same frequency as is emitted by the source.

Ex. III-4.8. *A train passes a railway station with a velocity of 40 m.p.h. continuously whistling at a steady frequency of 256 per sec. Find the frequency apparent to a stationary observer on the platform (a) when the train is approaching, (b) when it is receding. Velocity of sound = 1120 ft./sec.* [C. U.]

Solution : 40 m. p. h. $= \frac{176}{3}$ ft./sec.

(a) Since the source approaches the stationary observer, the apparent frequency is by eqn III-4.8.1

$$n' = n\frac{c}{c-u} = 256 \times \frac{1120 \times 3}{3360 - 176} = 256 \times \frac{1120 \times 3}{3184} = 270.1 \text{ per sec.}$$

where u is the velocity of the source.

(b) Since the source is receding, the apparent frequency, is

$$n'' = \frac{nc}{c+u} = 256 \times \frac{1120 \times 3}{3536} = 243.2 \text{ per sec.}$$

Ex. III-4.9. *A spectroscopic examination of light from a certain star shows that the apparent wavelength of a certain spectral line is 5001 Å, whereas the observed wavelength on earth for the same line is 5000 Å. Find the direction and speed of movement of the star in relation to the earth.* (C. U.)

Solution : Since the apparent wavelength is larger, i.e., the apparent frequency is smaller, it is a case of recession, i.e., the star must be moving away from the earth. Let c be the velocity of light, λ the true and λ' the apparent wavelengths, and n and n' corresponding frequencies. Then

$$\frac{5000}{5001} = \frac{\lambda}{\lambda'} = \frac{n'}{n} = 1 - \frac{v}{c}.$$

$$\therefore \quad \frac{v}{c} = \frac{1}{5001} \text{ or } v = c/5000 \text{ nearly} = 1.5 \times 10^{5} \text{ m/s}$$

Ex. III-4.10. *An engine approaches a bridge at 5 ft./sec. while sounding a whistle of 500 c.p.s. The sound is reflected from the bridge. Find the frequency of the beats heard by an observer seated in the engine if the velocity of sound be 1100 ft.*

Solution **:** The observer is receiving two notes, one directly from the engine, the other after its reflection from the bridge. The frequency of the sound received direct is unchanged.

To find the frequency of the reflected sound, the simplest plan is to consider the sound "image" of the whistle as the source. Since the bridge behaves as a plane reflector, this "image" is formed as far behind the reflector as the source is in front of it, the line joining them being perpendicular to the reflector. Since the reflector is fixed the image moves in a direction opposite to that of the source with equal velocity. Hence the "image" of the whistle approaches the observer with a velocity of 2×5 ft./sec. For this relative velocity of approach the apparent frequency $n' = n(1 + V/c) = 500(1 + 10/1100)$. This beats with the source frequency of 500. Hence the number of beats $= 500 \times 10/1100 = 4.6$ per sec.

Ex. III-4.11. *Two observers A and B both have sources of sound of frequency 500. If A remains stationary while B moves away with a velocity of 6 ft/sec find the number of beats heard by A and B. (Vel. of sound = 1100 ft./sec.).*

Solution : Beats heard by A : In this case the observer (A) is stationary and the source (B) is moving away. Hence by eq. III-4.8.2

$$n' = n\frac{c}{c+u} = 500 \times \frac{1100}{1106} = 497.29.$$

$\therefore$ Frequency of beats/sec $= 500 - 497.29 = 2.71$.

Beats heard by B : Here the source (A) is stationary and the observer (B) is moving away. Hence

$$n' = n\frac{c-u}{c} = 500 \times \frac{1094}{1100} = 497.27$$

$\therefore$ Beats heard by $B = 500 - 497.27 = 2.73$/sec.

III-5

REFLECTION
REFRACTION
DIFFRACTION

III-5.1. Impact of sound waves on surfaces of discontinuity and obstacles. When sound waves travelling in a homogeneous medium fall on a surface separating two media, (*i*) a part of the wave-train is thrown back into the original medium in the same form and travels with the same velocity while (*ii*) the rest passes into the second medium with a change in velocity and *generally* also with a change in direction. The first phenomenon is known as **reflection** and the second as **refraction**. The same thing happens at a surface separating regions of different densities inside the same medium.

When sound waves meet a rigid obstacle, their behaviour is determined by the size of the obstacle relative to the wavelength. Three cases may arise :

(*i*) When the obstacle is very large compared with the wavelength, the waves are **reflected** back into the medium with the formation of a shadow region behind the obstacle.

(*ii*) As the object diminishes in size and becomes comparable with the wavelength, the waves bend round its corners increasingly and encroach into the shadow appreciably. The phenomenon is called **diffraction**. In sound it is quite noticeable but not in light.

(*iii*) When the obstacle is smaller than the wavelength, it serves as a source of new waves spreading out in all directions. This is known as **scattering**. Small inhomogeneities in a medium also cause scattering. It is more noticeable in light causing brilliant reddish sunrise and sunset.

III-5.2. Reflection. Like all waves, sound waves are reflected under proper conditions. Reflection of sound plays an important part in a number of phenomena. Echoes, rolling of thunder, reverberations in auditoria etc., speaking tubes, stethoscopes, and wind pipes provide examples of reflection of sound.

For regular reflection, the dimensions of a surface must be large and its irregularities small compared with the wavelength. Since sound waves are long we require large reflectors to produce regular reflection. A big

enough brick wall can reflect sound but not light, because its irregularities are small compared with the wavelengths of sound but quite large for those of light. Light is reflected diffusely from reflectors that reflect sound regularly. *Ultrasonics* are reflected easily from small reflectors.

A surface of limited dimensions does not reflect all wavelengths equally. Shorter waves are more strongly reflected.

The geometrical laws of reflection of sound waves are the same as those of light waves. But this is not ordinarily appreciated because the sound waves are long and it requires large reflectors to reflect sound in the same way as light. Sound is reflected from walls, hills, rows of trees etc. The most familiar example of the reflection of sound is the *echo* (§III-5.3).

Formerly, big halls were sometimes provided with large parabolic sound reflectors, called **sound boards**, behind the dias. A speaker standing there was at the focus of the reflector. The sound rays were rendered parallel by reflection at the board. So they could travel over large distances before dying out. In churches, concert halls and show houses the back walls and the ceilings are suitably curved so as to concentrate sound by reflections. One such, stood in the vanished Senate Hall of our Calcutta University.

Other applications of sound reflection. To transmit sound over short distances without much loss of intensity a **speaking tube** may be used. Such tubes were generally installed in large buildings and steamships. Essentially, a speaking tube is a long metal pipe of small diameter with a funnel shaped end called the mouthpiece. It collects the sound waves, which pass into the tube. They cannot spread out but travel down the tube, being *successively* reflected at the walls of the tube. (fig. III 5.1.). Finally they leave the tube at the other end. The intensity of sound is thus maintained without much loss.

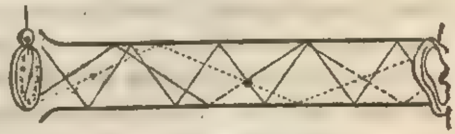


Fig. III-5.1.

Physician's **stethoscope** is an illustration of this principle. A little beyond the collecting funnel the tube bifurcates into two branches which separately go to the two ears.

III-5.3. Echoes. A sound heard by reflection and clearly distinguished from the original, is called an **echo**. For its formation, an echo requires a suitable reflector such as an extended wall, a cliff etc., whose

irregularities are small compared with the wavelength of the sound. To hear the sound and its echo *separately* the reflector must be at a sufficient distance, which may be calculated as follows :

A sensation of sound persists in the brain for almost 1/10th of a second after it is heard. Hence to recognise a sound and its echo as separate, *at least* this interval must pass between them. In our country, the velocity of sound under ordinary conditions (a mean temperature of 30°C and a fair amount of moisture in the air), is about 350 m/s. In 1/10th of a second sound travels 35 m in the air. Thus to hear distinctly the echo of a sound of *short duration*, the distance between the source and the reflector must be at least 17.5 m. The source and listener are supposed to be close together.

The rumbling and rolling of thunder is due to the echoing of a peal of thunder from a number of reflecting surfaces such as clouds, rocks, mountain sides, forests, surface of separation between air currents, etc. The continuous rumbling is due to the fact that echoes reflected from different sources enter the ear at intervals of less than 1/10th of a second.

In big halls it is often noticed that a continuous *rolling* of sound persists for some time after a loud sound has ceased. This is known as **reverberation** and is due to multiple reflections from the walls. At every reflection there is loss of some energy due to absorption. If absorption at the walls is small, the sound waves last long enough before they become inaudible, and give rise to the **reverberation.** Open windows are very effective in reducing reverberation, since sound waves incident on them pass into the outer atmosphere and are completely lost to the room. It is to prevent these reverberations and increase the absorption of undesired sounds that large halls and cinema houses are carpeted and part of the walls thickly cushioned with soft, sound-absorbing materials.

Echoes are used by whalers in Arctic waters to locate an iceberg (or land) in a dense fog at night. For this purpose a whistle is sounded or a short sharp sound made. By timing the interval between the signal and the receipt of the echo an iceberg may be roughly located and the boat steered clear of it.

If a sharp sound is made near a flight of steps, the reflections from adjacent steps will reach the listener in a regular succession. This may be fast enough to produce a musical note, the *echelon* echo.

Ex. III-5.1. *A ship A is at anchor at a distance of 1000 yds from a vertical cliff. Another ship B is in between A and the cliff. When B gives a short blast on its*

siren the sound is heard twice at A at an interval of 3 secs. Find the distance between the ships if sound travels 1100 ft/sec. [**Lond. U.**]

Solution : The first sound heard at A comes directly from B. The second one is the echo from the cliff of the siren blast at B. If x ft. be the seperation between the two ships, the distance of B from the cliff is $(3000-x)$ ft. At A, the first sound is heard $x/1100$ s after the siren sounds on B. The second sound is heard at A after the sound travels from B to the cliff and back to A. Hence at A it is heard $[2(3000-x)+x]/1100$ secs after the blast. Then from the given data

$$\frac{2\,(3000-x)+x}{1100}-\frac{x}{1100}=3 \text{ secs.}$$

$$\therefore\quad \frac{6000-2x+x-x}{1100}=3 \text{ secs or } x=1350 \text{ ft.}$$

Ex. III-5.2. *A man standing between two parallel hills fires a gun and hears two echoes, one $2\frac{1}{2}$ seconds and the other $3\frac{1}{2}$ seconds after the firing. If the velocity of sound is 330 metres/sec, find the distance between the two hills. How long will it take him to hear the third echo?*

Solution : The first echo is received after $2\frac{1}{2}$ seconds. Hence the sound has been reflected $1\frac{1}{4}$ seconds after the firing. In that time the sound travels $330\times1\frac{1}{4}=412\text{·}5$ metres. Hence the distance of one of the hills is 412·5 metres. Similarly the distance of the other hill is $330\times3\frac{1}{2}\times\frac{1}{2}=577\text{·}5$ metres. So the seperation of the two hills is $412\text{·}5+577\text{·}5=990$ metres.

The first and the second echoes will again be re-echoed from the second and the first hills respectively. For this the sound must altogether travel the distance between the hills twice. Hence the third echo will be heard $2\times990\div330=6$ seconds after the firing. Note that this echo is formed by one reflection at each of the two hills.

III-5.4 Echo depth-sounding.

Depths in the sea are often measured by timing the reflection of a sound from the sea-bed. There are many compact devices used for the purpose. Though they differ in details they employ the same principle, which is as follows : A sharp sound, now-a-days, generally a *strong pulse of ultrasonic waves*, is produced near a hydrophone I (a device similar to a microphone, fig III-5.2) submerged in water. The hydrophone responds twice to the sound, once to the original one and again to the waves reflected from the sea-bed. The time interval between the two events is

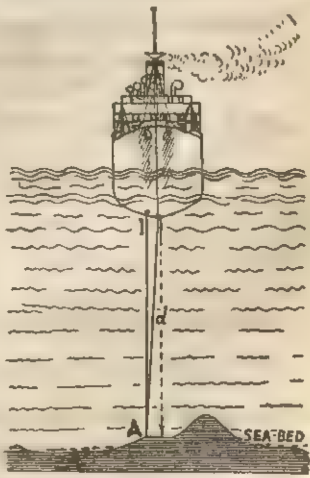


Fig III-5.2.

automaticaly recorded on a device which also measures short time intervals accurately. If the time between the two responses on the hydrophone is t and the velocity of sound in sea-water is c, then the depth at that place is $d=\frac{1}{2}ct$.

Reflection of **ultrasonic pulses** from a near or remote surface of discontinuity has found many important uses. Depth sounding, location of wrecks in seas, shoals of fish and submarines are some of the other ones. The recent applications are in the field of medicine for diagnosis. For this purpose ultrasonics are much better than X-rays. They can present the movement of the heart on a screen, or the movement of a child in the mother's womb. Detection of brain tumours has become an easy matter using an ultrasonic device. Detection of flaws in metal castings is easier with ultrasonic pulses than with X-rays. The records are said to be *ultrasonographs*.

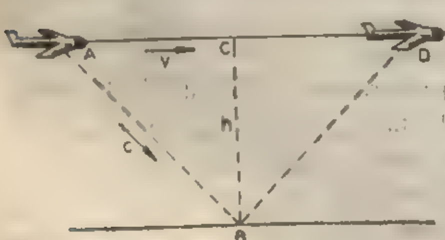
Ex. III-5.3. *An observer at a certain distance from a cliff notes that the interval between the sound he makes and its echo is 3 secs. He walks 550 ft. nearer and the corresponding interval reduces to 2 secs. Find (a) the velocity of sound, (b) the observer's original distance from the cliff.* [Lond. U.]

Solution : Let the distance between the cliff and the observer be x ft. and the velocity of sound c ft/sec. Then

$$2x/c = 3 \text{ secs, and } 2(x-550)/c = 2 \text{ secs.}$$

Solving for x and c we get $x = 1100$ ft/sec. and $x=1650$ ft.

Ex. III-5.4. *An echo completely repeats a word of six syllables. Find the minimum distance of the reflector if sound travels 1120 ft./sec.* [Gau. U.]



Solution : A monosyllable requires 1/5th of a second to utter and its impression to die. For a six-syllable word the corresponding time is 6/5 secs. During this interval sound travels 1344 ft. Hence the reflector must be at half that distance, i.e. 672 ft.

Ex. III-5.5. *A pilot while flying parallel to the ground with a velocity v fires a pistol and receives the sound reflected from the ground hears ts later. Find his height from the ground.*

Solution : The paths of the sound is shown in the above figure. Clearly the height h is $\sqrt{AB^2 - AC^2}$. Now $AB = ct/2$ and $AC = v.t/2$

$$\therefore\ h = \tfrac{1}{2}t\,(c^2 - v^2)^{\frac{1}{2}}$$

Problems: (1) A pilot flying horizontally at 120 m.p.h. fires a gun and hears the echo 3s later. Find the height of the plane, given the velocity of sound = 1120 ft/s. (*Ans.* 1659 ft) [H. S. '81]

(2) A person 112 ft. from a vertical wall uttered 6 syllables in succession. Will he hear the echoes of all syllables. If not, which one will it be ? c = 1120 ft/s

(*Ans.* Last syllable) [J. E. E. '73]

(3) A drummer standing in front of a large hill finds that he does not hear the echo when the drumming rate rises to 40 a minute. He approaches the hill by 90 m and finds the echo again disappearing when the drumming rate rises to 1 per second. Find the velocity of sound and the initial distance of the drummer from the hill. (*Ans.* 360 m/s, 270 m) [I. I. T. '74]

(4) The echoes from a pair of parallel hills for a gunfire from a certain point in the intervening valley are heard successively 2 and 4 s after the firing. Find the width of the valley. Is it possible to hear the subsequent echoes at the same instant and at the same point ? If so, after what time ? c = 330 m/s.

(*Ans.* 1080m ; Yes ; 6 s) [I. I. T. '73]

(5) A sharp tap in front of a flight of stairs produce a musical sound. If c = 340 m/s and each step $\frac{1}{4}$ m deep find the frequency of sound. (*Hint.* $n = c/2d$).

III-5.5. Refraction of sound. When sound waves fall on a surface separating two media, a part of it enters the second medium and travels with a different speed, When the incidence is oblique a change in the direction of traveloccurs. If i is the angle of incidence and r the angle of refraction (considered as in light) then

$$\frac{\sin i}{\sin r} = \frac{c_1}{c_2} = \mu \qquad \text{(III-5.5.1)}$$

where c_1 and c_2 are the speeds of sound in the first and second media respectively. μ is the *refractive index* of the second medium relative to the first. Sound rays bend away from the normal in the medium in which the speed is higher. The medium in which the speed is slower corresponds to the optically denser medium of light. Very little of the sound energy travelling in air enters a solid or liquid medium, because most of the energy is reflected at the boundary due to large difference in the ρc values of the two media The product ρc is called the *characteristic impedance* of a medium, where ρ is medium densitys c, the sound velocity in that medium.

Sound, like light, may be refracted by prisms and lenses of suitable materials. The geometrical relations are the same, but owing to the largeness of scale such experiments are inconvenient to perform and interpret.

To demonstrate the refraction of sound waves a thin rubber balloon is partially inflated with CO_2 gas and placed between a Galton's whistle a high-frequency source of sound and a sensitive flame, a detector of high-frequency sounds. Keeping the whistle several feet away from the balloon the flame is gradually moved away. At one particular position of the flame, for a given position of the source with respect to balloon, the flame roars violently. This is because the denser gas in the balloon makes it behave as a convergent lens, and the positions of the source and the flame represent *acoustic conjugate foci.* The acoustical image however is not so well defined as in light. Filled with H_2 gas the balloon would behave as a divergent lens, as sound moves faster in hydrogen than in air.

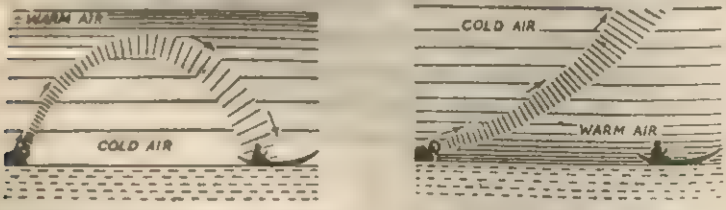
A. Total Internal Reflection. In passing from a medium in which the speed of sound is lower, the sound rays bend away from the normal. For a particular angle of incidence, called the critical angle (θ), the angle of refraction is 90°. Then from Snell's law we have $\sin\theta = \mu$ where μ is the ratio of the lower speed to the higher. For sound travelling from air to steel,

$$\sin\theta = \mu = \frac{c_1}{c_2} = \frac{332}{5200} \text{ whence } \theta = 4^\circ \text{ (nearly).}$$

Since all 'sound rays' incident on steel from air at a greater angle must suffer 'total internal reflection', the speaking tube (fig. III-5.1) can transmit sound with little loss over a long distance.

The speeds of sound in air and water are 330 m s and 1450 m/s nearly. This gives a critical angle of about $13\frac{1}{2}^\circ$. Sound rays in air incident on water at a larger angle will be totally reflected.

B. Refraction by air of varying temperature. A sound on shore may be heard from a good distance on a boat on an expanse of water at



(a) (b)

fig. III-5.3

night; but under the same conditions it may not be heard in daytime.

This is due to rafraction, arising out of variation of temperature at different altitudes. At night the air just above the water is cooler, but higher up it is warmer. Hence, the sound travels faster in the upper layers than in the lower ones. As a result, the upper part of the wave-front precedes the lower part (fig. III-5.3) and the waves bend downwards. This phenomenon may be compared with a superior mirage (see LIGHT). In daytime the entire phenomenon is reversed as the air in contact with the water is warmer. In this case, the sound waves bend upwards and do not reach the boat.

C. **Refraction due to wind**. Sound from a source is better heard in the direction in which the wind blows than against it. The whistle of a train or the tolling of a bell from a temple or a church is heard when the wind is from its direction, while it may be quite inaudible at other times. When wind blows, the upper layers of air move quickly than the

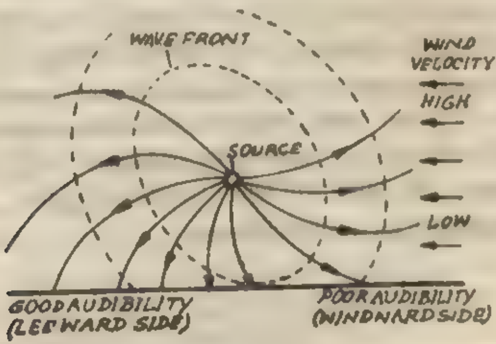


Fig. III-5.4

lower since the latter are retarded by trees, houses etc. on the ground. The wavefront of the sound moving in the direction of the wind therefore bends downwards. Moving against the direction of the wind they bend upwards (fig. III-5.4)

Reports of very loud explosions may be heard at distances where the ground wave does not reach. This is due to total reflection from layers of air high up in the atmosphere where the temperature of air begins to increase. In this way the existence of an atmosphere at a height of 25 to 40 miles has been established. The effect is similar to the formation of superior mirage as in fig, III-5.3(a)

III.5-6. Diffraction of sound. Whenever a sound wave passes the edge of a large obstacle it is found to bend round the corner. It is fortunate that it does so. Otherwise, we would not hear a sound if the source were screened. The bending of the waves round the edge of an obstacle is known as **diffraction**,

When sound waves fall on an aperture whose dimensions are comparable with the wavelength, the waves spread out into the region behind the screen. The spreading occurs over a hemisphere if $a < \lambda$, a being the opening of the aperture and λ the wavelength. As a/λ increases, the spreading diminishes. When a is several times the value of λ, we get a directed beam of sound. The same results hold if a represents the dimensions of a source of sound, such as a loudspeaker or a vibrating membrane.

In music the wavelengths range from about 10 metres to 3 cm (32 cps to 10,000 cps). If a sound issues through an aperture 1 foot (30 cm) wide, high frequency waves will spread out (be diffracted) very little, while low frequency sounds will be propagated in all directions behind the screen.

When a loudspeaker of the above diameter (30 cm) reproduces music, short waves do not spread much laterally, while long waves do. So a listener situated well off the axis of the loudspeaker will not hear the higher frequencies. The music will sound unnatural to him.

All the above effects are due to diffraction.

III-6
SUPERPOSITION OF WAVES

III-6.1. Principle of superposition. A very simple principle, first enunciated by Huyghens in connection with light waves and called the **principle of superposition**, can be used to explain the effects we find, when two sound waves, are superposed on (i.e. fall on) each other. It states that *when two or more wave-trains are superposed, the resultant displacement is equal to the vector sum of the individual displacements, provided they are small.* The principle, though strictly valid for waves of infinitely small amplitudes, is nearly true in all ordinary cases. *The principle also holds for light waves.*

Many important phenomena in sound arise due to superposition of *sonic* waves (waves of audible sound) from two different sources or two wave-trains derived from the same source. In discussing them by the principle of superposition, we *assume* that

(1) Passage of waves through one part of a medium is not affected by the simultaneous passage of other waves through the same part of the medium. For we notice that, after the waves have crossed the region of overlapping, they maintain the frequency, amplitude and other characteristics unchanged. (When two persons talk simultaneously, the voice of one is not modified by the voice of other.) This is, as you should recognise, a consequence of the Principle of Independence of vectors.

(2) The resultant displacement is the *vector sum* of those due to individual wave-trains.

The principle applies to all cases regardless of the directions of wave propagation or the frequency of the individual wave-trains. But cases of practical importance are those where the waves have equal or nearly equal frequencies and same or slightly inclined directions. Three such cases are important to us at present, namely,

(*i*) **Beats**, which occur when two wave-trains of *nearly equal frequencies and amplitudes moving in the same direction* overlap.

(*ii*) **Stationary** or **standing waves**, which occur when *two identical wave trains moving in opposite directions* overlap.

(*iii*) **Interference,** which occurs when two wavetrains of *same frequency and equal or nearly equal amplitudes overlap, their phase diff at a point retaining a fixed value.*

III-6.2. Beats. When two notes of nearly equal frequencies and amplitudes are sounded together the *resultant intensity rises and falls at regular intervals.* This phenomenon of rise and fall of sound is known as **beats.** A beat consists of one rise and one fall of the sound intensity.

Take two tuning forks of the same frequency and load a prong of any one of them with a drop of wax, thereby lowering its frequency. Excite both the forks at the same time. Beats will be heard. Beats may also be heard by pressing the two keys to the extreme left of a harmonium while playing it.

A. Perception of beats. To *hear beats clearly* the following conditions should be satisfied :—

(*i*) The *difference in frequency* of the two notes should be *small.* When this difference exceeds 6 per second the effect on the ear begins to be unpleasant. Beyond 10 per second the beats become too fast to be clearly distinguished.

(*ii*) The *difference in amplitude* of the two notes should be *small.* Loudness of a note depends on the amplitude of vibration. In the case of a large difference in amplitude the beats will not be clearly recognised as the weaker sound will not appreciably affect the louder one.

(*iii*) To produce a clear effect the two sources should be of the *same kind* (*i.e.*, the two notes should have the *same wave-form*).

When there is a large difference in frequency or amplitude, beats are physically present though they may not be clearly distinguished by the ear.

B. Beat frequency. The *number of beats per second is equal to the difference of the frequencies of sources.* To prove it in a general way, let the two sources have frequencies n and $n' = n+m$, where m is small compared with n. The corresponding wavelengths are λ and λ', both having the same velocity c. At a given moment, let the displacements be in phase at a particular point a (fig. II-6.1). They will also be in phase at a distance l at b fig. II-6.1 when the shorter wave has gained one wavelength over the longer. If ν be the number of longer waves contained in the distance l, then we shall have

$$l=\nu\lambda=(\nu+1)\lambda' \quad \text{or} \quad \nu=\frac{\lambda'}{\lambda-\lambda'}$$

Since both waves are travelling with the same velocity c, it is also the velocity of the resultant maxima (or minima). Therefore the number of maxima (or minima) received by a stationary observer in one second is the number lying within a distance c. Since the distance between two successive maxima (or minima) is l, their number within a distance c is

$$\frac{c}{l} = \frac{c}{v\lambda} = \frac{c}{\lambda}\left(\frac{\lambda - \lambda'}{\lambda'}\right) = \frac{c}{\lambda'} - \frac{c}{\lambda} = n' - n = m \qquad \text{(III-6.2.1}$$

So the number of beats per second equals the difference in frequencies of the two sounds.

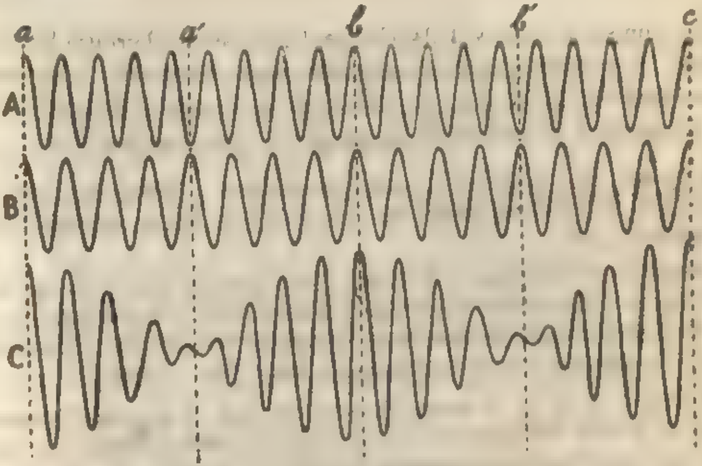


Fig. III-6.1

[A and B are sound waves of two notes at a given moment. C is the resultant waves formed by the superposition of A and B. The broken lines in the figure indicate places where the displacements in the two waves are in the same phase or in opposite phases.]

C. **Uses of beats.** Since beats are produced by small differences in frequency of two sources, they provide a sensitive means of detecting whether two notes are in unison or not. This test is used for tuning musical instruments. In almost all acoustic measurements of frequency, beats are used to find the exact matching of notes.

(i) **Frequency determination**. If the frequency of one of the two notes producing beats is known, the frequency of the other can be determined by counting the number of beats. Let two tuning forks have respective frequencies n and n' of which n is known. Provided n and n' are nearly equal, easily distinguishable beats will occur. Let their number per second be m. Then $n' = n \pm m$.

To determine whether n' is greater or smaller than n, count the nnmber of beats after putting a little wax on one of the prongs of the unknown fork thereby lowering the frequency of that fork. *If the number of beats increase, the required frequency is lower than the known one, i.e.,* $n'=n-m$. If the number of beats decreases the case is the reverse, *i.e.*, $n'=n+m$.

If an increase in mass of a tuning fork A increases the number of beats which it forms with another fork B, then the frequency of A is lower than that of B.

The number of beats will also change if the mass is *decreased* by filing the fork. *A decrease in mass increases the natural frequency of the fork. If decreasing the mass of a fork A decreases the number of beats* it formed with another fork *B*, then *the frequency of A is lower than that of B.*

Ex. III 6.1. *The tuning forks A and B, the frequency of B being 512, produce 5 beats per sec. A is filed and the beats are found to occur at shorter intervals. Find the frequency of A.*

Solution : Let the frequency of the fork *A* be n per sec. Then from the data $n = 512 \pm 5$. Filing the fork *A* decreases its mass and so increases its frequency. Since beats are occuring faster than before, the number of beats has increased and hence the frequency of *A* must be greater than that of *B*, *i.e.*, it is 517.

Ex. III-6.2. *A set of 24 tuning forks is arranged in a series of increasing frequency. If each fork produces 4 beats with the preceding one and the last fork sounds the octave of the first, calculate the frequencies of the first and the last forks.*

[And. U.]

Solution : Let the frequency of the first fork be n. Then that of the last fork will be $2n$. Now, the first fork produces 4 beats with the second, 8 with the third, 12 with the fourth and hence 23×4 or 92 beats with the last one. So between the first and the last is also *double* that of the first, the frequency of the first is 92 and that of the last is 184.

III-6.3. Stationary or Standing waves. Stationary or standing waves are formed when two wavetrains of the same kind, having the same frequency and velocity, and equal (or nearly equal) amplitudes are superposed on each other while moving in *opposite* directions. They can often be seen when the medium is bounded on all sides. If a regular wavetrain is generated in such a medium the wavetrain is reflected at the boundary and is superposed on the outgoing wavetrain. If the reflection at the boundary occurs at normal incidence, and the amplitude is not appreciably reduced, a clear, stationary wave pattern can be seen. We shall discuss this kind of wave pattern.

A. Characteristics of stationary waves. Look at fig. III-6.3. as you go through the following. Check the statements with the figure.

(i) The amplitude of vibration changes from place to place in medium and has a maximum value at certain points. The points of maximum amplitude are called **displacement antinodes** (or simply, *antinodes*).

(ii) At certain points the medium undergoes no displacement at all. Such points of no displacement are called **displacement nodes** (or simply, *nodes*).

(iii) The distance between successive nodes or successive antinodes is equal to half the wavelength ($\lambda/2$) of the superposed waves.

(iv) Along the common line of propagation, nodes and antinodes alternate with each other at separations of $\lambda/4$.

(v) All particles lying between adjacent nodes move in the same direction at the same time. They reach maximum or zero displacement all at the same time. They have therefore the same phase. The amplitude increases from zero at a node to a maximum at the antinode. The portion of the medium lying between two nodes is called a *loop*.

(vi) Particles in neighbouring loops vibrate in opposite phases.

(vii) A particle of the medium executes a complete vibration in the same time in which a component wave advances by one wave-length. The frequency of a particle is therefore the frequency of a component wave.

The reason why the disturbance in the medium is called a stationary wave is that the form of the disturbance, *i.e.*, the wave form, remains confined in space and does not advance through the medium as it does in a progressive wave. *The pattern looks as if a wave-train has been suddenly arrested in the course of its motion.*

B. Demonstration of stationary waves. Stationary waves can be demonstrated as follows :

(i) **Water waves.** Take some liquid in a conical bowl and give it a sharp tap. The water surface will show a circular wave pattern, which will remain steady if the diameter of the liquid surface is appropriate. This may be adjusted by pouring in more liquid. The pattern looks just as if a wave-train has been suddenly arrested in the course of its motion.

(ii) **Melde's experiment.** Stationary waves formed by the transverse vibration of a string under tension can be very elegantly shown by the following experiment due to Melde (Fig. III-6.2). An electrically maintained tunning fork of relatively low frequency (say 64 Hz) is clamped to a table as shown. One end of a string is attached to one of the prongs while the other end is attached to a pan after passing over a

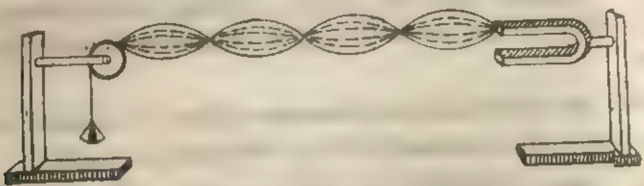


Fig. III-6.2.

smooth pulley. The fork is so placed that it vibrates *perpendicular to the length of the string*, which is the so-called *transverse* arrangement. The pan is then loaded and the fork excited. By adjusting the load on the pan or the length of the string the whole string may be made to vibrate transversely in one segment. If the load is reduced to one-fourth the string will vibrate in two segments : if the load is reduced to 1/9th the string will vibrate in three segments.

The vibrating fork sends transverse waves along the string. These are reflected at the other end and are superposed on the incident waves. The pattern becomes steady when the length of the string is an integral multiple of half the wavelength of the waves produced.

If an electrically maintained fork is not available, an electric bell may be used in its place. The string is tied to the hammer of the bell.

C. **Formation of stationary waves.** We have stated earlier that stationary waves are formed by the superposition of two identical wave trains moving in opposite directions in the same region of a medium. So the individual displacements produced at a given point in the medium at a given instant are given by

$$y_1 = a \sin (2\pi/\lambda)(ct-x), \text{ and } y_2 = a \sin (2\pi/\lambda)(ct+x) \quad \text{(III-6.3.1.)}$$

The line of propagation is along the x-axis. The two equations represent the waves travelling along the +ve and −ve directions of x respectively. In the equations, a is the amplitude, λ the wavelength, c the velocity and x the distance of the point from an arbitrary origin,

which generally is a point on the line of propagation where the waves meet in phase. The resultant displacement of the particle due to superposition of the two waves is, therefore,

$$y=y_1+y_2=a\sin(2\pi/\lambda)(ct-x)+a\sin(2\pi/\lambda)(ct+x)$$
$$=2a\sin(2\pi/\lambda)ct\cdot\cos(2\pi/\lambda)x^*.$$

Putting $A=2a\cos(2\pi/\lambda)x,$ (III-6.3.2)

we have $y=A\sin(2\pi/\lambda)ct$ (III-6.3.3)

This represents a simple harmonic motion of the same frequency as the waves but of an amplitude $A=2a\cos 2\pi x/\lambda$. The amplitude depends only on the position x of the particle. It is not a progressive wave as the final expression for the phase angle does not contain any term of the nature $(ct-x)$.

The amplitude $A=2a\cos 2\pi x/\lambda$ changes as x changes. When $\cos 2\pi x/\lambda=0$, then $A=0$. For this we must have

$$\frac{2\pi x}{\lambda}=(2n+1)\frac{\pi}{2}\text{ where }n=0,1,2,3,\text{ etc.}$$

$$\text{or,}\quad x=(2n+1)\frac{\lambda}{4}. \qquad \text{(III-6.3.4)}$$

∴ When $n=0$, $x_1=\lambda/4$
$n=1$, $x_2=3\lambda/4$
$n=2$, $x_3=5\lambda/4$, etc.

Such points, where the amplitude of displacement is zero, are called **displacement nodes** (points marked N in fig. III-6.3). Obviously, the distance between two successive nodes is equal to $\lambda/2$.

When $\cos 2\pi x/\lambda=\pm 1$, the amplitude is a maximum and has the value $2a$. For this we must have

$$\frac{2\pi}{\lambda}x=n\lambda\text{ where }n=1,2,3\text{ etc.}\quad\text{or,}\quad x=\tfrac{1}{2}n\lambda. \qquad \text{(III-6.3.5)}$$

Putting value for n we have $x_1=\lambda/2$, $x_2=2\lambda/2$, $x_3=3\lambda/2$ etc. These points of maximum amplitude are called **displacement antinodes** (marked A in Fig. III-6.3). As in the case of nodes, the distance between two successive antinodes is also $\lambda/2$. The distance between a node and the next antinode is $\lambda/4$. The amplitude of vibration thus varies from 0 to $2a$ over a distance $\lambda/4$.

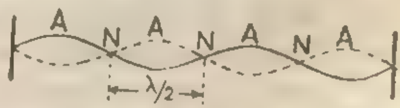


Fig. III-6.3

* From the trigonometrical relation $\sin(A+B)+\sin(A-B)=2\sin A\cos B$.

III-6.4. Comparison of progressive and stationary waves.

Progressive wave	Stationary wave
(1) Progressive waves are due to the continuous periodic vibration of a portion of the medium. So long as the waves do not meet any boundary, they are progressive.	(1) Stationary waves are due to the superposition of two identical progressive wave-trains moving along a line in opposite directions through a medium.
(2) The phase of one particle is transmitted to the next along the line of propagation. In this way the wave-form progresses forward. The velocity of wave-motion is determined by the elastic properties of the medium.	(2) The wave-form does not advance through the medium. It is confined to the portion of the medium where the oppositely moving progressive waves overlap.
(3) Each particle of the medium executes the same vibration about its equilibrium position with the same frequency.	(3) The amplitude of the particles is not the same everywhere. At the nodes the amplitude is zero, at the antinodes it is maximum. They are all executed with the same frequency.
(4) Along the line of propagation there is difference of phase between the vibrations of particles. This phase difference is proportional to the distance between the particles.	(4) All the particles between two nodes vibrate in the same phase. Particles on two sides of a node vibrate in opposite phases.
(5) With the progression of the wave every point of the medium undergoes the same change of pressure or density.	(5) Change of pressure or density is not the same at every point of the medium. It is a maximum at the (pressure) nodes and a minimum at the (pressure) antinodes, [Pressure nodes correspond to displacement antinodes and pressure antinodes to displacement nodes.]

III-6.5. Interference. If two waves of the same frequency and amplitude are superposed, the resultant displacement will be double that due to a single source if the vibrations are in phase. If they are in

opposite phases the resultant displacement is zero. Since intensity varies as the square of the amplitude, the effect produced in the first case will be four times that due to either of the waves, but zero in the second case. Thus the combined action of two sound waves may produce a greatly increased effect at places and silence at others. This phenomenon is known as *interference of sound.* There is no annihilation of energy in this case, only a redistribution. Energy removed from places of silence is concentrated at regions of increased sound.

Conditions of interference. To obtain *continuous silence* at a point in a medium under the action of two sound waves the following conditions must be fulfilled :

(1) The waves must have the same frequency and amplitude.

(2) The angle between the displacements due to the two waves at the given point must be zero or small.

(3) The waves must *continue to arrive* at the given point in opposite phases, *i.e.*, the sources must be *coherent.*

If these conditions are fulfilled at a point there will be no displacement and hence no response in a detector placed there. The first two conditions can be satisfied by sound waves from two seperate sources. To fulfil the third condition is more difficult. Ordinarily, no constant phase relation exists between two waves arriving at a point from two independent sources. Such sources are said to be *non-coherent.* They will be called *coherent* when there is a constant phase relation between them. The sources may vibrate in the same phase or have a constant phase difference. Waves from coherent sources continue to reach a place with a definite phase difference. So to fulfil the last of the above conditions sound waves from the same source may be made to arrive at the point by *different routes*, thereby acquiring the required phase difference.

Experimental investigation of interference of sound.

(1) To demonstrate interference of sound, a **Quincke's tube** (Fig. III-6.4.) is taken. It consists of a U-tube BAE with two side tubes at B and E. Another U-tube CD of smaller diameter can slide in and out of the larger one as indicated. The communication between the two limbs of BAE can be cut off by pushing in a shutter at A. The source of sound (T) is placed in front of the side tube at B and the detector (R) in front of the side tube at E.

The sound divides at B; the two parts follow the paths BAE and $BCDE$ and reunite at E. The phase difference between these two superposing sound waves depends on the difference of path lengths BAE and $BCDE$, since any *phase difference* = $(2\pi/\lambda)\times$ *path difference.* A path difference between the two routes can be introduced by sliding the portion CD. If this path difference

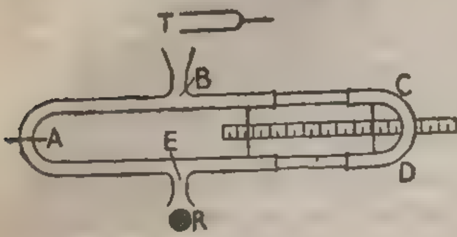


Fig. III-6 4

equals an odd multiple of half wavelengths [i.e., $(2n+1)\ \lambda/2$ where $n=0, 1, 2, 3, \ldots$ etc.] the waves will meet at E in opposite phases so long as sound is produced. The detector R will give no response. But if the path difference is zero or an integral multiple of λ, the waves reach E in phase and produce a strong response in the detector.

Let both paths be initially equal. Then the path difference is zero and the sound at E is a maximum. If now the sliding tube is slowly drawn out the response slowly diminishes and reaches a minimum when the path difference equals $\lambda/2$. Beyond that the response gradually grows to a maximum when the path difference measures $2.\lambda/2$, falls off again till the difference is $3.\lambda/2$, grows again till the difference becomes $4.\lambda/2$, and so on.

The conditions of interference are completely satisfied here. Since the wave train from a given source is divided, the two interfering systems have the *same frequency and practically the same amplitude.* They start obviously with the same phase but develop a phase difference by following paths of different lengths. So when they meet at E they do so with a constant phase difference, which may be made into π or any odd multiple thereof by adjusting the path difference. In this way, a compression due to one wave may be made to meet a rarefaction due to the other.

III-6.6. Relation between Path difference and Phase difference :

Let A and B (Fig. III-6.5) represent a pair of coherent point sources of waves and RQ a straight line at some distance from them. Let P be a point on it such that the path difference in reaching it from two sources is AP and BP which we put as r_1 and r_2.

The displacement of the particle at P due to disturbance from A at an instant is

$$y_1 = a \sin \frac{2\pi}{\lambda}(ct - r_1) \quad \text{and} \quad y_2 = a \sin \frac{2\pi}{\lambda}(ct - r_2)$$

is the displacement at P at the same instant due to the disturbance from

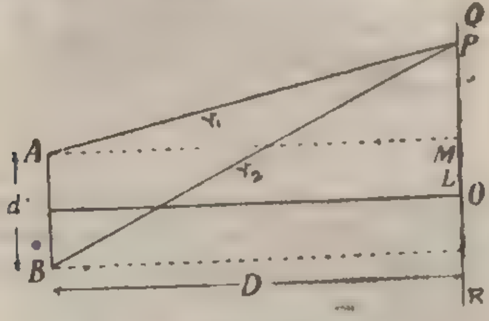


Fig. III-6.5.

B. Then the **phase difference** of vibrations at P due to superposition of waves is

$$\frac{2\pi}{\lambda}(ct - r_1) - \frac{2\pi}{\lambda}(ct - r_2) = \frac{2\pi}{\lambda}(r_2 - r_1)$$

Then **Phase difference** $= \frac{2\pi}{\lambda}$ **Path difference** (III-6.6.1)

If now $(r_2 - r_1) = \lambda/2,\ 3\lambda/2,\ 5\lambda/2 \ldots$ we have phase differences $\pi, 3\pi, 5\pi \ldots$ etc, so that the vibrations cancel out producing no displacement. If again the path difference is any integral multiple of λ, phase difference becomes that of 2π so that vibrations re-inforce each other doubling the amplitude.

So P will not at all be disturbed if $(BP - AP)$ happenes to be any odd multiple of half wavelengths and suffer double the individual displacement if $(BP - AP)$ is any even multiple of $\lambda/2$.

Interference due to reflection at oblique incidence. A number of cases of interference of sound occurs when direct waves and waves reflected at oblique incidence are superposed. For example, it has been noticed that as a boat approaches a high cliff on which a fog siren is sounding, it passes through alternate regions of maximum and minimum sounds. [Fog-sirens warn ships off the cliffs obscured in a fog.] Let F be the siren placed on a cliff and B be the boat approaching it (fig. III-6.6). The boat-

man receives the sound via two paths Fa and FPa. Suppose the difference between these two paths is equal to an odd multiple of $\lambda/2$. Then there will be silence at a. Again, if at b the path difference $(FQb—Fb)$ is

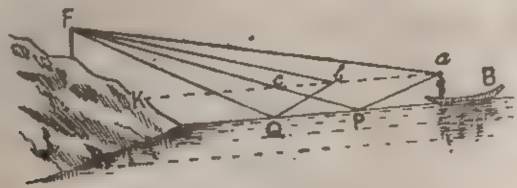


Fig. III-6.6.

an odd multiple of $\lambda/2$, there will be silence at b; similarly at c. Midway between a and b, and also between b and c the path-dirence will be an even multiple of $\lambda/2$. There will be reinforcement of and hence loud sounds at such places.

III-6.7. A. Interference of light. The phenomenon of interference, as we shall illustrate here, is a case of diffraction and superposition. Briefly,

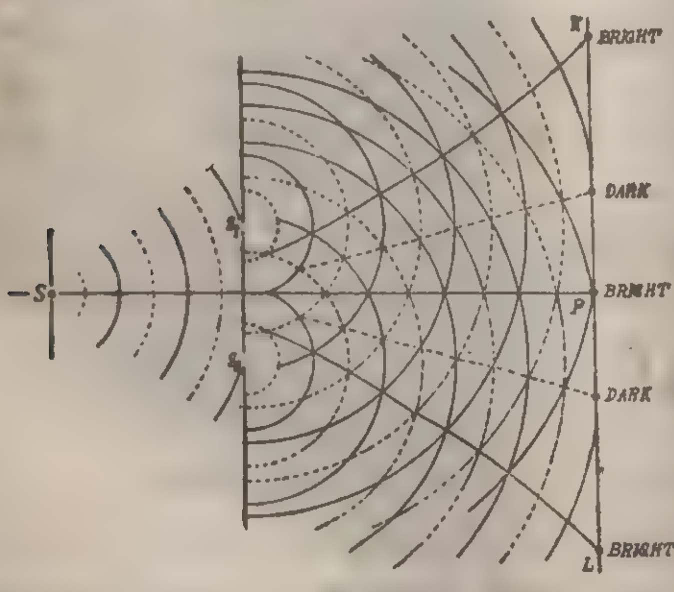


Fig. III-6.7.

we can say that when two identical wave trains, proceeding in about the same direction, *continue* to be so superposed that at some places they are

always in phase and at some other places they are always out of phase, there will be no vibration where the waves are out of phase and maximum vibration where they are in phase. This is *interference*. It is a property of all waves. We have discussed it above for sound waves.

Fig. III-6.7 shows an arrangement to produce interference of light. A strong beam of monochromatic light (say, sunlight filtered through red glass) falls on a narrow aperture S. Due to diffraction the beam passing through S spreads out and falls on two other small apertures S_1, S_2. The latter are so small that the waves passing through them spread out almost in the form of hemispherical waves and are superposed on each other over a region.

In the figure, the continuous lines represent surfaces (or wavefronts) on which the displacements are maximum. The broken lines represent surfaces (or wave-fronts) of minimum displacement—both at a given instant. The intersections of the continuous lines as also of the discontinuous lines centred on S_1 and S_2 represent superposition of vibrations in the same phase. These points of intersection mark paths along which vibrations are strongest.

The intersections of a continuous line centred on one of the sources S_1 or S_2 *with* a discontinuous line centred on the other source mark the places of superposition of two identical waves in opposite phases. There will be no vibration at these places.

If a screen KL is placed to receive the light coming from S_1 and S_2, the illumination on it will not be uniform. Instead, we shall find alternate bright and dark bands on the screen. These are called *interference fringes*. It corresponds to *alternate loud* sounds and *no* sound in the interference of sound waves.

Identical waves mean waves of the same frequency and amplitude. As S_1 and S_2 are always illuminated by the same wave from S, S_1 and S_2 always vibrate in phase. Hence in the superposed region the two superposed waves have a constant phase difference, that is they are coherent. This is the essential for producing interference effects.

Two sources of light are said to be *coherent* if the waves emitted by them are always in the same phase or always have a constant phase difference. *Coherence is the most fundamental of conditions of interference* and hence the repeatitions when discussing waves.

B. Fringe width. The distance W between two successive bright or dark fringes on the receiving screen (KL, fig. III-6.7) is called the *fringe width.* Its value is given by

$$W=\frac{D}{d}\lambda \tag{III-6.8.1}$$

where D = the distance between the receiving screen (KL) and the plane of the sources (S_1 and S_2),

d = the distance between the sources S_1 and S_2,

λ = the wavelength of the incident light wave.

By measuring W, D and d, λ can be determined.

C. Intensity. Maxima and Minima. Let I_1 and I_2 denote respectively the intensities due to the two sources at the centre of a bright fringe; the resultant intensity is $I=I_1+I_2$.

It is a maximum. As we move from here towards the nearest dark fringe the intensity gradually diminishes. At the centre of the dark fringe, the resultant intensity is a minimum and is given by

$$I=I_1 \sim I_2 \text{ (i.e., the difference of } I_1 \text{ and } I_2).$$

I_1 and I_2 are proportional respectively to the breadth of the corresponding source.

LIGHT WAVES

III-7.1. Light as a wave phenomenon. The reasons why light is considered as a wave phenomenon are that it shows all the properties of waves. These are (i) reflection, (ii) refraction, (iii) interference, (iv) diffraction, (v) scattering and (vi) polarization. It also shows Doppler effect, though it requires delicate apparatus to detect that. Moreover, light moves with a definite speed in a given medium. Light also shows the effects of superposition, and stationary waves can be formed with it.

You are already familiar with reflection and refraction of light. Interference has been discussed in Sec III-6.6, and., diffraction in Sec. III-5.4.

Polarization (Sec. III-7.5) of light shows that light waves are *transverse* in nature.

Between the 17th and 19th centuries there was no agreement among scientists as to the nature of light. Newton considered light as particles, while Huyghens suggested that they were waves. The question was settled by Maxwell (1831-1879). From theoretical considerations he came to the conclusion that light was a wave phenomenon. His conclusion was confirmed experimentally by Hertz (1857-1894).

Medium for light waves. Light reaches us from the sun and stars,

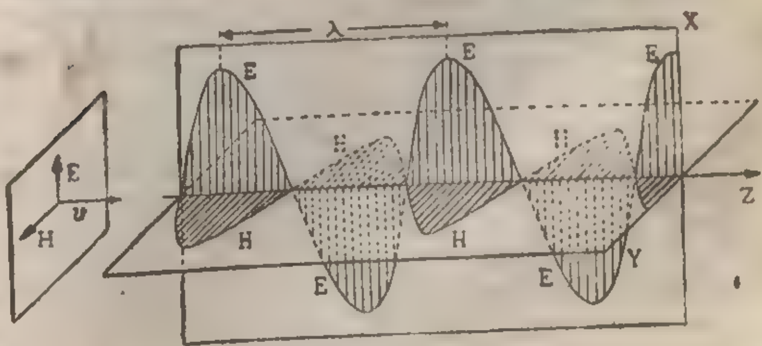


Fig. III-7.1

which are millions of kilometres away from us. Between them and the earth there is no material medium. The region is a void (vacuum). So

light waves do not require any material medium for transmission. They are *non-material waves.*

III-7.2. What is it that vibrates in light ? This was answered by Maxwell. The vibrations are those of an electric vector **E** and a magnetic vector **H.** They are respectively the electric intensity and the magnetic intensity. **E** and **H** are at right angles to each other. Both lie in a plane perpendicular to the direction of propagation of light waves (fig. 7.1). Hence the transverse character of light. Light waves are ***electromagnetic waves.***

Early in the history of light an imaginary medium, called the *Aether* (Ether), was postulated to provide a medium for light waves. Einstein (1879-1955) has proved it to be unnecessary. The existence of aether can neither be proved nor disproved. It is better not to think of it.

When we speak of light vibrations, we are really speaking of the vibration of the electric intensity vector **E**, for it affects our retina.

III-7.3. Electromagnetic spectrum. An electromagnetic wave is an oscillating electric force travelling through space, accompanied by a similarly oscillating magnetic force in a plane at right angles to it. Such waves travel in vacuum with the same velocity as light, irrespective of the wavelength (or frequency) of the waves.

The wavelength range extends from about 10^6m to 10^{-14}m. The frequency range extends from about 10^2 to 10^{22} Hz.

Waves in the different ranges have different names, though waves with different names have no sharp dividing lines with respect to wavelength or frequency. Some names and approximate ranges are given below.

Name	*Range (in m)*	*Range (in Hz)*
Power	$10^7 - 10^5$	$10^1 - 10^3$
Radio	$10^5 - 10^{-2}$	$10^5 - 10^{10}$
Microwaves	$10^1 - 10^{-3}$	$10^9 - 10^{11}$
Infrared	$10^{-3} - 7\times10^{-7}$	$10^{11} - 10^{14}$
Visible	$4\times10^{-7} - 7\times10^{-7}$	$4{\cdot}3\times10^{14} - 7{\cdot}5\times10^{14}$
Ultraviolet	$4\times10^{-7} - 2\times10^{-9}$	$10^{15} - 10^{17}$
X-rays	$10^{-8} - 10^{-12}$	$10^{16} - 10^{20}$
γ-rays	$10^{-10} - 10^{-14}$	$10^{18} - 10^{22}$

Standard broadcasting frequency is around 10^6 Hz. TV broadcasting is around 10^8 Hz and radar, 10^{10} Hz.

Essential similarities and dissimilarities in the properties of light and sound waves.

Similarities. Both show such wave properties as (a) reflection, (b) refraction, (c) interference, (d) diffraction and (e) scattering. Both move with different speeds in different media, though their speeds are widely different.

Dissimilarities.—Light waves can be polarized, but not to the sound waves. This shows that light waves are transverse in nature. Sound waves are longitudinal. Light travels far faster and its waves are far shorter than sound waves.

Light waves can pass through vacuum, but not so the sound waves. Sound waves are longitudinal, elastic waves in a material medium. Light waves are non-material electromagnetic waves. They consist of vibrations of an electric intensity vector **E** and a magnetic intensity vector **H**. **E** and **H** are perpendicular to each other. Both are perpendicular to the direction of propagation of light.

Frequency of audible sound lies in the approximate range of 20 to 20,000 Hz. Frequency of visible light lies in the approximate range 4.3×10^{14} to 7.5×10^{14} Hz.

Basic similarity and dissimilarity between Light waves and Radio waves. Both are electromagnetic waves and therefore have all the wave properties in common. The most prominent difference is that in wavelength. While the average wavelength of light is about 5×10^{-7}m, the range of wavelength of radio waves is about 10^{5} to 10^{-2}m.

Another point of difference is in the mode of generation. Light waves are generated by transition of electrons from one energy level to another. The process occurs inside atoms and molecules. Radio waves are generated by oscillating electrons in the radio antenna.

An accelerated charged particle can produce either radio waves or light, each under appropriate conditions.

Since energy in electromagnetic waves is proportional to frequency, light waves have much more energy than radio waves.

No electromagnetic radiation other than light can create the sensation of sight.

III-7.4. Velocity of light. Light has been found to have the tremendous velocity of about 3×10^{10} cm/s ($=3\times10^{8}$ m/s $=3\times10^{5}$ km/s)

in vacuum. In a material medium such as glass, water, air, etc. it travels with a lower speed. This speed depends on the colour of light. If the material medium has a refractive index μ for light of a particular colour, the speed c of light of that colour is c/μ.

A Danish astronomer Romer was the first to determine (1675) the velocity of light. He did so by making observations on the eclipses of one of Jupiter's satellites during two halves of a year. The first successful experiment from observations on the earth was done by Fizeau (1849). Later Foucault (pronounced Foo-ko), Michelson and others devised different methods for measuring the velocity of light (in vacuum) more and more accurately. Michelson (1852-1931), an American physicist, spent about fifty years of his life on such experiments. He was awarded the Nobel prize for his work. The best value he got for the velocity of light in vacuum was

$$c=299,\ 796\pm4 \text{ km/s.}$$

The error is a little over one part in a hundred thousand.

After the Second World War, great improvements were made in the methods of measuring length and short time intervals very accurately. Also very strong beams of light, visible and invisible, could be produced, (The former is known as 'laser' and the latter as 'microwaves'.) With their help many scientists determined c more and more accurately. Considering all accurate measurements upto 1964, scientists decided that the best value is

$$c=299\ 792{\cdot}5+0{\cdot}3 \text{ km/s}$$

The error here is one part in a million (a value 10 times better than Michelson's).

The advanced nations did not stop there. Their national laboratories are still carrying on the work. In 1972, the scientists of the National Bureau of Standards (U.S.A.) found

$$c=299,\ 792{\cdot}4562 \text{ km/s.}$$

They claim that the error in the value is about one hundredth of the previous lowest error.

Importance of determination of velocity of light. The velocity of light *in vacuum* is Nature's speed limit. No material particle can exceed, or even reach, this limit. This is a law of Nature, and is not due to any inability on our part to apply a large enough force.

The velocity of light in vacuum is the same under all circumstances. It is not affected by the motion of the sources or of the observer. This fact is the foundation stone of Einstein's theory of Special Relativity.

An accurate determination is also of importance in measuring distances. We can get a beam of light (in fact, microwaves) reflected from the object whose distance we want to find. When the velocity of light is known, a knowledge of the time interval between emission and reception back (of the microwaves) will give us the distance. It should be noted that the velocity of light in vacuum does not depend on wavelength. So, long radio waves, shorter microwaves or even shorter visible light travels with the same speed in vacuum. Light beams are used for measuring distances of artificial satellites and of the moon. Time intervals of the order of 10^{-8} s are no longer difficult to measure.

III-7.5. Polarization of light. Let us first see what polarization means and how it distinguishes transverse waves from longitudinal waves.

Refer to fig. III-7.2 AD is a stout string, or better, a fine spiral spring. Whichever we use, we shall call it a string. Its D end is fixed. The A end can be vibrated in any direction in a plane perpendicular to AD.

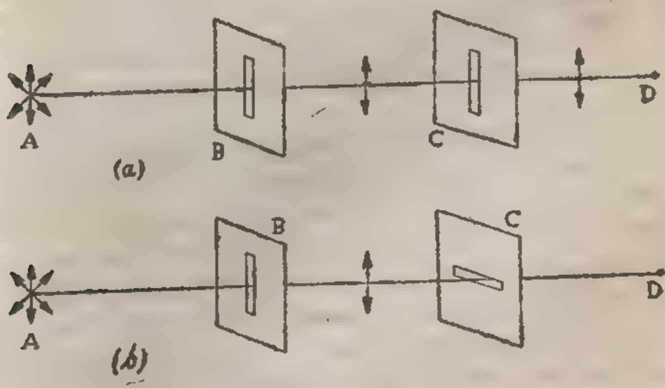


Fig. III-7.2.

The string passes through two narrow slits B and C in two parallel metal sheets. Either slit can be rotated in the plane of the slit in which it lies. Note that these planes and slits are perpendicular to the string AD.

First, let us have the slits parallel to each other. Vibrate the end A parallel to the slit B. These vibrations will pass through both B and C and reach D (fig. III-7.2. *a*). But if you now turn slit C in its own plane till it is perpendicular to B, no vibrations will pass through C. D will not feel any effect due to vibration of A. There will be vibrations between B and C, but none beyond C.

Now, periodically pull the end A longitudinally (parallel to AD). Longitudinal vibrations will pass along AD and reach D. Turning of the slits B and C will not stop the longitudinal vibrations from reaching D.

We thus find that propagation of transverse vibrations can be stopped by two slits at right angels. But such an arrangement will not stop the propagation of longitudinal vibrations.

If the end A were vibrated at an angle to the length of the slit B, the component of the vibrations in the direction of the slit will pass through B. If the direction of vibration of A is rapidly changed, B will allow only the component parallel to it to pass through. On passing B, transverse vibrations are all confined to one plane. This effect is called **plane polarization** of transverse vibrations. The slit C allows the component of such vibrations in its own direction to pass through. By turning C gradually this component is reduced in strength. When C is perpendicular to B, no vibration passes C. Thus B produces plane polarization while C detects it. We may call B the **polarizer** and C the **analyser.**

In the case of *light waves*, certain crystals play the same part as the slits in the above case of mechanical waves. Tourmaline is such a crystal. Take a thin plate of tourmaline and allow a light beam to pass through. Light is slightly reduced in intensity in passing through the crystal. But this may go unnoticed. The light beyond the tourmaline plate is plane polarized. This can be detected by another similar tourmaline plate. A naturally occuring tourmaline crystal has a certain clearly defined direction which we call its *axis*. When we place a second tourmaline crystal beyond the first, the first one acts as polarizer and the second as analyser of light. With the axes of both parallel, light passes through. As you rotate the second in its own plane, the light that passes falls in intensity. When the axes of the two are perpendicular to each other, no light passes the second.

This phenomenon which occurs in the case of light is known as *polarization of light.* It establishes the fact that light waves are transverse in nature.

There are many other ways in which polarization of light can be demonstrated. In the case of sound waves we cannot produce polarization. Sound waves are longitudinal,

III-7.6. Light rays and diffraction. We imagine a light ray as a line (without breadth) which marks the path of travel of a wave from one point to another. In a homogeneous medium rays are straight lines. Light waves are said to travel in straight lines in such media (rectilinear propagation of light).

Let us see what we find when we try to realize a ray of light in practice. Refer to Fig. III-7.3. S is a small source of light. A is a diaphragm whose aperture can be gradually reduced and B is a receiving screen. Let the aperture in A be large. Join S to the points marking the boundary of the aperture in A and produce the lines to meet B. The illuminated region on B will, *for all practical purposes*, be confined to the

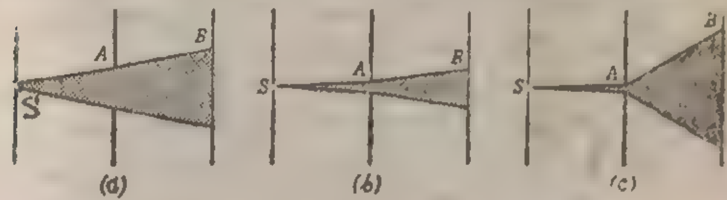


Fig. III-7.3.

region so marked (fig. III-7.3b), But as the aperture is gradually reduced in size, the light on B will gradually spread beyond the geometrical limit marked in the same way as before (fig. III-7.3b). When the aperture is very much reduced the spreading of light on B becomes much greater (fig. III-7.3c). The narrower we make a beam, the more it spreads beyond the aperture. This phenomenon is **diffraction**, a property of waves of all kinds.

When the width of the beam equals the wavelength of light the spreading beyond the aperture covers a hemisphere.

III-7.7. Validity of geometrical optics. If a ray of light cannot be realized in practice, how can we then accept the results of geometrical optics which is based on the ray concept? We have seen in the last section that *when the light beam has a width large enough compared with*

the wavelength of light, the ray concept is good enough as an approximation (see fig. III-7.3a). The mean wavelength of visible light may be taken as 6×10^{-5} cm. So a lens or mirror 1 cm wide is large enough to apply the ray concept. Even then, if you examine small regions close to the geometrical boundary of a shadow, or a focus, you will find small departures from simple geometrical laws. (These are due to diffraction.)

In optical instruments the mirrors, prisms and lenses used are very large comared with the wavelength of light. Hence their action may be described in terms of rays to a good approximation.

Q. *Explain why rectilinear propagation of light as assured in geometrical optics is only approximately true.* (*S. S. Q.*)*

Ans : Light waves (like all waves) have the property of bending round corners. This phenomenon is known as *diffraction*, In passing through an apertureo f breath a, the extent of bending is given by θ where $\sin\theta=\lambda/a$, λ is the wavelength of light, the mean value of which may be taken as about 6×10^{-5} cm. So an aperture of only 1 cm diameter will cause an angular spreading of 6×10^{-5} radian = about $\frac{1}{8}$ of arc. Such small spreading cannot be detected with our eyes. The beam appears, for all practical purposes, to move in a straight line.

In geometrical optics, apertures of beams that we use, are of this order of magnitude. Hence light beams appear to travel in straight lines.

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* S. S. Q. indicates it questions supplied by the Samsad as specimans, the like of which a student should know.

III-8
PHYSIOLOGICAL SOUND

III-8.1. Musical sound and noise. Helmholtz, the nineteenth century scientist (1821-1894), was a physicist and physiologist at the same time. He carried out a vast amount of research on the sensations of light and sound. On the basis of sensations sound was divided into two classes, namely, (i) musical sound and (ii) noise. *A sensation is not a physical quantity and cannot be measured.* Its study therefore does not properly belong to the realm of physics. Helmholtz sought to associate the various characteristics of different sensations of sound with different physical quantities. The above classification was therefore based on the following definitions :

(i) A sound that is continuous and periodic in nature is called *musical sound* (or *note*).

(ii) A *noise* is a sound of short duration, sharp and abrupt in nature, or one which changes character continuously if it persists.

The basis for the classification was not the pleasantness of hearing in the first case or unpleasantness in the second. A musical sound may be unpleasant to hear ; a noise may in some cases produce a pleasant sensation in the ear. The definition of noise has since been changed. *A noise now means an unwanted sound.* If a good piece of music is diverting your attention from the work you are earnestly engaged in, it will be noise to you.

III-8.2. Characteristics of musical sound. On the basis of sensation, three characteristics are ascribed to musical sound. They are (i) loudness, (ii) pitch and (iii) quality. All the three are judgements of the brain and none is a physical quantity.

(i) **Loudness** is the intensive attribute of an auditory sensation in terms of which sounds may be ordered on a scale extending from soft to loud. It corresponds to 'brightness' in light.

(iii) **Pitch** is that attribute of auditory sensation in terms of which sounds may be ordered on a scale extending from low to high, such as a musical scale. It corresponds to colour in optics and may be called

'musical colour'. A noise, according to *the old definition*, is characterized by the lack of a definite pitch.

(iii) **Quality** (or *timbre*) of a musical sound is that attribute of auditory sensation in terms of which a listener can judge that two sounds similarly presented and having the same loudness and pitch are dissimilar. We can easily distinguish a note sounded on a flute from the same note played on a harmonium or sitar. The characteristic of the sound sensation by which we differentiate them is *quality*. It is by quality that we recognize the voice of a friend or any person we know.

III-8.3. Physical factors on which loudness, pitch and quality depend.

(i) **Loudness and intensity.** Loudness depends primarily upon intensity ; but the two are not the same. Intensity is the rate of flow of sound energy per unit area. Intensity is a physical quantity and can be measured. But loudness is a physical sensation and cannot be measured. But loudness is a physical sensation and cannot be measured like a physical quantity. Intensity is proportional to the square of the amplitude and frequency; and is directly proportional to the density of the medium.

Loudness appears to be approximately proportional to the logarithm of the intensity.

[At low intensities, loudness depends much on frequency. At low brightness, the eye is most sensitive to yellow-green light ($\lambda = 5{\cdot}5 \times 10^{-5}$ cm). Similarly, at low loudness, the ear is most sensitive to frequencies of about 3000 – 3500 Hz.

Intensity is a physical quantity measured in ergs per sq. cm per second in the cgs system. Loudness is a physical sensation, not measurable like a physical quantity. Still, in view of its importance in connection with measurement of noise level, a unit, called the 'phon', has been used to express loudness level. Loudness level is measured in phons by the average sensation a sound produces in many men.

(ii) **Pitch and frequency.** Pitch depends primarily on the frequency of the sound. But it is also dependent to some extent on the intensity and the wave-form. If the intensity of a pure sound (sound of one frequency only ; wave form is a sine curve) of fixed frequency is increased, its pitch changes. But if the frequency is greater than 1000 Hz, there is is not much change of pitch with intensity.

Pitch and frequency are not the same. Pitch is a sensation while frequency is a measurable physical quantity. It is unfortunate that the word 'pitch' be used in the same sense as 'frequency'. In some elementary

books or in some question papers you may find the expression 'determination of pitch' where determination of frequency of a sound is meant. Determination of real 'pitch' is a very complicated affair, and is never undertaken in an ordinary sound laboratory. (Pitch is expressed in a unit called the '*mel*'. It has no connection with the *hertz* : but we cannot go into all these here.)

(iii) **Quality and wave-form.** Quality depends on the nature of vibration of the source, which may be simple or complex. A source executing S.H.M. produces a note of a single frequency, which is called a **tone.** Ordinarily, the vibration of a source is much more complex and has several frequencies at the same time. A musical sound, which we shall call a **note,** generally consists of several tones. The tone of the lowest frequency in a note is called its **fundamental** ; those of higher frequencies are called **overtones** or **upper partials.** Overtones having frequencies that are integral multiples of the fundamental frequency are called **harmonics.** The fundamental and overtones originating from the same source blend together into a single musical note. *The quality of a musical sound is due to the presence of overtones in addition to the fundamental* (Helmholtz). The number, nature and relative intensitie of the overtones determine the quality of the musical sound. The wave-form of a sound wave is also determined by exactly these quantities. So, we may say that *quality is determined by the wave-form.* The wave-forms of a note of the same fundamental frequency, but from different instruments, are shown in fig III-8.1.

A pure tone corresponds to a single colour in light ; a complex musical note corresponds to a mixture of colours. Since, in light, a mixture of colours is called a 'hue' we may say that 'quality' in sound corresponds to 'hue' in light. By analogy with light, we may say :

Loudness in sound corresponds to brightness in light, pitch to colour and quality to hue.

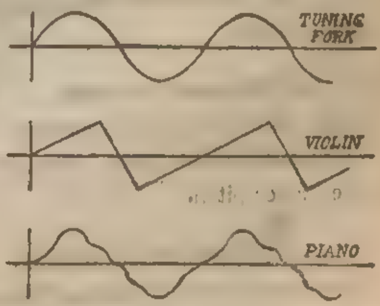


Fig. III-8.1

III-8.4. Graphical representation of the characteristics of a musical note. Since sound is wave-like in nature, all the characteristics it produces must have their physical

counterparts in the characteristics of waves. Now, a regular wave is characterised by (i) an amplitude, (ii) a frequency or wave-length, and (iii) a wave-form or time-displacement curve. Broadly speaking, **the amplitude, frequency and wave-form of a sound wave are associated respectively with the sensations of loudness, pitch and quality.** Simple harmonic wave-forms correspond to pure tones, while mixed tones, *i.e.*, notes, have more complex wave-forms.

A. **Difference in loudness.** In fig. III-8.2(*a*), both curves are sine curves of the same frequency. They have the same pitch and quality, but their amplitudes differ. They will be found to differ in loudness.

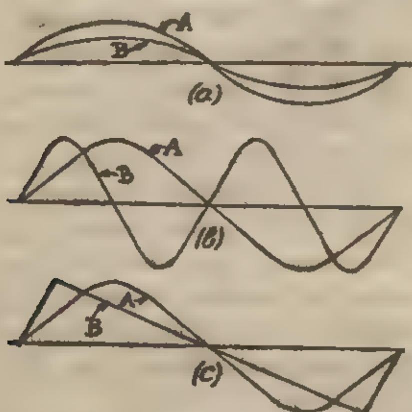


Fig. III-8.2

Other things remaining the same, the larger amplitude will produce the louder sound.

B. **Difference in pitch.** In fig. III-8.2*b*, we have two sine curves which differ only in frequency but not in amplitude. The tones they represent differ in pitch. The curve with the higher frequency has the the higher pitch.

C. **Difference in quality.** In fig. III-8.2*c*, the amplitude and frequency of the curves A and B are the same ; but their wave-forms are different. The sounds they represent will have about the same loudness and pitch but their qualities will be different. The sine curve A

represents a pure tone ; it has a single frequency. The curve B has a fundamental frequency equal to the frequency of A ; but it also contains overtones. The wave form depends upon the frequencies and relative amplitudes of the fundamental and the overtones present.

III-7.5. Noise. Noise has been defined as 'sounds not desired by the person receiving it'. The definition is clearly subjective. What is music to one man may be noise to another. A musical sound is *periodic* in nature, but a noise is *not*. Some noises seem to have a periodicity ; but this is often due to some secondary effect connected with the noise.

The most disturbing fact about noise is its loudness. Noise diverts attention and causes annoyance. In a factory, noise reduces the efficiency of a worker. Noise also affects adversely the human nervous system. Reduction of noise is becoming a more and more important problem of modern civilization.

III·9

VIBRATION OF STRINGS

III-9.1. Introduction.

In the world of music, stringed instruments play a dominant role. Musical sound is produced in them by transverse vibration of strings kept under tension. A point of a string is displaced from its position of rest by plucking, bowing or striking. The string then vibrates either as a whole or in segments emitting a musical note of definite pitch and quality. Theoretically a string is defined as "a perfectly uniform and perfectly flexible filament of solid matter stretched between two fixed points—in fact an ideal body, never actually realised in practice but closely approximated to by most of the strings employed in music" (Lord Rayleigh). The actual string always possesses some amount of rigidity, the effect of which diminishes as its length-to-diameter ratio increases. *The transverse vibration of an ideal string is affected by tension only* and not by rigidity.

III-9.2. Transverse displacement waves on a string : A string under tension resists any tendency to displace any point on it from the position of rest. This resistance supplies the restoring force when the

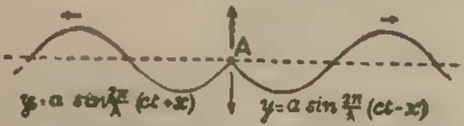


Fig. III-9 1.

displacement occurs. If a point on a long thin stretched string is displaced at right angles to its length and then released, the point begins to vibrate, and starts a twin displacement wave (fig. III-9.1) on either side of it which travel along the string. These waves are reflected from the fixed ends and interfere with the incident waves forming a stationary wave pattern. The string then vibrates either as a whole or in a number of segments depending on its tension and mass per unit length. The frequency of the note emitted depends on the mode of vibration. We shall discuss the velocity of wave propagation, mode of vibration and frequency of the note emitted, one by one.

III-9.3. Velocity of transverse waves along a string. Consider a very long flexible string (ab ; fig. III.9.2a), of mass m per unit length, stretched horizontally by a force of T dynes at each end. Suppose a sudden jerk at the end b produces a 'hump' AB, of length δl, on the string

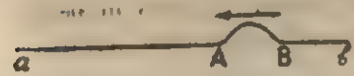


Fig. III-9.2a

Fig. III-9.2b

and that it travels from b to a with a velocity v. The tension T is supposed not to be changed by the hump.

The velocity v may be calculated by a simple method. Let an opposite velocity v from left to right be impressed on the whole string, so that the hump appears stationary to a stationary observer. Every point of aA as it comes to the hump will glide through the hump and travel on in the direction Bb (Fig. III-9.2b).

Suppose the hump AB (fig. III-9.3.) forms a circular arc of radius R, so that it subtends an angle $\delta l/R$ at the centre. Thus $\delta l/R = 2\phi$. At the

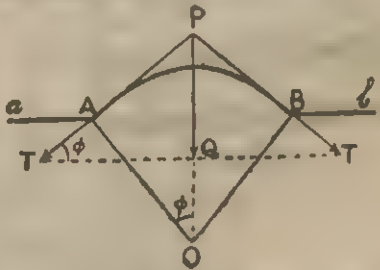


Fig. III-9.3

points A and B two tangential forces, each equal to the tension T, are acting. Each of them has a component PQ directed towards the centre O of the arc and of magnitude $T \sin \phi$. When the arc AB is very small $\sin \phi - \phi$ and each component $= T\phi$.

$\therefore$ Total force towards the centre $= 2T\phi = T\delta l/R$. Here we have a mass $m.\delta l$ moving with a velocity v along the arc of a circle of radius R. The necessary centripetal force is then $m.\delta l v^2/R$ and this is supplied by the components of the tension directed towards O. So we have

$$m\delta l v^2/R = 2T\phi = T\delta l/R$$

$$\text{or} \quad v^2 = T/m \quad \text{or} \quad v = \sqrt{T/m} \qquad \text{(III-9.3.1)}$$

III-9.4. Modes of transverse vibration of strings, and their frequencies.

Normally a vibrating string is rigidly fixed at its ends. We assume that no displacement can occur there and the travelling displacement wave is fully reflected with a reversed phase. The result is a stationary wave system of the same frequency as the travelling wave with two nodes at the two fixed ends. As there may be any number of nodes in between, various wavelengths, and hence frequencies of vibration, are possible.

If l is the length of the string then the simplest mode of vibration (fig. III-9.4 ; top) occurs when the string vibrates as a whole. Then obviously $l=\lambda_1/2$ where λ_1 is the longest possible wavelength. The next

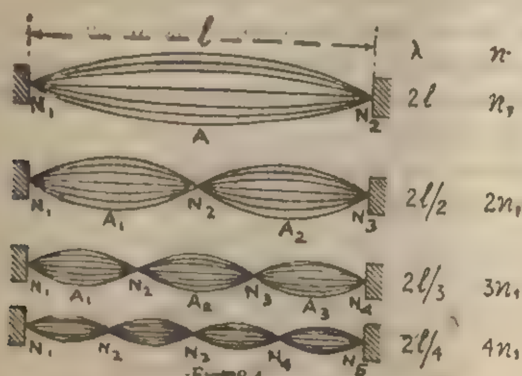


Fig. III-9.4

mode of vibration occurs when the string vibrates in two segments (fig. III-9.4 ; second from top). The corresponding wavelength $\lambda_2=l$. In the third mode, the string vibrates in three segments so that $l=\frac{3}{2}\lambda_3$, i.e., $\lambda_3=2l/3$. When the string vibrates in four segments the corresponding wavelength is $\lambda_4=2l/4$.

III-9.5. A. Fundamental frequency of a vibrating string. The simplest possible mode of vibration of the string is one in which there are only two nodes in the whole string. The antinode is then in the centre of the string. In this mode the entire string moves to and fro in a single segment (fig. III.9.4. Top). The frequency with which this vibration is executed is called the **fundamental frequency.** Since the distance

between two adjacent nodes is half the wavelength, the length l of the string is related to the wavelength λ_1 of the fundamental frequency by the relation $l = \frac{1}{2}\lambda_1$ or $\lambda_1 = 2l$.

If the fundamental frequency is n_1, then we shall have $n_1\lambda_1 = c = \sqrt{T/m}$. Since $\lambda_1 = 2l$, we get

$$n_1 = \frac{1}{2l}\sqrt{\frac{T}{m}} \tag{III-9.5.1}$$

B. Other modes of vibration. If the string vibrates in p loops or segments, each loop will have a length $l/p = \frac{1}{2}\lambda_p$ where λ_p is the corresponding wavelength. Thus $\lambda_p = 2l/p$. If the corresponding frequency is n_p, then $n_p\lambda_p = c = \sqrt{T/m}$,

$$\text{or} \quad n_p = \frac{1}{\lambda_p}\sqrt{\frac{T}{m}} = \frac{p}{2l}\sqrt{\frac{T}{m}} \tag{III-9.5.2}$$

This gives us all the characteristics of transverse vibration of stretched strings (See next Sec.). In this equation p is any integer, 1, 2, 3, etc. When $p=1$, the note emitted by the string is called its **fundamental** or **first harmonic.** It will appear from the equation that the possible notes from the string have frequencies which are integral multiples of the fundamental note. For each value of p, the string vibrates in a characteristic way, that is, with p loops. Each such way of vibration is called a *mode* of vibration.

Theoretically, a string has infinite modes of vibration. These are known as the **natural modes** of vibration of the string.

To make the string vibrate only in a particular mode is extremely difficult. This can be done by resonance with a vibration of that particular frequency. When a string actually vibrates, several modes of vibration are simultaneously present. Their number and nature are determined by the way the string was excited. The emitted note is complex and has several frequencies at the same time. Besides the fundamental, there will be overtones. Since the overtones in this case have frequencies which are integral multiples of the fundamental, they are *harmonics*. The harmonic having $p=2$ is called the *second harmonic*, $p=3$ is the *third harmonic*, and so on.

C. Quality. The particular harmonics and their relative strengths determine the **quality** (Chap. III-8) of the note from the string. If a vibrating string is touched at a point distant d from one end where $d = l/p$, p being an integer, the vibration of that point stops. So it becomes a node, and only those frequencies can be present which have a

node at d. These are the harmonics p, $2p$, $3p$, etc. If touched in the middle ($d=l/2$), only $p=2, 4, 6$, etc. harmonics, *i.e.*, the even harmonics only, can be present. All odd harmonics will be suppressed. If $d=l/3$, harmonics with $p=3, 6, 9$, etc. can be present. The higher harmonics are generally weaker. In stringed musical instruments the quality of the emitted note can be changed in this way, that is, by touching the vibrating main string at different points.

III-9.6. Laws* of transverse vibration of strings. These were discovered by the French mathematician Mersenne in 1636. Eq. III-9.5.1 was established theoretically by Taylor in 1715. Mersenne's laws are stated as follows : (If you write down Eq. III-9.5.1 you can state the laws immediately.)

(*i*) The frequency of vibration varies inversely as the vibrating length when the tension and mass per unit length remain constant. This is known as the **law of length.** In symbols,

$$n \propto 1/l \text{ when } T \text{ and } m \text{ are constant.}$$

(*ii*) The frequency of vibration varies directly as the square root of tension when the length and mass per unit length of the string remain constant. This is the **law of tension.** In symbols,

$$n \propto \sqrt{T} \text{ when } l \text{ and } m \text{ are constant.}$$

(*iii*) The frequency of vibration varies inversely as the square root of mass per unit length provided the length and the tension of the string remain constant. This is the **law of mass.** In symbols,

$$n \propto \sqrt{1/m} \text{ when } T \text{ and } l \text{ are constant.}$$

If the string is of circular cross section we have linear density m = area of cross-section $\times$ density $=\frac{1}{4}\pi d^2\rho$ where d is the wire diameter and ρ the density of the material.

The expression for frequency (Eq. III-8.5.1) then becomes

$$n=\frac{1}{2l}\sqrt{\frac{T}{\frac{1}{4}\pi d^2\times\rho}}=\frac{1}{l.d}\sqrt{\frac{T}{\rho\pi}} \qquad \text{(III-8.6.1)}$$

* Results stated as 'laws of transverse vibration of strings', 'laws of simple pendulum', 'laws of falling bodies', etc, were experimentally established in the early stages of the development of physics. Mathematical deductions came at a later stage. The so-called laws automatically follow from the respective mathematical relations.

Hence of a *uniform wire of circular cross-section*, we may put the law of mass in the following forms :

(*iii a*) **Law of diameter**. Length, tension and material remaining the same, frequency varies inversely as the diameter.

(*iii b*) **Law of density**. Length, tension and diameter remaining the same, frequency varies inversely as the square root of density.

III-8.7. Sonometer. A useful instrument based on the vibration of strings is the sonometer. Most of the laboratory experiments on strings are performed with the sonometer or monochord.

A. Description. A sonometer (fig. III-9.5) consists of a hollow wooden box with a thin horizontal wire, attached to a peg, stretched on the top of it. The wire passes over a grooved pulley (P) and carries a hanger on which weights (w = mg) can be placed so as to apply a known tension to the wire. Wooden bridges, B, $B\pi_2$ B_3 are placed beneath the wire for adjusting the vibrating length of the wire. To measure these lengths a horizontal

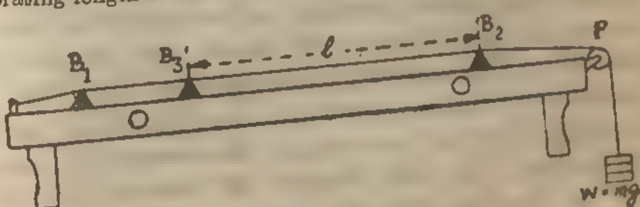


Fig. III-8.5 (a).

scale (the lower fig) is sometimes fixed on the box below the wire. Very often an auxiliary wire is provided alongside the main wire, stretched from another peg and passing over a second pulley (the lower fig). It also carries a hanger for weights. With only one string the apparatus is a *monochord*.

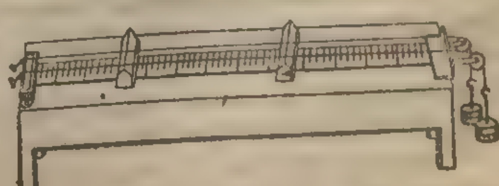


fig. III-8.5 (b)

The frequency of the emitted note from a given sonometer wire can be altered by either (*i*) changing the distance between the two bridge

while keeping the tension constant or (*ii*) by keeping the distance fixed and changing the tension.

B. Ex. : *In all experiments with the sonometer, the wire is required to vibrate in unison with the frequency of the source to be investigated.* To find when resonance occurs, place the tuning fork on the sonometer box and a small paper rider on the vibrating length Alter either the distance between the bridges or the tension on the string till the rider is thrown off. This happens when the natural frequency of vibration of the string agrees with that of the sounding body. At resonance, the string vibrates with a large amplitude and consequently throws off the rider. When there is a slight mistuning it can be detected by the beats that occur between the two sounds. One of the bridges may then be adjusted till the beats disappear. An experimenter with a good musical ear can, without the use of a paper rider, obtain this matching, or "tuning" as it is technically called, by listening.

C. **Action of box holes.** Note that in the sonometer, as in the violin, the sound comes mainly from the vibration of the air in the wooden box. Holes in the sides of the box enable energy from the vibrating air inside the box to pass into the air outside. The vibrations of the string are communicated through the bridges to the top board of the sonometer box. This is thrown into forced vibration. Its vibration throws the air inside the box into forced vibration. Progressive waves come out of the box through holes in its sides. That is how we hear the sound of the vibrating air. *The string itself is a poor radiator of sound energy.*

D. **Action of fork.** What happens when the stem of a vibrating fork is pressed on the board of a sonometer? We have noted in fig. III-4.1 that the stem vibrates longitudinally when the prongs are set into (transverse) vibration. The up-and-down movement of the stem throws the board into forced vibration. These are communicated to the string through the bridges. That is how a vibrating fork affects the string.

The forced vibration of the board gets its energy from the fork. So the vibration of the fork lasts for a shorter time than when not pressed on the board. At resonance, the rate of energy transfer is quickest. Hence when resonance occurs, the vibration of the fork diminishes fastest.

Example. III-9.1. *A wire 50 cm long is under a tension of 25 kg. The mass of the wire is 1·44 g. Find the frequencies of the fundamental and the second harmonic.*

Solution : Here $T=25$ kgf $=25\times 1000\times 980$ dyn ; $m=1{\cdot}44$ g/50 cm For the fundamental $l=50$ cm Substituting these values in Eq. III-9.5.1, we get the fundamental frequency $=291{\cdot}6$/s. The second harmonic has twice this frequency.

Ex. III-9.2. *Two wires of the same material and diameter have lengths 54 cm and 36 cm and the same frequency. If the tension in the first is 9 kg, what is the tension in the second ?*

Solution : Here $n=\frac{1}{2l_1}\sqrt{\frac{T_1}{m}}=\frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$ or, $\sqrt{T_2}=\frac{l_2}{l_1}\sqrt{T_1}$.

Putting values, we get $T_2=4$ kgf.

Ex. III-9.3. *The tension in a wire is 10 kg. Another wire of the same material has double the length and diameter of the first. What should be the tension in the second so that its fundamental may be an octave higher than that of the first ?*

Solution : Let l_1, d_1 and l_2, d_2 be the lengths and diameters of the two wires respectively. ρ is the same for both. We have $l_2=2l_1$ and $d_2=2d_1$. If n_1 is the frequency of the first, n_2 will be equal to $2n_1$. Therefore $n_1=\frac{1}{2l_1}\sqrt{\frac{10\text{ kgf}}{\pi\rho d_1^2}}$ and $n_2=2n_1=\sqrt{\frac{T}{\pi\rho d_2^2}}$. From these we get $T=640$ kgf.

Ex. III-9.4. *Find the velocity of transverse waves in a string under a tension of 0·23 kg when its length is 65 cm and the mass 0·52 g. If it emits its fundamental what is the frequency ? ($g=981$ cm/s²)*

Solution : Tension in *absolute units* $=0{\cdot}23\times 1000\times 981$ dyn

Mass per unit length $=0{\cdot}52/65=0{\cdot}008$ g/cm

$\therefore$ Velocity of the wave $c=\sqrt{T/m}$

$$=\sqrt{\frac{0{\cdot}23\times 1000\times 981}{0{\cdot}008}}=5311\text{ cm/s}$$

Wavelength of the fundamental $=2\times 65$ cm

$\therefore$ Frequency of the fundamental $=c/\lambda=5311/130=40{\cdot}85$ Hz.

Ex. III-9.5. *A steel string 100 cm long is struck at a point 25 cm from one end. What are the possible modes of vibration and which harmonics will be missing ? If the vibrations were stopped at 25 cm, which harmonics will be present ?*

Solution : Those notes of vibration of the string in which it has a node at the point struck, will not be excited. The distance of the point struck is $25/100=\frac{1}{4}$ fraction of the total length. Hence the 4th, 8th, 12th etc. harmonics will have nodes at that point. They will not be excited.

In the second case, vibrations with a node at 25 cm can be present, *i.e.*, the 4th, 8th, 12th etc. harmonics.

Ex. III-9.6. *Two identical wires stretched under the same tension of 5 kg-wt emit notes in unison which are of frequency 300/s One wire has its tension increased by 100 gm-wt. Find the number of beats heard when they are plucked.*

Solution : Original tension of both the wires in absolute units

$= 5 \times 1000 \times 981$ dynes.

Subsequent increased tension of one of them $= 5{\cdot}1 \times 1000 \times 981$ dynes.

When sounding in unison the frequency of the second wire was 300.

$$\therefore \quad 300 = \frac{1}{2l}\sqrt{\frac{5 \times 1000 \times 981}{m}}$$

At the higher tension the frequency of the second wire is

$$n' = \frac{1}{2l}\sqrt{\frac{5.1 \times 1000 \times 981}{m}}$$

$$\therefore \quad \frac{n'}{300} = \sqrt{\frac{5.1 \times 1000 \times 981}{5 \times 1000 \times 981}} = \sqrt{\frac{5.1}{5}}$$

$$\therefore \quad n' = 300\sqrt{\frac{5.1}{5}} = 303.$$

Hence the number of beats per.sec is $n' - 300 = 3$.

Ex. III-9 7. *Two wires of the same material have lengths in the ratio 2 : 3. If their diameters are the same what must be the ratio of their tensions for the shorter wire to give a note an octave higher than the longer ?*

Solution : Let the lengths of the two wires be $2x$ and $3x$, their frequencies n and n', and the tensions T and T' respectively. Since diameters and materials are the same the linear densities are equal.

Hence $n = \frac{1}{4x}\sqrt{\frac{T}{m}}$ and $n' = \frac{1}{6x}\sqrt{\frac{T'}{m}}$

As n is an octave higher than n', we have $n = 2n'$.

$$\therefore \quad \frac{1}{4x}\sqrt{\frac{T}{m}} = \frac{2}{6x}\sqrt{\frac{T'}{m}} \quad \text{or} \quad T/T' = 16/9.$$

III-10

VIBRATION OF AIR COLUMNS

III-10.1. Stationary vibrations in Air Columns. We have seen above how a vibrating string generates stationary waves. The frequency of the vibrations or waves is the frequency of any one of the *natural modes* of vibration of the string. They can persist for long in a string. Such a wave must satisfy two end-conditions (known as *boundary conditions*). The boundary conditions of a stretched string fixed at both ends are that there must be two nodes at the two ends since the ends are rigidly fixed and there can be no motion of the string at either end.

Exactly similar considerations apply to the vibration of an air column in a pipe, open at one or both ends. Like the string it has *natural* (or *characteristic*) modes of vibration. The frequency of such a vibration is a *natural frequency*. It is the frequency of a stationary wave which can persist in the air column. To do so the wave must satisfy boundary conditions at the two ends of the pipe. So, our arguments in finding the natural frequencies of an air column will be similar to those in the case of strings.

But the waves here are *longitudinal* in character. The air column is enclosed in a pipe, and is excited by blowing at one end. The compression and rarefactions started by the blowing, travel along the air column, and are reflected at the other end. Stationary waves are produced by the interference between the incident and reflected wave-trains passing through the air column in opposite directions, as discussed below.

We shall consider the air column to be contained in (i) a pipe *closed at one end and open at the other* or (ii) a pipe *open at both* ends : they are called a **closed pipe** and an **open pipe** respectively. It will be *assumed* that (a) the diameter of the pipe is small compared with the length of the pipe and the wavelength of sound, and (b) the walls of the pipe are rigid.

III-10.2. Closed pipe. A vibrating tuning fork, when held over the open end of a pipe, the other end of which is closed, sends a longitudinal

wave-train down the pipe. It is reflected from the closed end which is a rigid wall. Superposition of the two identical but oppositely-directed wave-trains produces stationary waves in the air column, generally of *small amplitude*.

Since the close end is rigid, air particles at this end cannot move. So a node occurs at the closed end. Again, at the open end the air particles have the maximum freedom of movement. So the displacement there will be a maximum, so as to produce an antinode at the open end. Stationary waves which satisfy these two boundary conditions can persist in the tube. Therefore, the natural modes of vibration of an air column in a closed pipe are such that **there is a displacement node at the closed end and an antinode at the open end.** Stationary waves which fulfil this condition can have various wavelengths as shown below.

We know that the separation between a node and an antinode is $\lambda/4$ or an odd multiple thereof. Hence, *the air column in a closed pipe will so vibrate as to produce only those waves that would make the pipelength equal to an odd multiple of* $\lambda/4$. Hence the relation between the pipe-length will be

$$l=(2m-1)\,\lambda_m/4 \text{ or } \lambda_m=4l/(2m-1) \qquad \text{(III-10.2.1)}$$

where $m=1, 2, 3$ etc.,

If c be the velocity of the sound waves produced, then *the frequency of the emitted tone* will be

$$n_m=c/\lambda_m=(2m-1)c/4l \qquad \text{(III-10.2.2)}$$

Fig. III-10.1

A. Fundamental in a closed pipe. If in the above eqn. we put $m=1$ then we have $n_1=c/4l$ (III-10.2.3)

This is the **fundamental tone** or the **first harmonic**, the lowest frequency that the given pipe can emit. Putting $m=1$ in eqn. III-10.2.1 we get the relation between the length of the pipe and the wavelength produced, which is $\lambda_1=4l$. This is the *longest wave* produced by the pipe.

Physically it represents the simplest mode of vibration with a node and an antinode at the two ends and no other in the pipe.

The mode of vibration of an air column in a closed pipe emitting the fundamental is shown in fig. III-10.1. The amplitude of vibration of air particles gradually falls off from the open end towards the closed one. The *entire column alternately moves towards the node* (closed end) *and away from it.* The pressure variation is a maximum at the displacement node and a minimum at the antinode. Fig. III-10.1 (b) shows the disposition of air layers throughout the length of the pipe when the compression at the closed end is highest.

[We may imagine the amplitude at any place in the pipe to be represented by a horizontal line drawn at the place and confined between the broken lines in fig. III-10.1 *(a)* In this and subsequent figures though the amplitudes are indicated as if the vibrations are transverse, it must be remembered that they are *longitudinal in nature and parallel to the pipe axis.*]

B. Higher harmonics of a closed pipe. In the mode of vibration next in simplicity, there is another pair of node (n_1) and antinode (a_1)

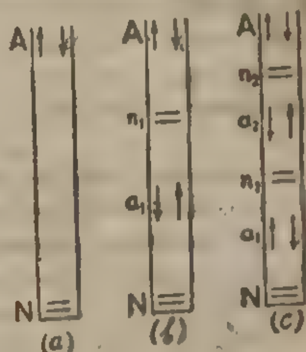


Fig. III-10.2

between the ends of the pipe (fig. III-10.2 *b*) is λ_2 then putting $m=2$ in eqn. III-10.2.2 we have, the next frequency as

$$n_2 = c/\lambda_2 = 3c/4l = 3n_1. \qquad \text{(III-10.2.4)}$$

This is the **first overtone** or the **third harmonic.** *The frequency is thrice that of the fundamental.* The value of the corresponding wave-length is obtained by putting $m=2$ in eqn. III-10.2.1 and $\lambda_2 = 4l/3$.

If there be two pairs of nodes and antinodes between the ends of the pipe (fig. III-10.2c) then there will be five quarter-wavelengths in the length of the pipe, *i.e.*, $l = 5\lambda_3/4$ or $\lambda_3 = 4l/5$. The corresponding frequency will be

$$n_3 = c/\lambda_3 = 5c/4l = 5n_1 \qquad \text{(III-10.2.5)}$$

This is the same as obtained by putting $m=3$ in eqn. III-10.2.2. This is the **second overtone** or the **fifth harmonic.**

Putting $m=4, 5$ etc. in succession in Eq. III-10.2.1 and III-10.2.2 we can find the wavelengths and frequencies of the higher overtones. The corresponding notes are the 7th, 9th etc. harmonics. This analysis from Eq. III-10.2.2 shows that **only the odd harmonics can be obtained from a closed pipe.**

As in a string, it is difficult to excite only one mode of vibration; more than one are almost always present. The particular overtones excited and their intensity relative to the fundamental depend on the mode of excitation. The **quality** of the emitted note depends upon the overtones present and their intensities relative to that of the fundamental.

III-10.3 Open pipe. To find the natural modes of vibration and the frequencies of an open pipe, we determine the kinds of stationary waves that satisfy the boundary conditions at the ends of the pipe. Remember, in a stationary wave the particle displacement is zero at a node. It gradually increases as we move away from the node and reaches a maximum at the nearest antinode. Thereafter, it falls to zero again at the next node.

In an open pipe, the air particles at both the ends have the maximum freedom to move. Hence **only those stationary waves can persist in an open pipe which have maximum displacement antinodes, at the ends.** This is the basis to calculate the natural modes and the corresponding frequencies of vibration of an open pipe. The natural modes of vibration in an open pipe will be those that have two displacement antinodes at its two ends.

Now, the distance between two antinodes is a multiple of $\lambda/2$ and given by $m\lambda_m/2$ where m is an integer 1, 2, 3, etc. Hence the relation between the pipe-length and the wavelength will be given by

$$l=m\lambda_m/2, \text{ or } \lambda_m=2l/m \qquad \text{(III-10.3.1)}$$

and the frequency of a note will be

$$n=c/\lambda_m=m.c/2l. \qquad \text{(III-10.3.2)}$$

As in a closed pipe or vibrating string, it is very difficult to excite one mode only. Depending on the method of excitation, several hormonics, besides the fundamental, are simultaneously present. Quality of a note is determined as in the other related cases.

Fundamental and harmonics in an open pipe. The simplest possible mode of vibration of air in an open pipe is that in which there is

no antinode in the tube except at the two ends. Denoting the corresponding wavelength by λ_1 we shall have $l = \lambda_1/2$ or $\lambda_1 = 2l$. The frequency n_1 corresponding to the wavelength is

$$n_1 = c/\lambda_1 = c/2l \qquad \text{(III-10.3.3)}$$

This is the **fundamental.** It can also be obtained by putting $m = 1$ in Eq. III-10.3.2. A single node will be in the middle of the pipe. The amplitude and disposition of the air layers throughout the length of the pipe are indicated in fig. III-10.3(*a*) and (*b*).

Putting $m = 2$ in Eq. III-10-3.1 and III-10-3.2 we obtain

$$l = \lambda_2 \text{ and } n_2 = c/\lambda_2 = c/l = 2n_1. \qquad \text{(III-10-3.4)}$$

Fig. III-10.3

This is the next mode of vibration and the emitted tone is the **first overtone** or the **second harmonic.** Its frequency is twice that of the fundamental. Physically, there are three antinodes and two nodes in the tube as shown in Fig. III-10.4(*b*).

Putting $m = 3$ in Eq. III-10.3.1 and III-10.3.2 we get $l = 3\lambda_3/2$ and $n_3 = c/\lambda_3 = 3c/2l = 3n_1$. (III-10.3.5)

This is the **second overtone** or the **third harmonic.** It has a frequency thrice that of the fundamental. In this case four antinodes and three nodes occur in the pipe (fig. III-10.4(*c*)).

Putting $m = 4$, 5, 6 etc in Eq. III-10 3.1 and III-10 3.2, we get the wavelengths and frequencies of the higher harmonics.

In the open pipe also, a number of overtones are simultaneously present along with the fundamental, whenever it is sounded. In contrast with the closed pipe, **all harmonics, odd as well as even, may be given out by an open pipe.**

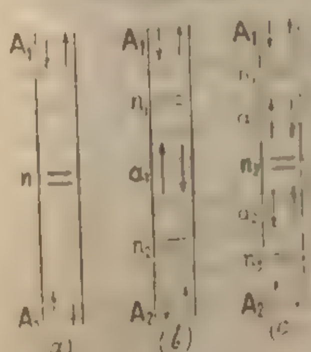


Fig. III-10.4

In all modes of vibration, the portions of the air column on two sides of a node simultaneously move towards the node or away from it. Fig. III-10.3(b) represents the relative positions of normally equidistant air layers as they all move towards a node. The arrows in Fig's III-10.4 represent this kind of motion.

III-10.4. Comparison between closed and open pipes.

Closed pipe	Open pipe
1. Must have a displacement antinode at the open end and a displacement node at the closed.	1. Must have displacement antinodes at both ends and displacement node at the middle.
2. Possible wavelengths $\lambda_m = 4l/(2m-1)$ where $m=1, 2, 3,$ etc.	2. Possible wavelengths $\lambda_m = 2l/m$ where $m=1, 2, 3,$ etc.
3. Possible frequencies $n_m = c/\lambda_m = (2m-1)\, c/4l$.	3. Possible frequencies $n_m = c/\lambda_m = mc/2l$.
4. For the fundamental $m=1$, $\lambda_1 = 4l$, $n_1 = c/4l$.	4. For the fundamental $m=1$, $\lambda_1 = 2l$, $n_1 = c/2l$.

Therefore, *for pipes of same length the fundamental of an open pipe is an octave above that of the closed one.* To produce the same fundamental, an open pipe should have double the length of the closed one.

Closed pipe	Open pipe
5. The *first overtone* has $m=2$, so that $\lambda_2 = 4l/3$, $n_2 = 3n_1$. This is the *third harmonic.*	5. *First overtone* has $m=2$, $\lambda_2 = l$ and $n = 2n_1$. This is the *second harmonic.*
6. Second overtone has $m=3$, $\lambda_3 = 4l/5$, $n_3 = 5n_1$. It is the fifth harmonic.	6. Second overtone has $m=3$, $\lambda_3 = 2l/3$, $n = 3n_1$. It is the third harmonic.
7. **Only odd harmonics can be produced.**	7. **All harmonics, both odd and even, can be produced.**

Fig. III-10.5

Fig. III-10.6

Fig.s III-10.5 and III-10.6 show the number, position etc. of the nodes and antinodes in the two types of pipes as each emits its first and second overtones respectively. Every successive overtone has one node and one antinode more than the preceding overtone.

Example.III-10.1. *A closed organ pipe 24.5 cm long is in unison with a tuning fork of frequency 256. If its length is increased by 2 mm, find the number of beats produced when the fork and the pipe are sounded together.* [**And. U.**]

Solution : Assuming that the pipe is sounding its fundamental we have from the given data $256 = c/(4 \times 24.5)$
where c is the velocity of sound. Then $c = 256 \times 4 \times 24.5$
When the length is increased, the frequency becomes

$$n = c/4l = 256 \times 4 \times 24.5/(4 \times 24.7) = 253.9 \text{ or } 254 \text{ (approx.)}$$

$\therefore$ The number of beats $= 256 - 254 = 2$ per sec.

Ex. III-10.2. *Two organ pipes, one closed and the other open, are respectively 2.5 ft. and 5.2 ft long. When sounded together 4 beats are heard per second. Find the velocity of sound.* **[Pat. U]**

Solution : If n and n' are the fundamental frequencies of the closed and open organ pipes respectively, then

$$n = c/4l, \quad n' = c/2l \quad \text{and} \quad n - n' = 4.$$

$$\therefore \quad n = c/(4 \times 2.5) = c/10 \text{ and } n' = c/(2 \times 5.2) = c/10.4.$$

$$\therefore \quad c\left(\frac{1}{10} - \frac{1}{10.4}\right) = 4 \text{ or } c = 4 \times 10 \times 10.4/0.4 = 1040 \text{ ft/s.}$$

Ex. III-10.3 *A swimmer sends a sound signal from a lake bottom to the top. 5 beats are heard with the fundamental of a 20 cm long closed tube. Find the wave-length of sound in water given that velocity of sound in air and water are 360 m/s and 1500 m/s respectively.* [I. I. T '74]

Solution : Let n be the frequency of the sound signal. Then that of the closed pipe will be $n \pm 5$. So we have

$$n' = n \pm 5 = C_A/4l = 360/4 \times 0.2 = 450/s$$

$$\therefore \quad \lambda = C_W/n' = 1500/455 \text{ or } 1500/445 = 3.3 \text{ m or } 3.37 \text{ m}$$

Ex. III-10.4 *A 25 cm long string with a mass of 2.5 g under tension produces 8 beats with a closed pipe 40 cm long, when the string sounds its first overtone and the pipe its fundamental. On decreasing the tension beat frequency falls. Find the tension in the string if sound speed in air is 320 m/s.* [I. I. T. '82]

Solution : $n_S = \frac{2}{2l}\sqrt{\frac{T}{m}} = \frac{2}{50}\sqrt{\frac{T}{2.5/25}} \quad n_P = \frac{c}{4l} = \frac{320}{4 \times 0.4} = 200/s$

From the given condition $n_P = 200 \pm 8$

But as decreasing the tension lowers n_S and beat frequency also falls we conclude $n_S > n_P$ *i.e.*, $n_S = n_P + 8$

$$\therefore \quad 208 = \frac{1}{25}\sqrt{10\,T} \quad \therefore \quad 10\,T = (208 \times 25)^2 \quad \text{or} \quad T = 26.04 \text{ megadynes.}$$

III-10.5. Vibrating air columns as sources of sound. When we speak of a source of sound, we understand that, it is a vibrating body which generates sound waves in air. Air columns in closed or open pipes can vibrate with characteristic frequencies, and they are stationary vibrations. How can they serve as sources of sound, giving rise to progressive waves ?

Stationary vibration of air columns in closed or open pipes can be generated in more than one way. A common method is to blow across the mouth of the pipe. This blowing sets up waves within the pipe. These are reflected repeatedly from the two ends of the pipe (so long as the blowing persists), forming stationary waves. Though it is surprising, a large part of the wave inside a pipe is reflected back into the pipe at each incidence *at an open end*. A small part of these incident waves there pass out through the opening and spread out as progressive spherical waves. This is how an open or closed pipe serves as a source of progressive waves and hence is a source of sound.

III-10.6. Determination of the frequency of a fork or Velocity of Sound by resonance with an air column. The frequency of a fork may be determined by setting up resonance with it of a column of air of adjustable length. The frequency can then be determined from the length of the column if the velocity of sound in the air column is known. An open pipe which has one end immersed in water works as a closed pipe. The length of the enclosed air column can be altered by varying the depth of immersion.

To find the frequency of the fork, the vibrating fork is held at the

fig. III-10.7

open mouth of the tube which has been sunk to the maximum depth.

The tube is then slowly raised along with the vibrating fork (fig. III-10.7). At a particular position of the tube the air in the tube will be thrown into resonant vibration by the fork and emit a fairly loud sound. Since we started from a very short length the note emitted by the air column must be its fundamental. If c is the velocity of sound in the air column, l the length of the resonant air column and n the frequency, then from Eq. III-10.2-3. we get $n = c/4l$.

The *velocity* so determined *is that in moist, saturated air at the temperature of the room.* Velocity increases with temperature and moisture

Knowing c and measuring l we can find n, the frequency of the fork. If we know n, we can, from the relation $c = 4ln$, find c. Thus the velocity of sound in the most air of the tube can be determined in the laboratory.

If the pipe is filled with a gas other than air, the velocity of sound in the gas can be found by the same method.

End Correction. Lord Rayleigh, has shown theoretically that if stationary waves are generated in a tube, the antinode is not formed exactly at the open end of the tube but lies a little beyond it. If the radius of the tube is r, the antinode will be at a distance $0.6r$ away from the open end.

Therefore, if the actual length of the tube is l, the distance between the node at the closed end and the antinode at the open end is $(l + 0.6r)$. To correct for this effect at the end we should write

$$n = c/4(l + 0{\cdot}6r) \qquad \text{(III-10-6.1)}$$

Ex. III-10.5 *An open pipe, 30 cms long, and a closed pipe, 23 cms long, both of the same diameter, are each sounding its first overtone which are in unison. What is the end-correction of the pipes ?* [Lond. Inter]

Solution : When the open pipe is sounding its first overtone, the length of the vibrating air-column will be $30 + 2e$ since *there are two ends.* If the wave-length is λ, then

$$\lambda = c/n = 30 + 2e \qquad \text{(A)}$$

where c is the velocity of sound and n the frequency of the first overtone of the open pipe.

When the closed pipe is sounding its first overtone, the length of the vibrating air column must be $23 + e$. Since the tube is sounding its first overtone, the vibrating length will be $3\lambda/4$.

$$\therefore \quad \frac{3\lambda}{4} = 23 + e \text{ or } \frac{3c}{4n} = 23 + e. \qquad \text{(B)}$$

Since the frequencies are the same, we get from relations (A) and (B)

$$23 + e = \tfrac{3}{4}(30 + 2e).$$

$$\therefore \quad 4e + 92 = 90 + 6e \text{ or } e = 1 \text{ cm}$$

Alternative method. There is an alternative method of finding n or c by eliminating the end effect. If we keep on increasing the length of the air column beyond the first resonance, a second resonance will occur when the length of the tube is nearly three times its previous value. The tube then emits its first overtone or third harmonic.

Let the wavelength of the note emitted by the fork be λ. If l_1 is the length at the first resonance, then

$$\lambda/4 = l_1 + 0{\cdot}6r = b_1 + e \qquad \text{(A)}$$

It l_2 be the length of the vibrating column at the second resonance then

$$3\lambda/4 = l_1 + 0{\cdot}6r = b_2 + e \qquad \text{(B)}$$

[The matter will be clearly understood if we imagine the closed end, *i.e.*, the water surface in the tube, to lie at N_2 in fig. III-10.6 (left) at the time of the first resonance, and at N_1 at the second.]

Subtracting (A) from (B), we have $\lambda/2 = l_2 - l_1$

$$\therefore \quad c = n\lambda = 2n(l_2 - l_1) \qquad \text{(III-10.6.2)}$$

This relation is free from end correction.

A simple form of the resonance tube is shown in fig. III-10.7. A more elaborate apparatus is shown in fig. III-10.8. It consists of a metal tube

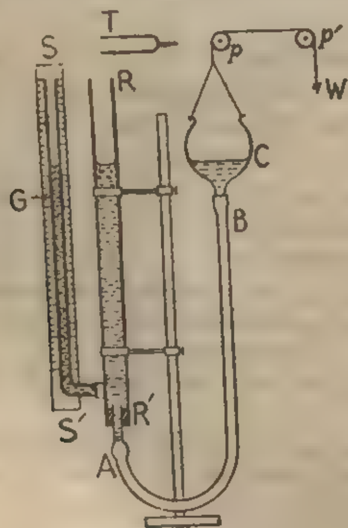


Fig. III-10.8

RR' about 5 cms in diameter and 1 metre long. A rubber tubing AB connects it to a metal container C. The container and the tube are partly

filled with water as shown. The level of water in the tube can be changed by raising or lowering C. A number of strings from the periphery of the container is tied together to a stout string W which passes over two pulleys p and p'. C can be raised or lowered by pulling or slackening this string. A narrow glass tube G is connected by a side tube to RR' and stands in front of a scale SS'. The tops of G and RR' are at the same level so that the length of the air column can be read off from the scale. To carry out an experiment, a sounding tuning fork T is held over RR' and the water level is adjusted by manipulating W till resonance is obtained. Several readings for resonance are taken both when the length of the air-column is decreasing and when it is increasing.

Experiment. To determine the velocity of sound in air. The container is raised so that the length of the air-column is very small. Now, holding a vibrating fork of known frequency over the tube, C is slowly lowered so that the length of the air column in RR' slowly increases. At a particular length the air in the tube will resonate producing a loud sound. When the sound is loudest, the length of the air-column is read off from the water-level in G. C is lowered a little more and then raised to find the resonating length. The observations are repeated.

The mean gives the length for the first resonance and the tone emitted by the air-column is its fundamental tone.

The container is then lowered continuously till a second resonance is obtained. The readings for the resonating lengths, both when C is being raised and lowered, are taken as before and the mean value obtained. This length gives out the first overtone which is the third harmonic.

III-10.7. Effect of temperature and humidity on the frequency of an air column. The frequency n of the note emitted by an air column is given by $n=(2m+1)\,c/4l$ or $n=mc/2l$ according as the column is closed or open. Here $m=1, 2, 3,\ldots$etc., $c=$ velocity of sound in air and l the length of the column. Factors which affect c will also affect n. So if the temperature increases, n will increase as the square root of the absolute temperature ; n will also increase when the air in the column is more moist, as the density of moist air is less than that of dry air.

Ex. III-10.6 *A fork of frequency 250 produces resonance when the length of air column in a tube is 31 cms and again, when it is 97 cms. Find the velocity of sound in air and the diameter of the tube.* [U. P. B.]

Solution : The difference between successive resonant lengths is half the wavelength. Therefore, the velocity is $c = 2n(l_2 - l_1) = 2 \times 250 \times (97-31)$ cm/sec $= 330$ metres/sec. Now from eqns A and B above we get.

$$e = \tfrac{1}{2}(l_2 - 3l_1) = \tfrac{1}{2}(97-93) = 2 \text{ cms.}$$

$$\therefore \; d = e/0.3 = 6.7 \text{ cms.}$$

Ex. III-10.7 *A tuning fork held over a resonance tube shows resonance with air columns 24 cms and 74.1 cms long. Find the frequency of the fork, the end correction of the tube and the room temperature, if the velocity of sound is 340 metres/sec, at room temperature and 330 metres/sec at 0°C.* **[Utk. U.]**

Solution : From Eq. III-10-6.1, the frequency of the fork is

$n = c/2(l_2 - l_1) = 34000/\{2 \times (74.1 - 24)\} = 34 \times 10^3/(2 \times 50.1) = 339.3$ per sec.

The end correction $e = \frac{1}{2}(l_2 - 3l_1) = \frac{1}{2}(74.1 - 72) = 1.05$ cms.

To find the room temperature, we apply Eq. III 4-6.3 $c'/c = \sqrt{T'/T}$.

Hence $$\frac{273+t}{273} = \left(\frac{340}{330}\right)^2, \text{ or } 1 + \frac{t}{273} = \left(1 + \frac{1}{33}\right)^2 = 1 + \frac{2}{33}$$

neglecting $(1/33)^2$ which is very small.

$$\therefore \; t = 2 \times 273/33 = 16.5°\text{C}.$$

Problems : (1) A closed pipe, 25 cm long, is filled in turn with (*a*) hydrogen, (*b*) air and (*c*) oxygen, under the same conditions of temperature and pressure. The densities of the gases, proportional to their molecular weights, are in the ratio 1 : 14·4 : 16. If the velocity of sound in air 330/ms, find the frequencies of the fundamentals in the three gases. [**Ans.** 330 m/s, 1252 m/s, 313 m/s.]

(2) The frequency of a note emitted by an open air column at 10°C is 100 Hz. What will be the value at 30°C ? [**Ans.** 103·5 Hz.]

(3) Two open organ pipes give 6 beats per second when sounded together in air at 10°C. What will be the number of beats at 24°C ? [**Ans.** 6·15 per second.]

EXERCISES

III-1. (Simple Harmonic Motion)

[A] Essay type questions :

1. Define simple harmonic motion in two ways. What are the characteristics of S.H.M. ?

2. From the definition of Simple harmonic motion, derive expressions for (i) the displacement of the particle, (ii) its velocity and (iii) its acceleration. Under what conditions will they assume maximum and minimum values ?

3. Establish the equation of S.H.M. Prove that the total of kinetic and potential energy at any instant in an S.H.M. is constant. **(H. S. '81, '82)**

4. How does energy in S.H.M. depend on the mass of the particle and the amplitude and frequency of the motion ? At which points in S.H.M. are the kinetic energy and the potential energy of the particle a maximum ? Where are they zero ?

5. (a) What do you understand by phase in S.H.M. ? What are phase angle and phase difference ?

(b) Explain how you can compose graphically two collinear S.H.M.'s of same period and phase but of different amplitudes ? What will happen, if the phases are opposite ? **(H. S. '81)**

6. Give a precise definition of S.H.M. and discuss how far the motion of the simple pendulum is simple harmonic. **(J. E. E. '76)**

7. Show that motion of each of the following is simple harmonic and find the time period :

(i) loaded vertical elastic spring,
(ii) cylinder floating vertically in a liquid,
(iii) liquid in a U-tube,
(iv) gas enclosed in a cylinder under a piston,
(v) body suspended by a wire,
(vi) horizontal bar magnet in earth's magnetic field,
(viii) body dropped in the hole bored along the diameter of earth.

[B] Short answer type questions :

8. A copper ball suspended on a spring performs vertical oscillations. How will the period of oscillations change

(a) if an aluminium ball of the same radius is attached to the spring instead of the copper one ?

(b) if the copper ball is immersed in a nonviscous liquid of density one-tenth that of copper ?

9. Give some examples of motion that are approximately simple harmonic. Why are motions that are exactly simple harmonic very rare ?

10. A point mass m is suspended at the end of a massless wire of length l and cross-section A. If Y is Young's modulus for the wire, obtain the frequency of oscillation for its simple harmonic motion along the vertical line. (I.I.T. '78)

11. A spring has a force constant k, and a mass m is suspended from it. The spring is cut in half and the same mass is suspended from one of the halves. Is the frequency of vibration the same before and after the spring is cut ? How are the frequencies related ?

12. Suppose we have a block of unknown mass and a spring of unknown force constant. Show how we can predict the period of oscillation of this block-spring system simply by measuring the extension of the spring produced by attaching the block to it.

13. You have a light spring, a metre scale and a known mass. How will you find the time period of oscillation of the mass attached to the spring without using a clock ? (I.I.T. '74)

14. Predict by qualitative arguments whether a pendulum oscillating with large amplitude will have a period longer or shorter than the period for oscillations with small amplitude.

[C] Numerical problems :

15. Write down the equations of harmonic motions with amplitude of 0·1 m, a period 4 s and an initial phase zero. If the initial phase is 45°, then what will be its equation ? [40·1 sin $(0{\cdot}5\pi t+\pi/u)$ m.]

16. The equation of motion of a point is given as $x=2\sin\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)$ cm. Find (i) the period of oscillation, (ii) the maximum velocity of the point and (iii) its maximum acceleration. [4s, 3·14 cms^{-1}, 4·93 cm^{-2}]

17. The equation of motion of a point is given as $x=\sin\frac{\pi}{6}t$. Find the moments of time at which the maximum velocity and acceleration are attained. [0, 6, 12s etc, 3, 9, 1 s etc.]

18. Write down the equation of a harmonic oscillatory motion if the maximum acceleration of the point is 49·3 cms^{-2}, the period of oscillation 2s and the displacement of the point from equilibrium at the initial moment 25 mm. [5 sin $\left(\pi t+\frac{\pi}{6}\right)$ cm.]

19. The initial phase of an harmonic oscillator is zero. When the point deviates by 2·4 cm from the position of equilibrium, its velocity is 3 cms^{-1} and by 2·8 cm when it is 2 cms^{-1}. Find the amplitude and period of this oscillation. [3·1 cm, 4·1s.]

20. The initial phase of a harmonic oscillator is zero. After the lapse of what fraction of the period will the velocity of the point be equal to half its maximum velocity ? [1/6. T]

21. Show that the when a particle in S. H. M. is at a distance of $\sqrt{3}/2$ times its amplitude, its velocity is half the maximum velocity.

22. What is the displacement of a particle in S. H. M. When its kinetic and potential energies are equal ? [amplitude $\div \sqrt{2}$]

23. What is the ratio between kinetic and potential energies of a particle in S. H. M. for the moments of time (i) $t=\frac{T}{12}$s, (ii) $t=\frac{T}{8}$s, (iii) $t=\frac{T}{6}$s ? The initial phase of oscillation is zero. [3, 1, 1/3]

24. What is the relationship between kinetic and potential energies of a particle in S. H. M. for the moments when the displacement of the point from the position of equilibrium is (i) $x=\frac{a}{4}$, (ii) $x=\frac{a}{2}$, (iii) $x=a$, where a is the amplitude of oscillations. [15, 3, 0]

25. Show that if the displacement of a particle moving at any time is given by an equation of the form $x=a \cos \omega t+b \sin \omega t$, the motion is simple harmonic. If $a=3, b=4$ and $\omega=2$, find the period, amplitude, maximum velocity and maximum acceleration of the motion. **(Madras)**

[**Hint :** Show that $f=-\omega^2 x$] [3·142, 5, 10, 20]

26. A particle executing an S. H. M. has a maximum displacement of 4 mm and its acceleration at a distance of 1 mm from its mean position is 3 mms^{-2}. What will be the velocity at a distance of 2 mm from the mean position ? [6 mms^{-1}]

27. A body executing S. H. M. has an amplitude of 10 cm and time period of 1·s. Calculate the time taken by the body to travel a distance of 5 $\sqrt{3}$ cm from its rest position. **(Patna)** [0.25 s]

28. A steel strip, clamped at one end vibrates with a frequency of 20 Hz and an amplitude of 5 mm at the free end, where a small mass of 2 g is positioned. Find (i) the velocity of the end when passing through the zero position, (ii) the acceleration at maximum displacement, (iii) the maximum kinetic and potential energies of the mass.

[0·628 ms^{-1}, 79 ms^{-2}, 3·9 $\times 10^{-4}$ J, 3·9 $\times 10^{-4}$ J]

29. A small coin is placed on a horizonal platform connected to a vibrator, the amplitude of which is 0.08 m and is kept constant as the frequency is increased from zero. At what frequency the coin will be heard chattering ? Take $g=9·8$ ms^{-2}. [1·8 Hz]

30. A particle is moving with S. H. M. in a straight line when the distance of the particle from the equilibrium position has the values x_1, and x_2 and corresponding velocities u_1 and u_2. Show that the period is $2\pi[(x_2{}^2-x_1{}^2)/(u_1{}^2-u_2{}^2)]^{\frac{1}{2}}$.

31. A block is placed on a horizontal platform which is moving vertically with S. H. M. of amplitude 10 cm. Above a certain frequency, the thrust between the particle and the platform would become zero at some point in the motion. What is the frequency and at what point in the motion does the particle lose contact with the platform ? [1·6 Hz]

32. A particle which is performing an S.H.M. of period T about the centre O and it passes through a point P with velocity v in the direction OP. Prove that the time which elapses before its return to P is $(T/\pi) \tan^{-1} (vT/2\pi.OP)$.

33. A block on a horizontal surface is moving with an S. H M. of frequency 2 oscillations per second. The coefficient of static friction between the block and plane is 0·50. How great can the amplitude be if the block does not slip along the surface ? [3·1 cm.]

34. The vibration frequency of atoms in solids at normal temperatures are of the order 10^{13}/s. Imagine the atoms to be connected to one another by springs. Suppose that a single silver atom vibrates with this frequency and that all the other atoms are at rest. Then compute the force constant of a single spring. One mole of silver has a mass of 108 g and contains $6·02 \times 10^{23}$ atoms. [$2·34 \times 10^{-18}$ dyn cm^{-1}]

35. An automobile can be considered to be mounted on a spring as far as vertical oscillations are concerned. The springs of a certain car are adjusted so that the vibrations have a frequency of 3·0 per second. What is the spring's force constant if the car weighs 3200 lb ? What will the vibration frequency be if five passengers, averaging 160 lb each, ride in the car ? [$11·4 \times 10^{3}$ lb wt/in, 2·8 Hz.]

36. The end of one prong of a tuning fork which executes S. H. M. of frequency 1000 per second has an amplitude of 0·40 mm. Neglect damping and find (i) the maximum acceleration and maximum speed of the end of the prong and (ii) the speed and acceleration of the end of the prong when it has a displacement 0·20 mm.

[$1·6 \times 10^{4}$ ms^{-2}, 1·5 ms^{-1}, 2·2 ms^{-1}, $7·9 \times 10^{3}$ ms^{-2}.]

37. When a metal cylinder of mass 0·2 kg is attached to the lower end of a light helical spring the upper end of which is fixed, the spring extends by 0·16 m. The metal cylinder is then pulled down a further 0·08 m. (i) Find the force that must be exerted to keep it there if Hooke's law is obeyed. (ii) The cylinder is then released. Find the period of vertical oscillations, and the kinetic energy the cylinder possesses when it passes through its mean position. [1·0 N, 0·8 s, 0·045 J.]

38. A helical spring gives a displacement of 5 cm for a load of 500 g. Find the maximum displacement produced when a mass of 80g is dropped from a height of 10 cm on to a light pan attached to the spring. [5 cm.]

39. A test tube of weight 6g and of diameter 2 cm is floated vertically on water by placing 10g of mercury at the bottom of the tube. The tube is depressed by a small amount and then released. Find the time period of oscillation. **(Bihar).** [0·4527 s.]

40. The rise and fall of the tide at a certain harbour may be taken to be simple harmonic, the interval between successive high tides being 12 hr 20 m. The harbour entrance has a depth of 10 m at high tide and 4 m at low tide. Find how soon after a low tide, a ship drawing 8·5 m can pass through the entrance. [4 hr. 6 m. 40 s.]

41. A cylindrical wooden block of cross-section 15 cm^{2} and mass 230 g is floated over water with an extra weight 50g attached to its

bottom. The cylinder floats vertically. From the state of equilibrium, it is slightly depressed and released. If the specific gravity of wood = 0·3 and $g = 980$ cm s^{-2}, find the frequency of oscillation of the block. [0·79.]

42. A uniform U-tube is filled with water to a height of 30 cm. When water level is pushed in one arm through a small distance and then released, find the time period of oscillation of the water column. : **(U. P., Delhi**) [1·098 s]

43. A bask of volume 500 cc. has a narrow necked tube of area $\pi/4$ cm^2. The flask contains a gas enclosed by a small ball at the neck. The mass of the ball is 8g and the whole vessel is thermally isolated from the surrounding. If the ball is pushed a small distance and then released, the ball is found to move up and down. If the ratio of specific heats of the gas is 1·4, find the period of oscillation. The atmospheric pressure is 10^6 dyn/cm^2. [0·338 s]

44. A particle of mass m is attached to the mid-point of a wire of length l, stretched between two fixed points. If T be the tension in the wire, find the frequency of the lateral oscillations. $\left[\frac{1}{2\pi}\sqrt{\frac{T}{ml}}\right]$

45. Two charges $+Q$, $+Q$ are placed at a distance of $2a$ from each other. Another charge q is at the mid-point. If the charge q is displaced to either of the sides, find the time period of oscillation. If the charge q be negative and it is laterally displaced by a small amount, find the period of oscillation. $\left[2\pi\sqrt{\frac{a^3m}{4Qq}},\ 2\pi\sqrt{\frac{a^3m}{2Qq}}\right]$

46. The period of vibration of a magnetic needle in the earth's magnetic field is 2s. It is broken into two identical halves. Calculate the period of vibration of each half. [1s.]

47. If a small magnet makes 12 oscillations per minute in a field of 0·37 oersteds what additional field will be necessary for it to make 20 oscillations per minute at the same place ? (0·66 oersted)

48. A straight frictionless tunnel is bored through the earth from one point of its surface to another. If an object is dropped into the tunnel find the period of oscillation of the object. ($g = 981$ cms^{-2} and radius of the earth $= 6·38 \times 10^6$ m.) (84·4 s)

49. If the mass of the spring m is not negligible but is small compared to the mass M of the object suspended from it, show that the period of motion is $T = 2\pi\sqrt{(M + m/3)/k}$.

III.-2. (Vibrations)

[A] **Essay type questions :**

1. Explain the terms periodic motion, period, frequency and amplitude of vibration. Give two examples of periodic motion.

2. What are meant by free and damped vibrations. Explain the nature of damped vibration with an example.

3. Distinguish between free and forced vibrations, giving examples.

4. Distinguish between forced vibration and resonance, giving examples. What do you mean by sharpness of resonance ?

[B] **Short answers type questions :**

5. Give one example of each of transverse, longitudinal and torsional vibrations.

6. When the base of a vibrating tuning fork is pressed against a table the intensity of the sound is very much increased. Explain how this extra energy is obtained and show that the principle of conservation of energy is not violated. (J. E. E. '75)

7. Why do soliders break steps while marching over a wooden bridge ?

8. Why are a number of strings fixed in stringed instruments ?

9. Why are stringed musical instruments provided with a hollow box ?

10. 'Empty vessel sounds much'—Explain the statement.

11. Why is the string of a sonometer made of a thin wire while the speaker of a radio-receiver is made of a thick paper cone ?

12. Why are damping devices often used in machineries ?

III.-3. (Wave Motion)

[A] **Essay type question :**

1. What do you understand by a wave ? What are transverse and longitudinal waves ? Explain with illustrations. Compare a transverse wave with a longitudinal wave.

2. Take any material wave as an example and illustrate with its help what happens in wave motion as also the nature of wave motions.

4. What is meant by compressions and rarefractions in a longitudinal wave ? In which directions do particles of the medium move in two such portions of a wave ?

4. Discuss the characteristics of wave motion.

5. What is a simple harmonic wave ? What do you understand by the amplitude, period, time, frequency and wavelength of such a wave ?

Find the relation between wavelength and wave velocity.

6. Explain the terms periodic wave and wave-form. What importance would you attach to wave-form ?

7. What is meant by a plane, progressive harmonic wave ? What is the importance of such a wave ?

Establish an equation for particle displacement in such a wave and explain the meanings of the symbols used.

8. Show how the equation $y=a \sin 2\pi n\left(t-\frac{x}{v}\right)$ and $y=a \sin 2\pi n\left(t+\frac{x}{v}\right)$ represent two waves moving in opposite directions along the X-axis. Explain the symbols used in equations.

9. Take any of the equations in the above question and show that displacement is repeated at intervals of time $T = 1/n$ and distances v/n.

10. State the common properties of waves and briefly explain their nature.

11. What is a wave-front ? How are rays and wave-fronts related ?

[B] **Short answer type questions :**

12. What advances in a progressive wave ?

13. What kind of motion does a particle of the medium execute in a longitudinal wave ?

14. While a longitudinal wave can move through all kinds of material media, a transverse wave can move through only a solid. Why is it so ?

15. Where does the energy of wave motion come from ?

16. What relation does a periodic wave bear to a simple harmonic wave ?

17. At a point inside a liquid medium a to and fro periodic motion (a disturbance) along the x-direction is impressed. Explain briefly using a neat sketch how this disturbance will be propagated along the x, y and z directions and the type of elasticity modulus involved in each. **[J. E. E. '79]**

18. Does the velocity of a wave also give the velocity of the particles of the medium ?

19. Is an oscillation a wave ? Explain.

20. A wave transfers energy. Does it transfer momentum ?

[C] **Numerical problems :**

21. A tuning fork vibrates 200 times per second. If the velocity of the waves generated in air by these vibrations have a wave velocity of 340 ms^{-1}, what are the periodic time and wavelength of the wave ? **[1/200 s, 1·7 m]**

22. If the frequency of a fork vibrating in water is 256 Hz and the velocity of longitudinal waves in water is 1024 ms^{-1}, how many vibrations will the fork execute before the waves move over 100 m ? [25]

23. A body vibrating with a constant frequency sends waves 10 cm long through a medium A and 15 cm long through another medium B. The velocity of the wave in A is 90 cms^{-1}. Find the velocity of the waves in B.

24. What is the difference of phase between the oscillations of two points at a distance of 10 and 16 m respectively from the source of oscillations ? The period of oscillations is 0·04 s and the velocity of their propagation 300 ms^{-1}. [π rad]

25. The displacement from the position of equilibrium of a point 4 cm from a source of oscillations is half the amplitude at the moment $t = T/4$. Find the length of the wave.

26. The speed of electromagnetic waves in vacuum is 3×10^8 ms^{-1}. (i) Waves lengths in the visible part of the spectrum range from about

4×10^{-7} m in the violet to about 7×10^{-7} m in the red. What is the range of frequencies of light waves ? (b) The range of frequencies for shortwave radio is 1·5 megacycles/s to 300 megacycles/s. What is the corresponding wave length range ? (c) X-ray wavelength range extends from about 5×10^{-9} m to $1\cdot0\times10^{-11}$ m. What is the frequency range for X-rays ?

[$7\cdot5\times10^{14}$ to $4\cdot3\times10^{14}$ Hz, 200 to 1 m, 6×10^{16} to 3×10^{19} Hz]

27. Write the equation for a wave travelling in the negative direction along the x-axis and having an amplitude 0·010 m, a frequency 550 vib/s and a speed 300 ms^{-1}.

$\left[\cdot01 \sin \frac{11\pi}{3}(300t - x) \text{ m}\right]$

28. A piece to cork is floating on water in a tank. When ripples pass over the water surface, the cork moves up and down. What will be the maximum velocity of the cork if the velocity of the wave is 0·2 ms^{-1}, wave length of the wave is 15 mm and amplitude of the wave is 5 mm ?

[0·42 ms^{-1}]

29. The equation of a wave is $y=5 \sin\left(\frac{t}{0\cdot04}-\frac{x}{50}\right)$ with lengths expressed in cm and time in second. Find (i) wave length, (ii) amplitude, (iii) frequency and (iv) velocity of wave. Also calculate the maximum velocity and acceleration of the particle of the medium.

[50 cm, 5 cm, 25 Hz, 1250 cms^{-1}, 785·5 cms^{-1}, 125000 cms^{-2}.]

30. A wave is represented by $y=0\cdot25\times10^{-3} \sin(500\,t-0\cdot025\,x)$ where y is in cm, t in seconds and x in metres. Determine (i) the amplitude, (ii) the period, (iii) the angular frequency, (iv) the wave length. Find also the amplitudes of particle velocity and particle acceleration.

[$0\cdot25\times10^{-3}$ cm, (ii) $\pi/250$ s, 500/s, 80π cm, 0·125 cms^{-1}, 62·5 cms^{-2}.]

III.-4. (Sound Waves and Velocity)

[A] Essay Type Questions.

1. How would you support the statement that sound waves are longitudinal, elastic waves in a material medium ?

Which elastic coefficient is concerned in the propagation of sound through air ?

2. What is the advantage of a tuning fork as a source of sound ? Describe its mode of vibration. What effect does loading or filing of an arm of a fork have on its frequency ?

3. Explain how sound waves propagate through air.

4. What is the relation between the velocity of sound in a gas and its pressure and density ?

Newton did not get the correct result when he used the relation. How did Laplace improve the relation ? State Laplace's argument.

5. What are the effects of (a) pressure, (b) temperature, (c) density and (d) humidity on the velocity of sound in air ? Explain.

6. What is Doppler effect? Illustrate it with an example. If the source advances towards an observer what will be the apparent frequency of the waves? If the observer advances towards the source then what will be the modified frequency?

7. Calculate the modified frequency of the waves if both the source and the observer are moving. What happens when there is no relative motion between the source and observer?

8. Can Doppler effect be detected in the case of light waves? Explain with examples. Mention some applications of Doppler effect in astrophysics.

[B] **Short answer type questions:**

9. The speed of sound is the same for all wavelengths—Explain.

10. During a thunderstorm, sound of thunder is heard much after the lightning flash is seen—Explain.

11. If a sound is made at one end of a long hollow iron tube, two sounds are heard at the other end. Explain

12. Sound waves are longitudinal in nature—give some reasons in support of this statament.

13. What are infrasonic and ultrasonic waves?

14. What is the special characteristic of a tuning fork?

15. Is there a Doppler effect for sound when the observer or the source moves at right angles to the line joining them?

16. A satellite emits radio waves of constant frequency. Describe how the sound changes as the satellite approaches, passes overhead, and recedes from the detector on the ground.

17. Why is a tuning fork made with two prongs? Would a tuning fork be of any use of its normal purpose if one of the prongs is sawn off?

18. How is it that we can hear the sound of coming train distinctly by applying one ear on the railway line, but we cannot hear the same sound through air?

19. In a sports-meet, the timing of a 100 m straight dash is recorded at the finish point by starting an accurate stop watch on hearing the sound of the starting gun fixed at the starting point. When will the time recorded be more accurate, in summer or in winter?

20. The velocity of sound is generally greater in solids than in gases at N.T.P. Why?

21. What would you hear if you were to move away from a source of sound with the speed of sound?

22. Why the vibration of a simple pendulum is not audible to the human ear? But that by mosquito or fly-wings are?

23. Write down the expression for the velocity of sound in terms of r.m.s. velocity of gas molecules.

[C] Numerical problems :

24. Calculate the velocity of sound in air at N.T.P. (the ratio of specific heats for air = 1·4 ; density of air at N.T.P. = 0·001293 g/cm^3). [332·5 ms^{-1}]

25. Young's modulus for steel is $2·14 \times 10^{12}$ dyn cm^{-2} and its density is 7·8 g cm^{-3}. Find the velocity of propagation of sound through a steel bar. How would the temperature of the bar affect the result ? [5237 ms^{-1} Increase]

26. If the root mean square velocity of the molecules of a diatomic gas is 461 ms^{-1}, find the velocity of sound. ($\gamma = 1.41$) [315 ms^{-1}]

27. The temperature of the upper layer of the atmosphere cannot be measured with a thermometer, since it will not get into thermal equilibrium with the environment owing to the low density of the gas. For this purpose use is made of a rocket with grenades which explode at a certain altitude. Find the temperature at an altitude of 20 km from the earth's surface if the sound produced by an explosion at an altitude of 21 km is detected 6·75 s after that produced by an explosion at an altitude of 19 km. (velocity of sound at N.T.P. = 330 ms^{-1}) [-55°C]

28. Calculate the velocity of sound in air at 10°C when the pressure of the atmosphere is 76 cm. The velocity of sound in air at N.T.P. = 332·5 ms^{-1} [338·6 ms^{-1}]

29. Assuming that the velocity of sound in air at N.T.P. is 1080 ft/s, find the velocity at 50°C and 70 cm pressure. [1185 ft s^{-1}]

30. The wavelength of the note emitted by a tuning fork of frequency 512 is 66·5 cm in air at 17°C. If the density of air at N.T.P. is 1·293 gcm^{-3}, calculate the ratio of the two specific heats of air. [1·39]

31. A sound wave emitted by a source at one end of an iron tube is 950m long and two sounds are heard at the other end at an interval of 2·5s Find the velocity of sound in iron, assuming that the velocity of sound in air at N.T.P. is 332·5 ms^{-1}. [3161 ms^{-1}]

32. A thunder is heard 6 s after a lighting flash. Calculate the distance at which the lightning occured if the mean temperature of air be 25°C (velocity of sound in air at 0°C = 332 ms^{-1}). [2081·64 m.]

33. A stone is dropped into a well 78·4 m deep. 4·23 s later the sound of the stone splashing the water is heard. Calculate the velocity of sound. Take g = 980 cms^{-2}. [341 ms^{-1}.]

34. Compare the velocity of sound in argon and carbon dioxide at 27°C and under a pressure of 76 cm of mercury. The molecular weights of argon and carbon dioxide are 40 and 44 respectively and ratio of specific heats of argon and carbon dioxide are $\frac{5}{3}$ and $\frac{4}{3}$ respectively. [$\sqrt{11/8}$]

35. A stone is dropped from the top of a tower and the sound is heard 4·4 s later. Find the height of the tower. Velocity of sound in air = 1000 fts^{-1}, $g = 32$ fts^{-2}. (H. S. '79) [272·2 ft.]

36. Two trains are travelling towards each other at speeds of 72 kmh^{-1} and 54 kmh^{-1} respectively. The first train whistles emitting

a sound with a frequency of 600 Hz. Find the frequency of the sound which can be heard by a passenger in the second train (i) before the trains meet, (ii) after the trains pass. The velocity of sound is 340 ms^{-1}. [666 Hz; 542 Hz]

37. A man on a seashore hears the hooting of a ship. When neither is moving, the sound has a frequency of 420 Hz. When the ship moves towards the man, the frequency of sound he hears is 430 Hz. When the ship moves away from the man, the frequency is 415 Hz. Find the speed of the ship in the first and second cases if the velocity of sound is 338 ms^{-1}. [28·3 kmh^{-1}, 14·7 kmh^{-1}]

38. A bullet flies with a velocity of 200 ms^{-1}. How many times will the height of the tone of its whistling change for a man standing still, past whom the bullet flies? The velocity of sound is 330 ms^{-1}. [4 times]

39. A bat flies perpendicular to a wall with a speed of 6 ms^{-1} emitting an ultrasonic sound with a frequency of $4·5 \times 10^4$ Hz. What two frequencies can be heard by the bat and why? The velocity of sound is 340 ms^{-1}. [$4·50 \times 10^4$ Hz, $4·66 \times 10^4$ Hz]

40. A motor car is fitted with two horns, which differ in frequency by 298 vibrations per second. If the car sounding both the horns is moving at 30 m.p.h. towards a person, who is at rest, calculate the change in difference in frequencies of the notes heard by him. (velocity of sound in air is 1120 fts^{-1}.) [299 c.p.s.]

41. A policeman on duty at a crossing challenges a motor driver for crossing the speed limit of 72 kmh^{-1} by detecting a change of 20 vibrations in the horn note of frequency 128 as the car passes him. Is the policeman correct? (velocity of sound 330 ms^{-1}) [Yes, the car crossed the speed limit.]

42. The whistle of an engine moving at 30 mi hr is heard by a motorist driving at 15 mi hr and estimated to have a pitch of 506. What must be the actual pitch of the whistle to the nearest whole number when (i) the two are moving in opposite directions but approaching each other, (ii) the two are moving in the opposite directions but away from each other, (iii) the two are moving in the same direction, the motorist being behind the engine, (iv) the two are moving in the same direction, the motorist being ahead of the engine. Velocity of sound is 1200 fts^{-1}. (Agra) [473, 538, 509, 491 Hz]

43. A vibrating tuning fork, tied to the end of a string 6 ft long, is whirled round in a circle. It makes 120 revolution per minute. Calculate the difference of the frequency between the highest and the lowest note heard by an observer situated in the plane of rotation. Velocity of sound = 1100 fts^{-1}. (Bombay) [0·01377]

44. A spectroscopic examination of light from a certain star shows that the apparent wavelength of a certain spectral line is 5001 Å, whereas the observed wavelength of the same line produced by a terrestrial source

5000Å. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ?

[Receding with a velocity of 6×10^6 cms^{-1}.]

45. Could you go through a red light fast enough so as to have it appear green ? Would you get a warning for speeding ? (Wave length of red and green light are 6800×10^{-8} cm and 5400×10^{-8} cm respectively and speed of light $= 3 \times 10^{10}$ cms^{-1}.) [No.]

III.-5. (Reflection, Refraction, Diffraction)

[A] Essay type questions :

1. What geometrical laws of reflection do sound waves obey ? What is the disadvantage of demonstrating these laws ?

State any practical application of reflection of sound.

2. What is an echo ? To hear to echo of a sound clearly, the reflector must be at a minimum distance. Why is it so ? How can you measure the depth of water with echoes ?

3. Distinguish between reflections and reverberation of sound. Give an example of reverberation and state how it is caused.

4. What are the geometrical laws of refraction ? Why do we say that in the case of sound, air is an acoustically denser medium than water ?

[B] Short answer type questions :

5. A sound emitted from the bank of a river at night can be heard farther on the water than a similar sound emitted in the day time. Explain why.

6. When there is a wind, a sound is carried farther down the wind than in the opposite direction. Example why.

What relation has this phenomenon with the refraction of sound ?

7. Why does the sound is an empty hall appear louder than when filled with people ?

8. Why can't we hear echo in small rooms ?

9. How is reverberation minimised in a modern cinema hall ?

10. Is it always true that a reflected sound has the same frequency as the original direct sound ?

11. The wall of a room can reflect sound waves but not light waves ; on the other hand a small mirror can reflect light waves but not sound waves. Explain why.

12. Why do we hold our palm in a curved fashion near our ear to catch a sound distinctly ?

13. Why are the roofs of public auditoria meant for meetings bent like arches ?

14. The music reproduced by a loudspeaker is not natural to a person if the listener is situated well off from the axis of the loudspeaker. Explain why ?

15. Why does the sound of thunder is rumble and rolling ?

16. Explain the kind of sound waves necessary for revealing minute details of the ocean bed by depth sounding method. (J. E. E. '75)

[C] **Numerical problems :**

17. A explosion occurred under water and at the same moment a siren was sounded from a ship. The ship was 3850 ft. away from a mountain. If the velocity of synd in air be 1100 ft s^{-1}, find the time of hearing the echo. If the echo in water is heard after 11/7 s, find the velocity of sound in water. [7S, 4900 ft s^{-1}]

18. A man runs towards a cliff at the rate of $4ms^{-1}$ and fires his pistol. At the instant of firing the man is 2.49 km away from the cliff. Find when and where the man will hear the echo. [14·82 s, 2430·72 m]

19. The velocity of sound in air is 340 ms^{-1} and that in water is 1500 ms^{-1}. What will be the critical angle of refraction from air to water ? [13° nearly]

20. An engine approaching a tunnel whistles and the driver hears an echo after an interval of 20 s. Ten minutes later the echo is heard after an interval of ·6 s. How far is the engine now from the tunnel and what is its speed ? Velocity of sound in air = 332 ms^{-1}.

[2665·1 m, 1·11 ms^{-1}]

21. A gun is fixed on a sea-shore in front of a line of cliffs. A man standing 300 ft away from the gun and equidistant from the cliffs notices that the echo takes twice as long to reach him as does the direct report. Find the distance of the gun from the cliffs. (Cal. Univ.) [225 ft.]

22. An echo repeats 4 syllables. Find the distance of the reflecting surface if it takes 1/5th of a sound to pronounce and hears one syllable distinctly. The velocity of sound is 1120 ft s^{-1}. (Pat. Univ.) [448 ft.]

23. From a vertical cliff two men stand at equal distances. The distance between the two men is 300 ft. One man fires his pistol and the second man hears the direct sound after t s and the reflected sound after 2 t s. If the speed of sound is 1100 ft s^{-1}, find the distance of separation of the men from the cliff. [260 ft.]

24. A rifle shot is fired in a valley between two parallel mountains. The echo from one mountain is heard after 2s and the echo from the other mountain is heard 2s later. What is the width of the valley ? Is it possible to hear the subsequent echoes from the two mountains simultaneously at the same point ? If so, after what time ? (I.I.T. '73)

[1080 m, possible, 6 s.]

25. Some explosives are kept at a depth of x from the sea level. At the same level but a certain distance away, a hydrophone is kept. When the explosives burst, the hydrophone receives two sounds after intervals of t_1 and t_2 from the time of explosion. Show that the depth of the ocean is $x+\frac{v}{2}(t_2{}^2-t_1{}^2)^{\frac{1}{2}}$ where v is the velocity of sound in water.

26. Find the refractive index of the second wave on the boundary between air and glass. Young's modulus for glass is $6{\cdot}9\times10^{10}$ N/m^2,

the density of glass $2.6\ g\ cm^{-3}$, and the air temperature is $20°C$. (Velocity of sound in air $= 340\ ms^{-1}$) [0.067]

27. What is the depth of a sea measured by means of an echo sounder if the time between the moments the sound is produced and received is 2.5 s? The coefficient of compression of water is 4.6×10^{-10} m^2/N and the density of sea water is $1030\ kg\ m^{-3}$. [1810 m.]

III.-6. (Superposition of waves)

[A] **Essay type questions :**

1. What is meant by the principle of superposition of waves? Describe two phenomena which are due to superposition of two sound waves.

2. What are beats? What conditions must be fulfilled so that beats may be heard clearly?

How do beats enable one to determine the frequency of a fork?

3. Show that the number of beats per second is equal to the difference of frequencies of the superposed sound waves.

4. What are stationary waves? Describe their characteristics. Compare stationary and progressive waves.

5. Calculate mathematically the positions of nodes and antinodes in a stationary wave.

6. Under what circumstances are stationary waves formed? Describe an experiment to demonstrate stationary waves.

7. What is interference of waves? Give an example of the interference of sound waves.

[B] **Short answer type questions :**

8. Two identical tuning forks emit notes of the same frequency. Explain how you might hear beats between them.

9. How a stringed instrument is 'tuned'?

10. How are beats utilised in detecting poisonous gas in a mine?

11. What are the conditions that must be satisfied for clear audibility of beats?

12. Distinguish between the formation of an echo and the formation of a stationary wave by reflection, explaining the general circumstances in which each is produced.

13. Explain the conditions necessary for the creation of stationary waves in air.

14. Mention the difference between a progressive wave and a stationary wave. (H. S. '82)

15. If the two waves differ only in amplitude and are propagated in opposite directions through a medium, will they produce standing waves? Are there any nodes?

16. When two waves interfere, does one alter the progress of the other?

17. When waves interfere, is there a loss of energy ?

18. What are the conditions necessary for interference of waves ?

19. Why don't we observe interference effects between the light beams emitted from two flashlights or between the sound waves emitted by two tuning forks ?

20. Two sound waves of amplitude 5 and 2 cm and frequencies 100 and 102 Hz travel in the same direction. Discuss the experience of an observer receiving these waves.

21. In a two-slit experiment using blue light, state the effects the following produce on the appearance of the interference fringes :

(i) The separation of the slits is decreased.
(ii) The screen is moved closer to the slits.
(iii) The source is moved closer to the two slits.
(iv) Red light is used in place of blue light.
(v) One of the two slits is covered up.
(vi) The source slit is made wider.
(vii) While light is used in place of blue light.

22. A ringing sound is heard when a sharp sound is produced near a flight of stone steps—why ?

[C] Numerical problems :

23. A tuning fork of unknown frequency makes three beats per second with a standard fork of frequency 354 Hz. The beat frequency decreases when a small piece of wax is put on a prong of the first fork. What is the frequency of this fork ? [357 Hz.]

24. A source of sound of frequency 1000 vibrations per second moves away from you toward a cliff at a speed of 10 ms^{-1}. (i) What is the frequency of the sound you hear coming directly from the source ? (ii) What is the frequency of sound you hear reflected of the cliff ? (iii) What beat frequency would you hear ? (Speed of sound in air is 330 ms^{-1}.) [970 vib/s, 1030 v/s, zero]

25. A source S and a detector D of high-frequency waves are at a distance d apart on the ground. The direct wave from S is found to be in phase at D with the wave from S that is reflected from a horizontal air layer at an altitude H. The incident and reflected rays make the same angle with the reflecting layer. When the layer rises a distance h, no signal is detected at D. Find the relation between d, h, H and the wavelength λ of the waves. [$\lambda = 2\sqrt{4(H+h)^2+d^2} - 2\sqrt{4H^2+d^2}$]

26. An engine approaches a bridge at 10 ft s^{-1} while sounding a whistle of frequency 500 Hz. The sound is reflected from the bridge. Calculate the frequency of the beats heard by a stationary observer behind the engine. Velocity of sound = 1100 ft s^{-1}.) [9 beats per sec.]

27. Sound waves from a tuning fork reaches an observer by two different paths. When one path is greater than other by 17 cm, no sound is heard by the observer. When the path difference is 34 cm, sound is heard by the observer and when 51 cm no sound and so on. Explain the phenomenon and find the frequency of the fork if the velocity of sound in air is 340 ms^{-1}. [1000 Hz.]

28. Sound waves from a vibrating body reach a point by two paths. When the paths differ by 8 cm or 24 cm, there is silence at the point. Calculate the frequency of the body if the velocity of sound in air is 332 ms^{-1}. [2075 Hz.]

29. A tuning fork of unknown frequency when sounded with another of frequency 256 Hz gives 4 beats and when loaded with a certain amount of wax, it is again found to give 4 beats. Find the unknown frequency. (I. I. T. '74) [260 Hz.]

30. A set of 65 tuning forces is so arranged that each gives three beats per second with the previous one. Between the first and the last stands the octave of the first. Find the frequencies of the first and last forks. [192 ; 384 Hz.]

31. A sound wave of frequency 165 is incident normally on a wall and retraces its path on reflection. At what distance from the wall, the amplitudes of vibration of air particle will be (i) maximum and (ii) minimum. Velocity of sound in air=330 ms^{-1}.

[nodes will be at the distance of 0, 1, 2, 3 meters and antinodes at distances 0·5, 1·5, 2·5 etc. frym the wall.]

32. Three tuning forks of frequency $n+4$, n and $n-4$ respectively are sounded together. Assuming their amplitudes to be equal, show that the number of beats per second produced by them is 4.

33. In a Young's slits experiment the Separation of ten bright fringes is 7·5 mm when the wavelength used is 6200 A°. The distance from the slits to the screen is 80 cm. Calculate the distance between the two slits. [0·8 mm]

34. Two slits are 0·3 cm apart and are illuminated by a source of wavelength 5900 A°. Fringes are obtained at a distance of 30 cm from the slit. Find the width of the fringes. [0·0059 cm.]

35. A sharp tap made in front of a flight of stone steps produce a ringing sound. If velocity of sound in air is 330 ms^{-1} and each step is 16·5 cm deep, find the frequency of sound. [1000 Hz.]

III.-7. (Light waves)

[A] Essay type questions :

1. What reasons can you give to show that light is a kind of wave ? Give a rough value and, as far as possible, an accurate value of velocity of light in vacuum.

2. What vibrates in light waves ? Does light require any material medium for propagation ?

3. What is meant by polarization of waves ? Briefly explain how polarization decides whether a wave is longitudinal or transverse. To which of these kinds do light waves belong ? Why do you think so ?

4. If light is a kind of wave, and waves have the general property of bending round corners, how can we accept the results of geometrical optics ? Geometrical optics is based on the assumption that light travels in straight lines in a homogenous medium.

[B] Short answer type questions :

5. Does light waves require any material medium for transmission ?

6. What is it that vibrates in light ?

7. What are the essential similarities and dissimilarities in the properties of light and sound waves ?

8. What are the basic similarity and dissimilarity between light waves and radio waves ?

9. What is the importance of determination of velocity of light ?

10. Light waves can be polarized. Can sound waves be ?

11. If light can bend around an obstacle (diffraction), why can't we see around a wooden partition ?

III.-8. (Physiological sound)

[A] **Essays type questions :**

1. Distinguish between musical sound and noise. What are the characteristics of musical sound ? Define them. Are they physical quantities ?

2. State with what physical characteristics of a sound wave the loudness, pitch and quality of a musical note are principally concerned ?

3. Explain clearly the terms note, tone, fundamental, overtone and harmonic.

4. Distinguish between pitch and frequency of a note.

5. Distinguish between loudness and intensity of sound wave.

[B] **Short answer type questions :**

6. Is it true that a tuning fork emits musical sounds ?

7. Explain why a musical sound can not always be distinguished clearly from a noise.

8. Does intensity of sound depend upon the medium ?

9. What are the factors that makes the voice of your one friend from that of the other different ? How can you explain the roar of a lion and buzzing of a mosquito from the point of view of the characteristics sound ? [J. E. E. 172]

10. How does the same notes from different musical instrument differ ? (I. I. T. '70)

11. All harmonics are overtones but all overtones are not harmonics —Explain the statement.

12. When a saw starts cutting a log a high-pitched sound is produced, but the pitch falls as the saw cuts into the wood—Explain why ?

13. Why does the loudness of a sound decrease as the listener moves away from the source ?

14. Higher pitch means smaller wavelength—Explain.

15. Even a blind man can be easily identify the sound coming from a violin and piano although the tunes have the same pitch and loudness —How ?

16. To hear the voice of a person over a telephone we bring our eass in contact with the earpiece of the telephone but the sound of a loud-speaker can be heard from a great distance—Explain.

17. Discuss the factors that determine the range of frequencies in your voice and the quality of your voice.

18. What is the nature of the wave motion in which (a) the amplitude is same at all points, but the phase varies with position, (b) the phase is same at all points, but the amplitude varies with position ?

III.-9. (Vibration of strings)

[A] Essay type questions :

1. Derive an expression for the velocity of transverse waves along a stretched string.

2. If a string under tension is fixed at both ends, how will stationary waves be formed in it ?

If such a string vibrates in one or more segments what will be the relation between the frequency, the length of the wire, its tension, etc ? Write the relation in the form of an equation and explain it.

What is meant by the fundamental frequency of a vibrating wire ? What is meant by harmonics ?

3. State the laws of transverse vibration of strings.

Describe a sonometer. How would you verify experimentally the laws of transverse vibration of strings with its help ?

4. How would you compare the frequencies of two tuning forks by a sonometer ?

[B] Short answer type questions :

5. l is the length of a string under tension, m is an integer $=1, 2, 3$ etc. What harmonics will be present in the vibrating string if l/m is a (i) node, (ii) antinode ?

6. If the diameter of a vibrating string is doubled, material and length remaining the same, what change should be made in the tension to keep the frequency the same ?

7. What purpose do the holes in sonometer's sides serve ?

8. Why is the sonometer box made hollow ?

9. How are the vibrations of the fork transmitted to the wire in a sonometer ?

10. At what point should a stretched string be plucked to make its fundamental tone most prominant ?

11. A string is struck at the midpoint. Explain what harmonics will be present and which are absent.

12. Why is a stringed musical instrument mounted on a hollow wooden box ?

13. Explain how by touching lightly at suitable points, the quality of the sound emitted by a vibrating string can be modified.

14. How the fundamental frequency emitted by a stretched string will be modified if (i) if length is doubled, (ii) if tension is increased four times.

[C] Numerical problems :

15. A 50 cm long wire weighs 1·25 g. Its tension is 25 kg. What will be the frequency of a fork in unison with the wire ? [313 Hz.]

16. A stretched wire is 30 cm long and 0·02 cm in diameter. Its fundamental frequency is 200 Hz. Another wire of the same material and tension is 20 cm long and 0·025 cm in diameter. What is the fundamental frequency of the second wire ? [240 Hz]

17. When the length of a stretched wire under the same tension is 70 or 75 cm, it produces 6 beats per second with a given fork. What is the frequency of the fork ? [174 Hz.]

18. Two tuning forks produces 4 beats per second. They are respectively in unison with stretched wires of lengths 96 and 97 cm, their material, diameters and tension being the same. What are the frequencies of the forks ? [384 Hz, 388 Hz.]

19. The diameter of a steel wire is 1·20 mm. If the velocity of sound wave in the string is 5·0 ms^{-1}, find the tension. Density of steel $= 7·7$ gm cm^{-3}. [2·222 kgf.]

20. A rope weighing 0·05 kgm^{-1} is stretched at a tension of 245 N between two points 30 m apart. If the rope is plucked at one end, how long will it take for the resulting disturbance to reach the other end ? [0·43 *S*.]

21. An addition of 24 kg to the tension of a string, changed the frequency of the string to three times the original frequency. What was the original tension ? **(Calicut '75)** [3 kgf.]

22. A wire under tension vibrates with a frequency of 450 Hz. What would be the fundamental frequency if the wire were half as long, twice as thick and under one-fourth the tension. **(P. U.)** [225 Hz]

23. A sonometer wine 100 cm long resonates with a certain weight. On adding 100 g to the weight the length was increased by 2 cm in order to restore the tuning. What is the initial weight in the pan ? **(Delhi '75)** [4·975 kgf.]

24. A wire 100 cm long and of mass 1 g is making 256 vibrations per second under a tension supplied by a brass weight hanging vertically. On immersing the weight in water the vibrating length of the wire has to be shortened by 5 cm to regain its original pitch ? What is the density of brass ? **(Mysore '75)** [10·23 g cm^{-3}]

25. Four violin string, all of the same length and material, but of diameters in the ratio 4 : 3 : 2 : 1 are to be stretched so that each gives a note whose frequency is 3/2 times that of the preceding string. If the stretching force of the first string is 2·048 kgf, calculate the tension in the other strings. [2·592 kgf, 2·592 kgf, 1·458 kgf.]

26. The length of a sonometer wire is 1 m and the tension 5 kgf. By how much should the length be altered to keep the frequency unchanged if tension be increased by 0·2 kgf ? **(Mysore '72)** [0·02 m.]

27. In Melde's experiment when the tension is 100 g and the fork vibrates at right angles to the direction of the string, the latter is thrown

in four segments. If now the fork is set to vibrate along the string, find what additional load will make the string vibrate in one segment.
(**Vikram.**) [300 gf.]

28. A thin steel wire has been stretched so that its length is increased by 1%. If the distance between the bridges is 100 cm, calculate the frequency of the wire. Young's modulus of steel $=2{\cdot}0\times10^{12}$ dyn cm^{-2} and density $=7{\cdot}8$ g cm^{-3}. (**Allahabad**) [252·5 Hz.]

29. A wire 100 cm long and of mass 1 g vibrates with a frequency of 256 Hz under a tension supplied by a brass weight of density 8·7 g.cm^{-3} hanging vertically, On immersing the weight in water, the vibrating length of the wire has to be shortened to maintain the same frequency. Calculate the length of the vibrating wire. [94·08 cm.]

30. A steel wire 0·8 mm in diameter is fixed to a rigid support at one end and is wrapped round a cylindrical tuning peg 5 mm in diameter at the other end, the length of wire between the peg and the support being 60 cm. Initially the wire is straight under a negligible tension. What will be the frequency of the wire if it is tightened by giving the peg a quarter of a turn ? Density of steel $=7800$ kg m^{-3} and Young's modulus 2×10^{11} N m^{-2}. [341·1 Hz.]

31. Two similar wires vibrate transversely in unison. When the tension in one is increased by 2·01 per cent and two wires vibrates simultaneously, three beats are produced per second. Find the original frequency of vibration of the two strings. (**C. U.**) [300 Hz.]

32. Two 60 cm long identical sonometer wires are stretched by the same tension to give a note of frequency 300. By how much should the length of one of them be altered to give five beats per second ?
(**Gorakhpur**) [0·0102 m.]

33. A movable bridge divides a sonometer wire into two parts which differ in length by 1 cm and produce 4 beats per second when sounded together. If the whole length of the wire is 100 cm, find the frequencies of parts. (**Delhi**) [202 Hz, 198 Hz.]

34. The movable bridge of a sonometer is adjusted so that 4 beats per second are heard when the string is sounded simultaneously with a tuning fork. The length of the vibrating portion of the string is found to be 60 cm. When the bridge is moved so as to lengthen the string by 1 cm, 4 beats per second are again heard. What is the frequency of the fork ?
[484 Hz.]

35. A certain tuning fork is found to give 2 beats per second in conjugation with a streched string vibrating transversely under a tension of either 10·2 or 9·9 kgf. Calculate the frequency of the fork. [268 Hz.]

36. A wire having a density of 0.05 gm cm^{-3} is stretched between two rigid supports with a tension of $4{\cdot}5\times10^{7}$ dynes. It is observed that the wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire. (**I. I. T. '71**) [214·3 cm.]

37. A sonometer wire of length 76 cm is maintained under a tension of 40 N and an alternating current is passed through the wire. A horse-

shoe magnet is placed with its poles above and below the wire at its middle point and the resulting forces set the wire in resonant vibration. If the density of the material of the wire is 8800 kgm^{-3} and the diameter of the wire 1 mm, find the frequency of the alternating current. [50 Hz.]

38. Two wires of radii r and $2r$ respectively are welded together end to end. The combination is used as a sonometer wire and is kept under tension T, the welded point is midway between the two bridges. What would be the ratio of the number of loops formed in the wires such that the joint is a node when stationary vibrations are set up in the wires. **(I. I. T. '76)** [1 : 2]

39. A uniform circular loop of string is rotating clockwise in the absence of gravity. The tangential speed is v_0. Find the speed of the waves travelling on this string. [v_0]

40. Two ends of a wire are rigidly fixed to two clamps l apart. The cross-section of the wire is A, tension T, Young's modulus Y and the coefficient of linear expansion is α. If the temperature of the wire is decreased by θ°, how many times will the frequency increase?

$$\left[1+\frac{\alpha\theta YA}{2T}.\right]$$

41. Two wires are fixed on a sonometer. The tensions are in the ratio 8 : 1, the lengths in the ratio 36 : 35, the diameters in the ratio 4 : 1 and the densities are in the ratio 1 : 2. Find the frequency of beats produced if the note of the higher pitch has a frequency of 360 per second. **(I. I. T. '68)** [10 per second.]

42. If the fundamental frequency of a string 60 cm long is n, where would you place a bridge under the string to produce notes of frequency n_1 for one part and n_2 for the other, so that the intervals from n to n_1 and from n_1 to n_2 shall be the same. **(Lond Univ.)** [37.1 cm from one end.]

43. A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of a rope. What is the wavelength of the pulse when it reaches the top of the rope? **(I. I. T.)** [0.12 m.]

44. A steel wire of length 1 m, mass 0.1 kg and of uniform cross-sectional area 10^{-6} m^2 is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the transverse waves are set up by plucking the string at the middle, calculate the frequency of fundamental mode of vibration. (Given for steel, coefficient of linear expansion $=1.2\times10^{-5}/°C$ and Young's modulus $=2\times10^{11}$ N/m^2) **[I. I. T. '84]** [11 Hz]

45. The ends of a string are rigidly fixed. If the temperature of the string falls by 10°C, calculate the change in its tension from the following data:

Area of cross-section of the string = 0.01 cm^2. Coefficient of linear expansion of the material of the string = $16\times10^{-6}/°C$.

Young's modulus of the material of the string = 20×10^{11} $dyne/cm^2$.

If this causes the frequency of transverse vibration of the string to increase to double its previous value, what was the original tension? (J. E. E. '80) [32×10^5 dyne, 10.67×10^5 dyne.]

III.-10. (Vibration of air column)

[A] Essay type questions :

1. Find the possible frequencies of an air column in a closed pipe. Show that only odd harmonics can be present.

2. Find the possible frequencies of an air column in an open pipe. Show that all harmonics, odd and even, can be present.

3. Describe the nature of vibration of an air column in a closed and open tube when each emits the fundamental.

How will the frequency of the fundamental change when one end of an open tube is suddenly closed?

4. Compare the modes of vibration of air column in open and closed tubes of the same length.

5. We speak of wavelengths in connection with the vibration of an air column in a closed or open pipe.

Explain which progressive or stationary waves these wavelengths refer to.

How can a vibrating air column be considered a source of sound?

6. How would you determine the velocity of sound in air with the help of a closed tube and a fork of known frequency? Is it the value in dry air and at 0°C? If not, what are the conditions of temperature and moisture at which the value has been determined?

[B] Short answer type questions :

7. Discuss the statement that the vibrations of an air column are stationary vibrations.

8. At the room temperature the air column in a closed brass tube is in unison with a fork. Why should beats appear between the sources as the tube is heated gradually? How will the number of beats be affected by a greater increase of temperature?

9. How do the air layer vibrates on two sides of (a) a node, (b) an antinode in the air column of an open tube?

10. How the frequency of the fundamental be changed if the temperature of the tube increases?

11. How the frequency of a note from a pipe be changed if a wider pipe is used?

12. The bugle has no valves. How then can we sound different notes on it? To what notes is the bugler limited? Why?

13. A tube can act like an acoustic filter, discriminating against the passage through it of sound of frequencies different from the natural frequencies of the tube. The muffler of an automobile is an example. Explain how such a filter works. How can we determine the cut-off frequency, below which frequency sound is not transmitted?

14. If a pitcher is filled with water in the dark it can be easily said whether the pitcher is full, by hearing the sound—Explain.

15. Why a sharp sound is emitted when we blow through the hole of a key ?

16. What will happen if one mouth of an open organ pipe is suddenly closed ?

17. How will the pitch of an organ pipe be affected with change of humidity ?

18. Sound emitted by an open organ pipe is more musical than that emitted by an organ pipe closed at one end—Explain (H. S. '60)

19. What is the relation in the fundamental frequencies emitted by an open tube and a closed tube of same length ?

20. Draw the wave pattern set up in a resonance tube open at both ends when the length of the tube is equal to the wave length and also when the length is 3/2 times the wave length. (J. E. E.)

21. Why does an open pipe emit a note an octave higher than the fundamental when it is blown vigorously ?

22. How will the frequency of fundamental emitted by an open organ pipe change if one open end is partially closed ?

23. What is the ratio of the lengths of an open and a closed pipe when the frequencies of the fundamental emitted by them are identical ?

24. Explain what will happen to the pitch of the fundamental note emitted by an open organ pipe when air is replaced by CO_2 ?

25. What is end correction ? Why it is necessary ?

[C] Numerical problems :

26. The air column in an open tube emits a fundamental note of frequency 256 Hz. If the velocities of sound in air and in coal gas be respectively 350 ms^{-1} and 500 ms^{-1}, what will be the frequency of the fundamental and its wavelength when the tube is filled with coal gas ?
[366 Hz, 1·37 m.]

27. A tube 100 cm long and 2 cm in diameter, is completely full of water. A fork of frequency 510 Hz is sounding near its upper mouth while water is being gradually let out at the bottom. The velocity of sound in the air of the tube is 340 ms^{-1}. Explain why there will be resonance when the air column is about 17·3 cm, 50·6 cm and 83·6 cm long.

28. An open organ pipe has a length of 2 m. Neglecting end-corrections find the freqnency of its fundamental and first overtone when it is (a) open, and (b) closed at one end, taking the velocity of sound in air as 340 m/s. [85, 170 ; 42·5, 127·5 Hz.]

29. Calculate the resonance frequencies of air at 25°C and 750 mm of Hg in a narrow pipe one metre long and closed at one end. Ratio of sp. heats of air = 1·4 and density of air at N. T. P. = 0·001293 g cm^{-3}. **(Mysore '74)** [86·5, 259·5. 432·5...Hz.]

30. A closed pipe, 25 cm long, resounds when full of oxygen, to a given tuning fork. Find the length of a closed pipe, full of hydrogen, which will resound to the same tuning fork. Velocity of sound in oxygen = 320 ms^{-1} and that in hydrogen = 1280 ms^{-1}. **(Mysore)** [100 cm.]

31. Two narrow organ pipes of the same length, but one closed and the other open are sounded together. If the second overtone of the closed pipe differ by 150 vibrations per second from the first overtone of the open pipe, calculate the fundamental frequency of the closed pipe. [150 Hz.]

32. Two closed pipes of the same diameter produces notes which are an octave apart. If lengths of the pires are 62 and 30 cm respectively, what is the end correction of the pipes ? [1 cm.]

33. A closed brass pipe emits a note of frequency 256 at 0°C. What will be the frequency of the note, if the temperature rises to 27°C ? Co-efficient of linear expansion of brass = $1·85 \times 10^{-5}$ per °C. **(Madurai '75)** [268H z.]

34. If the pitch of the note omitted from a metal organ pipe is to be independent of temperature show that the ratio of the cubical expansion of air to that of the metal is 2/3.

35. A train of sound waves of amplitude 0·001 cm is propagated along a narrow pipe, and is reflected without loss of amplitude from the open end. If the wavelength is 36 cm what is the amplitude of vibration at a point 33 cm inside the pipe ? [0·001732 cm.]

36. The shortest length of a resonance tube closed at one end which resounds to a fork of frequency 256 is 30·0 cm. The corresponding length for a fork of frequency 480 is 12·0 cm. Calculate the end correction for the tube and the velocity of sound in air. [8·57 cm, 395 ms^{-1}.]

37. A tuning fork *A* is in resonance with an air column 32 cm long and closed at one end. When the length of this air column is increased by 1 cm, it is in resonance with another fork *B*. When *A* and *B* are sounded together, they produce 8 beats per second. Find the frequencies of the forks. [264, 256 Hz.]

38. Two closed pipes give 4 beats per second when sounded together at 15°C. Calculate the number of beats at 40°C. Velocity of sound in air at 0°C is 332 ms^{-1}. [4·2 per second.]

39. A string 25 cm long and having a mass of 2·5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string the beat frequency decreases. If the speed of sound in air is 320 ms^{-1}, find the tension in the string. **(I. I. T. '82)** [27·04 N.]

40. When the tuning fork and a column of air are sounded together they produce 4 beats per second at 15°C, the fork giving the lower note. When the temperature falls to 10°C, 3 beats per second is produced. Find the frequency of the fork. [108 Hz.]

41. A certain organ pipe at 15°C is in unison with a steel wire of diameter 0·50 mm vibrating transversely in its fundamental mode under a tension of 5 kgf, the vibrating length being 34·6 cm. Describe and explain what will be heard if the temperature of the air in the pipe rises 2°C, other conditions remaining constant, and the pipe and the wire are sounded together. Density of steel 7·8 g cm^{-3}. **(London)** [1 beat/s.]

42. *AB* is a cylinder of length 1 m filled with a thin flexible diaphragm *C* at the middle and the two other thin flexible diaphragms *A* and *B* at ends, the portion *AC* and *BC* contains hydrogen and oxygen respectively. The diaphragm *A* and *B* are set into vibrations of the same frequency. What is the minimum frequency of these vibrations for which the diaphragm *C* is a node. velocity of sound in hydrogen is 1100 ms^{-1} and that in oxygen is 300 ms^{-1}. **(I. I. T. '80)** [1650 Hz.]

43. The fundamental note emitted by a stretched wire vibrating transversely between two bridges 60 cm apart is in unison with that of an organ pipe when the temperature is 15°C. Find approximately the change in length between the briges necessary to restore unison when the temperature of the air in the pipe rises to 30°C, the tension in the wire remaining unchanged. [1·57 cm.]

44. Two similar organ pipes when sounded together give 7 beats per second. If their lengths are in the ratio 50 to 51, calculate the frequencies. **(Kanpur '71)** [357 and 350 Hz.]

45. An organ pipe is sounded with a tuning fork of frequency 256 Hz. When the air in the pipe is at a temperature of 15°C, 23 beats

occur in 10 s, the tuning fork giving the higher note. What change in temperature is needed to bring the pipe and the fork in unison ? [5·2°C.]

46. A pop gun consists of a tube 25 cm long, closed at one end by a cork and at the other end by a tightly filling piston. The piston is pushed slowly. When the pressure has risen to 1·5 atmospheres the cork is violently blown out. Calculate the frequency of the 'pop' caused by it ejection. Velocity of sound in air is 340 ms^{-1}. [S 10 Hz.]

47. A long cylindrical tube is being filled up by water at a uniform rate from a tap. An observer found that a tuning fork of frequency 300 is producing resonance with the air column in the tube after every 100 second. Calculate the volume in cm^3 of the water supplied per second. The radius of the tube = 10 cm and the velocity of sound = 330 ms^{-1}.
(J. E. E. '76) [172·7 cm^3s^{-1}.]

48. A string 25 cm long and having a mass of 2·5 g is under tension. A pipe closed at one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound the air 320 ms^{-1}, find the tension in the string. (I. I. T. '82) [27×10^5 dyn.]

49. An organ pipe open at both ends, 2·5 ft long, produces 5 beats per second with a similar pipe slightly shorter in length and the same number of beats with a tuning fork at 0°C. Calculate the number of beats produced by the shorter pipe with the tuning fork at 22°C. Velocity of sound at 0°C is 1100 ft s^{-1} and at 22°C it is 1144 ft s^{-1}. [9]

50. A tube closed at one end is closed at the other end by a vibrating diaphragm which may be assumed to be a displacement node. It is found that when the frequency of the diaphragm is 2000 Hz, a stationary wave pattern is set up in the tube and the distance between adjacent nodes is then 8 cm. When the frequency is gradually reduced the stationary wave pattern disappears but another stationary wave pattern reappears at a frequency of 1600 Hz. Calculate (i) the speed of sound in air, (ii) the distance between adjacent nodes at a frequency of 1600 Hz, (iii) the length of the tube between the diaphragm and the closed end, (iv) the next lower frequency at which a stationary wave pattern will be obtained. [320 ms^{-1}, 10 cm, 40 cm, 1200 Hz.]

PART IV

Heat

HEAT

HEAT AND TEMPERATURE : RECAPITULATION

IV-1. Heat. We derive the sensations of warmth and cold through our sense of touch. When a vessel containing cold water is placed over a fire, it becomes warm, then hot and finally begins to boil. *The external agent which turns a cold body hot is called heat.* When a body is heated we say that *heat* has been added to it.

All bodies contain heat. However cold a body is, it contains some heat. Water in a lake, pond or river is colder than boiling water ; ice is colder than water in a lake. If ice is added to water in a tumbler it melts. The water becomes colder than before, but remains warmer than ice. We explain it by saying that some heat has passed from the water into the ice and melted and warmed it. The water in the tumbler, which was colder than boiling water undoubtedly contained heat.

Even ice contains heat. Liquid air is much colder than ice. When a vessel containing liquid air is placed over ice, liquid air boils like water over a fire. Heat passes from ice into the liquid air.

The same agent 'heat' is responsible for both the sensations of heat and cold. When heat enters our body from the object touched, we feel the sensation of warmth. When heat leaves our body, we feel the sensation of cold. *Faster* the heat *enters* our body from an object *hotter* it appears. *Faster* the heat *leaves*, *cooler* the object appears to the touch.

IV-2. Temperature. We have said that all bodies contain heat. Our sensation of warmth or cold does not depend on the total amount of heat that a body contains. When a quantity of cold

water is poured into a vessel containing hot water, the total amount of heat in the water of the vessel is larger than before, but the water appears colder than it was.

If the sensation of warmth is not to depend on the total quantity of heat in a body, it must depend on some other factor. This factor is called *temperature*. In trying to define temperature we observe that when two bodies are kept in contact, heat may pass from one body into the other. One of the bodies gets colder and the other warmer. The former loses heat to the latter. This behaviour of bodies helps us in defining temperature. As an elementary definition we may say—

The temperature of a substance is a number which expresses its degree of hotness on some chosen scale.

Heat flows from a body at a higher temperature to one at a lower temperature. When the temperature is the same for two bodies heat ceases to flow from one to the other. (Compare the flow of water between two levels).

If we take a cupful of water out of a bath tub, the water in the cup has the same temperature as that in the tub. But as the tub may contain a thousand cupfuls of water, the total heat in the tub is much greater than that in the cup.

IV-3. Difference between heat and temperature. The points of difference between heat and temperature may be stated as follows:

(i) Heat is a form of energy. Temperature is a thermal state which determines the direction in which heat will flow. Heat always flows from a body at a higher temperature to a body at a lower temperature.

(ii) Temperature is not determined by the amount of heat in a body. (Consider a bucketful of water, and a cupful taken out of it. Both have the same temperature; but the water in the bucket contains many times the heat contained in the cup.)

(iii) If equal amounts of heat are added to different bodies, their temperature rise, in general, will be different.

(iv) Temperature may be compared with the water level in a vessel and heat with the amount of water in it. Addition of heat raises the temperature just as addition of water raises the water level. When two vessels with different water levels are connected together, water flows from the one having the higher level to the one having the lower level. The amount of water in either vessel does not determine the direction of flow.

Strictly speaking, it is not proper to speak of the heat in a body' Later we shall learn that 'heat is energy in transit due to temperature difference'. The amount of heat we can get from a body depends on the temperature of the other body in contact.

Different forms of Heat : We recognise three forms namely— **Sensible, Latent** and **Radiant Heat.** When on heating a body its *temperature changes but not its state of aggregation*, the heat is said to be *sensible* i.e. directly perceptible to the senses of touch.

When addition of heat *changes the state of aggregation* i.e. solid to liquid or that to gas, heat is said to be *latent* i. e. hidden ; for then the *tempereture does not change* and the effect of heat is not recognised by sense of touch.

Heat is found to flow without the presence of any medium, without contact across vast distances, e. g. from the sun to all the planets, in the form of electromagnetic waves as light does. This is the *radiant* form of heat.

IV-4. Effects of heat. Heat raises temperature ; the other changes are direct effects of the rise of temperature. Almost all physical properties are affected more or less by heat. It can also bring about chemical changes. Most chemical actions take place faster at higher temperatures. This also applies to life processes ; but life cannot continue at high temperatures. In the study of heat, temperature is the most fundamental quantity. It is in fact the *fourth* fundamental indefinable in addition to those, mass, length and time (See ; O-1.5)

Principles of Thermometry : *As indicated above* temperature

cannot be directly measured; we measure only the temperature difference in terms of (i) a temperature scale and (ii) changes in other properties namley,

(1) change in length of a solid in the form of a coiled filament

(2) change in volume of a liquid or gas under constant pressure in a glass container

(3) change in pressure of a gas or vapour of constant volume

(4) change of electrical resistance of a metallic wire

(5) change of thermo-emf of a thermocouple of a pair of dissimilar metals

(6) change of magnetic susceptibility of cortain metallic salts.

Note that so many diverse properties of matter change with addtion of heat when temperature changes. Any of these property-changes can be also related with rise in temperature by definite mathematical relations. The relations can be listed as follows :—

$L_t = Lo\,(1+\alpha t)$, $V_t = V_o(1+\gamma t)$, $P_t = P_o(1+\gamma' t)$, $R_t = R_o(1+\alpha_r t)$, $E = at + bt^2$, $X_m^{\cdot}T = \text{Const.}$

where L_t, V_t, P_t R_t refer to length, volumen pressure and resistance at a temperatures *above absolute zero*, α, γ, γ', and α_r constants peculiar to the substances used; E is the emf developed, a b constants peculiar to materials used, χ the magnetic susceptibility.

Any of these properties can be used as a *thermometric property* and the substance as *thermometric substance*. However we shall concentrate on *liquid-in-glass thermometry* where a liquid is the thermometric substance and increase in volume with temperature, the thermometric property. Whenever occasion arises we shall briefly touch upon the other forms of thermometers.

IV-5. Thermometers. *A thermometer is a device for measuring temperature.* To measure a temperature we fix upon some property of a substance which changes regularly with change in temperature.

The most common form of thermometer is the *mercury-in-glass*

thermometer. Mercury is the thermometric substance. Expansion of mercury with temperature is the thermometric property we use for measuring temperature. Its expansion with rise in temperature is fairly large. Strictly speaking it is the *difference of expansion between mercury and its glass container* that we utilize. We may therefore, say that in a mercury thermometer, the change in length of the mercury column in the glass tube is the thermometric property, for the cross-section is taken to remain constant.

Some other liquids notably *alcohol*, are used as thermometric substances. For low temperature measurements pentane is used where both mercury (F.P. $-39°C$) and alcohol (F. P. $-130°C$) cannot be used.

Properties desirable for a Thermometric liquid :—

(i) *Long Range* : The liquid should have a low enough freezing point and high enough boiling point so that a large temperature range may be measured. High boiling point ensures a low vapour pressure, a desirable property.

(ii) It should have *low specific thermal capacity* such that in *rising* through a given temperature range it absorbs as little heat as possible from the body in touch.

(iii) It should have *high heat conductivity* so that conduction and consequent temperature rise may be fast.

(iv) It must have *large coefficient of volume expansion* so that for a small temp rise there is an appreciable expansion of it. Also that the expansion is *regular* i.e. *uniform*.

(v) The liquid should be *opaque* as to be easily visible through glass, should be *easily available* in the pure state and *cheap*.

(vi) It should have *low vapour pressure*.

Mercury fulfills most of these requirements but it has *high density* which makes its thermal capacity *per unit volume* and hydrostatic pressure exerted in the bulb large ; So it absorbs appreciable amount of heat when the thermometer bulb (as in sensitive thermometers) is large and inflates the bulb when the

thermometer is held vertical (recall $p=h\rho g$ and Pascal's law). Further, as mercury does not wet glass it moves jerkily i.e. *not uniformly* and mercury is *not cheap*, now-a-days.

Its alternative, **alcohol has a lower F. P.** but also a lower B. P. So its vapour peressure is large. Its density is low but so is its couductivity. Its expansivity is larger so is its specific heat capacity. It is cheaper but transparent (so needs to be coloured). It wets glass and hence moves smoothly but sluggishly.

Mercury thermometer. Description. In view of the above points mercury is the most widely used liquid-in-glass thermometers.

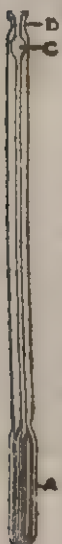


Fig. IV-1.1

It consists of a (i) thin walled large bulb (A) attached to a (ii) thick walled fine capillary with a (iii) smaller closed bulb (C) at the top (fig. IV-1.1). The bulb and some length of the capillary is full of mercury. The large bulb accomodates a greater quantity of it so as to allow a greater volume expansion ; its wall is thin to allow quick passage of heat. The capillary is thin to allow a change in volume of mercury appear as a large change in length. Its thick wall gives mechanical strength and inhibits loss of heat from the capillary. The small bulb at the top is a protective device to guard against accidental overheating ; if that occurs mercury may shoot up beyond the top of the capillary and break it open ; but with the bulb present, mercury flows into it and does not press up to the top.

IV-6. Scales of temperature. To give a temperature a numerical value, a scale of temperature is necessary. So we take two temperatures fixed by Nature—the melting point of ice and the boilling point of pure water, both at a pressure of one standard atmosphere. The former is called the *ice-point* and the latter, the *steam point.* In preparing a temperature scale, these two tempearatures are given arbitrary values. The most used

temperature scale is the *centigrade scale.* In it, the ice point is given the value of $0°C$ and, the steam point, the value of $100°C$, The temperature interval between these two fixed polnts is called the *fundamental interval.* In the centigrade scale the fundamental interval is divided into 100 equal parts, called degrees. The scale is extended above and below the two fixed points. Temperatures below $0°C$ (the ice-point) are marked negative, such as, $-1°C. -20°C$, etc.

In 1948, the International Organization for Standards directed the use of the word 'Celsius' for 'centigrade' in honour of Anders Celslus, a Swede, the inventor of the scale in1742. The symbol for a degree on this scale continues to be $°C$, but we now call it 'degree celsius'.

On the Fahrenheit scale, the ice point is given the value of 32°F and the steam point, the value 212°F. The fundamental interval is thus divided into 180 equal parts or degrees. Since a difference of 180 fahrenheit degrees equals a difference of 100 celsius degrees, we have (fig. IV-1.2)

1 fahrenheit degree $= \frac{100}{180} = \frac{5}{9}$ celsius degree.

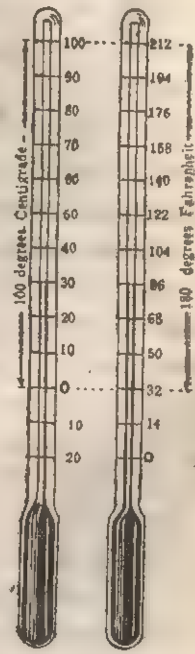


Fig. IV-1 2

IV-7. How to express a value of temperature and a difference of two temperatures in symbols. What will you understand when we write $20°C$ or 20°F ? Obviously, it means a temperature of $20°C$ or 20°F. But how shall we express a difference of temperature of $40°C-20°C=20$ celsius degrees, in symbols ? Many authors prefer to indicate the difference as 20 celsius degrees or $20C°$, putting the degree mark after the symbol C. (Similarly, 20 fahrenheit degrees or 20F°.) Unless we use the word 'difference', it is better to write $20C°$

or 20C deg. There is, however, *no uniformity* of writing in this regard. But the point is worth remembering.

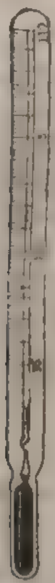


Fig. IV-1.3

IV-8. **Conversion from celsius to faherehelt reading and vice versa.** Suppose a temperature reads c on the celsius scale and f on the fathrenheit scale. f is $(f-32)$ F° away from the ice-point (0°C).

Since $1\ \text{F}^\circ = \frac{5}{9} C^\circ$, $(f-32)\text{F}^\circ = \frac{5}{9}(f-32)\ C^\circ$.

Hence c is $\frac{5}{9}(f-32)$ celsius degrees away from the ice-point.

$$\therefore\ c = \frac{5}{9}(f-32) \text{ and } f = \frac{9}{5}c + 32. \quad \text{(IV-1.6.1)}$$

Though India uses the celsius scale, our clinical or Doctor's thermometers were being graduated in fahrenheit degrees. Recently, however, we have changed over to the celsius degree. The normal body temperature is taken as 37°C or 97.4°F. (See fig IV-1.3). Note the small range of graduations. Human life cannot servive for long above or below these limits and hence graduations beyond is unnecessary.

Temperature scales and Inter conversions. Apart from **celsius** and **fahrenheit** scales you know the **Kelvin** or absolute scale. **Rankine** scale used by engineers in the U.S.A may be taken as the fahrenheit counterpart of the kelvin scale. On it the abolute zero comes out to be -459.7°F and each of its degree rise corresponds to 1°F.

The temperature difference between freezing and boiling points of pure water under standard atmospheric pressure is said to be the **Fundamental Interval.** Remember whatever the scale of temp considered

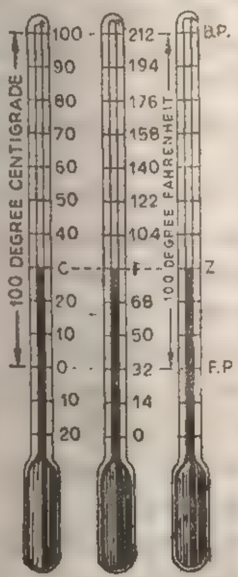


Fig. IV-1.4

The ratio $\frac{\text{Temp Reading—Lower fixed point}}{\text{Fundamental Interval}}$ is constant.

Hence we shall have for the any scale of temp

$$\frac{C-O}{100-O}=\frac{F-32}{212-32}=\frac{K-273}{373-273}=\frac{R-492}{672-492}=\frac{Z-F.P.}{B.P-F.P.}$$

where Z represents temp on any arbitrary scale (including the so-called faulty thermometers) whereas B. P, the boiling temp and F.P. the freezing point of pure water under standard atmospheric pressure. Mention of pressure *is a must*, for remember that a change of pressure by 1 cm of mercury changes boiling point of water by about O·27°C. Fig IV-1.4 seeks to correlate the inter-relations between the different temp scales. The following worked out problems would clarify the method.

Examples IV-1. 1. *The freezing point is marked 20° on a thermometer and the boiling point 150°. What reading would it give for a temp of 45°C ?* **[Tripura H. S. '81]**

Solution : Let Z be the required reading. Then

$$\frac{Z-20}{150-20}=\frac{C-0}{100-0} \text{ or } \frac{Z-20}{130}=\frac{45}{100} \text{ or } Z=20+45\times 1{\cdot}3=78{\cdot}5^\circ,$$

Ex. IV-1. 2. *The upper and lower fixed points of a thermometer are marked 140° and 20° respectively. What would it read for a temp of 92°F ?*

Solution : Let Z be the required reading. Then

$$\frac{Z-20}{140-20}=\frac{92-32}{212-32} \text{ or } \frac{Z-20}{120}=\frac{60}{180} \text{ or } Z=60^\circ F$$

Ex. IV-1. 3. *A faulty thermometer reads 1° in melting ice and 96° under normal atmospheric pressure. Find the correct temp when it reads 39°, both in centigrade and Fahrenheit scales, the bore of the thread and graduations being uniform.* **[Dac. U]**

Solution : Let C and F be the required readings. Then

$$\frac{C-0}{100-0}=\frac{F-32}{112-32}=\frac{39-1}{96-1}$$

Or, $\frac{C}{100}=\frac{38}{95}$; $\frac{F-32}{180}=\frac{38}{95}$ $\therefore$ $C=40^\circ$ and $F=104^\circ$

IV-1 9 Modern ideas about heat and temperature. What we have said so far about heat and temperature is rather old story. Our modern views of these quanties are somewhat different.

Heat is now defined as *energy in transit from one place to another due to difference of temperature between the places.* Temperature is no longer a scale, but is a fundamental physical quantity like Length or Mass. Just as the metre is the unit of length or the Kilogram the unit of mass, the **Kelvin** (symbol K) is the unit of temperature. *It is defined as 1/273·16 fraction of the temperature difference between the absolute zero and the triple point of water.* **Triple point** of water is the temperature at which ice, water and water vapour can coexist. It is taken to be O·1 K above the normal melting point of ice.

We no longer speak of absolute temperature as *degree* Kelvin (°K). It is merely Kelvin (K). *The degree sign has been dropped.* We do not even speak of an absolute scale of temperature, but only of temperature in Kelvin. We of course use the celsius scale for convenience. But the proper recognized scale is the International Practical Temperature Scale. Unfortunately, we cannot go into it here. We cannot here go into a more detailed discussion of the modern concepts. They will come up in due course.

IV 1 10. More about Thermometers and Temperature :

A. **Sensitive thermometer :** Smaller the temp difference a thermometer can detect more sensitive it is said to be. To achieve that a large expansion for a small temperature rise is required. Hence for a sensitive thermometer

(i) the bore of the capillary must be fine

(ii) the thermometric substance must have a large coefficient of volume expansion.

iii) the amount of liquid i. e. the volume of the bulb should be large.

B. **Fast recording thermometer :** Such one should be indicating the required temperature in the shortest possible time. To achieve that

(i) Amount of liquid and so the bulb should be small.

(ii) Capillary should be wider.

(iii) Conductivity and specific heat capacity per unit volume of the liquid should be respectively high and low.

So the requirements for the two A and B are contradictory.

C Six's thermometer : This is a device by which the maximum and minimum temperatures attained during a day can be recorded. It utilises both mercury and alcohol and is widely used in Meteorological observatories.

Description. The bulb of the thermometers B (Fig IV-1.5) is at the end of a long capillary U tube which carries a smaller bulb D at the other end. The bulb B and part of the stem down to A are filled with alcohol. This is the real thermometric part of the instrument. A column of mercury extends from A to C in the stem. It acts as an index. Above C the stem contains alchohol which also fills part of D, the rest of which contains alchohol vapour, providing room for expansion of the liquid. Above the mercury thread in each limb rests a small steel dumb-bell I_1 and I_2 carrying a light spring each pressing against the inner wall.

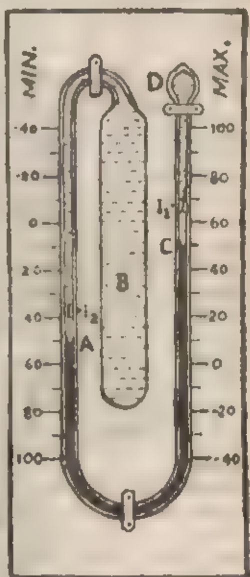


Fig. IV-1.5

Action : As temp falls, alcohol in B contracts. Pressure of alcohol vapour in D pushes the mercury column up the left hand limb, along with the steel pointer I_2. As temperature rises, alcohol in B expands and pushes down the merchury column leaving the steel pointer held in position by its spring. Its lower end records the minimum temp attained.

As alcohol expands in B it pushes up the mercury column in the right hand limb along with its steel index I_1. Its lower end records thus the maximum temp attained.

To set the thermometer afresh the steel indices are brought down to the mercury tops by a magnet.

D. Range of Temp : You know that Nature has put a lower limit to the temp possible which is the absolute zero, today

recognised as 0. Nothing in the universe can be lower than this (−273°C) temperature.

There seems to be no upper limit to high temperature. The temp of the solar surface is estimated at about 6000°C. In its interior the temp, is believed to be about 20 million deg C. The core of a bursting nuclear bomb or hydrogen bomb may attain a temperature of a few million deg for a while, while the interior of the hottest stars may be at hundreds of millions of degrees We know nothing hotter than this but a precise measurement is quite out of question. We append below some melting points and liquefaction points under std. atmospheric pressure in the **celsius scale**.

Material	Melting Point
Solid CO_2	−78°
Mercury	—39°
Ice & Salt (4 : 1)	—17°
Ice	0°
Tin	232°
Lead	330°
Silver	960°
Gold	1063°
Copper	1083°
Iron	1545°
Platinum	17 0°
Tungsten	3400°

Gases	Liquifies at
Steam	100°
NH_3	—34°
NO_2	—34°
SO_2	—8°
Oxygen	—183°
Nitrogen	—196°
Hydrogen	—253°
Helium	—269°

E. Summary of Thermometric Principles, Substances and Ranges :

Thermometer	Substance	Property	Range (in °C)
1. Bimetallic strip (IV-3)	Solid filament of diff. metals	Relative Expansion in length	—4 to 500
2. Liquid-in-glass (IV-1)	(a) Mercury (b) Alcohol	Expansion in Vol.	—38 to 350 —80 to 60
3. Vapour Pressure (IV-8)	Vapour of volatile liquid	Vapour Pressure	—150 to −272
4. Gas Thermometer (a) Const. Vol (b) Const. Press (IV-5)	Hydrogen Helium	Increase in Press " " Vol	—260 to 1600 —183 to 600
5. Resistance Vol II (IV-2)	Platinum	Increase in Resistance	—200 to 1200
6. Thermocouple Vol II. (IV-4)	Pt-Rh and Cu-Constantan	Rise in thermo-emf	—200 to 1600
7. Pyrometer (Radiation)	Tungsten wire	Change in brightness (Stefan's Law)	800 up to any value
8. Magnetic material	Paramagnetic salt	Magnetic susceptibility (Curie's Law)	Below —270

CALORIMETRY

IV-2.1. Heat is a measurable Quantity

In the foregoing chapter we have learnt how to measure **Temperature**; in this we seek to measure the other fundamental quantity, **Heat**. In the early days, heat was wrongly identified with a fluid called *Caloric* which was supposed to be in every type of matter; which on addition was supposed to raise the temperature of a body and on extraction, lower the same. Measurement of caloric was named Calorimetry.

Heat is now-a-days defined as the *energy transferred from one body to another because of temp difference*. This provides the basic principle of Calorimetry—*When a hot body is brought in intimate contact with a cooler body, provided no heat enters or leaves the closed system, heat lost by the hot body is equal to that gained by the cold body.* Flow of heat stops when the two reaches the same temperature just as flow of liquid from a higher to a lower level stops when the liquid attains the same level. In the former, thermal eqilibrium, in the latter, mechanical equilibrium is reached when the flow stops. To know this equilibrium temperature is the quest of all calorimetric measurements and can be reached from the knowledge of thermal properties of the bodies involved.

IV-2. 2. Units of heat. Before we can measure a quantity we require a unit. The unit of heat is defined as the quantity of heat required to raise the temperature of unit mass of water by unity. This gave rise to three different units, *viz.*

(i) the **calorie**, which is the unit of heat in the **cgs** system. *It is the quantity of heat required to raise the temperature of one gram of water by one degree celsius. A kilocaloric* (also called a *large calorie*) is 1000 calories.

(ii) the **British thermal unit** (abbreviated **Btu**), which is the unit of heat in the **fps** system. It is defined as the *heat required to raise the temperature of one pound of water by one degree fahrenheit.*

A commercial unit of heat is the **therm.**

1 therm = 100,000 or 10^5 Btu

(iii) the **pound degree-centigrade unit** or *centigrade heat unit* (abbreviated CHU), a hybrid unit which *raises the temperature of one pound of water by* $1°C$.

(iv) the **Joule** is the unit of heat in the **SI** or **MKS** system. It is the same as the unit of work or energy in that system.

It is clear from the definitions that the heat required to raise m grams of water through $t°C$ is $m \times t$ calories, while H calories will raise the temperature of m grams of water by *skt*. We are thus led to the following relations between the units :

$$1 \text{ Btu} = 1 \text{ lb} \times 1°F = 453.6\text{g} \times \tfrac{5}{9}°C = 252 \text{ calories ;}$$

$$1 \text{ lb}°C = 1 \text{ lb} \times 1°C = 453.6\text{g} \times 1°C = 453.6 \text{ calories.}$$

$$1 \text{ Joule} \doteq 0.24 \text{ cal.}$$

With improvement in the accuracy of measurement, we gradually came to know that water does not require the same quantity of heat (minimum at 37°C) for a one degree rise at different temperatures. So, in defining the heat units it became necessary to state at what temperature this rise should take place. The present *calorie* (symbol, cal) was defined as :

The calorie is the quantity of heat required to raise the temperature of one gram of pure water from 14·5° C to 15·5° C under a pressure of one atmosphere. It is called the *15°-calorie.*

West Germany has passed a law (1970) to the effect that the joule (J) should be used instead of the calorie even for domestic purposes—such as in stating the calorific value of food or fuel.

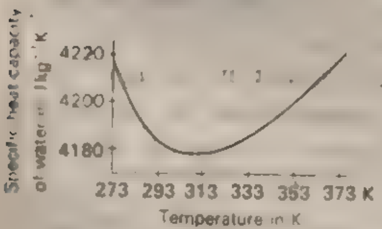


Fig. IV-2.1

We shall presently learn that specific heat capacity or specific heat (as called formerly) of water is the energy required to raise unit mass of it through unit rise in temp. In fact a calorie or a Btu is that. The

15°-cal is equal to 4·1855 J as told above, but at other temperatures the value is different as shown in fig. IV-2.1. where values along the ordinate gives $JKg^{-1}K^{-1}$ i.e. joules required to raise 1 kg. of water through 1 K. plotted against temp in K.

IV-2.3. Specific heat. The specific heat is the quantity of heat required to raise the temperature of unit mass of a substance by one degree. In the cgs system, *the specific heat of a substance is the heat in calories required to raise the temperature of* 1 *gram of the substance through* 1°*C*. The unit in which to express specific heat is *calories per gram per°C.* In symbols, we write it as cal/g °*C* or cal g^{-1} $°C^{-1}$. For 'cal', you may find J in some books.

The term **specific heat capacity** is now preferred to the old term 'specific heat'.

We shall generally use cgs units. From the definition of specific heat it follows that if *s* be the specific heat of a substance, heat required to change the temp. of 1 gm of a substance by 1°*C* is *s* calories ; heat required to change the temp. of *m* gm of a substance

by 1°*C*. = *ms* calories ;

Heat (*Q*) required to change the temp. of *m* gm of a substance

by *t*°*C* = *mst* calories ;

In symbols, $$Q = mst \qquad \text{(IV-2.3.1)}$$

Heat gained = mass × specific heat × temperature rise.

Heat lost = mass × specific heat × temperature fall.

Eq. IV-2.3.1. is the *fundamental equation* in calorimetry.

We may arrive at the same result more elegantly from experimental results as follows, which provides a definition of sp. heat capacity also. Let *Q* be the amount of heat supplied to a mass *m* of a substance to raise its temperature by *t*. Then from experiments we find that

(1) $Q \propto m$ when rise of temp *t* is const

(2) $Q \propto t$ when mass *m* remains constant.

i.e. for (1) you take different masses of a given material and

heat them through same temp rise ; for (2) you take a given mass of the given material and heat it through different temp intervals. Combining the two we get by theorem of Joint Variation,

$H \propto mt$ when both m *and* t vary,

$\therefore$ $H = s.m.t$ where s is the constant of proportionality, a const for a given material, called the *specific heat capacity.* So $s = H/mt$ i.e. cal/g/$C°$ or Joules/kg/K—heat required to raise unit mass through unit temp difference.

Remember, specific heat (capacity) of water in cgs system is cal/gm/$C°$, in SI system 4185 J/kg/K.

These relationships also apply to the other units. Thus if s is the specific heat given as a number, Q will be in Btu if m is in lb, and t in °F. When m is in lb and t is in °C, Q will be in centigrade heat units (lb °C).

Table : Specific heats (in calories per gram per °C)

Substance	Sp. heat	Substance	Sp. heat	Substance	Sp. heat
Aluminium	0·210	Nickel	0·109	Glass	0·12—19
Copper	0·091	Platinum	0·032	Ice	0·502
Gold	0·030	Silver	0·056	Marble	0·22
Iron	0·105	Tin	0·054	Castor oil	0·508
Lead	0·030	Zinc	0·092	Olive oil	0·47
Mercury	0·033	Brass	0·088	Turpentine	0·42

Example IV-2.1. If the specfic heat of iron is 0.1 cal/g°C how much heat will 100g of iron require to be heated from 30°C to 100°C ?

Solution : From equation IV-2.3.1 we have

heat gained = mass × sp. heat × temp. rise

$$100\text{g} \times 0.1 \frac{\text{cal}}{\text{g°C}} \times (100-30)°\text{C} = 700 \text{ cal.}$$

Ex. IV-2.2. Express a specfic heat of 0.1 cal/g°C in fps units.

Solution : Let 0.1 cal g^{-1} °C^{-1} = x Btu lb^{-1} °F^{-1}.

$$\text{Then } x = 0.1 \frac{\text{cal}}{\text{Btu}} \times \frac{\text{lb}}{\text{g}} \times \frac{°\text{F}}{°\text{C}}$$

$$= 0.1 \times \frac{1}{252} \times 453.6 \times \frac{5}{9} = 0.1$$

N. B. In the same way you can show that this specific heat is also equal to 0.1 C.H.U/lb°C. See the example on p.13 in the Chapter 0-1. However the equality does not hold in the SI system. There the above value 0.1 will be 418.5 $JKg^{-1}K^{-1}$,

Ex. IV-2.3. A Kg of water is heated from 30°C to 100°C and 300 lb of water from 92°F to 212°F. Which one requires more heat ? [H.S. '65]

Solution : Heat required in $(1) = 10^3 \times 1 \times (100-30) = 30$ Kcal

$$\text{in } (2) = 3 \times 1(212-92) = 360 \text{ Btu}$$
$$= 360 \times 252 = 90,720 \text{ Cal}$$
$$= 90.72 \text{ Kcal.}$$

Hence more heat is required in the second case

Ex IV-2.4. If the sp. heat of ice is 0.5 cal/g °C how much heat will 4 kg of ice give up in cooling from 0°C to 10°C – 10°C ?

Solution : Since heat lost = mass × sp. heat × fall in temp., we have

$$\text{heat lost} = 4000 \text{ g} \times .05 \frac{\text{cal}}{\text{g°C}} \times 10°\text{C} = 20 \text{ K cal.}$$

Ex. IV-2.5. In cooling from 100°C to 20°C a mass of 50 g of brass gives up 360 calories of heat. Find the specific heat of brass.

Solution : If s is the required specific heat we have from relation IV-3.2:1

$$360 \text{ cal} = 50 \text{ g} \times 80°\text{C} \times s$$

$$\therefore s = \frac{360}{50 \times 80} \cdot \frac{\text{cal}}{\text{g°C}} = 0.09 \frac{\text{cal}}{\text{g°C}}$$

IV-2.4. Experiment to show difference in specific heats : That different substances differ in the values of their specific heats may be demoustrated by a simple experiment. Take a number of spherical balls of different materials but of the *same mass*. Heat them together in boling water and piace them on a thick sheet of paraffin. It will be seen that different balls penetrate to different

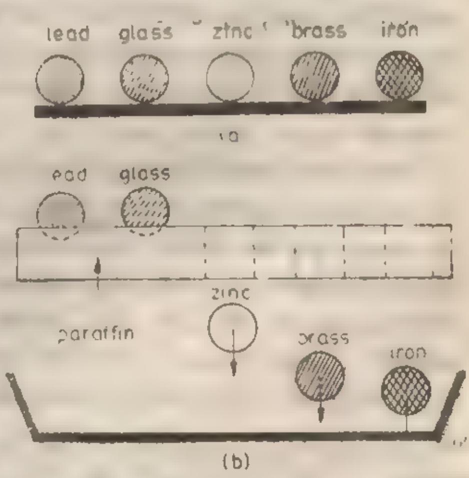


Fig. IV- 22

depths within the wax. The reason lies in the difference in the heats given out by the balls in cooling from the temperature of boiling water to melting wax. The ball that gives up more heat melts more wax and penetrates deeper. Heat given out by any of them is equal to its mass × specic heat × fall in temperature. Since the mass and the fall in temperature are the same for all, their specific heats must be different.

Fig IV-2.2 shows the state of affairs; clearly glass a non-metal and lead a metal has low specific heat capacities whereas iron has the most. That would be borne out from the above table also.

Effects of high specific heat of Water : *In all but the SI system* water has the *specific heat of unity.* The table and worked out sums above show that is *the highest possible value*, much higher than most.

So to raise some quantity of water through 1° rise much greater amount of heat than others is necessary; also for a drop of 1° in temp much more heat than others is released. So it can act as a sort of *heat bank*, a good coolant as well as a good heater. Hence its use in cooling motor car radiators and steam engines as in hot water bags to revive a sinking patient or room heaters in cold countries.

This large specific heat of water helps in mitigating large daily and seasonal variations of temperatures in islands and sea-shores Sea heats up less slowly than land and also cools less slowly. This is the reason for land and sea breezes maintaining the equability of temperatures near the sea. During the winter the earth is at perihelion (*peri*—close, *helios*—the sun) and receives more heat than in summer. For the very fortunate tilt of the earth-axis, the sun in winter is overhead the southern hemisphere which is mostly oceans. Hence over-all rise in temp is much less than would have happened for land. Marine life is also benefited greatly by this small temp variation and large store of heat in the seas and oceans.

The same characteristic makes water an undesirable calorimetric liquid.

IV-2.4. **Thermal capacity** (or heat capacity) and **water-equivalent.** The **thermal capacity** *of a body is the heat required to raise the temperature of the body by* 1°C. If m be the mass of the body in grams and s the specific heat of its material (in cal/g°C) it requires $Q=mst$ calories for a rise of t°C. Then its heat capacity C=heat required for 1C° rise of temperature$=Q/t=ms$ calories. In symbols, we that have

$$C=ms \text{ cal/°C} \qquad \text{(IV-2.4.1)}$$

It is clear from the above that **specific heat** (s) **or, specific heat capacity** is the **thermal** (or **heat**) **capacity of unit mass.**

Water equivalent *of a body is the mass of water in grams which will be heated through* 1°C by *the heat that raises the temperature of the body itself by* 1°C. If m is the mass of the body in gm and s its specific heat, the body requires ms calories in order that it may be heated through 1°C. Since 1 calorie heats 1 g of water by 1°C, ms calories will heat ms grams of water by 1°C. Therefore, the *water equivalent W of the body is ms grams.*

$$W=ms \text{ grams} \qquad \text{(IV-2.4.2)}$$

From equations IV-2.4.1. and IV-2.4.2 we see that the numerical values of the thermal capacity and the water equivalent of a body are the same, but they are expressed in *different units.*

When the water equivalent W of a body is known, it follows from the definition of this quantity that the heat Q which the body requires for a rise of t°C is

$$Q=Wt \qquad \text{(IV-2.4.2)}$$

since the specific heat of water is unity.

[The use of the term 'water equivalent' is no longer encouraged. The term 'heat capacity' is used instead]

Ex. IV-2.6. Two substances have densities in the ratio 2 : 3 and their specific heats respectively 0.12 and 0.09. compare the thermal capacities of their unit volumes. [C.U./I.Sc., H.S. '83]

Solution : $\frac{m_1 s_1}{m_2 s_2} = \frac{VP_1 s_1}{VP_2 s_2} = \frac{2}{3} \cdot \frac{0.12}{0.09} = \frac{8}{9}$.

Thermal capacity of a body is *ms* and we take same volume of them.

Problem : Sp. gr. of a certain liquid A is 0.8 and that of another. B, is 0.5 It is found that heat capacity of 1.5 litres of A equals that of 1 litre of B, compare their specific heats.

(Ans : 5 : 12) [**Pat U.**]

Ex. IV-2.7. Equal volumes of mercury and glass have the same heat capacity (a plus pt. for mecury in glass thermometers). Find the sp. heat of a piece of glass of sp. gr 2.5 if the specific heat and sp. gr for mercury is 0.0333 and 13.6 respectively. [**Pat. U.**]

Solution : Masses of *v* cc of glass and mercury are 2.5 *v*g and 13 6 *v*g. From given condition we have

$$V \times 2.5 \times s = V \times 13.6 \times 0.0333 \text{ or } s = 0.181.$$

Ex. IV-2.8. An iron saucepan contains 100g of water at 25°C. 50g of water at 60°C is poured into it ; the final temp attained is 35°C. Assuming no loss of heat find the water equivalent of the saucepan and sp. heat of iron if its mass is 233g. [**H. S. '60**]

Solution : Let W be the required water equivalent. Then from the relation Heat lost = Heat gained we get

$$50 \times 1 \times (60 - 35) = (100 + W)(35 - 25) \text{ or } W = 25\text{g}.$$

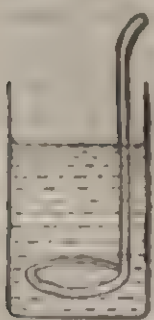


Fig. IV-2.3

Since again water equiv = Thermal capacity and sp. heat = Thermal capacity of unit mass we have $s = W/m = 95/238 = 0.105$ cal/g/°C.

IV-2.5. The fundamental principle of calorimetry. In most experiments on calorimetry bodies at different temperatures are brought into intimate contact inside a calorimeter. It is a cylindrical *copper* * vessel (Fig. IV-2.3) containing a liquid and a stirrer for stirring the liquid. Heat flows from the hotter to the colder bodies until all of them reach a common or

* A material of high heat conductivity and low specific heat capacity.

equilibrium temperature. The method is known as the **method of mixtures.**

We *assume* that (1) no heat enters or leaves the calorimeter after the bodies have been brought into contact (2) no chemical action takes place between the bodies or with the calorimeter, (3) nor does the solid dissolve. Then, from the principle of conservation of energy, we may say that

Heat lost by the warmer bodies=heat gained by the colder bodies.

This is the *fundamental principle of calorimetry.* Note the conditions to be fulfilled.

Precautions. No heat is to enter or leave the calorimeter. The calorimeter must be so designed as to eliminate loss or gain of heat by conduction, convection and radiation.

A. Precautions in design. The calorimeter must be of a *highly conducting* material so as to achieve equality of temp all through and quickly. To minimize heat loss by *conduction* the calorimeter is placed on a thermal insulator, such as cork or felt. It is better however to support it on spikes of cork (fig. IV-2.4) or strips of cardboard set on edge or to suspend it by fine threads. *Convection* is reduced by surrounding the calorimeter with an outer vessel and packing dry cotton wool or felt in between and to cover it by a wooden lid. The lid is provided with

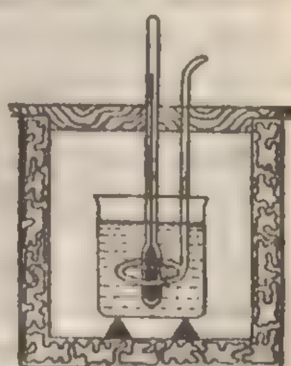


Fig IV-2.4

holes to allow the thermometer and stirrer to pass in. To reduce *radiation loss* the calorimeter is highly polished. The heat lost by radiation—rather by convection and radiation—can however be determined experimentally and allowed for in calculating the results.

B. Precautions in method. Besides, precautions such as

(i) prolonged heating of the solid so that it may attain a steady temperature (ii) quick transference of the body into the calorimeter so that the body may not lose heat on its way down (iii) continuous stirring of the liquid so that it may attain a uniform temperature (iv) screening the calorimeter from other sources of heat and (v) avoidance of splashing of the liquid while dropping the solid into it, should be taken.

Water is not suitable as a calorimetric liquid. Its specific heat is much higher than that of any other liquid. So the rise in temperature of water for a given supply of heat will be much less than that of other liquids. The percentage accuracy with which a temperature difference can be measured diminishes as this difference itself does; with water as the calorimetric liquid the result is likely to be less accurate than if some other liquid, such as an oil, were used.

Properties desirable for a calorimetric liquid. From above discussions, we may list them as follows :—The liquid must

(i) have low specific heat capacity

(ii) have high boiling point so that evaporation be small

(iii) be chemically inert i.e. will not chemically react with or dissolve the solids or the material of the container

(iv) be a good conductor so that equality of temp throughout is quickly attained

(v) have high fluidity so as not to stick to the calorimeter.

Oils fulful the requirements except the last two better than water. *Aniline* a pure chemical with medium sp heat (0.6) and high boling point 184 °C, is now regarded as the best calorimetric liquid.

IV-2.6. Determination of specific heats of Solids and Liquids (Method of Mixtures)

To determine the former a liquid of known sp heat and for the latter a solid of known sp. heat is required. The experimental procedure is exactly the same. The solid and the liquid must not interact. Remember in all experiments *a hot body is put in*

intimate thermal contact with a cold liquid and heat flows till thermal equilibrium is reached.

To carry out the experiment the solid is first weighed and then suspended inside a **steam heater** (fig IV-2.5) which is a hollow double-walled cylindrical chamber, steam circulating between the walls It enters through a pipe and leaves through another. The hollow space C is stoppered by a cork above. Through holes in it are suspended the experimental solid by a thread close beside the bulb of a thermometer. The lower opening may be closed by a movable lid. The heater and the lid can slide up and down an

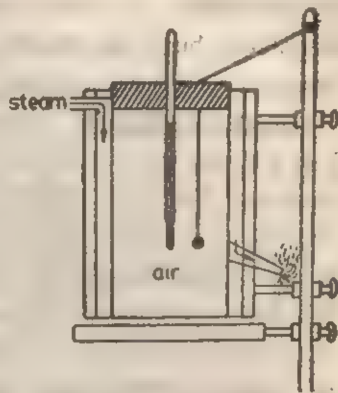


Fig. IV-2.5

upright with a large base on which may be placed the calorimeter to receive the heated solid. Steam is raised in a boiler and passed through a rubber tubing to the inlet.

Steam is passed for a long time till the thermometer shows a steady reading close to 100°C. In the mean time the metal calorimeter is weighed with the stirrer, dry and empty and again with water a *little* more than that required to completely immerse the solid. It is next well-stirred, placed inside the outer box and then with a *sensitive* thermometer the temp is noted, placed well away from the heater.

The calorimeter is placed on the base of the heater, the lid moved out and the hot solid quickly dropped inside the water which is continuously stirred and the rising temp noted at half-minute intervals till it reaches a maximum and then begins to fall. This is the common final temp which should not be much higher ($<10^{\circ}C$) than that of the surrounding.

As you have noted before, many precautions are necessary to minimise heat loss from the calorimeter and its contents.

IV-2.7. Calorific value of fuels. Different samples of fuels are categorised by the amount of heat that can be obtained by burning a given mass of one, completely. For example, when it is stated that the calorific value of a certain sample of coal is 10^4 Btu per pound, it means that when one pound of the sample is completely burnt we would get 10,000 BThU of heat. Calorific value of the other *fossil fuel*—oil, can be similarly expressed.

Bomb calorimeter. It is the device to determine this quantity. It is a strong steel container with a small hole closed by a gas-tight cover and filled with oxygen at high pressure. A small quantity of the fuel powdered and dried is placed inside it and the so-called bomb is kept completely immersed in water in a large calorimeter. The fuel is fired by a wire placed inside and carrying electric current. The heat produced by combustion passes into the water of which the initial and final temp are noted. The mass of the powder, that of water in the calorimeter as well as water equivalents of the bomb and the calorimeter being previously known, the required calorific value can be determined from this rise of temp.

IV-2.8. Basic equation of the method of mixture. All calorimetric problems based on the method of mixtures can be solved with the help of **one equation only**. The equation is established below.

Let the mass of calorimeter+strrier	$=m_1$ g
specific heat of its material	$=s_1$ cal/g°C
the mass of the solid (to be heated)	$=m$ g
its specific heat	$=s$ cal/g°C,
mass of liquid in the calorimeter	$=m'$ g, (this may be water)
specific heat of liquid in calorimeter	$=s'$ ($s'=1$ for water)
water equivalent of the calorimeter	=mass of cal. × sp. heat of its material $=W$ g,
initial temp of liquid in calorimeter	$=t_1$°C,
initial temperature of the hot body	$=t_2$°C,
final common temperature	$=t$°C.

Here *the body loses heat while the calorimeter and its contents gain heat.*

Heat lost by the body $=ms\,(t_2-t)$ cal ;

Heat gained by liquid in the calorimeter $=m's'(t-t_1)$ cal ;

Heat gained by the calorimeter $=W(t-t_1)$ cal ;

$$\therefore\quad ms(t_2-t)=(m's'+W)\,(t-t_1) \qquad \text{(IV-2.8.1)}$$

From this equation any of its unknown quantities, such as s, s', W_2 etc., may be determined, when all other quantities are known.

Ex IV-2.9. Sp. heat of a water-soluble solid. 10g of common salt are heated to 97°C and dropped into a calorimeter containing oil of turpentine If the mass of the oil is 125g, its specific heat 0.43, and temperature 32°C, find the specific heat of common salt when the water equivalent is 15 g and the final temperature 35°C.

Solution : Let s be the required specific heat.

Heat lost by common salt$=10\times s\times(97-35)$ cal.

Heat gained by the calorimeter and oil

$=(15+125\times0.43)\,(35-32)$ cal.

$\therefore\quad 10\times s\times62=(15+125\times0.43)\times3$

whence $s=0.333$ (in cal g^{-1} °C^{-1}).

Ex. IV-2.10. Determination of a high temperature with a calorimeter. An iron ball weighing 50 g is, heated in a furnace and dropped into 240 g of water at 30°C contained in a vessel of water equivalent 10g. If the temperature rises to 50°C, find the temperature of the furance, given that the specific heat of iron$=0.1$.

Solution : Let t be the required temperature

Heat lost by the ball$=50\times0.1\times(t-50)$ cal.

Heat gained by water and the vessel$=(10+240)(50-30)$ cal.

$\therefore\quad 5(t-50)=550\times20$ or $t=1050$°C.

This is a method of *pyrometry* (*pyros* – fire)

Ex. IV-2.11. Determination of the specific heat of a liquid A piece of glass weighing 100 g is heated to 95°C and dropped into olive oil contained in a calorimeter. The mass of the oil is 120g,

the mass of the calorimeter 150 g and its specific heat 0.1. If the temperature rises from 30°C to 45°C, find the specific heat of the oil, given that the specific heat of glass is 0.22.

Solution : Heat lost by glass $=100\times0.22\times(95-45)=1100$ cal.

Heat gained by the calorimeter and the oil

$=(150\times0.1+120s)(45-30)=225+1800s$ cal.

$\therefore\ 1800s+225=1100$ or $s=0.486$.

Ex. IV-2.12. Determination of water equivalent of a calorimeter. A calorimeter contains 70.2 g of water at 15.3°C. If 143.7 g of water at 36.5°C are mixed with it the common temperature is 28.7°C. Find the water equivalent of the calorimeter.

Solution : Let the water equivalent be W g.

Heat gained by the calorimeter $=W(28.7-15.3)=13.4\ W$ cal.

Heat gained by 70.2g of water in being heated from 15.3°C to 28.7°C $=70.2\times13.4$ cal $=940.68$ cal.

Heat lost by the warm water $=143.7(36.5-28.7)=1120.86$ cal.

$\therefore\ 13.4W+940.68=1120.86$ whence $W=13.4$ g

Ex. IV-2.13. Calculation of common temperature. 50g copper are heated to 98°C and dropped into a calorimeter containing 100g of water at 30°C. If the water equivalent is 10g what is the final temperature ? Specific heat of copper $=0.09$.

Solution : Let the final temperature be t°C.

Heat lost by copper $=50\times0.09\times(98-t)$ cal.

Heat gained by the calorimeter and water $=(10+100)(t-30)$ cal.

$\therefore\ 110(t-30)=50\times0.09\times(98-t)$ whence $t=32.7$°C (approx).

How to handle thermal units properly. Handling units is never a problem when, in an equation, you write all the quantities with the unit symbols after their numerical values. If the unit for any quantity is not given, its value will automatically come out from the equation ; you *just treat the unit symbols as algebraical quantities.* This applies not only to thermal problems, but to all.

To illustrate it, let us treat the last solved example above in the way stated. Take specific heat not as a ratio but in cal g^{-1}°C^{-1}.

Then, heat lost by copper $= 50g \times 0{\cdot}09$ cal $g^{-1}{}^\circ C^{-1} \times (98-t)^\circ C$
$= 50 \times 0{\cdot}09 \times (98-t)$ cal (as before).

Heat gained by calorimeter and water $=(10g+100g)$ $(t-30)^\circ C$ $\times 1$ cal $g^{-1}{}^\circ C^{-1}$ because heat gained is mass of water $\times$ sp. heat of water $\times$ temp. rise, and the specific heat of water is 1 cal $g^{-1}{}^\circ C^{-1}$

Equating these you get t (in °C).

Ex. IV-2.14. A hot solid weighing 70g is dropped in a calorimeter of water equivalent 10g and containing 116g of water. Find the sp. heat of the solid if its fall in temp is 15 times as great as the rise in temp. of water. [J E E. 67]

Solution : From the relation Heat lost=Heat gained, we have

$m_1 s_1 \times$ fall in temp=(mass of water+water equiv) $\times$ rise in temp

Or, $70 \times s \times 15t = (116+10)t$ Or, $s = 126/(70 \times 15) = 0.12$

Ex. IV-2.15. An alloy contains 60% of Cu and 40% of Ni. A piece of it weighing 50g is heated to 80°C and dropped in a calorimeter of water equivalent of 10 g containing 90 g of water at 10°C. Find the final temp of the mixture ; given sp.ht for Cu=0.09 and for Ni=0.11. [H S '78]

Solution : Let m be the mass of Cu in the alloy piece and that of Ni is $(50-m)$. Then we have

$$[m \times 0.09 + (150-m) \times 0.11]\,(80-t) = (10+90)(t-10)$$

Now $m = 50 \times 60\% = 30g =$ mass of Cu.

So mass of Ni $=(150-m) = 20g$

$$\therefore \frac{(30 \times 0.09 + 20 \times 0.11)}{100} = \frac{t-10}{80-t} \quad \text{Or } (0.027+0.022)=0.049$$

$$\therefore \quad t = 13.27^\circ C$$

Ex. IV-2.16. Temp of equal masses of three different liquids A, B and C are 12°C 19°C and 28°C. Mixture of A and B results in a final temp of 16°C ; that on mixing B and C produces a temp of 23°C. Find the resultant temperature when A and C are mixed.

[I I.T. '76]

Solution : Let their specific heats be S_A, S_B, S_C. Now from the given data

(1) $m \times S_A \times (19-16) = m \times S_B \times (16-12)$ $\therefore$ $S_A/S_B = 3/4$

(2) $m \times S_C \times (28-23) = m \times S_B \times (23-19)$ $\therefore$ $S_B/S_C = 5/4$

$\therefore$ $\frac{S_A}{S_B} \times \frac{S_B}{S_C} = \frac{S_A}{S_C} = \frac{15}{16}$. Let the required temp be t

$\therefore$ $m \times S_C \times (28-t) = m \times S_A \times (t-12)$

$\therefore$ $\frac{28-t}{t-12} = \frac{S_A}{S_C} = \frac{15}{16}$ Subtracting both sides from 1 we have

$\frac{t-12-28+t}{t-12} = \frac{1}{16}$ Or, $t-12 = 16(2t-40)$

Or, $31\ t = 640-12 = 628$ Or. $t = 20.25°C$

Ex. IV-2-17. Three liquids of equal masses at temp $t_1°$, $t_2°$ and $t_3°C$ of sp. heats s_1, s_2, s_3 are thoroughly mixed. Find the common temp if $t_1 > t_2 > t_3$.

Solution : Let T be the common temp. obtained by mixing the first two liquids where $t_1 > T > t_2$. Then

$ms_1\ (t_1 - T) = ms_2\ (T - t_2)$ Or, $T = \frac{s_1 t_1 + s_2 t_2}{s_1 + s_2}$

Again let the third liquid be mixed when finally the common temp be T'. where $t_3 > T' > T$. Then we shall have

$$ms_1(T'-T) + ms_2(T'-T) = ms_3(t_3 - T')$$

Or, $T'(s_1 + s_2 + s_3) = s_3 t_3 + T(s_1 + s_2) = s_3 t_3 + \frac{s_1 t_1 + s_2 t_2}{(s_1 + s_2)}\ (s_1 + s_2)$

$$= s_3 t_3 + s_1 t_1 + s_2 t_2$$

$$\therefore\ T' = \frac{s_1 t_1 + s_2 t_2 + s_3 t_3}{s_1 + s_2 + s_3}$$

Ex. IV-2.28. A steel ball of mass 10g and sp. heat 0.1 is dropped quickly from a furnace into a *thick* copper ($s = 0.09$) vessel of mass 200g at 50°C. The whole is then dropped in a calorimeter

of water equivalent ·20g and containing 180g of water at 20°C. Thermometer in it records a maximum temp of 26°C. Find the furnace temp and also find by calculation whether there will be any local boiling [J.E.E. '78]

Solution : Let the furnace temp be t°C. Then

Heat lost by (the ball+copper vessel) = heat gained by (water+calorimeter)

$$\text{Or, } 10\times0.1(t-26)+200\times0.09(50-26)=(180+20)(26-20)$$

$$\text{Or, } (t-26)+18\times24=200\times6 \quad \therefore \quad t=794°\text{C}.$$

Again let t' be the temp of the copper vessel when the ball from the furnace is dropped into it : We then have

$$10\times0.1(794-t')=200\times0.09(t'-50)$$

$$\text{Or, } 794-t'=18(t'-50) \quad \text{Or, } t'=89{\cdot}2°\text{C}$$

Since the vessel temp is less than 100°C, there will be no local boiling when the vessel is dropped in the water of the calorimeter.

Note : Had the ball been dropped in the calorimeter directly without the *auxiliary* copper vessel which has absorbed 1200 calories of heat, heating there would have been much greater, temp rise leading to much evaporation and loss of water. Further had the auxiliary vessel been thin it would have absorbed less heat and so greater amount of heat would have gone into calorimeter possibly producing local boiling.

Ex. IV-2.19. 60 cu ft of water in a bath is heated by a gas burner. Calorific value of the gas is 600 Btu per cu ft and it costs 95 p. per 1000 cu ft. Find the cost of heating the bath water from 55°F to 100°F assuming 70% of the heat goes to water. [J.E.E. '81]

Solution : Mass of given water = 60 cu ft × 62.5 lb/cu ft

Rise of temp = (100 − 55°) = 45°F

$\therefore$ Heat required = 60 × 62.5 lb × 45°F = 60 × 62.5 × 45 Btu

Let the volume of the gas burnt be V cu ft. So heat from the burner is $V \times 600$ Btu/cu ft. From the given datum

$$70\% \times V \times 600 \text{ Btu/cu ft} = 60 \times 62.5 \times 45 \text{ Btu}$$

$$\therefore \quad V = 401.8 \text{ cu ft.}$$

$\therefore$ Cost of this volume of gas would be $\frac{401.8}{1000} \times 95 = 38$ p.

Ex. IV-2.20. **Methods of calorimetry.** Numerous methods are available for making calorimetric measurements. They may be broadly divided into two classes.

A. Thermometric calorimetry. Here changes in temp are recorded. This may be further sub-divided into,

(i) *Method of mixtures*: All our preceding discussions fall under this head.

(ii) *Electrical calorimetry*: Here heat energy is supplied electrically which makes manipulations much easier and calculating the energy input far more precise.

A heating coil is embedded in the metallic solid or immersed in the experimental liquid. If the heating current is I and flowing for t s under a P. D. of V volts then the electrical energy supplied is VIt joules and heat developed is VIt/J cal where J is the mechanical equivalent equal to 4.2J/cal. Since the current can be controlled very precisely and no solid to be heated for long and transferred to the liquid gingerly, the experiment can be controlled closely and made to yield precise results.

The variation of sp. heat of water as shown in fig 4-2.1, was obtained from such electrical experiments.

(iii) *Method of Cooling*: Newton had established that provided not much heat is lost by conduction of heat through air, rate of heat loss from a body follows a definite law.

Newton's Law of Cooling: It states that when a hot body cools in air without appreciable conduction of heat the *time rate of*

cooling is proportional to the difference in temperature of the body and that of its surroundings. Newton however expressly stated the law to be valid when the body is *not in still air but in a uniform current of air.* In symbols.

$$-\frac{dH}{dt} \propto (\theta_B - \theta_s) = K\theta \qquad \text{(IV-2,9.1)}$$

where θ_B and θ_s represent respectively the instanteneous temperatures of the body and that of the surrounding. The relation will be deduced from Stefan's Law later. The $-$ve sign indicate a *fall* in temperature (fig. IV-2.6)

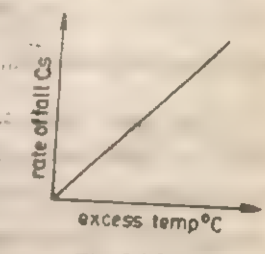
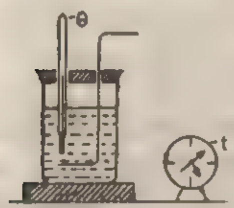
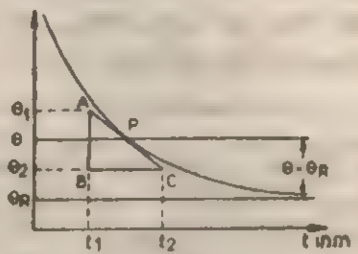


Fig. IV-2.6



Apporotus

Fig. IV-7.(a)



Results

Fig. IV-7.(b)

Determination of specific heat capacity of a liquid : A weighed quantity of it is allowed to cool in a calorimeter and a cooling (i.e. fall of temperature with time) curve (fig. IV-2.7a and 7b) is drawn from temp readings of the liquid every half-minute. The liquid is then replaced by an *equal volume* of warm water for cooling through the *same range of temp.* During cooling both the liquid should be well-stirred.

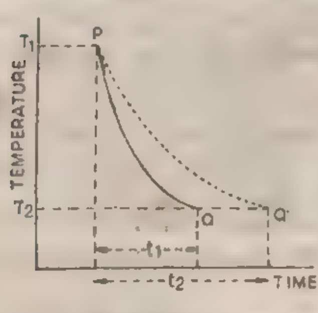


Fig. IV-2.7(c)

From the curves (Fig. IV-2.7c) time-intervals t_1 and t_2 taken by the same volumes of liquid and water to fall through the same temp range from T_1 to T_2. If W is the waterequivalent of the calorimeter, m the mass of liquid.

and M that of water then applying Newton's law to the cooling of the two liquids we arrive at the relation

$$\frac{W+ms}{W+M}=\frac{t_l}{t_w}$$

where s is the only unknown.

Ex. IV-2.20. Temperature of a calorimeter fell from 30.2°C to 29.8°C in 2 min. and again from 27.7°C to 27.3°C in 2 min. 40s. Find the temp of the enclosure;

Soluiton : Let t be the required temp. The rate of cooling in the first case is 0.4/120 and the average temp $\frac{1}{2}(30.2+29.8)=30°C$ and the temp excess $(30-t)$

$$\therefore \quad 0.4/120=K(30-t)$$

K being the variation const. For the second case we have similarly, $0.4/160=K(27.5-t)$

$$\therefore \quad t=20°C.$$

Ex. IV-2.21. Some liquid is found to cool from 353K to 337K in 5 min., to 325K in 10 min. What the temp will be after 15 min. and what is the room temp ? [J.E.E. '86]

Solution : (a) Let θ be the required room temp. The average temp during the first interval is $\frac{1}{2}(353+337)=345K$ and that during the second interval $\frac{1}{2}(337+325)=331K$.

$$\therefore \quad 16/5=K(345-\theta)$$

$$\text{and } 12/5=K(331-\theta) \quad \text{Or, } 4/3=\frac{345-\theta}{331-\theta} \quad \text{Or, } \theta=289K$$

$$\text{Again } K=\frac{16}{5\times(345-289)}=\frac{16}{5\times 56}$$

(b) If T min. be the required temp then

$$\frac{325-T}{T}=\frac{16}{5\times 56}(325-289) \quad \text{Or, } T=314.7\ K$$

B. Latent heat calorimetry : You know that to change a solid to a liquid or the latter to a gas, heat is to be supplied but

the temperature does not change. So is the heat described as *latent*. Specific latent heat is fixed for a given substance and the amount of heat to change 1g of it from solid to liquid (latent heat of fusion) or from liquid to vapour (latent heat of vaporisation). Hence it is possible to measure a quantity of heat by noting the mass changing from one form to another by the heat supplied, and so to measure specific heat capacity of a solid.

1. **Black's Ice Calorimeter :** It acts on the principle that when heat is applied to ice at 0°C a quantity of ice melts which is proportional to the amount of heat supplied.

The calorimeter is a roughly cubical block of ice with its upper face sloping away gently from a wide deep central hole which may be covered with another piece of thick ice. Water that may have collected inside the hole is soaked out by soft rags. A small but heavy piece of the experimental solid is heated up inside a steam heater till it attains the steam temperature, and quickly dropped into the hole and covered up. After sometime, the upper block is removed, the *ice-cold* water that has formed by the heat transferred from the piece to the surrounding ice is quickly soaked up by a piece of *previously weighed* blotting paper and as quickly weighed again. (Quickness prevents loss of cold water by evaporation). The difference gives the mass m' of ice melted ; the heat used up is $m'L$ where L is the specific latent heat of melting of ice. This equals $ms(100-0)$=heat lost by the hot body of mass m and specific heat capacity s, 100 and 0 being the steam and ice temperatures respectively.

$$\therefore\ ms.100=m'L \quad \text{Or,}\ s=m'L/100m$$

No radiation loss of heat occurs nor a thermometer is required. Clearly the experiment is quite crude.

2. **Joly's steam calorimeter :** In this arrangement as in the last, latent heat released due to condensation of steam on the experimental solid is utilised to find the specific heat capacity. Again, the specific latent heat of condensation of steam can be found from the same experiment, with a solid of known sp. heat.

The calorimeter (fig. IV-2.8) is a metal chamber A into which steam enters through T and leaves through E. Through a hole H passes into A a fine wire suspended from one arm of a sensitive balance, carrying a small metal pan P. The experimental *solid* piece B is placed on P and weighed. Steam is now admitted into A where it condenses on the solid, the pan and the inside walls. This mass of water adds to the weight of B and P and raises them to steam temp. In a preliminary expt the mass of water condensed on P for the same temp rise is found out.

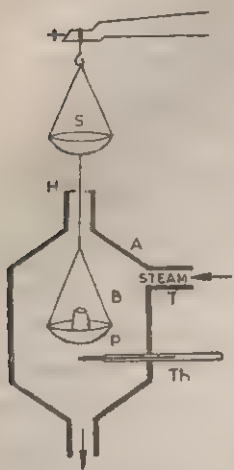


Fig. IV-2.8

If m be the mass of steam condensed on B to raise it from room temp t to steam temp, M the mass of B and s the specific heat capacity of its material and L_s the specific latent heat of condensation of steam then

$$Ms(100-t)=mL_s$$

very similar to the expression in Black's Ice calorimeter. But unlike it, Joly's steam calorimeter with a few more refinements yield high accuracy in results.

Ex. IV-2.21. A 15 g iron piece at 113.6°C is dropped in a cavity of ice block of which 2.5 g melts. Find the sp. heat of iron if L is 80 cal/g. [C.U.]

Solution : we know that $Mst=mL$

Or, $s=mL/Mt=2.5\times80/15\times113.60$

$=0.11$

Ex IV-2.22. When a piece of metal of mass 48.5 g at 10.7°C is exposed to a stream of stem 0.762 g of steam is found to condense on it. Find the sp. heat of the metal. ($L_s=540$ cal/g) [I.I.T. '68]

Solution : Note that the problem runs on the same line as outlined above. So if the sp. heat of metal be taken as s then we have

$$Ms(100-t)=mL,$$

Or, $48.5\,s\,(100-10.7)=0.762\times 540$

Or, $s=0.095$

Problem : A copper ball 56.32 g in mass and at 15°C is exposed to dry steam at 100°C. Find the mass of water collected till the ball reaches 100°C (s for Cu$=0.093$ and $L_s=540$ cal/g)

(*Ans.* 0.83g.) [**Dac. U.**]

Ex. IV-2.23. Ice and water at 0° have densities of 0.916 g/cc and 1g/cc respectively. A metal piece of mass 10g at temp of 90°C is dropped into a mixture of ice and water. Some ice melts and the mixture contracts by 0.1cc at 0°C. Find the sp. heat of the metal, given $L=80$ cal/g. [**I.I T. '64**]

Solution : Vol of 1 g of ice at 0°C$=1/0.916$ cc and 1 cc of water weighs 1g. So on melting, 1 g of ice contracts by $(1/0.916-1)$ or 84/916 cc. So for 0.1 cc of contraction (which is solely due to melting of ice) the mass of melted ice is

$$0{\cdot}1\times\frac{916}{84}=\frac{91.6}{84}\text{ g}$$

So heat required to melt it must have come from the solid

$$m_m\times s\times 90=m_i\times L$$

$$\therefore\quad 10\times s\times 90=\frac{91.6}{84}\times 8\ \therefore\ s=0.097$$

Specific heat of Gases : None of the above methods are suitable for measuring sp. heat of gases, for a gas has two sp. heats one at constant pressure, the other at constant volume. Different amounts of heat are required to heat 1 g of gas through 1°C under these two different conditions. If the mass of gas taken is 1 mole then difference in its specific heat becomes equal to the molar gas const R_0.

$$C_p-C_v=R_0$$

when C_p and C_v are expressed in joules. We shall return to this discussion in the last chapter.

However we shall not discuss any method of determining them.

EXPANSION OF SOLIDS

IV-3-1. Some demonstration experiments. When a solid is heated it expands; when it is cooled it contracts. The expansion or contraction is so small that it cannot be seen with the unaided eye. It therefore requires carefully designed experiments to demonstrate the effect.

Solids expand on heating. (i) Expansion of a solid on heating

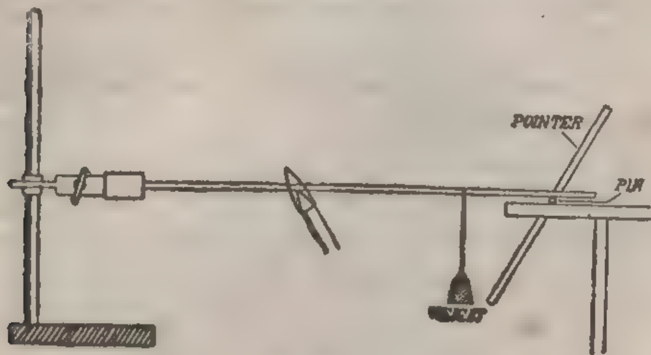


Fig. IV-3.1(a)

can be easily demonstrated with the arrangement shown in fig. IV-3.1(a). One end of a thin metal rod is clamped to a stand.

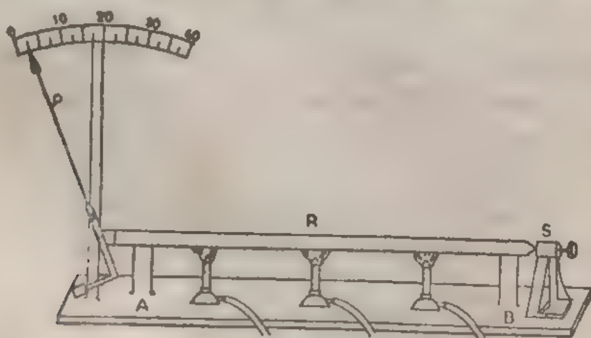


Fig. IV-3.1(b)

The free end is placed on a pin which carries a light pointer. A weight is hung from near the free end of the rod so that the rod

presses on the pin and keeps the pointer in position. The rod is heated by running a flame along it. It will be seen that the pointer is deflected from its position. When the rod is allowed to cool the pointer regains its original position. The explanation lies in the expansion of the rod due to heating and subsequent contraction on cooling.

(ii) **Fergusson's expt.** Fig IV 3.1(b) provides a more sophisticated demonstration of the same. Rods (*R*) of same length and cross-section may be placed horizontally on grooves cut on a pair of stout iron cylinders *A* and *B*. A screw *S* prevents expansion of the beam to the right. To the left the rod pushes against the short arm of a vertical lever *P* pivoted at *O*. Its long arm can move over a graduated scale. The rod can be heated from below by a row of burners.

With rise in temp. the tip of the pointer moves to the right indicating expansion of *R* to the left.

(iii) **Gravesand's Ball and Ring experiment :** Fig. IV-3.2 shows a ball *A* which, when cold, just passes through the ring *B*, but does not do so when heated.

(iv) **Expansion of wire when heated electrically.** A metal wire about 0.5 mm in diameter is hung from a hook and stretched vertically by a small weight. An electric current of several amperes is passed through the wire so that it becomes red hot. As the wire is heated the weight descends rapidly. It rises when the current is stopped. This provides a vivid demonstration of the expansion of a wire on heating.

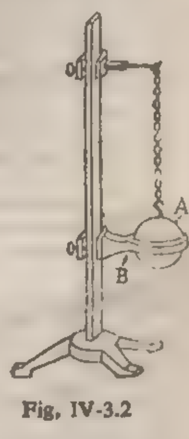


Fig. IV-3.2

To show the effect elegantly an iron wire about 2 m long is fastened to a hook *A* at one end (IV-3.3a) and to a weight *W* at the other ; in between the two points the wire passes over three pulleys *B*, *C*, *D*. A large battery passes a strong current through it from

end to end. A pointer P is attached to D which moves over a circular scale.

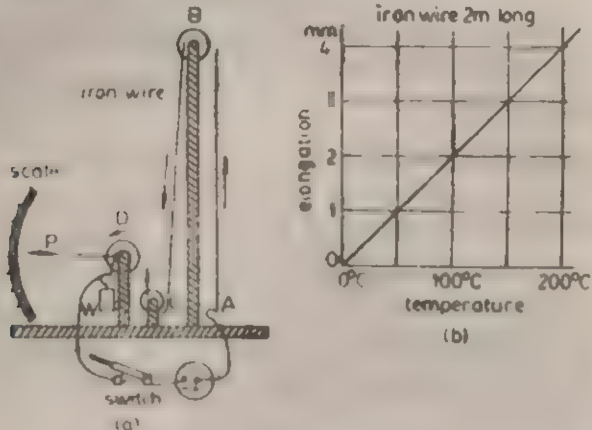


Fig. IV-3.3

With rise in temp the wire lengthenes turning the pulleys as indicated as also P and W descends. The *elongation with rise in temp* is shown in the adjoining diagram and note that the relation is a linear one (fig. IV-3.3b).

Different solids expand differently. Different solids expand by different amounts under identical conditions. For demonstration purposes a *composite bar* of aluminium and iron with a wooden

Fig. IV-3.4(a)

Fig. IV-3.4(b)

handle is shown in Fig IV-3.4(a). On heating it bends with iron inside.

Two metal strips of exactly equal length are riveted together [fig. 3.4(b)]. One is of brass and the other of iron. On heating,

this compound bar bends with the brass or the copper on the outside. On cooling it straightens again. The bending is due to the fact that one strip becomes longer than the other on heating. Brass or copper expands more than iron under the same conditions.

IV 3.2. **Co-efficient of linear expansion.** When a solid is heated it expands in all directions; but very often we are concerned with its expansion only in one direction, viz., its length. We speak of this as the *linear expansion* of the solid.

Let l_1 be the length of a bar at a temperature t_1, and l_2, its length at temperature t_2. Experiment shows that the elongation i.e., *increase in length*, $(l_2 - l_1)$, is approximately proportional to (i) the *original length* l_1 and to the (ii) *rise in temperature* $(t_2 - t_1)$ We may, therefore, write,

$$l_2 - l_1 = \alpha l_1 (t_2 - t_1) \qquad \text{(IV-3.2.1)}$$

where α is a small constant of proportionality whose *value depends on the material.* It is called the coefficient of linear expansion of the material. It should properly be called the *mean coefficient of linear expansion* between the temperatures t_1 and t_2. Note that elongation being proportional to temp rise their relation is linear as seen in fig IV-3.3b. Writing then,

$$\alpha = \frac{l_2 - l_1}{l_1 (t_2 - t_1)} \qquad \text{(IV-3.2.2)}$$

We may define α as follows: *The coefficient of linear expansion is the change in length per unit length per degree rise of temperature.*

Coefficient of linear expansion (abbreviated C.L.E.)

$$\alpha = \frac{\text{Increase in length}}{\text{Original length} \times \text{rise in temperature}} = \frac{\delta l}{l.\delta t} \qquad \text{(IV-3.2.3)}$$

It follows that if l_1 is the length at a temperature t_1 of a piece of material of coefficient of linear expansion α, its length l_2 at temperature t_2 will be given by.

$$l_2 = l_1 \{1 + \alpha (t_2 - t_1)\} \qquad \text{(IV-3.2.4)}$$

It is easy to see that α *is independent of the unit of length, but is dependent on the temperature scale.* In equation IV-3-2 2, $(l_2 - l_1)/l_1$ represents the change in length per unit length and is a pure number. In mentioning the value of α we must say if the change in length was caused by 1°C or 1°F rise of temperature. α, therefore, depends on the scale of temperature adopted.

The meaning of a statement like "the coefficient of linear expansion of brass is 19×10^{-6} per °C" should be clearly understood. It means that for each 1C° rise of temperature a rod of brass expands by 19×10^{-6} part of its original length. In other words

for 1C° rise of temperature a 1 cm long brass rod increases by 19×10^{-6} cm ;
,, ,, ,, ,, 1 metre ,, 19×10^{-6} metre ;
,, ,, ,, ,, 1 yard ,, 19×10^{-6} yard ;
,, ,, ,, ,, l units ,, $19 \times 10^{-6} \times l$ units.

For a t°C rise in temperature a rod of length l units will increase by $l\alpha t$ units where α is expressed in "per °C" unit.

Values of α for some substances. The following table gives the values of the linear expansion of some common substances on the celsius scale. They are mean values between room temperature and steam point.

Substance	α per °C	Substance	α per °C
Aluminium	25.5×10^{-6}	Brass	18.9×10^{-6}
Copper	16·7 ,,	Bronze	17·7 ,,
Iron (cast)	10·2 ,,	Glass (crown)	8·5 to 9·7 ,,
,, (steel)	10·5 to 11·6 ,,	,, (flint)	7·8 ,,
Platinum	8·9 ,,	,, (pyrex)	3 ,,
Silver	18·8 ,,	Invar (64 Fe, 36 Ni)	0·9 ,,
Zinc	26·3 ,,	Quartz (fused)	0·5 ,,

The International Prototype Metre, made of an alloy of 90% Pt and 10% Ir, has a coefficient of expansion of $8{\cdot}7 \times 10^{-6}$ per °C. The Imperial Standard Yard, made of 32 parts of copper, 2 parts of zinc and 5 parts of tin, has a coefficient of 17.7×10^{-6} per °C.

Example IV-3.1. A piece of copper wire has a length of 200 cm at 10°C. Find its length at 100°C given $\alpha = 17 \times 10^{-6}$ per °C.

Solution : Here rise of temperature=90C°.

Now, expansion of 1 cm for 1C° rise$=17 \times 10^{-6}$ cm,

∴ Expansion of 200 cm for 90C° rise$=200 \times 90 \times 17 \times 10^{-6}$ cm$=0 \cdot 306$ cm.

∴ The length of the wire at 100°C=200·306 cm.

Alternatively, we could apply equation IV-3.2.4 For this equation

$$l_1=200 \text{ cm}, t_1=10°\text{C}, t_2=100°\text{C}, \alpha=17 \times 10^{-6} \text{ per °C}$$

$$\therefore \; l_2=l_1\{(1+\alpha(t_2-t_1)\}=200(1+17 \times 10^{-6} \times 90)=200.306 \text{ cm}.$$

Ex. IV 3.2. A piece of steel has a length of 30 inches at 15°C, At 90°C its length increases by 0.027 in. Find the coefficient of linear expansion of steel.

Solution : Here rise of temp$=90-15=75$ C°.

Expansion of 1 inch for a temperature rise of 1 C°

$$=0 \cdot 027/(30 \times 75)=12 \times 10^{-6} \text{ in.}$$

∴ The coefficient of linear expansion$=12 \times 10^{-6}$ per °C.

Or, apply formula IV-3-2.1.

Ex. IV-3.3. A rod of iron and one of zinc ($\alpha'=29 \cdot 8 \times 10^{-6}$/°C) each 2m long are heated to 50°C when the Zn rod is 0.181 cm longer. Find α for iron. (C.U.)

Solution : Let dashed symbols represent Zn and plane ones iron. Then $l_2'=200(1+29 \cdot 8 \times 10^{-6} \times 50)$

and $\quad l_2=200(1+\alpha . 50)$

and $\quad l_2'-l_2=0.181=200 \times 29 \cdot 8 \times 10^{-6} \times 50-200 \times 5' \times \alpha$

$$=(29.8 \times 10^{-6}-\alpha) \times 10^{+4}$$

$$\therefore \; \alpha=29.8 \times 10^{-6}-18.1 \times 10^{-6}=11.7 \times 10^{-6}/°\text{C}$$

Ex. IV-3.4. One end of a steel rod 61 cms long is fixed and the other end presses against an end of a lever 10 5 cm from the fulcrum. On heating through 500°C the rod turns the lever through 2°. Find the increase in length and α of the material (Pat. U.)

Solution : Fig IV-3.1 (b) gives you an idea.

Now $2° = \frac{2\pi}{360} \times 2 = \frac{\pi}{90}$ rad = arc/radius = arc/10.5

The arc = increase in length = $\frac{\pi}{90} \times 10 \cdot 5 = \frac{22}{7} \times \frac{10.5}{90} = 0.366$ cm

$\therefore \alpha = \frac{0.366 \text{ cm}}{610 \times 500/°C} = 12 \times 10^{-6}/°C.$

Problem : Two equal bars one of iron ($\alpha = 12 \times 10^{-6}/°C$) and the other of brass ($\alpha = 18 \times 10^{-6}/°C$) each 80 cm long are joined together at one end and a needle 1 mm in dia and carrying a pointer is clipped between their free ends. When the bars are heated the pointer rotates through 10°. Find the temp range of heating.

(Ans 29.1°C) **[Oxf. & Camb]**

***IV 3 3. Cause of Thermal Expansion.** It is clearly due to the increase in the average separation between the atoms in the solid. In Fig II-5.4 we have already studied the relation between potential energy U and separation r of molecules. The same is shown again in fig IV-3 5. The curve is asymmetrical in shape.

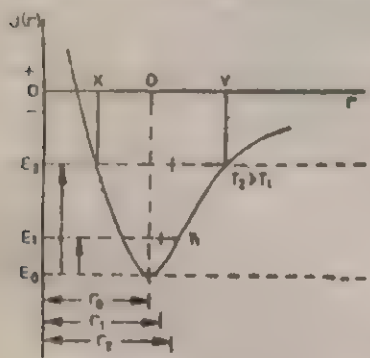


Fig. IV-3.5

When the vibrating molecules are at equilibrium separation r_0 potential energy is a minimum (E_0). When they come closer together, *strong repulsive force* takes over, as we find the P. E curve rising steeply ($F = -dU/dr$). As they move further apart, *weaker attractive* forces (gentler slope of the curve) come into play.

At a given temp T_1 the molecules have a given energy of vibration, atomic separation periodically changes from a maximum to a

* For the more inquisitive student

minimum the average separation $r_1 > r_0$ and the equilibrium point shifts to the right. At a still higher temp T_2, we find $r_2 > r_1 > r_0$ and the equilibrium point shifts more, and thus the solid as a whole expands.

Thus the *asymmetry in the P. E.–Separation curve* is found to be responsible for thermal expansion.

IV-3.3. Determination of the coefficient of linear expansion. In all solids the expansion is so small that we must either magnify the change in length before measuring it or use some delicate measuring instrument. There are many forms of apparatus of which we describe one which uses a micrometer screw, and is known as *Pullinger's apparatus* (fig IV-3.5).

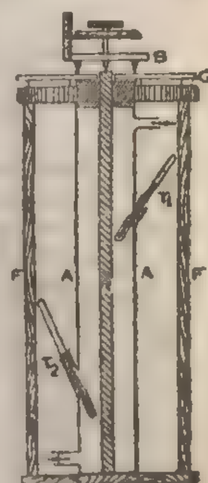


Fig. IV-3.6

The experimental rod *R*, about a metre long, stands vertically in a tube *A* mounted on the frame *F*. A has two thermometers T_1, T_2 inserted in *A* to measure the temperature of *R*. It has also side tubes for inlet and outlet of steam and is surrounded by felt or some other non-conductor of heat. The top of *R* passes through a cork in *A* and through a hole in a glass plate *G* which rests horizontally on the frame. A spherometer rests on *G*. After the initial length *l* of the rod has been measured it is placed in position and the central leg of the spherometer brought into contact with the top of *R*. (This adjustment may be accurately made by connecting one end of an electric circuit containing an electric bell with a leg of the spherometer and the other end with *R*. When the contact between the spherometer and the rod *R* is established the bell starts ringing.) The spherometer reading is taken. The central leg may now be raised without removing the spherometer. The initial temperature t_1 of the rod is read off from T_1 and T_2.

Steam is then allowed to pass through *A*. Temperature of *R* as

indicated by T_1 and T_2 rises and finally becomes steady. When it has been steady for some time the central leg of the spherometer is again brought into contact with R and its reading taken. The difference in the two readings of the spherometer gives the expansion x of the rod. The final temperature t_2 of the rod is noted from T_1 and T_2. We have all the data to find the coefficient of expansion α. It is given by

$$\alpha = \frac{x}{l(t_2 - t_1)}$$

IV-3.5. Thermal stress. Force of expansion or contraction is enormous. The breaking bar experiment (fig. IV-3.7) demonstrates that an enormous force is brought into play when a solid contracts or expands. A stout iron rod fits across two slots in a heavy iron stand. The rod has a hole through one end and a screw at the other. A cast iron pin (about 5 mm in diameter) is passed through the hole. The rod is then heated and clamped tightly while hot by means of the screw. When the rod is allowed to cool it is found that the cast iron pin snaps owing to the enormous force of contraction. A slight modification of the rod (using the inner hole) enables us similarly to demonstrate the force of expansion.

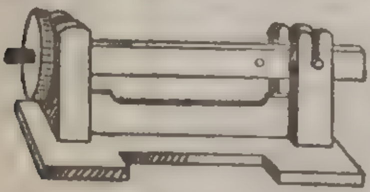


Fig. IV-3.7

To find the force we consider the stress developed as the rod expands on heating. When a force lengthenes a rod of length l of cross sectional area A of material with Young's modulus Y, by δl, we know from definition of Y, that

$$Y = \frac{F/A}{\delta l/l} \quad \text{Or,} \quad F = YA\,\frac{\delta l}{l} \qquad \text{(IV-3.5.1)}$$

From eqn. IV-3.2.3. we find for a temp. rise of t

$$(\delta l/l = \alpha t \quad \text{or} \quad F/A = Y(\delta l/l) = Y\alpha t \qquad \text{(IV-3.5.2)}$$

i.e. *thermal stress is independent of the dimensions* (A, l) of the rod *but depends on its material* namely Y and α, Young's modulus and coefficient of linear expansion of the material, and temp rise.

Ex-IV-8 5. Two rods of different metals $(\alpha_1\ Y_1\ ;\ \alpha_2, Y_2)$ of same cross section A but different lengths are held end to end between two massive walls and raised in temp by $T°$. Find the force developed between the two and rod lengths at the higher temp. Assume that rods do not bend, cross-section does not alter, walls remain vertical. [I.I.T. '75]

Solution : Let F_1 and F_2 be the required forces. Now

$$F_1 = Y_1\ A\frac{\delta l_1}{l_1} = Y_1 A\frac{l_1\alpha_1 T}{l_1} = AT\frac{l_1\alpha_1}{l_1/Y_1}\quad \therefore\quad F_2 = AT\frac{l_2\alpha_2}{l_2/Y_2}$$

$$\text{Mutual force } F = F_1 = F_2 = \frac{AT\,(l_1\alpha_1 + l_2\alpha_2)}{l_1/Y_1 + l_2/Y_2}$$

$$= \frac{AT\ Y_1 Y_2\ (l_1\alpha_1 + l_2\alpha_2)}{l_1 Y_2 + l_2 Y_1}$$

Again had there been no mutual force the length of the first rod would have become $l_1(1+\alpha_1 T)$ but due to the opposite expansion of the other rod it suffers a fractional diminution in length by an amount Fl_1/AY_1

$$\therefore\quad (l_1')_T = l_1\left[1 + \alpha_1 T - \frac{YT(l_1\alpha_1 + l_2\alpha_2)}{l_1 Y_2 + l_2 Y_1}\right]$$

Similarly for $(l_2')_T$.

Problems (1) A composite rod is made by joining a copper rod 30 cm long ($Y = 13\times 10^{10}\,\text{N/m}^2$, $\alpha = 17\times 10^{-6}/°\text{C}$) end to end to another rod of same cross-section so as to make a rod 1m long at 25°C. At 125° the increase in length totals 0.191 cm. If held between massive walls increase in length is prevented though temp rises. Find Y and α for the material of the other rod. [I I.T. '79]

[**Ans.** $Y = 11.1\times 10^{10}\ \text{N/m}^2$, $\alpha = 20\times 10^{-6}/°\text{C}$]

[*Hint*: [illegible]

$= (\delta l_1 + \delta l_2) = 0.191$

[illegible]]

2. A steel wire 1 mm² in cross-section is held just taut between two supports at [illegible]. Find the tension developed at [illegible] given $Y = 2.1 \times 10^{11}$ N/m² and $\alpha = 12 \times 10^{-6}$ [illegible]. Ans. [illegible] N

3. A steel rod of [illegible] cm² is [illegible] cm long. What stretching force elongates the rod by the same amount as by heating it through [illegible] °C? Given $Y =$ [illegible], $\alpha =$ [illegible].

Ans. 1600N, [I.I.T. '71]

IV.3.3. Differential Expansion. In [illegible] (see fig. IV.3.[illegible]) we saw that [illegible] arises from different expansions of different metals. Let us take a pair of rods AB and CD (fig. IV.3.[illegible]) of different metals (α_1 and α_2) of lengths l_1 and l_2 rigidly connected [illegible] at a temp. t_1 so that

$$\delta_1 = l_2 - l_1$$

At a higher temp. t_2 their lengths would be

$$l_1' = l_1 \{1 + \alpha_1 (t_2 - t_1)\}$$

and $$l_2' = l_2 \{1 + \alpha_2 (t_2 - t_1)\},$$

Fig. IV.3.3

The new difference δ_2 between the lengths would be

$$\delta_2 = l_2' - l_1' = [illegible]$$

$$= \delta_1 + (l_2 \alpha_2 - l_1 \alpha_1)(t_2 - t_1)$$

By [illegible] choice of materials the lengths of [illegible] be made [illegible] following uses—

A. [illegible] It is used in compensating [illegible] pendulum [illegible] and [illegible]. This is achieved by making $l_1 \alpha_1 = l_2 \alpha_2$ or [illegible] (IV.3.[illegible]).

i.e. taking the rod lengths in the inverse ratio of the coefficients of linear expansion of their materials.

B *The distance BD extends or shrinks* The shorter bar CD is made of *invar* (expansion with temp negligible) so that the relative expansion of BD with temp becomes much greater than a bar of length d_2.

If the longer bar is of [illegible], BD rapidly shrinks with rise in temp. This is the *principle of thermostats* to control temp of various devices.

Ex IV 3 6. A platinum rod 1.3 m long has one of its ends fixed rigidly to a cross piece along with a zinc rod placed parallel to it. If they are heated to the same range of temp and yet the difference in length between the two rods does not change find the length of the other. Given [illegible]

Solution : From equation [illegible] we have

$$l_{zn} = \frac{9 \times 10^{-6}/°C}{[illegible]} \quad [illegible]$$

Problem : Find the length of a brass and a [illegible] at all temp their separation [illegible]

[illegible]

C Thermostat : It is [illegible] electric switch which closes when the temp reaches a certain temp and opens when it reaches another. [illegible] Fig. IV [illegible]. If the temp is low the bimetallic strip is straight and makes electrical contact between A and B. [illegible] and [illegible] the [illegible] the [illegible] circuit [illegible] for the [illegible] element [illegible] as in a box. As an example we consider an automatic electric iron [illegible]. When the temp has risen to the desired value, the strip has bent away far enough to break the electrical contact at A and B, [illegible]

Fig. IV [illegible]

the circuit. When temp falls the strip straightens out and re-establishes the circuit again making the circuit.

The *starter* supplied with your modern tube-light is a thermostat. This device is also an element for automatically controling the temp of your bath-tub water. A *fire alarm* is also a thermostat in series with an electric bell but with the the invar-side facing the contact-side. The contact here is kept broken. On heating the strip bends towards the contact and on touching sets the bell ringing to give the alarm. They may be installed in warehouses, in holds of ships or in places where a fire may break out without being quickly discovered.

We describe below a *gas thermostat* in which heating is controlled by the differential expansion of two metals as in bimetallic strips.

Refer to fig IV-3.9(b). The gas flows to the oven through a valve *V*, which has an invar stem. The stem is attached to the closed end of a brass tube which projects into the top of the oven.

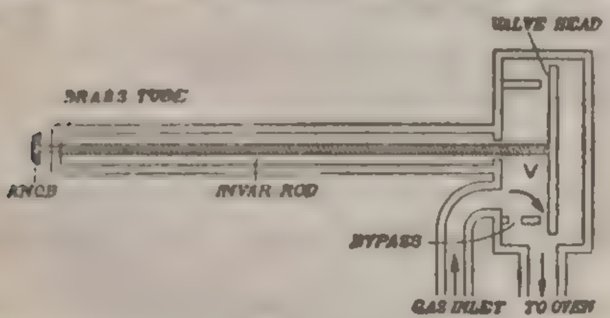
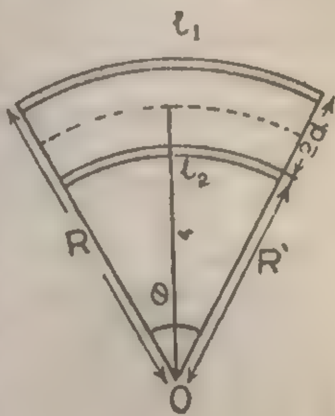


Fig. IV-3.9(b)

As the gas burns and the oven heats up, the brass tube expands. But the expansion of invar is negligible. So the valve head moves to the left and partially closes the opening through which the gas flows. If the oven cools, the brass contracts and the valve head moves to the right, increasing the gas flow. A rotating knob attached to the invar rod controls the opening of the valve. The position of the knob sets the temperature of the oven.

The rate of supply of gas which will maintain the oven at a constant temperature is thus controlled by the differential expansion of brass and invar.

Ex. IV-3.7. Two metal strips each of length lo and breadth d are so riveted at temp t that their edges coincide. Coefficients of their linear expansions are α and α' $(\alpha > \alpha')$. It bends into an arc when the temp rises by δt. Find the radius of curvature of the bent bimetallic strip.



Solution **:** Refer to the fig. Let the strip lengths at the higher temperature be l_1 and l_2 where $l_1 = lo\,(1+\alpha\delta t)$ and $l_2 = lo\,(1+\alpha'.\delta t)$.

Let the strips bend into arcs of radii R and R'. Then

$$l_1 = R\theta = lo\,(1+\alpha\delta t) \text{ and} \qquad \text{(A)}$$

$$l_2 = R'\theta = lo\,(1+\alpha'.\delta t) \qquad \text{(B)}$$

Then obviously $\theta\,(R-R') = lo\,(\alpha-\alpha')\,\delta t$

$$\text{Or, } \theta = \frac{lo\,(\alpha-\alpha')\,\delta t}{R-R'} = \frac{lo\,(\alpha-\alpha')\,\delta t}{2d}$$

Again adding (A) and (B) wet get

$$\theta\,(R+R') = 2lo + lo\,(\alpha+\alpha')\,\delta t$$

$$\therefore\ R+R' = \frac{2lo + lo.\,\delta t\,(\alpha+\alpha')}{\theta}$$

The mean radius of the bent strip must be then

$$R = \tfrac{1}{2}\,(R+R') = \tfrac{1}{2}.\,\frac{2lo + lo.\,\delta t\,(\alpha+\alpha')}{lo\,(\alpha-\alpha')\,\delta t/2d}$$

$$= \tfrac{1}{2}.\,\frac{lo\,[\,2+(\alpha+\alpha')\,\delta t\,]}{lo\,(\alpha-\alpha')\,\delta t} \times 2d$$

$$= \frac{2d}{(\alpha-\alpha').\,\delta t} + \frac{(\alpha+\alpha')\,d}{(\alpha-\alpha')}$$

Problem : Two strips of iron and brass are riveted to form a straight bar of thickness 0.1 cm. If it is heated to 100°C find the radius of curvature of the bent bar. (Ans : 167 cm)

Ex. IV-3.8. An iron and a copper rod differ in length by 2 cm at 50°C and also at 450°C. Find their lengths at 0°C, α's being respectively $12\times10^{-6}/°C$ and $17\times10^{-6}/°C$. [J. E.E. '74]

Solution : Let l be the length of the iron rod at 50°C, so that of the copper rod is $(l-2)$ cm as its α is greater. Then

$$\frac{l}{l-2}=\frac{17}{12} \quad \therefore \; l=6.8 \text{ cm}$$

$$\therefore \; 6.8=l_0\,(1+12\times10^{-6}\times50) \quad \therefore \; (l_0)_{fe}=6.796 \text{ cm}$$

and $(l_0)_{cu}=4.796$ cm

Fig. IV-3.9

Bimetallic thermometers (fig. IV-3.10). The thermo-element here is a long thin bimetallic strip curled as shown. The strip has brass on outside and invar inside. On heating the strip curls up more, the pointer moving to the right. Though not much sensitive it is a small, compact device that you can very easily carry in your pocket.

IV-3.7. A. Some Advantageous Expansions :

(*i*) Steel plates of boilers, ships or buildings are held together by rivets. **Rivets** are fixed while hot. On cooling they contract and make a very firm joint (fig. IV-3.10).

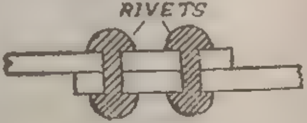


Fig. IV-3.10

(*ii*) Walls which have bulged outwards may be drawn in by passing iron bars through them across the building. To the extremi-

ties of the bars are attached iron plates screwed upto the walls with nuts. The bars are heated and the nuts and plates screwed up tightly against the wall. When the bars contract they draw the walls with them. You have just seen what tremendus forces they develope because of thermal contraction.

Iron rods in furnaces have a set of ends sealed into a brick wall but the other ends are kept free to expand so as not to damage the the structure because of thermal stresses developed.

(*iii*) Glass stoppers often stick to bottles, too tightly to be opened by ordinary means. Careful heating of the mouth of the bottle enlarges it, when the stopper comes out easily. If a metal cap sticks, the cap itself is to be heated.

B. Disadvantageous expansions and their remedies : Expansion of solids more often than not causes inconvenience and calls for suitable remedies.

( ) The **rails on railways** expand on hot days and contract at cold nights. The extremes of temperature may be fairly large. To allow for the expansion and contraction of the rails a small gap is always left between the ends of adjacent rails. The arrangement is shown in Fig. IV-3.11.

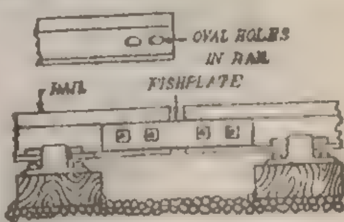


Fig. IV-3.11

The rails are joined by **fish-plates** bolted to the rails, Bolt holes are oval in shape and allow the rails to move in the direction of their length.

Ex IV-3.10. 390 miles separate Delhi from Allahabad. Find the total space left between the rails to allow for a rise in temp from 36°F in winter to 117°F in summer. ($\alpha = 12 \times 10^{-6}/°C$)

[All. U.]

Solution : Total exp$= l\alpha t = 390$ miles $\times 12 \times 10^{-6}$ °C $\times (117 - 36) \times \frac{5}{9}$°C$= 0{\cdot}2084$ m.

Problem : A gap of 0·5″ inch. separate rails 66′ ft. long at 10°C. At what temp will they join up ? (Ans 67.3°C)

[**C. U., Gau. U., H. S. '83. Tripura '79**]

In tramways however it is the common practice to weld rails together in order that the resistance to the flow of electric current through the rails to the earth may be the least. The rails are embedded in the road and surrounded by granite blocks and concrete. This gives the rails much protection against temperature changes. The rails of a railway are fully exposed.

(*ii*) The ends of a big steel bridge are not rigidly built into the brick work on which they rest. They are supported on rollers which allow them to expand or contract without damaging the masonry (fig. IV-3.12).

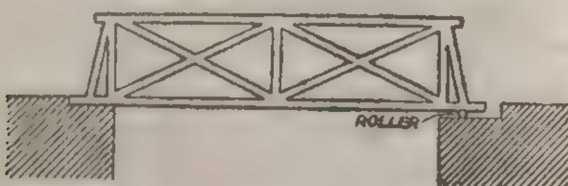


Fig. IV-3.12

Ex. IV-3.11 The steel span of a bridge is 0.5 km long and has to withstand a temp difference from 44°F to 116°F. If $\alpha = 10^{-5}/°C$ what allowance must be kept for its expansion ? [J.E.E. '62]

Solution : Since a temp diff 1°F is smaller by $\frac{5}{9}$ of 1°C temp diff, we have $\alpha_f = \frac{5}{9}\alpha$,

$\therefore$ Required Allowance $= l_2 - l_1 = l_1 \alpha_f . t = 5 \times 10^4$ cm.

$(116-44)°F \times 10^{-6} \times \frac{5}{9} °F = 5 \times 72 \times (5/9) \times 10^{-1}$ cm $= 10$ cm.

Problem : The Eiffel Tower in Paris is 335 m high and is made of steel ($\alpha = 12 \times 10^{-6}/°C$). How much taller is the Tower in summer (100°*F*) than in winter (0°F) ? (*Ans* 22.23 cm) [H S. '63]

(*iii*) **Cracking of glass.** When hot water is poured into a glass vessel with thick wall, it is likely to crack. The inner wall tends

to expand ; but as glass is a *bad conductor* of heat the outer wall remains cooler and does not expand. This sets up large stresses in the glass, and it cracks. The risk is less when the wall is thin or when the glass has a small coefficient of expansion. The linear coefficient for ordinary glass is about 9×10^{-6} per °C. Pyrex glass has a coefficient of 3×10^{-6} per °C. It can be heated in a flame without cracking. Fused silica (quartz) has such a small coefficient that a crucible made of it can be heated red hot in a furnace and then immediately chilled in water without cracking.

Heating a stone often cracks it. This is seen when a piece gets into a coal fire. The outer layers of the stone expand before the inner layers become hot. So they break away often with a loud report.

(*iv*) **Glass and metal seals :** Unequal expansion of glass and many metals makes it impossible to seal a metal wire such as copper or iron into a glass bulb. Platinum and some alloys have very nearly the same expansion coefficient as glass and are used for sealing. In modern electric lamps the leads which pass through the glass stem are made of an alloy of nickel and iron coated with copper. This alloy expands equally with the glass.

Ex. IV-3.12. To thread a cu wire (2mm dia. at 20°C) through a smaller hole in a carbon block for connection to the carbon anode in a transmitting valve the specimens had to be lowered to -80°C. Find the diameter of the hole at 20°C if α's for Cu and C are 17×10^{-6}/°K and $5\times^{-6}$/°K respectively. **(Oxf and Camb.)**

Solution : 1C° = 1K°. Let d be the required dia at 20°C.

Then d' at $-80°C = d\ [1-5\times10^{-6}\{20-(-80)\}] = 0.9995d$ cm

Similarly dia of cu wire at $-80°C = 0.2\ (1-17\times10^{-6}\times100) =$ 0.19966 cm

$\therefore$ $0.9995\ d = 0.19966$ cm or $d = 1.998$ cm.

(*v*) To allow for the **expansion and contraction of steam pipes** they are provided with expansion bends or loops as shown in

fig. IV-3.13. When the pipe expands or contracts the shape of the loop changes slightly. The pipes carrying oil through deserts and ice-fields are also fitted with such bends and loops to allow for their expansion and contraction. They may also be laid in a zig-zag, loosely on supports, or have telescopic joints, the clearances sealed with asbestos fibres.

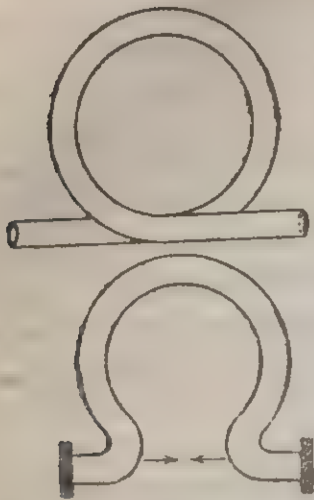


Fig. IV-3.13

(*vi*) A telegraph, telephone or a power line is fixed between poles loosely, so as to allow for its expansion and contraction. In summer it is longer and looser; in winter it is shorter and tensed.

(*vii*) The distance between the **graduations of a scale** increases in summer and diminishes in winter. The graduations are correct

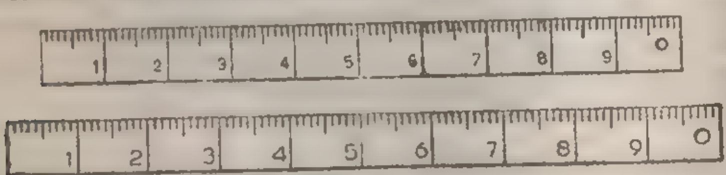


Fig. IV-3.14

only at the temperature of calibration. For high accuracy in readings a correction is therefore necessary.

Fig IV-3.15 shows the same steel ruler at two different temperatures. On heating, every dimension of it expands in the same proportion—the scale, the numbers, the hole and the thickness. However the expansions here are shown much exaggerated.

If the graduations be done at θ, at a higher temp θ_1 it will read less and at a lower temp the reading will be higher as you realise from the above fig as well as example IV-3.13

True reading at θ_1 is (l_2)=scale reading $(l_1)\times[1+\alpha(\theta_1-\theta)]$

Ex. IV-3.13. A brass scale is correct at 20°C. The length of a rod as measured by it at 30°C is 50 cm. What is the true length of the rod ? (α for brass $= 19 \times 10^{-6}$ per°C).

Solution : A distance of 1 cm on the scale at 20°C increases to $1 + 19 \times 10^{-6} \times 10 = 1{\cdot}00019$ cm at 30°C. Therefore the distance the scale reads as 1 cm when it is at 30°C. is really 1.00019 cm.

$\therefore$ The true length of the rod $= 50 \times 1.00095 = 50.0475$ cm.

Ex. IV-3.14. A steel scale is correct at 0°C. The length of a brass tube measured by it at 30°C. is 4·5 metres. What is the length of the tube at 0°C ? α of steel $= 11 \times 10^{-6}/C°$ and of brass $= 19 \times 10^{-6}/C°$.)

Solution : 1 metre on the scale at 30°C is really $1 + 30 \times 11 \times 10^{-6} = 1{\cdot}00033$ metres.

$\therefore$ The true length of the tube at 30°$C = 4.5 \times 1{\cdot}00030$ metres. If it is cooled to 0°C, its length would be $4.5 \times 1.00030\ (1 - 30 \times 19 \times 10^{-6}) = 4{\cdot}499$ metres nearly.

Problems. (1) A metre scale, made of steel, is correct at 10°C. What would be the correct distance between two consecutive cm-markings on this scale at 30°C and 60°C ?

A brass rod measured by the above scale at 30°C is found to be 5 metre long. What would be the correct length of the rod at 10°C ? α of steel $= 12 \times 10^{-6}/°C$. α of brass $= 18 \times 10^{-6}/°C$?

[H. S. 86]

(2) A steel tape is correct at 68°F. It measures a brass rod at 50°C and finds it to be 1.5 m. Find the true length at 15°C. $\alpha = 11.2 \times 10^{-6}/°C$

[H S. '78]

Ex IV-3.15. The brass scale of a barometer reads correctly at 0°C ($\alpha = 20 \times 10^{-6}/°C$). The barometer reads 75 cm at 27°C, What is the atmospheric pressure at 0°C ? (Ans 75.04 cm) [I.I.T. ' 77]

Solution : All you have to do is to find the scale reading at 0°C. Now $l_{27} = lo\,(1+\alpha t)$

$$=75\,(1+20\times10^{-6}\times27)=75\,(1+54\times10^{-5})=75.0405 \text{ cm}$$

Ex. IV-3.16. A steel meter scale is to be so prepared that the mm. intervals are to be accurate within 0.0005 mm. Find the maximum temp. variation allowable during the ruling of the mm mark (Given α for steel$=13.22\times10^{-6}$/°C) [I. I. T. '64]

Solution : From the given datum 1 mm mark ruled may be 1.0005 mm.

$$\therefore\quad 1.0005=1(1+13.22\times10^{-6}\times t)$$

$$\text{or,}\quad 0.0005=13.22\times10^{-6}\times t$$

$$\therefore\quad t=\frac{5\times10^{-4}}{13{\cdot}22\times10^{-6}}=37.8°\text{C}$$

Ex. IV-3.16. A screw of pitch 0. 5 mm is so mounted that it can move against one end of a 1 m long rod of which the other end is fixed. A circular scale attached to the screw has 100 div. At 20°C the pitch scale reading is a little above the zero mark and the circular scale is 92 div. At 100°C the linear scale is a little above 4 div and circular scale reads 72 div. Find α for the material of the rod. [I. I.T. '67]

Solution : The least count of the circular scale is 0.5/100 =0.0005 mm. Hence the expansion of the rod

$$e=(\,4\times0{\cdot}5+72\times0.005\,)-(0\times0.5+92\times0{\cdot}005\,)$$

$$=2+0.005\,(\,72-92\,)=2-0.1=1.90 \text{ mm}$$

$$\therefore\quad \alpha=\frac{1.90}{100\times80}=(\,1.9/8\,)\times10^{-3}=23.75\times10^{-6}/°\text{C}$$

IV-3.8. Compensated pendulums & Balance wheels of watches

A. The time period of a pendulum depends on its length. Since its length increases in summer, the time period also increases and the clock goes 'slow'. The reverse takes place in winter. Arrangement must be made in order that a clock or a watch may

keep correct time in all seasons. Such clocks or watches are called 'compensated'. *The effective length of a pendulum is the distance between the point of suspension and the centre of gravity of the suspended system.* If this length remains constant the pendulum keeps correct time. The pendulum bob is mounted on rods of two different materials in such a way that the downward expansion of one rod is equal to the upward expansion of the other. This is diagrammatically shown in fig. IV-3.15 Let l and l' be the lengths of the rod and a and a' their coefficients of linear expansion. Their expansions for $t°$ rise of temperature are lat downwards and $l'a't$ upwards. The distance of the centre of gravity of the pendulum from the point of suspension will remain unchanged if

Fig. IV-3.15

$$lat = l'a't$$

$$\text{or,} \quad l/l' = a'/a \qquad \text{(IV.3.8.1)}$$

So for unchanging separation between point of suspension and center of the bob *the lengths of rods should be in the inverse ratio of their a's.*

The coefficients of iron and zinc are 0·00001 and 0·000028 per °C respectively. Hence zinc rods used for compensating iron rods will have less than half the length of the latter.

B. The clock BIGBEN on the Parliament House in London uses zinc for compensation. Its pendulum is as shown in the fig. IV-3.16(a). *Smaller clocks* may have the pendulum

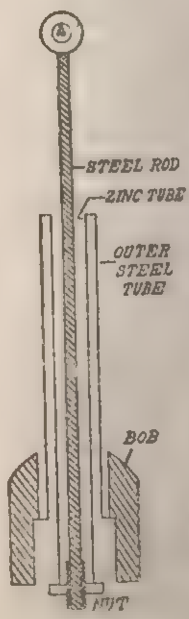


Fig. IV-3.16(a)

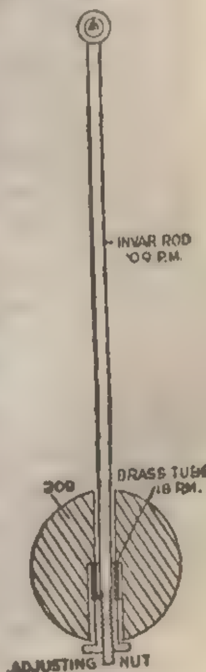


Fig. IV-3.16(b)

rod made of 'invar', an alloy of 64% iron and 36% nickel. (The name is derived from 'invariable'.) It has an expansion coefficient of $0{\cdot}9 \times 10^{-6}$ per °C (about 1/13 that of iron). The very small expansion of the invar rod is compensated by a brass tube [fig IV-3.16(b)].

C. **Compensated balance wheels of watches.** In watches the movement of the hands is controlled by the balance wheels which execute rotational oscillations under the elastic control of a steel spiral hair-spring. The time of oscillation depends on the effective distance, the *radius of gyration* of the rim of the wheel from the axis of rotation. The rim is made of two metals, the outer being more expansible. This bimetalic rim is made in two or three parts (fig. IV.3.17) each supported by an arm of the wheel.

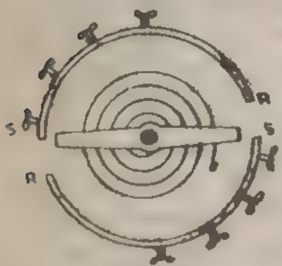


Fig. IV-3.17

The rim also carries small screw weights. When the temperature rises the arm expands, but the outer rim of more expansible metal curls in and brings the weights nearer to the axis, thus keeping the effective radius of the ring constant. With change of temperature the stiffness of the hair-spring changes and alters the time-period. The rim is arranged to compensate for this effect also. The rim may also be of invar.

Ex.IV-3 18. A compensated seconds pendulum has a zinc disc as the bob of an invar rod. Find the length of the rod and the radius of the disc. Given $g=980.7$ cm/s² and $\alpha_{zn}=26.3\times10^{-6}/°C$ and $\alpha_{in}=0.9\times10^{-6}/°C$.

Solution: It being a seconds pendulum ($T=2\pi\sqrt{l/g}=2s$) we have the effective $L=g/\pi^2=99.36$ cm $=l-l'$ where l is the length of the pendulum and l' the disc radius. Now from the relations $L=l-l'$ and $l/l'=\alpha'/\alpha$ we get

$$l=\frac{\alpha' L}{\alpha'-\alpha} \text{ and } l'=\frac{\alpha L}{\alpha'-\alpha}$$

$$\therefore \quad l=\frac{26.3\times10^{-6}\times99.36}{10^{-6}(26.3-0.9}=102.9 \text{ cm and } l'=l-L=3.52 \text{ cm}$$

D. Old compensated Pendulums : Graham's Mercury Pendulum The bob is a pair of glass cylinders in a frame with mercury and the rod is of iron. With rise of temp the latter extends downwards which is balanced by the upward expansion of mercury. The amount of mercury can be varied so as to maintain the separation between its C.G and the point of suspension of the rod invariable.

(b) **Harrison's Grid Iron Pendulum** [fig IV-3·18 a and (b)]. Since coefficients of linear expansion of brass and iron are in the ratio 3 : 2, two rods of the former and three of the latter, all of about the same length will have their total expansions equal and may be made to provide an invariable length, as indicated in the sketch (a). They are so connected in series alternately that while iron rods expand downwards the brass rods do so upwards, keeping the effective pendulum length constant. An actual arrangement shown in (b) contains 5 steel rods and 3 brass ones. Scrutinise and you will find that *effective expansion involves three of steel and two of brass rods*, only. The pendulum is clumsy and has long been discarded.

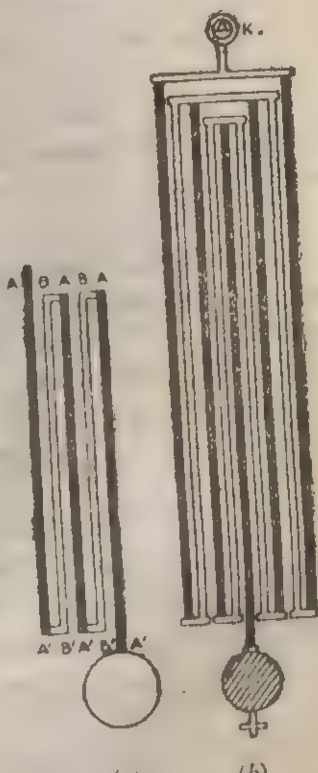


Fig. IV-3.18

It is described only to illustrate the differential expansion of different metals.

Ex IV 3·19. What must be the relative lengths of brass ($\alpha = 19 \times 10^{-6}/^{\circ}C$) and steel ($\alpha = 11 \times 10^{-6}/^{\circ}C$) rods used in a grid-iron pendulum? [Pat U. Gau. U]

Solution: If l and l' be the lengths of brass and iron rods then from the relation $l/l' = \alpha'/\alpha$ we have the required ratio to be 11 : 19

Ex IV-3.20. A grid iron pendulum consists of 7 iron and 6 copper rods. If each iron rod is 1·5 m long find the length of a cu rod given that $\alpha_{fe} = 12 \times 10^{-6}/^{\circ}C$ and $\alpha_{cu} = 17 \times 10^{-6}/^{\circ}C$.

Solution: Refer to fig IV-3.18(b). Note that here 4 iron and 3 cu rods are effective. Then $4l = 3l'$ and $l/l' = \alpha'/\alpha$.

$$\therefore \quad \frac{4 \times 1{\cdot}50}{3 \times l'} = \frac{17}{10} \quad \text{Or } l' = 1{\cdot}41\,\text{m}$$

Problem. A grid iron pendulum is made of 5 iron and 4 brass rods. If each brass rod is 50 cm long find that of each iron rod. [C. U.]

B. **Loss or gain of time due to change of temp of a Pendulum Rod :** With change of temp at a given place the length of a pendulum rod changes and so is of fixed length of 864000s, does its time period. Since a day with rise of temp. length increases and the pendulum loses. We can find the loss thus—

$$T = 2\pi\sqrt{l/g} \quad \text{and} \quad nT = 1$$

$$\therefore \quad n = \frac{1}{2\pi}\sqrt{g/l}$$

$$\text{or} \quad \log n = \log 1 - \log 2\pi + \tfrac{1}{2}(\log g - \log l)$$

where n is the frequency or the beats executed by the bob. Since n depends only on l in this case, we get on differentiation,

$$\frac{dn}{n} = -\frac{1}{2}\frac{dl}{l} = -\frac{1}{2}\frac{dl}{ldt}dt = -\tfrac{1}{2}\alpha dt \qquad \text{(IV-3.8.2)}$$

Where dn is the change in the number of beats and α the coefficient of linear expansion of the material of the pendulum rod.

Ex IV-3.21. An iron pendulum makes 86405 oscillations a day. At the end of the next day it loses 10s. If $\alpha = 17 \times 10^{-6}/^{\circ}C$ find the change in temp. [Pat U., All. U]

Solution : Here $dn = -10s$ $n = 86405$

$$\therefore \quad \frac{-10}{86405} = -\tfrac{1}{2} \times 17 \times 10^{-6} \times t \text{ Or } t = 19{\cdot}7°C$$

Ex IV-3·22. An iron pendulum ($\gamma = 36 \times 10^{-6}/°C$) keeps correct time at 20°C. If temp rises to 40°C how much will it gain or lose per day ? [I.I.T. '77]

Solution : Here $n = 86400s$. $\alpha = \gamma/3 = 12 \times 10^{-6}/°C$, $t = +20°C$

Then $dn = -\frac{1}{2} n \alpha t = -\frac{1}{2}\, 86400 \times 12 \times 10^{-6} \times 20$ s

$= -864 \times 12 \times 10^{-5}s = -80{\cdot}36s$ It will run slow.

Problem. A brass pendulum ($\alpha = 18 \times 10^{-6}/°C$) beats seconds at 30°C. What will happen if the temp is held const at 20°C ? (**Ans** : 7·71s fast/day) [C. U.]

IV-3.9. Coefficients of surface and volume expansion. When the length of a body changes with temperature, the area of its surface as well as its volume undergo changes.

A. Def : (1) The *coefficient of superficial expansion* is the change in area per unit area per degree rise of temperature. If S_1 and S_2 are the areas of any portion of the surface of a body at temperatures t_1 and t_2 respectively, then the coefficient of superficial expansion is

$$\beta = \frac{S_2 - S_1}{S_1(t_2 - t_1)} = \frac{\delta S}{S \delta t} \tag{IV-3·9·1}$$

$$\text{or,} \quad S_2 = S_1\{1 + \beta(t_2 - t_1)\} \tag{IV-3·9·2}$$

(2) The *coefficient of cubical expansion* is the change in volume per unit volume per degree rise of temperature. If V_1 and V_2 are the volumes of a body at temperatures t_1 and t_2 respectively, then the coefficient of cubical (or volume) expansion is

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} = \frac{\delta V}{V.\delta t} \tag{IV-3 9·3}$$

$$\text{or,} \quad V_2 = V_1\{1 + \gamma(t_2 - t_1)\} \tag{IV-3.9.4}$$

Like the coefficient of linear expansion, these coefficients also depend on the temperature scale only and are expressed in per °C or °F as unit.

The coefficients β and γ defined above are really the *mean coefficients* between t_1 and t_2.

B. Relations between α, β and γ :

Let a square of sides l cm be heated through 1C°. Each side becomes $l\,(1+\alpha)$ cm and the area $l^2\,(1+\beta)$ cm².

$\therefore\ l^2(1+\beta)=l^2(1+\alpha)^2$ or $1+\beta=1+2\alpha+\alpha_2$ or $\beta=2\alpha$ (α^2 being neglected as α is very small).

If a cube of sides l cm be heated through 1 $C°$, the sides become $(1+\alpha)$ cm and the volume is changed to $l^3\,(1+\gamma)$ cm³.

$\therefore\ l^3(1+\gamma)=l^3(1+\alpha)^3$ or, $1+\gamma=1+3\alpha+3\alpha^2+\alpha^2$ or, $\gamma=3\alpha$ (α^2, α^3 being neglected),

[Since α is very small, α^2, α^3 are smaller still. For example, if α is 0·00001, then $\alpha^2=0·0000000001$ and $\alpha^3=0·000000000000001$. We are, therefore, justified in neglecting α^2 and α^3 compared with α].

Thus $\alpha=\frac{1}{2}\beta=\frac{1}{3}\gamma$ (IV-3.9.5.)

The above relations can be very easily deduced by using calculus. Note that when changes are very small,

$$\beta=\frac{ds}{s\,dt}=\frac{d\,(l^2)}{l^2.dt}=\frac{2l.dl}{l^2.dt}=2.\frac{dl}{l.dt}=2\alpha$$

$$\text{and }\gamma=\frac{dV}{V\,dt}=\frac{d(l^3)}{l^3.dt}=\frac{3l^2.dl}{l^3.dt}=3\frac{dl}{l.dt}=3\alpha$$

$$\text{or }\alpha=\tfrac{1}{2}\,\beta=\tfrac{1}{3}\gamma$$

IV-3.10. Expansion of a hollow vessel. (a) A hollow vessel will expand as if it were solid. Its expansion will be outwards, not inwards. Inward expansion will reduce its surface area. But heating causes expansion of the surface area also.

(b) If an annular ring is heated, its inner diameter will increase. The circumference of the inner ring will undergo linear expansion with rise in temperature. Hence its diameter will also increase.

(c) A hollow cylinder of diameter d and lenth l will change its dimensions to $d(1+\alpha t)$ and $l(1+\alpha t)$ where α is the coefficient of linear expansion of its material and t the rise in temperature. The old volume $V=\frac{1}{4}\pi d^2 l$. The new volume $V_t=\frac{1}{4}\pi d^2\,(1+\alpha t)^2 l(1+\alpha t)$ $=\frac{1}{4}\pi d^2 l\,(1+\alpha t)^3=V\,(1+3\alpha t+3\alpha^2 t^2+\alpha^3 t^3)=V\,(1+3\alpha t)=V(1+\gamma t)$,

where $\gamma = 3\alpha$ = coefficient of cubical expansion. ($\alpha^2 t^2$ and $\alpha^3 t^3$ are ignored because of their smallness).

IV-3.11. Change of density with temperature. The density of a substance is its mass per unit volume. Since the volume of a given mass of a substance changes with temperature while its mass remains constant, the density undergoes a change with change of temperature.

Consider a body of mass m. Let V_1 and ρ_1 be its volume and density respectively at temperature t_1 and V_2 and ρ_2 the corresponding values at t_2. Then from the definition of density we get

$$m = V_1\rho_1 = V_2\rho_2 = V_1\{1+\gamma(t_2-t_1)\}\rho_2 \text{ from eqn IV-3.9.4}$$

Or,
$$\rho_2 = \frac{\rho_1}{1+\gamma(t_2-t_1)} \qquad \text{(IV-3.11.1)}$$

$$= \rho_1\{1-\gamma(t_2-t_1)\}, \text{ since } \gamma(t_2-t_1) \text{ is small.}$$

Writing t for $(t_2 - t_1)$ we have

$$\rho_2 = \rho_1(1-\gamma t) \qquad \text{(IV-3.11.2)}$$

if the values of ρ at two temperatures are known. These equations may be applied to find γ.

Ex IV-3.23. The density of lead at 0°C is 11.34 g/cm³. What is the density at 100°C, given that α of lead = 28×10^{-6} per °C ?

Solution : The coeff of volume expansion $\gamma = 3\alpha$

$$= 3\times28\times10^{-6} = 84\times10^{-6} \text{ per °C.}$$

From equation IV-3 11.2 $\rho_2 = \rho_1(1-\gamma t)$

$$= 11{\cdot}34\,(1-84\times10^{-6}\times100) = 11{\cdot}25 \text{ g/cm}^3.$$

Ex IV-3.24. Show that change in density $\Delta\rho$ for a rise in temp of Δt is given by $\Delta\rho = \rho\gamma.\Delta T$.

Solution : From eqn. IV-3 11.2 we find

$$\frac{\rho_1-\rho_2}{\rho_1} = \gamma t \qquad \therefore \quad \Delta\rho = \gamma\rho\Delta T$$

Ex IV-3.25. A uniform pressure p exerted on all sides of a cube reduces the volume at a temp t. By how much the temp is to be raised so as to restore the original volume with the pressure still on. Coeff of volume expansion is γ and modulus of volume elasticity K. [I.I.T. '78]

Solution : Let the change in volume be $-\delta V$ and required rise in temp δt. Now by definition.

$$K=\frac{p}{-\delta V/V} \quad \therefore \quad \frac{-\delta V}{V}=\frac{p}{K}. \quad \text{Now } \gamma=\frac{-\delta V}{V.\delta t}$$

Or $\delta t=\dfrac{-\delta V}{V\gamma}=\dfrac{p}{\gamma K}$. Let V be the initial and V' the diminished volume and t' the temp at which the volume is restored. Then

$$t'-t=\frac{V-V'}{\gamma V'}=\left(\frac{V}{V'}-1\right)\frac{1}{\gamma}$$

But $\dfrac{V-V'}{V}=1-\dfrac{V'}{V}=\dfrac{V}{K}$

Or $V'/V=1-V/K=(K-V)/K$ or $V/V'=K/(K-V)$

$$\therefore \quad t'=t+\left(\frac{V}{V'}-1\right)\frac{1}{\gamma}=t+\left(\frac{K}{K+V}-1\right)\frac{1}{\gamma}=t+\frac{V}{(\gamma K-V)}$$

EXPANSION OF LIQUIDS

IV-4.1. Real and apparent expansions of a liquid. A liquid has no shape of its own ; it readily takes up the shape of the container. We cannot therefore speak of linear or superficial expansion of a liquid. *Liquids have volume expansion only.* The expansion of liquids is roughly ten times that of solids.

The volume of a liquid varies in a complicated way with temperature. In general, the relation is of the form

$$V_t = V_o(1+at+bt^2+ct^3+\cdots)$$

where a, b, c are constants of fast diminishing magnitude.

For example, for mercury

$$V_t = V_o(1+1{\cdot}8144\times 10^{-4}t+7{\cdot}016\times 10^{-9}t^2+2{\cdot}86\times 10^{-11}t^3+\cdots)$$

Mercury is a liquid of expansion rather much more uniform than that of others, for here b is much smaller than a. See fig IV-4.2.

When a liquid is heated the container is also heated along with it and expands. The expansion of the vessel masks the expansion of the liquid and makes the latter appear smaller than its real value, i.e., the *apparent expansion* of a liquid (i.e., the expansion of a liquid that we see) is less than its *real expansion.*

To show the expansion of a liquid and that of the container. Take a litre flask filled with coloured water. Fit a cork to it through which passes a glass tube with a narrow bore (fig. IV-4.1). By pushing in the tube or pulling it out as and when necessary, the level of water in the tube may be made to stand at any desired height (say at A). Put an ink mark there. Now plunge the flask suddenly in hot water ; the water level in the tube sinks. The reason is that the flask has expanded due to heat, but no heat has yet entered the liquid. Later as the liquid heats up the water in the tube rises and

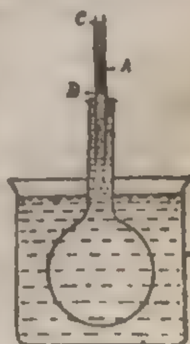


Fig. IV-4,1

moves past the former mark A, say upto C. Fig. IV-4.2. shows the expansion of different liquids for the same rise in temp.

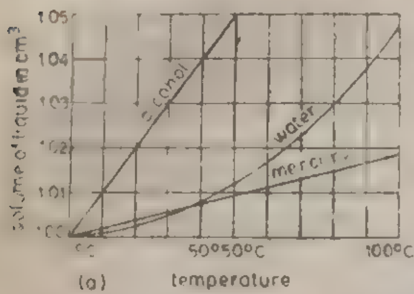


Fig. IV-4.2

IV-4.2. A. Relation between real and apparent expansion. In fig IV-4.1 let A be the initial level of water in the tube at a temperature t_1, B the position which the level would occupy if the vessel alone expanded to temperature t_2 without any expansion of the liquid and C the final position of the level when the liquid is also at t_2.

The apparent expansion of the liquid is AC while its true expansion is BC ; AB is the expansion in volume of the vessel, all for the same rise of temperature from t_1 to t_2.

From the figure $\quad BC = AC - AB$

Or, $\quad$ *the real expansion = the apparent expansion*
+ the expansion of the vessel.

B. Coefficients of real and apparent expansion. So for liquids we have two coefficients, the coefficient of *real* (or *absolute*) expansion and that of *apparent* expansion.

(i) The *coefficient of real* (or *absolute*) *expansion* (γ_r) is the *true* increase in volume per unit volume of the liquid per unit rise of temperatnre.

(ii) *The coefficient of apparent expansion* (γ_a) is its *observed* increase in volume per unit volume per unit rise of temperature. γ_a is the value of the coefficient of expansion as we find it when we ignore the expansion of the container. In finding γ_r, we have to consider the expansion of the container.

C. Relation between coefficients of real and apparent expansion.

Let V_o be the volume of a given mass of liquid at the lower temperature,

V be its *true volume* at a temperature t°C *higher* than the former,

V' be its *apparent volume* at the same *higher* temperature,

γ_r and γ_a be its coefficients of real and apparent expansion,

γ_v be the coefficient of volume expansion of the material of the vessel

Then $V - V_o =$ the real expansion of the liquid.

$V' - V_o =$ its apparent expansion,

and $V_o \gamma_v t =$ the expansion of the vessel.

Then Real expansion = Apparent expansion + Expansion of the vessel $\quad V - V_o = (V' - V_o) + V_o \gamma_v t.$

$$\text{or,} \quad \frac{V - V_o}{V_o t} = \frac{V' - V_o}{V_o t} + \gamma_v$$

By definition, the term on the left hand side of the equation is the coefficient of real expansion, and the first term on the right hand side is the coefficient of apparent expansion.

$$\therefore \quad \gamma_r = \gamma_a + \gamma_v \qquad \text{(IV-4,2,1)}$$

The initial temperature may be 0°C or have any other value. This shows that whether we consider the *zero coefficient* or the *mean coefficient*, we get the result that.

The coefficient of real expansion of a liquid = its coefficient of apparent expansion + the coefficient of volume expansion of the material of the vessel.

As for solids, if the initial temperature is 0°C, we call the coefficient the *zero coefficient*. If the initial and final temperatures are t_1°C and t_2°C, then the coefficient is the *mean coefficient* between t_1 and t_2. Note that the zero coefficient is also a mean coefficient—that between 0°C and t°C. It is not necessary to take V_o always as the value at 0°C, while the temperatures of observation are t_1° and t_2°, both different from 0°C.

If V_1 = volume of a mass of liquid at t_1°C,

V_2 = volume of the same mass of liquid at t_2°C,

then the mean coefficient γ between t_1° and t_2° is

$$\gamma = \frac{V_2 - V_1}{V_1(t_2 - t_1)} \quad \text{(IV-4.2.2)}$$

$$\text{or,} \quad V_2 = V_1\{1 + \gamma(t_2 - t_1)\}. \quad \text{(IV-4.2.3)}$$

γ, so defined, will be the *apparent* or *true* coefficient according as the volumes are the apparent or true volumes of the same mass

The following table gives the values of mean γ_r of some liquids over a small range around 18°C, unless otherwise stated. The values are per °C. The expansion coefficient generally increases with rising temperature.

Substance	γ_r	Substance	γ_r
Water (5° to 10°C)	$5 \cdot 3 \times 10^{-5}$	Mercury	18.1×10^{-5}
,, (10° to 20°C)	$15 \cdot 0 \times 10^{-5}$	Glycerine	47×10^{-5}
,, (20° to 40°C)	30.2×10^{-5}	Benzene	122×10^{-5}
,, (40° to 63°C)	45.8×10^{-5}	Sulphuric acid (pure)	56×10^{-5}
,, (60° to 80°C)	58.7×10^{-5}		
Chloroform	127×10^{-5}	Olive oil	70×10^{-5}
Pentane	155×10^{-5}	Alcohol (Ethyl)	108×10^{-5}
Ether	163×10^{-5}	Turpentine	96×10^{-5}

IV 4 3. Change of density of a liquid on heating. *Though expansion of the vessel masks the expansion of the liquid, it cannot affect the change in its density* with rise in temperature. The coefficient of real expansion bears a simple relation to the change in density and may be determined from observations on the latter.

Let V_0 be volume of a given mass m of a liquid at the lower temperature,

V be its *true* volume at a temperature t°C *higher* than the colder temperature,

ρ_0 be density of the liquid at the lower temperature,

ρ be its density at the higher temperature,

and γ_r = the *mean coefficient of real expansion* between the two temperatures.

Then $\rho_0 = \dfrac{m}{V_0}$; $\rho = \dfrac{m}{V}$ and $V = V_0(1+\gamma_r t)$;

so $$\rho = \frac{m}{V_0(1+\gamma_r t)} = \frac{m}{V_0} \cdot \frac{1}{1+\gamma_r t};$$

or $$\rho = \frac{\rho_0}{1+\gamma_r t} \qquad \text{(IV-4.3.1)}$$

When $\gamma_r t$ is small enough we may write by Binomial Expansion

$$\rho = \rho_0(1+\gamma_r t)^{-1} = \rho_0(1-\gamma_r t) \qquad \text{(IV-4.3.2.)}$$

Example IV-4.1. A glass hydrometer reads sp. gr. of a liquid as 0.92 at 45°C. What will it read at 15°C ($\gamma_l = 52.5 \times 10^{-5}$/°C and $\gamma_g = 2.4 \times 10^{-5}$/°C)? **(Pat. U.)**

Solution: In the cgs system sp. gr. and density are numerically the same. Let V and V' be the volumes of the hydrometer at 15°C and 45°C and ρ and ρ' be the densities of the liquid. Then

$V' = V(1+\gamma t)$ and $\rho' = \rho(1-\gamma_r t)$

Again the mass of V volume of the liquid $= V\rho$

Now $V = V'(1+\gamma t)^{-1} = V'(1-\gamma t)$ and $\rho = \rho'(1+\gamma_r t)$

$\therefore \quad V\rho = V'(1-\gamma t)\rho'(1+\gamma_r t)$

$$\therefore \quad \rho = \frac{V'}{V}(1-2.4\times10^{-5}\times30)\times0.920(1+52.5\times10^{-5}\times30)$$

$$= 0.9345$$

Ex. IV-4.2. A cylinder 20″ high floats vertically in mercury at 0°C. If the common temp rises to 100° by how much will the cylinder sink? Cubical expansion of mercury between 0°C and 100°C = 0.018153 and linear expansion of iron between 0° and 100°C = 0.001182. Sp. gr. of iron and mercury at 0°C are 7.6 and 13.6. **[Pat. U.]**

Solution: Note that total expansions between two temp are given, not the coefficients. Hence

$\gamma_{Hg} = 18.153 \times 10^{-3}/100°C = 18.153 \times 10^{-5}$/°C.

$\gamma_{Fe} = 3 \times 1.182 \times 10^{-3}/100°C = 3.546 \times 10^{-5}$/°C.

Let l and l' be the immersed lengths of the cylinder, A and A' its areas of cross-section at 0° and 100°C.

$(\rho_{\circ})_{Fe}=7.6\times 62.5$ lbs/cuft and $(\rho_{\circ})'_{Hg}=13.6\times 62.5$ lbs/cuft.

$\therefore\ (\rho_{100})_{Fe}=7.6\times 62.5(1-3.546\times 10^{-5}\times 100)$

$(\rho_{100})'_{Hg}=13.6\times 62.5(1-18.153\times 10^{-5}\times 100)$

By the law of floation we have

$$20\times A_{\circ}\times\rho_{\circ}\times g=l_{\circ}\times A_{\circ}\times(\rho_{\circ})'g \qquad (A)$$

and $20(1+100\alpha_{Fe}).\ A_{100}.\ \rho_{100}g=l_{100}A_{100}(\rho_{100})'g$ (B)

From (A) we get $l_{\circ}=\dfrac{20\rho_{\circ}}{\rho'_{\circ}}=\dfrac{20\times 7.6\times 62.5}{13.6\times 62.5}=11.176''$

From (B) we have $20(1+1.182\times 10^{-3})\rho_{\circ}(1-3.546\times 10^{-3})$

$$=l_{100}\rho'_{\circ}(1-18.153\times 10^{-3})$$

Or, $20(1+0.001182)\times 7.6\times 62.5(1-0.003546)$

$$=l_{100}(13.6\times 62.5)(1-0.018153)$$

$\therefore\ l_{100}=11.355''$. So the cylinder sinks by

$$(11.355-11.176)=0.179''.$$

Problem : The weights of 1 cc of water at 0°C and 4°C are 0.999874g and 1.00g. Find γ_r for water between 0°C and 4°C.

Ans. $(-3.15\times 10^{-6}/^{\circ}C)$ [H.S. '68]

(Compare the value in the foregoing table)

IV-4.4. Methods of Measuring the Coefficients of Expansion of a liquid. There are two broad classes—one for apparent coefficient the other for real expansion. The former may be called the *method of envelopes* and the latter the *method of balancing columns.*

A. Principle of method of Envelopes : The liquid is contained in a bulb shaped container. When heated, some of the liquid is allowed to overflow and is weighed as in a *weight thermometer* or it is forced into a graduated stem as in the *volume thermometer* i.e., a dilatometer.

(1) **Determination of coefficient of apparent expansion of a liquid. Weight Thermometer method.** This can be easily done with a specific gravity bottle.

(a) **Principle :** Let M_1 be the mass of the liquid required to fill the bottle at temperature t_1°C and M_2 the mass required to fill the same bottle at t_2°C. As expansion of the vessel is small compared with that of the liquid, disregard the expansion of the vessel and *assume that the vessel has the same volume at both temperatures.* If ρ_1 and ρ_2 are the densities of the liquid at t_1°C and t_2°C respectively, then the volume V of the vessel is given by

$$V=M_1/\rho_1=M_2/\rho_2$$

But by Eq. IV-4.3.1, we get $\rho_2\{1+\gamma(t_2-t_1)\}=\rho_1$.

Here γ is the coefficient of apparent expansion since we have *disregarded* the expansion of the vessel.

$$\therefore \quad \frac{M_1}{M_2}=\frac{\rho_1}{\rho_2}=\frac{\rho_2\{1+\gamma_a(t_2-t_1)\}}{\rho_2}=1+\gamma_a(t_2-t_1)$$

$$\text{or} \quad \gamma_a(t_2-t_1)=\frac{M_1}{M_2}-1 \quad \text{or} \quad \gamma_a=\frac{M_1-M_2}{M_2(t_2-t_1)} \qquad \text{(IV-4.4.1)}$$

M_1-M_2 is the mass of the liquid expelled from the vessel when it is heated from t_1° to t_2°C, and M_2 is the mass retained by the vessel at the higher temperature. Hence we may write

$$\gamma_a=\frac{\text{mass expelled}}{\text{mass retained}\times\text{temp. rise}} \qquad \text{(IV-4.4.2)}$$

(b) **The experiment** consists in

(*i*) Weighing a dry and clean specific gravity bottle when empty (Mass $=m_1$)

(*ii*) completely filling it with the liquid, inserting the stopper, wiping it dry and weighing again (Mass $=m_2$)

(*iii*) noting the temperature of the liquid in the bottle (t_1°C)

(*iv*) heating the bottle in a water bath which maintains a steady temperature (t_2°C) for several minutes. The extra liquid is expelled through the bore in the stopper.

(*v*) Noting the temperature of the bath (t_2°C).

(*vi*) Wiping the bottle dry, allowing it to cool, and weighing it again (Mass $=m_3$).

Then $m_2 - m_1 = M_1$ and $m_3 - m_1 = M_2$, Eq. IV-4.4.1 is then applied. This gives the coefficient of apparent expansion.

(c) To get the **coefficient of real expansion** of the liquid, we may then apply Eq. IV-4.2.1., if the coefficient of volume expansion of the bottle is known. Or else we can proceed as follows :

$$\frac{\rho_1}{\rho_2} = \frac{M_1}{M_2} \times \frac{V_2}{V_1} = \frac{M_1}{M_2}\left[1 + \gamma_g(t_2 - t_1)\right]$$

But from eqn. IV-4.3.2 we have $\rho_1/\rho_2 = 1 + \gamma_r(t_2 - t_1)$

Equating, $1 + \gamma_r(t_2 - t_1) = (M_1/M_2)[1 + \gamma_a(t_2 - t_1)]$

$$\text{Whence } \gamma_r = \frac{M_1 - M_2}{M_2(t_2 - t_1)} + \frac{M_1}{M_2}\gamma_g = \gamma_a + \frac{M_1}{M_2}\gamma_g \qquad \text{(IV-4.4.3)}$$

Fig. IV-4.3

The method is capable of high precision. It is said to be a *weight thermometer* for the temperature t_2 of a liquid bath can be accurately determined if it can be held constant. A weight thermometer in actual use is shown in fig. IV-4.3. To fill it with liquid initially it is to be heated and cooled alternately with its mouth kept dipped into a vessel of the liquid.

Ex. IV-4.3. A weight thermometer weighs 50g when empty, 163.13g when filled with glycerine at 0°C and 157.65g when filled with glycerine at 100°C. Find the coefficient of real expansion of glycerine, given that the coefficient of cubical expansion of glass is 27×10^{-6} per °C.

Solution : From equation IV-4.4.1 the coefficient of apparent expansion of glycerine is

$$\frac{\text{mass expelled}}{\text{mass retained} \times \text{temp. rise}}$$

$$= \frac{163.13 - 157.65}{(157.65 - 50) \times 100} = \frac{5.48}{107.65 - 10} = 50.9 \times 10^{-5}/^\circ C$$

∴ Coefficient of real expansion $= 0.000509 + 0.000027$

$= (50.9 + 2.7) \times 10^{-5}/^\circ C = 0.000536$ per °C.

Ex. IV-4.4. A vessel is completely filled with 500g of water and 1000g of mercury. 3.52g of water overflows on supplying 21200 cal of heat. Find γ for mercury. [I.I.T. '76]

[Given γ for water $=15\times10^{-5}/°C$, ρ for mercury and water 13.6g/cc and 1g/cc respectively and s for mercury $=0.03$ cal/gm/°C.]

Solution : Let the rise in temp of water and mercury be t. Then

$$500\times1\times t+1000\times0.03\times t=21200 \text{ cal} \quad \therefore \quad t=40°C$$

Expansion of water $=V\gamma_w t=500\times15\times10^{-5}\times40=3$ cc

Expansion of mercury $=(1000/13.6)\times\gamma\times40$ cc

$$\text{So total expansion}=3+\frac{4\gamma}{13.6}\times10^4 \text{ cc}$$

The volume of water expelled $=3.52/\rho'$

where ρ' is the density of water after a 40°C rise.

Now $\rho''=\rho(1-\gamma_w t)=1(1-15\times10^{-5}\times40)$

$$\therefore \text{ Volume of expelled water}=\frac{3.52}{1-6\times10^{-2}}=3.52(1.06)=3.54 \text{ cc}$$

Now total expansion of the liquids = volume of water expelled

$$\therefore \quad 3+\gamma.(4\times10^4/13.6)=3.54$$

$$\therefore \quad \gamma=\frac{0.54\times13.6}{4\times10^4}=18\times10^{-5}/°C$$

Problems : (1) If γ_a for mercury in glass be 1/6500, find the overflow of mercury from a weight thermometer containing 400g of it at 0°C when heated to 90°C ? (Ans. 5.47g) [C. U.]

(2) A glass bottle in melting ice contains 300g of mercury. How much mercury will overflow if it is maintained in boiling water at 76 cm. of Hg. ? (γ_a for mercury = 1/6500). Would the overflow differ if the atmospheric pressure falls considerably ?

(Ans. 4.55g ; Yes) [C. U.]

(2). **By Volume Thermometer or Dilatometer method.** (fig IV-4.4). It is a long glass bulb attached with a graduated stem

of narrow bore up which a liquid rises on heating say from V_o to V_t which can be directly read off. It is used chiefly for volatile liquids. Note that the liquid-in-glass thermometer is same as the dilatomer. With it

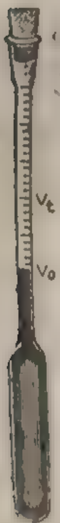


Fig IV 4.4

$$\gamma_a' = \frac{V_t - V_o}{V_o(t_2 - t_1)}$$

is the usual formula. To find γ_r, let us remember that V_t is really $V_t\,[1+\gamma_v(t_2 - t_1)]$ where γ_v is the volume coefficient of expansion of the material of the vessel and $V_t\gamma_v(t_2 - t_1)$ is the volume expansion of the vessel. So

Increase in volume

$$= V_t\,[1+\gamma_v\,(t_2 - t_1)] - V_o = V_o\,\gamma_r\,(t_2 - t_1)$$

$$\therefore\quad \gamma_r = \frac{(V_t - V_o) + V_t\gamma_v\,(t_2 - t_1)}{V_o(t_2 - t_1)}$$

$$= \frac{V_t - V_o}{V_o(t_2 - t_1)} + \frac{V_t}{V_o}\gamma_v$$

$$= \gamma_a + \frac{V_t}{V_o}\gamma_v \qquad \text{(IV-4.4.4)}$$

an expression very similar to that for the weight thermometer

Ex IV-4.5. : γ for mercury is 18×10^{-5}/°C and α for glass is 8×10^{-6}/°C. In a graduated tube mercury occupies 100 div. Find the rise in temp to cause the mercury occupy 101 divisions.

[Lond. Matric]

Solution : Let t be the required rise when the length of the mercury thread becomes $100\,(1+0.00018t)$

$$\therefore\quad 101 \text{ div} = 100\,(1+0.00018t) \text{ or } 1 \text{ div} = \frac{101(1+0.00018t)}{101}$$

But each division of the tube is $1+0.000008t$ div for t°C rise.

$$\therefore\quad \frac{100}{101}(1+0.00018t) = 1+0.000008t$$

$$\therefore\quad 100+0.018t = 101+101\times 8\times 10^{-6}\times t$$

$$\therefore\quad t\,(18\times 10^{-3} - 808\times 10^{-6}) = 1 \quad\text{or}\quad t = 58.2°C.$$

Ex IV-4 6. A long uniform capillary contains a 1m long mercury thread at 0 C. At 100°C the thread gets 16.5 mm longer. If $\gamma_r = 18.2\times10^{-5}$/°C find γ for glass. [H.S.'60]

Solution : If A_0 be the area of cross section of the capillary at 0°C and A_{100} of the same at 100°C then $A_{100}=A_0(1+\beta\,100)$ $=A_0(1+2\alpha.100)$ where α and β are linear and superficial coefficients of expansion of glass. Again the volumes of mercury at 0° and 100°C are respectivly $100\,A_0$cc and $101.65\,A_{100}$ cc.

$\therefore$ Expansion of mercury$=101.65\,A_{100}-100\,A_0$.

$$\therefore\quad \gamma_r=\frac{101.65\,A_0\,(1+2\alpha.100)-100A_0}{100\,A_0\times100}=18.2\times10^{-5}\quad\text{(given)}$$

$\therefore\quad 101.65-100+200\times101.65\alpha=1.82\quad$ or $\quad\alpha=8.3\times10^{-6}/°C$

Problem : The cross section (S) of a mercury thermometer remains constant and V_0 the volume of the bulb at 0 C is just filled with mercury at ice point. Find the length of the mercury column at t C (*Ans.* $l=V_0\,(\gamma-3\alpha)\,t/S$).

B. By the sinker or upthrust method (fig IV-4.5a) : A glass bulb weighted with lead shots is weighed in a liquid at two known temperatures t_1 and t_2. Now if m_1 is the *apparent loss in weight* of the bulb immersed in liquid at t_1 and m_2 that at temp t_2 then by Archemedes principle we have

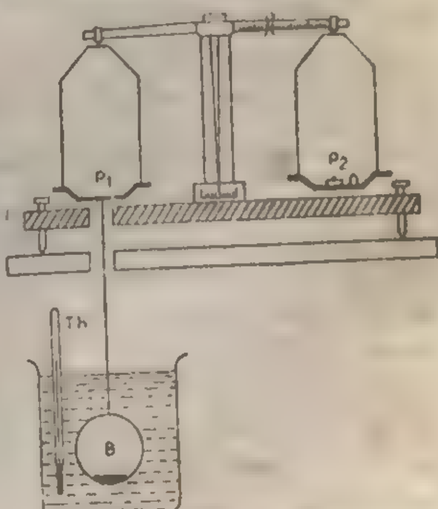


Fig. IV-4.5(a)

$m_1=v_1\rho_1$ and $m_2=v_2\rho_2$ where m_1 is the weight of the liquid displaced by the bulb at t_1 and m_2 that at t_2 C ; v_1 and v_2 are also the volumes of the bulb at the

two temp. Then we have as for the weight thermometer

$$m_1/m_2=(v_1/v_2)\,(\rho_1/\rho_2)$$

Then proceding exactly as there we get

$$\gamma_r=\frac{m_1-m_2}{m_2(t_2-t_1)}+\frac{m_1}{m_2}\gamma_v$$

Now we find m_1 and m_2 ; If the sinker weighs w_1g in air, w_2 in the liquid at t_1 and w_3 in the liquid at t_2 then clearly $m_1 = w_1 - w_2$ and $m_2 = w_1 - w_3$. Then in terms of these weights.

Fig. IV-4.(5b)

$$\gamma_r=\frac{W_3-W_2}{(W_1-W_2(t_2-t_1)}+\frac{W_1-W_2}{W_1-W_3}\gamma_v \qquad \text{(IV-4,4.5)}$$

and $$\gamma_a=\frac{W_3-W_2}{(W_1-W_2)(t_2-t_1)} \qquad \text{(IV-4.4.6)}$$

In this method a continuous set of records may be taken in the course of a single experiment and the mean coefficient between any two temperatures within the range determined.

The experiment can be easily carried out by a simpler hydrostatic balance as shown in fig IV-4.5 (b).

IV-4.5. Change in Temp and Hydrostatic Balance Expts :

A. Apparent loss in weight of a solid in liquids of changing temperatures : As density of a liquid falls with rise in temp the force of buoyancy or upthrust on an immersed solid diminishes and the solid appears heavier. At the same time, the solid expands displacing more liquid thereby raising the upthrust. The change in resultant upthrust is the difference of the two. As coefficient of volume expansion for a liquid is generally ten times that of a solid, *loss in weight with rising temperature, diminishes* for the solid.

The loss in weights of the solid at two temp is from above m_1 and m_2. We have seen

$m_2 = V_2\rho_2$ where V_2 = Vol of displaced liquid

= Vol of solid at t_2

$= V_1(1+\gamma t)\rho_1(1-\gamma_r t)$ where γ is volume coeff of the solid.

$= V_1\rho_1(1+\gamma t)(1-\gamma_r t) = m_1(1+\gamma t)(1-\gamma_r t)$

$\simeq m_1[1-(\gamma_r-\gamma)t]$ (IV-4.5.1)

neglecting the product $\gamma\gamma_r t$ which is of the order of $10^{-11}t$

So $m_1 > m_2$ and hence $W_3 > W_2$ i.e. apparent weight of a solid rises with temp.

Ex. IV-4.7. A glass piece weighs 47g in air, 31.53g in water at 4°C and 31.75°C at 60°C. Find the mean coefficient of cubical expansion of water between 4°C and 60°C when that for glass is 24×10^{-6}/°C [C. U.]

Solution : Here $m_1 = 47 - 31.53 = 15.47$g

and $m_2' = 47 - 31.75 = 15.25$g

∴ From eqn IV-4.5.1 We obtain

$15.25 = 15.57\,[1-(\gamma_r - 24\times10^{-6})\times56]$

Or $\gamma_r - 0.000024 = \dfrac{15.47-15.26}{56\times15.47} = \dfrac{0.21}{56\times15.47} = .0002424$

$\gamma_r = 0.000266$/°C $= 26.6\times10^{-5}$/°C

Alternatively, Vol of glass at 60°C = Vol of displaced water = 15.47g. Vol of glass at 60° = $15.47\,[1+24\times10^{-6}\,(60-4)]$

= 15.49 = vol of displaced water at 60°C

Again wt of displaced water at 60°C = 47 − 31.75 = 15.25 g

∴ ρ_w at 60°C = 15.25/15.49

Then $(\rho_w)_{60} = (\rho_w)_4\,\{1-\gamma(60-4)\}$

∴ $\dfrac{15.25}{15.49} = [1-\gamma 56]$ for $(\rho_w)_4 = 1$ g/cc

Or $56\,\gamma = 1 - \dfrac{15.25}{15.49} = \dfrac{0.24}{15.49}$

Or $\gamma = 0.24/(15.49\times56) = 27.6\times10^{-5}$/°C.

Ex IV-4.8. A metal piece weighs 46g in air and 30g in a liquid of sp gr 1.24 at 27°C. It weighs 30.5g at which temp the sp. gr of the liquid is 1.20. Find α for the metal. [I.I.T. '74]

Solution : Loss in weight at 27°C is 16g and at 42°C is 15.5g. So the volume of the solid at 27°C = 16/1.24 cc. So for the solid

$V_{42} = (16/1.24)\,[1+\gamma.15] = (16/1.24)\,[1+3\alpha\times 15]$

Again at 45°C loss in weight = 13.5g = wt of displaced liquid

$\therefore$ Vol of displaced liquid = 15.5/1.20 cc = vol of metal at 42°C

So we have $\frac{16}{1.24}(1+45\alpha) = \frac{15.5}{1.20}$ or $\alpha = 23.1\times 10^{-6}/°C$

Problem : A metal bob weighs 50g in air and 45 g in a liquid at 25°C and 45.1g in the same liquid at 100°C. Find γ for the the liquid if α for the metal is $12\times 10^{-6}/°C$. (**Ans.** $30.6\times 10^{-5}/°C$) [I.I.T. '73]

Ex IV-4.9. A sphere of diameter 7 cm and mass 266.5 g floats in a liquid bath. As temp rises the sphere begins to sink at 35°C. Find γ neglecting the expansion of the sphere if (ρ_0) of the liquid is 1.527 g/cc.

Solution : Vol of the sphere $= \frac{1}{6}\pi d^3 = \frac{1}{6}.\frac{22}{7}(7)^3 = 179.6$ cc When the sphere starts sinking at 35°C it must displace its own volume of the liquid of which the wt would be $179.6\,\rho_{35}$.

$\therefore$ $179.6\,\rho_{35} = 266.5\text{g} =$ the wt of the sphere.

Or $\rho_{35} = 266.5/179.6 = \rho_0\,(1-\gamma_r t) = 1.527\,(1-\gamma.35)$

Or $(1-\gamma.35) = \frac{266.5}{179.6\times 1.527}$ or $\gamma = 8\times 10^{-4}/°C$

B. Volume coefficient of a solid at different temp ranges can be found by the *specific gravity method* by weighing it in a liquid of known volume coefficient of expansion γ at different temp. The weight of the solid in air and the liquid at two different temp are taken from which its sp. gr at these two temps are found.

Let M = mass of solid, V_1, V_2 its volumes, ρ_1, ρ_2 its sp. gravities at temp t_1 and t_2, d_1 and d_2 densities of the liquid at these temp then $\rho_1 = M/V_1 d_1$, $\rho_2 = M/V_2 d_2$

or $\rho_1/\rho_2 = (V_2/V_1)\;(d_2/d_1)$

$= [1+\gamma_s\,(t_2-t_1)]\,[1+\gamma_l\,(t_2-t_1)]^{-1}$ (IV-4.5.2)

ρ_1 and ρ_2 the sp. gravities, are experimentally determined. From known value of γ_l we can find γ_s in the temp range (t_2-t_1).

IV-4.6. Determination of the coefficient of real expansion of a liquid : Dulong and Petit's method. This is also known as the *method of balancing columns.* The apparatus is represented in principle in fig. IV-4.6. The arms AB and CD of a U-tube are surrounded by a jacket each. The experimental liquid, say mercury, is poured into the U-tube. When the temperatures of the two arms are the same, mercury stands at the same level in both. Cold water is then circulated through one of the jackets and steam through the other. As the temperatures of the mercury columns change, the density changes also. Since under all circumstances the hydrostatic pressure due to one column is equal to that due to the other, the heights of the columns in the two limbs of the U-tube will be different when the temperatures are different. To prevent the flow of heat from one limb into the other the horizontal part AC of the U-tube is kept cold. The temperatures in the jackets are read off from thermometers T_1 and T_2.

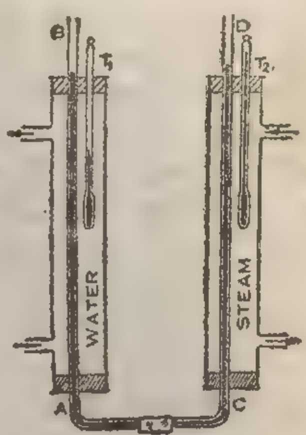


Fig. IV-4.6

Let t°C = difference temp. between the hot and the cold limbs.

h_0 and h = heights of the liquid levels in the cold and hot limbs respectively above the axis of the cross tube AC.

ρ_0 and ρ = densities of the cold and the hot liquid columns

Since the hydrostatic pressures due to the liquid columns must balance each other, we have

$$h_0\rho_0 g = h\rho g \quad \text{or,} \quad \frac{h}{h_0} = \frac{\rho_0}{\rho} = \frac{\rho(1+\gamma_r t)}{\rho} = 1+\gamma_r t.$$

$$\therefore \quad \gamma_r = \frac{h-h_0}{h_0 t} \qquad \text{(IV-4.6.1)}$$

As in all experiments, there are some *sources of error* in this experiment too. For example, the temperature in either limb is not constant and equilibrium is never established. Besides, the exposed columns of the liquid are at temperatures different from those inside the limbs. The arrangement was later modified to minimize such errors, Callendar and Moss putting six tubes in series.

The determination of γ_r for mercury is a matter of great importance. From a knowledge of this value γ_v for any container may be calculated. This is a better method than taking γ_v to be three times the coefficient of linear expansion, particularly for glass. Once γ_v of a container is known, it may be used to find γ_a for any other liquid. Hence γ_r for the latter may also be found out. Since mercury is the liquid used in pressure gauges and barometers, an exact knowledge of its density is necessary for the accurate determination of pressures.

Ex IV-4.10. A mercury column 76.35 cm long at 100°C balances another 75 cm long and at 0°C. Find γ_r. [H.S. 79]

Solution : $\gamma_r=\dfrac{h-h_o}{h_o t}=\dfrac{76.35-75}{75\times 100}=18\times 10^{-5}/°C$

Problem. A U-tube contains aniline, the limbs being surrounded one by melting ice and the other by steam. The colder limb is 31.8 cm long and the liquid heights in the two limbs differ by 2.7 cm. Find γ for aniline. (Ans. $8.5\times 10^{-4}/°C$)

IV-4.7. Two Related corrections. A. Barometer correction. In a barometer we measure the length of a mercury column by a metal scale. With rise in temp two-fold change occurs—(*i*) expansion of the scale graduations (see fig IV-3.15 and also the example of same number) and (*ii*) lowering of density and consequent increase in height for which corrections become necessary.

(*i*) *Scale correction* : If the scale reads correctly at t_o°C and the barometer reading taken at some other temp t, then the barometer reading h corresponds to a corrected height H given by

$$H=h\,[1+\alpha\,(t-t_o)] \qquad \text{(IV-4.7.1.)}$$

where α is the coefficient of linear expansion of the material of the scale.

A *nomograph* [fig (IV-4 7)] devised by Mehmeke provides a simple method of correcting the barometer reading for temp if the scale is graduated at 0°C. For example let the observed reading at 17°C be 751 mm. Join by a st line the corresponding points on the two scales. The intercept on the latter, 2.1 mm is the amount to be deducted from the observed height to reduce the reading to 0°C.

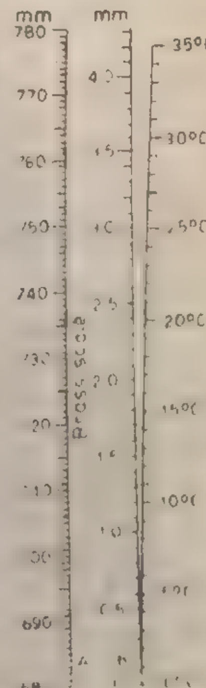


Fig. IV-4.7

Ex. IV-4.11. A barometer has a brass scale correct at 50°F. reads 754 mm at 40°F, what will be the true height at 32°F. Given $\alpha = 18\times10^{-6}/°C$, $\gamma = 18\times10^{-5}/°C$ [Utk. U.]

Solution : Since $1F° = (5/9)C°$ we have $\alpha_f = \frac{5}{9}\alpha_c = 10^{-5}/°F$ and $\gamma = 10^{-4}/°F$.

Here $t_0 = 50°F$

$$\therefore\ H_{32} = \frac{754[1+10^{-4}(40-50)]}{1+[10^{-4}(40-32)]} = \frac{754[1-10^{-3}]}{1.0008} = 753.32\text{mm}$$

Problem : Find the temp at which no temp correction would be necessary if the brass scale has been correctly graduated at 15°C. (**Ans.** -1.76°C)

(*ii*) *Density correction* : The corrected barometric pressure should be $H\rho g$ where ρ is the density of mercury different from ρ_0 the density at 0°C. We have to find H_0 the mercury height at 0°C that will exert the same pressure as the mercury height H does at t_0°C. Clearly then,

$H_\circ \rho_\circ g = H\rho g$. or $H_\circ / H = \rho/\rho_\circ = 1 + \gamma_r t$

$$\therefore \quad H_\circ = \frac{H}{1+\gamma_r t} = \frac{h[1+\alpha(t-t_\circ)]}{1+\gamma_r t} \qquad \text{(IV-4 6.2)}$$

If however $t_\circ = 0$°C then

$H = h(1+\alpha t)(1+\gamma_r t)^{-1} = h(1+\alpha t)(1-\gamma_r t)$

$= h[1-(\gamma_r-\alpha)t]$ (since $\gamma_r \alpha t^2$ is a small quantity). (IV 4.6 3)

(Apply this formula to solve the sum above)

B. Correction for Exposed stem of a thermometer. A thermometer when measuring the temp of a liquid bath has only its bulb inside but the rest of it outside the bath. Obviously its two parts are at different temp and so are the mercury in the bulb and the capillary. Some heat loss and so fall in temp would occur at the capillary ; so a correction becoms necessary.

Let a thermometer be so immersed in a liquid bath at t°C that n of its divisions are exposed to air. Let it read t' and let t_m be the mean temp of the exposed column. Let γ_a be the apparent coefficient of mercury in glass. To calculate $t-t'$ we take the volume of one degree division as the unit of volume. The exposed n divisions are heated from t_m to t then the expansion of mercury would be $n\,\gamma_a\,(t-t_m)$.

Hence $t-t' = n\,\gamma_a\,(t-t_m) = n\gamma_a\,(t'-t_m)$ [for $t'=t$] (IV-4.6.4)

Ex. IV-4.12. A mercury thermometer immersed wholly in boiling water reads 100°C. When the stem is so withdrawn that all the graduations above 0°C are at an average temp of 10°C, find the thermometer reading if $\gamma_a = 15.8 \times 10^{-5}$/°C.

Solution : Here $t=100$, $n=100$, $t_m=10$°C.

$100-t' = 100 \times 15.8 \times 10^{-5}(100-10) = 15.8 \times 10^{-3} \times 9 = 1.422$

$\therefore \; t' = 100 - 1.4 = 98.6$°C.

Note : If t is given, you can find γ_a.

IV-4.7 Anomalous expansion of water. A liquid expands when heated, but water at 0°C is an exception. *When heated from*

0°C water contracts as the temperature rises. This continues to about $4°C$, above which point a rise in temperature causes expansion. Water, therefore, has maximum density at about $4°C$. An exact knowledge of the temperature is important, since the litre has been defined as the volume of one kilogram of water at its maximum density.

Fig. IV-4.8(a)

A. Constant volume dilatometer. The anomalous expansion of water may be demonstrated by it. Fig. IV-4.8(a) represents a glass vessel with a tube of narrow bore fitted to it. The tube carries a scale. The vessel contains mercury which occupies *one-seventh* part of the volume of the vessel. The rest is filled with water, which rises to certain height in the tube.

In it the *space abve mercury inside the flask remains constant at all temp.* For, when the temp is raised, the vessel expands and the volume of the flask increases ; simultaneously mercury inside, expands reducing the empty volume within the flask. The volume of flask and that of mercury within it are so adjusted that their expansions (being opposed to each other just cancel out.

Amount of mercury in a const volume dilatometer can be calculated as follows: Glass has generally an α value of $9\times10^{-6}/°C$ so that its γ-value of $27\times10^{-6}/°C$ whereas that of mercury is $18\times10^{-6}/°C$. Let V be the volume of the flask and V' that of mercury inside. Let both of them be raised by $t°C$. Then by the condition of the problem the expansions are equal.

$V\gamma_g t = V'\gamma_m t$ or $V'/V = \gamma_g/\gamma_m = 27\times10^{-6}/18\times10^{-6} = 3/20$

i.e. mercury within the flask should have about 1/7th the volume of the flask.

Problems. 1) If a flask is made of a material of γ value of $27\times10^{-6}/°C$ and 3/20 of its volume is occupied by mercury of cubical expansion $180\times10^{-6}/°C$ show that the volume of the remaining space will not change with temp. [H.S. '63]

(2) What fraction of the inner volume of a glass vessel should be kept filled with mercury so that the remaining volume will be the same at all temp ? γ for glass $2.4 \times 10^{-5}/°C$ and γ for mercury $1.8 \times 10^{-4}/°C$. (**Ans :** $\frac{2}{15}$) [H.S. '82]

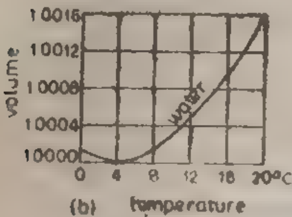


Fig. IV-4.8(b)

B. Working of the constant volume dilatometer and its findings : Since the cubical expansion of mercury is seven times that of glass the inner volume of the vessel (not occupied by mercury) will not change when the vessel is heated. It is kept in melting ice for some time, when it acquires the temperature 0°C. The position of the water column in the tube is noted. If the vessel is now slowly heated it will be found that the water column descends at first, showing that water at 0°C contracts on heating. This goes on till 4°C is reached. Beyond that temperature the water column ascends. The adjoining graph (fig IV-4.8b) shows the results of the experiment where temp is plotted along the x-axis and the volume along the y-axis. Volume of water diminishes as temp rises from 0°C till 4°C, and then it increases. It is because of such behaviour that *in the temp range* between 0°C and 10°C water cannot be used as a thermometric liquid.

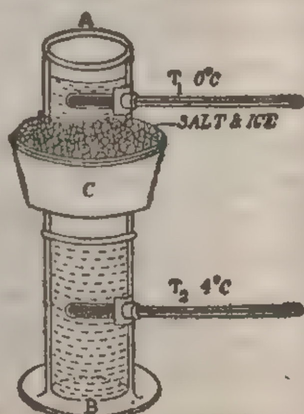
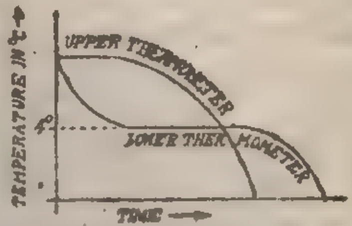


Fig. IV-4.9(a)

C. Hope's experiment. The anomalous expansion of water may be demonstrated and temperature at which the density of water is maximum may be determined by a simple experiment due to Hope. A tall glass vessel *AB* (fig IV-4.9.a) full of water has a freezing bath *C* of ice and salt round about its middle. Two

thermometers T_1 and T_2 are fitted into AB near about the top and the bottom. Their readings are noted before the freezing mixture is applied. As the freezing mixture cools the water in its neighbourhood, the water becomes cooler and denser and sinks to the bottom. The temperature recorded by the lower thermometer T_2 gradually falls until it reaches about 4°C. By this time the upper thermometer T_1 has shown practically no change. While the reading of T_2 remains steady at 4°C, that of T_1 begins to fall slowly at first, and then more rapidly until 0°C is reached. This shows that water is densest at about 4°C. Careful measurements show that the temperature at which the density of water is maximum is 3.98°C. The manner in which the two thermometers change with time after the freezing mixture has been applied, is reprsented in fig (IV-4.9 b).



IV-4.9(b)

* The reason for the behaviour of T_1 is as follows: After the water coulmn from the bottom B to the freezing bath C has reached the temperature 4°C, further cooling at C makes the water lighter. It practically stays where it is until with further coolIng ice begins to form. Ice being lighter than water rises to the top and melts thus gradually lowering the temperature of T'_1.

The density of water rises from 0.99987 g/cm³ at 0°C to exactly 1 at 4°C (by definition). It then diminishes, and at 8°C acquires the same value as at 0°C. The diminution continues; at 100°C the density is 0·9584 g/cm³.

D. **Effect of anomalous expansion of water on marine life.** As water cools below 4°C it expands and becomes lighter. When ponds, lakes, rivers and seas in very cold countries freeze in winter, marine life is saved from extinction by the anomalous expansion of water. As atmospheric temperature comes down the upper layers

of the lake cool, contract and sink to the bottom. This goes on until the whole of the lake acquires the temperature of 4°C. When the top layers cool further, the cooler water does not sink as it is lighter than the water below. With further cooling the top layers gradually freeze. Both ice and water are bad conductors of heat. So the lower layers are protected to a great extent against freezing by the upper layers of ice and cold water. The result is that water continues to exist at the bottom though a thickness of ice may have formed at the top. This enables marine life to continue. Fishes can then live in lakes, ponds and rivers even in extreme cold because the water of the bottom remains liquid.

EXPANSION OF GASES

IV-5.1. Introduction. Gases expand *much more* than solids or liquids for a given rise of temperature. Moreover, a gas expands when its pressure is reduced. In determining the effect of temperature on the volume of a given mass of gas, it is therefore necessary to keep the pressure of the gas constant. Such a restriction is not necessary for solids or liquids, as a change of pressure causes but an inappreciable change of volume in them. In addition mass of the gas taken, is relevant, for both the volume and pressure may change at the *same temp* for the *same mass* as Boyle's law tells you ; this does not happen for solids and liquids.

So to explain the behaviour of gases we have *four* parameters or variables to consider, pressure, volume, temperature and mass In stating the gas laws however, *mass is taken constant*. So in discussing them three relations arise taken two variables at a time out of the trio of them. You already know them—(1) *Boyle's* law connecting changes of volume with pressure at const temp (2) *Charles'* law connecting changes in volume with temp at const pressure and (3) *Regnault*'s or the pressure law connecting changes in pressure and temp at const volume.

Gases that obey them strictly, are called Ideal or Perfect gases. Most gases at low pressures and high temperatures obey them accurately.

In conection with expansion of gases vis-a-vis liquids, remember

(1) Gases expand much more than liquids for same rise of temperature, e.g. nearly 24 times that of Hg.

(2) They expand also with fall in pressure unlike liquids.

(3) They are highly by compressible while liqids are nearly incompressible.

(4) *All gases expand uniformly and almost equally* for the same rise in temperature i.e. coefficient of volume expansion is the same for all of them. Liquids do not so behave at all.

IV-5 2. Gas expansion on Heating. In discussing the expansion of gases it should be remembered that the **volume** V **of a given mass of a gas depends both on its temperature** t **and pressure** P. When the temperature is constant the relation between V and P is given by Boyle's law.

To show that gas expands when heated. Fit up a round bottomed flask with a rubber cork through which one end of a narrow glass tube projects a little into the flask. Invert the flask and introduce the other end into a beaker of water as in fig IV-5.1. On gently heating the flask with a flame, air bubbles will be seen to come out. The flame heats up the air inside the flask causing expansion of air within the flask, which escapes in the form of bubbies. On allowing the flask to cool the air contracts and some water is sucked up into the tube.

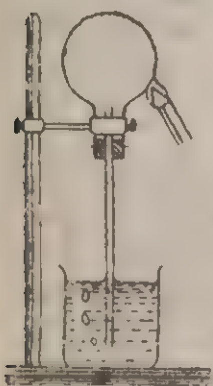


Fig. IV-5.1

The burner even is unnecessary. If you just grip the flask firmly with your palms the same will occur. You replace the air inside by other gases such as oxgen, nitrogen, hydrogen or CO_2 and repeat the procedure ; you will see the same thing

The height to which the water creeps up the tube on cooling gives you a rough measure of the expansion of the gas. Put a mark at the liquid top. *For any gas* inside the flask, if you heat by the same amount you will find the rise of water i e. *expansion is the same.*

Fig IV-4.2 shows you the first authentic thermometer recorded, the air thermometer devised by that iniator of experimental physics, Galeleo. His apparatus was the same as above, only a scale was

attached to the long stem. Note that graduations are in reverse, lower temp at the top and the higher towards the bottom. Expansion of air within the flask, pushes the indicator, the coloured water downwards (fig IV-5.2.)

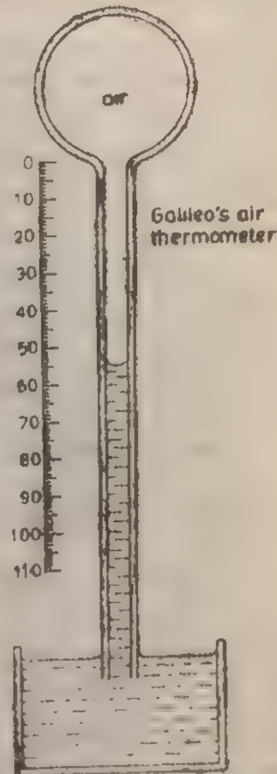


Fig. IV-5.2

IV-5.3 A. Charles' law. The relation between volume and temperature when the pressure is kept constant is the Charles' Law. **The coefficient (α_p) of increase in volume at constant pressure** is defined as the fraction of the volume at 0°C by which the volume of a given mass of gas increases for a rise in temperature of 1°C, the pressure remaining constant. If V_0=volume of a fixed mass of gas at 0°C, and V=its volume at t°C, both at the same pressure,

then $\alpha_p = \dfrac{V - V_0}{V_0 t}$ (IV-5.3.1)

which gives $V = V_0(1 + \alpha_p t)$. (IV-5.3.2)

Charles showed that *the coefficient of increase in volume of all gases at constant pressure is the same.* This statement is referred to as Charles' or Gay-Lussac's law. (Gay-Lussac discovered the law independently a few years after Charles.) α_p is often called the **volume coefficient** of expansion of gases.

According to Regnault, who measured the coefficient accurately'

$$\alpha_p = 1/273 \text{ per °C or } 0{\cdot}00366 \text{ per °C.} \qquad \text{(IV-5.3.3)}$$

In the light of Regnault's value for α_p **Charles' law** may be restated as follows :

For a fixed mass of any gas heated at constant pressure, the volume increases by 1/273 of its volume at 0°*C* for each celsius or kelvin degree rise in temperature.

B. Necessity of taking the volume at 0°C. For expansions of solids and liquids we do not start with volume at 0°C when we heat them from say t_1°C to t_2°C. For it is found that a *negligible error* is introduced thereby though for extreme accuracy one has to use the volume at 0°C. But for gases failure to do so introduces a sizeable error.

Let us take 91 cc of air at 0° and heat it to say 90°C and 150°C. We shall have.

$$V_{90}=V_0+V_0\cdot\frac{t}{273}=91\left(1+\frac{90}{273}\right)=121 \text{ cc}$$

$$\text{and}\quad V_{150}=V_0+V_0\frac{150}{273}=91+50=141 \text{ cc} \qquad \text{(A)}$$

Now if we heat V_{90} to V_{150} then we have.

$$V_{150}=V_{90}+V_{90}\cdot\frac{60}{273}=121+121\times\frac{60}{273}\simeq159 \text{ cc.} \qquad \text{(B)}$$

The volume in eqn (B) comes out much greater than in A and in the ratio 141/121 to 159/121. From this you realise that V_0 gives you a standard volume as the basis for computation. This is not required for solids ($\gamma=12\times10^{-6}$/°C for iron) or for liquids (γ for mercury $=180\times10^{-6}$/°C). For γ for a gas is 3660×10^{-6}/°C.

For comparison let us take a 3m long iron rod at 0.° Then

$$l_{100}=300(1+12\times10^{-6}\times100)=300.36 \text{ cm}$$

$$\text{and}\quad l_{150}=300(1+12\times10^{-6}\times150)=300.54 \text{ cm} \qquad \text{(C)}$$

If we now heat the rod from 100°C to 150°C we get

$$l_{150}=300.36\,(1+12\times10^{-6}\times50)=300.5402 \text{ cm} \qquad \text{(D)}$$

See how negligible is the error in (D) compared to (C)

C. Verification of Charles' Law : (1) Fig IV-5.3 shows a modern form of apparatus. B the bulb contains a mass of dry air. It

forms a part of a U-tube and graduated in cc. It is connected to a mercury reservoir R which can be raised or lowered as and when required. A current of steam may be passed through the water bath (W) surrounding the U-tube, to raise the temp of water and thereby of the air enclosed in B which can be read off from the thermometer T. The stirrer S is used to achieve same temp throughout the bath.

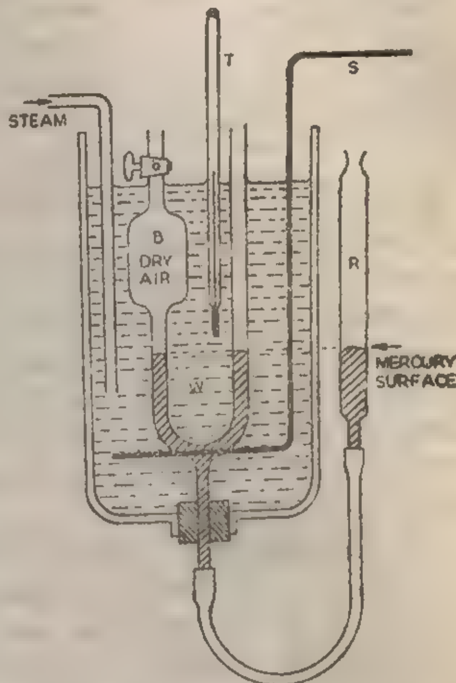


Fig IV-5.3

The room temp is read off and R is moved till the mercury levels in both arms of the U-tube and R the same. The air in B is then at the atmospheric pressure. The volume of air in B is also noted. Steam is then passed and water vigorously stirred. The mercury level in B starts lowering. When the bath temp increases by say, 10°C, mercury levels are made the same by raising R and the air volume in B read off. It is under constant pressure, the atmospheric pressure. Temp is quickly taken from T.

Several such pairs of readings are taken for temp (t) and volume V each time adjusting R so as to have the air in B under the atmosheric pressure.

V is then plotted against t and an *ascending straight line* APB is obtained (fig IV-5.4). This confirms Charles' law that the volume of a gas rises at a constant rate with rising temp when

pressure and mass of it are held constant. Extending BPA backwards we find it cutting the volume axis at C, OC giving us V_0. Now ON$=t$, PN$=V_t$ and so

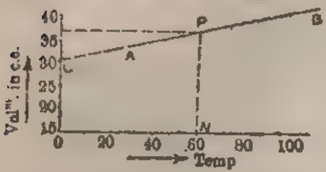


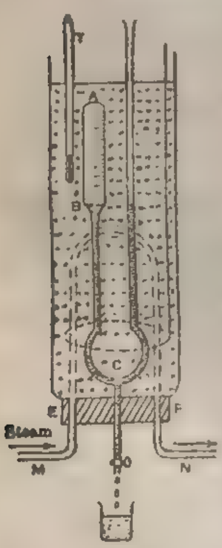
Fig. IV-5.4

$$\alpha_p = \frac{V_t - V_0}{t} = \frac{PN - OC}{ON}$$

You would find the value to be nearly 0.00366 or 1/273.

A bonus from the graph is provided, if you extend (i.e. extrapolate) BPAC further back, it will cut the temp-axis at a value of – 273 °C. This means that at that temp the volume of the gas disappears. This temp is said to be the **absolute zero** for it is absurd to think that simply by cooling a given mass of gas, it would disappear. The idea is elaborated in the next article. Before this temp is reached the gas will turn liquid and then solid thus no longer would obey the gas laws.

(2) The apparatus even now used in the laboratory for verifying Charles' Law is shown in fig IV-5.5(a). Instead of using the costly mercury, conc. H_2SO_4 is used as the indicator. As and when the need arises the acid is run out by opening the tap until the height of the liquid in both the arms are equalised. Incidentally H_2SO_4 being hygroscopic serves to keep the gas in the bulb dry. The experimental procedure is the same as above.



(3) A very simple but crude piece of apparatus to find α_p *directly* is shown in fig

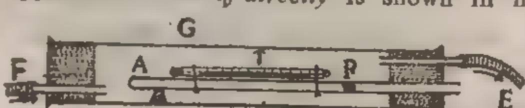


Fig. IV-5.5(a)

Fig. IV-5.5(b)

IV-5.5(b). A capillary tube about 50 cm long and 1 mm in bore is sealed at one end *after* a small pellet of mercury (P) or

coloured water is drawn into it. So that a definite mass of air is trapped inside at atmospheric pressure. The capillary is tied to a thermometer (T) and scale (not shown) and mounted horizontally in a wider glass tube closed at the two ends by corks. The corks have holes for an inlet (F) and outlet pipes (E) and also for the open end of the capillary to project out of the tube.

First ice-cold water is circulated through the tube and when the pellet reaches a steady position the readings of the sealed end of the capillary and the left end of the pellet are taken. The difference l_0 multiplied by S, the cross-section of the capillary gives the volume of trapped air at ice point. Steam is now passed through and a pair of similar readings are taken when things steady up, giving $l_{100}S$ as the volume of trapped air at the steam point. Both readings are taken under constant atmospheric pressure. Then,

$$\alpha_p = \frac{V_{100} - V_0}{V_0 \times 100} = \frac{(l_{100} - l_0)S}{l_0 S \times 100} = \frac{l_{100} - l_0}{100 l_0}$$

IV-5.4. The Absolute Scale of temperature : Since for a gas $\alpha_p = 1/273$ per °C, its volume V_t at any temperature t°C is given by

$$V_t = V_0(1 + t/273)$$

Thus the volume at 1°C is $V_1 = V_0(1 + 1/273)$,

the volume at -30°C is $V_{-30} = V_0(1 - 30/273)$ etc.

If α_p remains the same whatever be the temperature, then at a temperature of -273°C,

$$V = V_0(1 - 273/273) = 0 \qquad \text{(IV-5.4.1)}$$

At a temperature below -273°C the volume would be *negative.* Since we cannot imagine a negative volume, temperatures lower than -273° will have no sense to us. We therefore take the temperature at which the *calculated* volume of a gas becomes zero, to be the lowest temperature possible and call this temperature the **absolute zero.** Any scale of temperature measured from the absolute zero is called an **absolute scale of temparatnre.** On the

celsius (centigrade) scale, the absolute zero will be taken as $-273°C$. If will be $-459°F$ in the fahrenite scale.

The above is not however, a happy way of defining the absolute zero. When you reach a higher stage of study you will learn that *Nature has put a limit to the lowest temperature possible in the universe. This limit is the absolute zero.* Careful experiments show that the value is about $-273{\cdot}15°C$. The agreement of this value with the one at which the calculated volume of a gas becomes zero is *accidental* and is due to the fact that most permanent gases (such as H_2, He) behave practically as perfect gases (i.e., satisfy Boyle's law and Charles' law with a high degree of accuracy.)

The absolute scale of temperature in which there are 100 degrees between the ice point (0°C) and the steam point (100°C), i.e., the absolute scale in which each degree *interval* is equal to a degree interval on the celsius scale, is called the **Kelvin scale**. The readings on this scale were indicated as degrees Kelvin or in symbols as °K. Now we write merely *Kelvin*, with K as the symbol. The degree sign has been by international agreement dropped. It should be noted that

a difference of temperature of 1 K (or 1 °K)

=a difference of temperature of 1°C

For ordinary purposes, we take the absolute zero on the celsius scale as $-273°C$, the ice point on the Kelvin scale as 273K and any temperature $t°C$ as $(273+t)$ K, i.e.

$$t°C=(273+t)\text{ K}. \tag{IV-5.4.2}$$

The kelvin is now the SI unit of temperature just as the meter is the unit of length.

IV-5 5. **Another form of Charles' law.** From Charles' law, we have for a given mass of gas at constant pressure,

$$V=V_0(1+t/273)=V_0(273+t)/273 \tag{IV-5.5.1}$$

where V is the volume of the given mass at $t°C$ and V_0 that at 0°C.

Now t°C$=(t+273)$ K$=T$ K (say), while 0°C$=273$ K$=T_0$ K (say).

$\therefore$ From Eq. IV-5.5.1 $V=T(V_0/T_0)$

$$\text{or} \quad \frac{V}{T}=\frac{V_0}{T_0} \qquad \text{(IV-5.5.2)}$$

Since V_0 has a fixed value for a given mass, and T_0 is a constant, V_0/T_0 is also constant. We can therefore say that for a given mass of gas V/T is a constant, *i.e.*, *at constant pressure the volume of a given mass of gas is directly proportional to its absolute temperature.* This assertion may also be taken as the statement of Charles' law. In symbols, $V \propto T$ when P is constant.

Strictly speaking, the temperature T as defined here is called the temperature on the **perfect gas scale,** or briefly the *gas scale.* The kelvin scale is defined in Thermodynamics. The definition is different from the one given here. But the perfect gas scale and the kelvin scale have been found to be *identical.* So we need not differentiate between the kelvin scale and the perfect gas scale at this stage.

IV 5.6 **Boyle's law.** A. Gases are highly compressible. Consider a mass of gas confined in a cylinder by an air-tight piston. If the pressure on the gas is increased by pushing the piston inwards, the volume of the gas decreases. As the pressure is reduced, the volume increases.

This relation between the pressure (P) and volume (V) of a fixed mass of gas was discovered by Robert Boyle (1627-1691), an English philosopher. This is **Boyle's law** and may be stated as follows :

For a fixed mass of gas at a given temperature, the volume varies inversely as the pressure, *i.e.*, the product of the pressure and volume is constant.

In symbols, $\quad PV=\text{constant}=K \qquad$ (IV-5.6.1)

If P_1 and V_1 are the initial values of the pressure and volume and P_2 and V_2 the final values, we shall have.

$$P_1V_1=P_2V_2=K. \qquad \text{(IV-5.6.2)}$$

Example IV-5.1. The volume of a mass of gas at 740 mm pressure is 1250 cm^3. Find its volume at 760 mm, if the temperature is unchanged.

Solution : Let V be the new volume. Then from the relation $P_1V_1 = P_2V_2$ we have $V \times 760\ mm = 740\ mm \times 1250\ cm^3$ or $V = 1217\ cm^3$.

Ex. IV-5.2. A bubble of gas, 100 mm^3 in volume, is formed at a depth of 100 metres of water. Find its volume when it reaches the surface, the atmospheric pressure being 76 cm. Assume unchanged temperature.

Solution : Atmospheric preesure $= 76$ cm of mercury $= 76 \times 13.6$ cm of water $= 10.34$ metres of water. This is the final pressure P_2 of the bubble. The initial pressure $P_1 = (100 + 10.34) = 110.34$ metres of water. If V is the final volume, then $V \times 10.34$ meters of water $= 100\ mm^3 \times 110.34$ metres of water, whence $V = 1067\ mm^3$.

B. The **Value of the constant K** in the above equations depends upon the following factors—

(*i*) the **mass** of the gas taken, (*ii*) its **temperature** and (*iii*) the **units** in which P and V are expressed.

Ex IV.5.3. At a pressure of one atmosphere 32g of oxygen occupy a volume of 22.4 litres when the temperature is 0°C. Therefore for 32 g of oxygen at 0°C the product is

$PV = K = 22.4$ litre $\times$ 1 atmosphere $= 22.4$ litre-atmospheres.

Since 1 litre $= 1000\ cm^3$ and one atmosphere $= 1.013 \times 10^6$ dyn/cm^2, the same product is also equal to.

$22.4 \times 1000\ cm^3 \times 1.013 \times 10^6\ dyn/cm^2 = 22.69 \times 10^9$ erg.

Ex IV-5.4. If we consider 16g of oxygen at 0°C, the volume is 11.2 litres at a pressure of 1 atmosphere.

$\therefore$ for 16g of oxygen at 0°C the product

$PV = K = 11.2$ litre-atmospheres $= 11.35 \times 10^9$ erg.

Ex IV-5.5. At 27°C the volume of 16g of oxyen is 12.3 litres

if the pressure is 1 atmosphere. Under these conditions the product $PV = K = 12.3$ litre - atmospheres $= 12.46 \times 10^{9}$ erg.

At a given temperature, the product

$PV = K$ is directly proportional to the gas mass taken.

C. **Density and pressure.** Let ρ_1 and ρ_2 be the densities of a gas at pressures P_1 and P_2, and V_1 and V_2, the volumes of the same mass m of the gas at these pressures, the temperature remaining constant. Then

$m = V_1\rho_1 = V_2\rho_2$. By Boyle's law $P_1V_1 = P_2V_2$

Dividing the first relation by the second, we have

$$P_1/\rho_1 = P_2/\rho_2 \qquad \text{(IV-5.6.3)}$$

which means that *the density of a gas is proportional to its pressure, so long as the temperature remains constant.*

Ex IV-5.6. Find the mass of 1 cubic metre of air at 0°C and 19 *torr* pressure, given that the density of air at S.T.P. is 0.001293 g/cm³.

Solution : If ρ is the density at 19 mm Hg pressure, then

$$\frac{\rho}{19 \text{ mm Hg}} = \frac{0{\cdot}001293\text{g/cm}^3}{760 \text{ mm Hg}}$$

or $\rho = \frac{19}{760} \times 0{\cdot}001293\text{g/cm}^3$

$\therefore$ mass of 1cu. metre $= \frac{0{\cdot}001293}{760} \times 19 \frac{\text{g}}{\text{cm}^3} \times (100)^3 \text{ cm}^3 = 32\text{g}$

IV-5.7. A. Regnault's Law : Increase of pressure with rise of temperature at constant volume. If the volume remains constant, it follows from this law that $P_2 = P_1(1 + \alpha_v t)$ (IV-5.7.1)

from which we get $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ (IV-5.7.2)

i.e., the pressure of a perfect gas at constant volume is directly proportional to its absolute temperature. This shows that when a

given mass of gas at constant volume is heated, its pressure, increases. Experiment shows that *at constant volume the pressure of a given mass of gas increases by a constant fraction of its pressure at* 0°C *for each degree celsius rise of temperature.* This fraction is called the **pressure coefficient** (α_v) of the gas *at constant volume.*

If P_0=pressure of a given mass of gas at 0°C,

P=pressure at temperature t°C,

then the fractional increase of pressure=$(P-P_0)/P_0$, and the fractional increase of pressure per °C rise of temperature (i.e. α_v)

$$\alpha_v=(P-P_0)/P_0t \quad \text{or} \quad P=P_0(1+\alpha_v t)$$

It is further found from experiment that *α_v is approximately equal to* 1/273 *per °C for practically all gases.*

Ex IV-5.7. A gas is at 27°C. To what temperature must it be heated at constant volume so that the pressure is doubled ?

Solution : Let the initial pressure be P_1. The initial temperature is $T_1=27+273=300$ K. The final pressure $P_2=2P_1$; the final temperature T_2 in K. Therefore,

$P_1/300=2P_1/T_2$, whence $T_2=600\text{K}=600-273=327$°C.

B **To show that for a perfect gas the pressure and volume coefficients are equal ($\alpha_v=\alpha_p$).** Perfect gases satisfy the relation PV/T=constant. It may be shown that α_p and α_v for such gases are the same.

Suppose a fixed mass of a perfect gas has a volume V_0 and a a pressure P_0 at 0°C. If the pressure be kept constant while the temp changed to t°C, its volume V will be $V=V_0(1+\alpha_p t)$. Now keep the temperature consant at t°C. but increase the presure from P_0 to P so that the volume V diminishes and reaches the former value V_0. Then from Boyle's law we get $P_0V=PV_0$ (the temperature being t°C).

Hence, we shall have $P_0V_0(1+\alpha_p t=PV_0$

$$\text{or,} \qquad P=P_0(1+\alpha_p t) \tag{A}$$

The changes described above are such that the new temperature is t°C and the pressure is P, while the volume is unchanged. As the

pressure rises from P_0 at 0°C to P at t°C at constant volume, we have, from the definition of α_v,

$$P = P_0(1+\alpha_v t) \qquad \text{(B)}$$

Hence, from the relations (A) and (B) we find $\alpha_v = \alpha_p$. (IV-5.7.3)

Values of α_p and α_v for some gases are given in the table below. They are per °C. Values for some of them are shown graphically in fig IV-5.4.

Gas	α_p	α_v
Air	0.00367	0.00367
H_2	·00366	.00366
N_2	·00367	.00367
O_2	.00367	.00367
He	.00366	·00366
CO_2	·00374	·00372

As the intial pressure is reduced it is found that the values of α_p and α_v for all gases (even of CO_2) approach the same value. This *limiting value* is found to be 0.0036608 = 1/273·15 per°C.

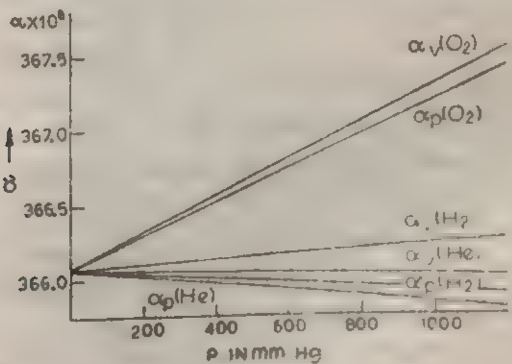


Fig. IV-5.5

C. **Verification of Regnault's Law and finding α_v**

The apparatus used for the purpose may be called a *constant volume gas thermometer*. It consists of a bulb B (fig. IV-5.6), connected by of a capillary tube to a wider glass tube A,

C is another glass tube of a relatively wide bore, open at both ends and serves as a mercury reservoir. A and C are connected together by a thick walled India-rubber tube, and may be raised or lowered on a vertical stand to which they may be clamped. By raising or lowering C, the level of mercury in A may always be brought upto a certain mark M near the junction of A and the capillary tube connecting it to B. This ensures that the experimental gas contained in B always has the same volume. A scale is fixed to the stand in between A and D, from which the difference between the mercury levels in A and D may be read off.

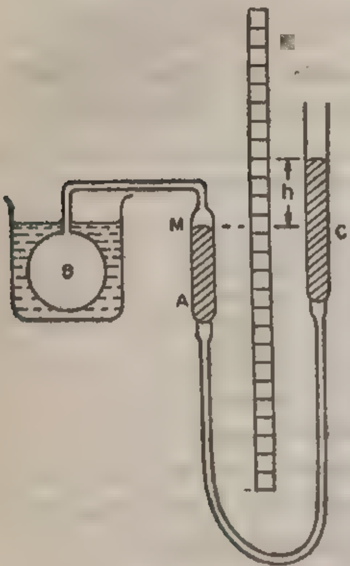


Fig. IV-5.6

To find the pressure coefficient α_v of the experimental gas in B at constant volumes, B is placed in a water bath whose temperature can be read off from a thermometer. Mercury level in A is brought upto M, and the height of the mercury level in D above that in A is noted. Let this value be h_1 cm. If P_0 is the atmospheric pressure in centimeters of mercury at the time of the experiment, the pressure of the gas in B is $P_0+h_1=H_1$ cm of mercury. Its temperature$=t_1$°C is that of the bath.

If the bath temperature be raised to a value t_2°C (about 8 to 10°C higher than t_1) and held constant there the gas in B expands and pushes the mercury column down A. C is raised till the mercury level in A comes upto C again. The difference in height between the mercury levels in A and C is noted as before. Let the value be h_2 cm. The gas pressure in B is $P_0+h_2=P_2$ cm. of mercury.

If H_0 is the pressure of the gas in B at 0°C,

Then $P_1 = P_0(1+\alpha_v t_1)$ and $P_2 = P_0(1+\alpha_v t_2)$

$$\text{or} \qquad \frac{P_1}{P_2} = \frac{1+\alpha_v t_1}{1+\alpha_v t_2}$$

$$\text{whence} \qquad \alpha_v = \frac{P_2 - P_1}{P_1 t_2 - P_2 t_1} \qquad \text{(IV-5.7.4)}$$

Alternatively, the bath temperature may be held constant at different values. The mercury level in A is brought upto M each time and the corresponding difference in height of the mercury levels in A and C noted. The total gas pressure in B may then be plotted against the temperature and P_0 found by extrapolation from the graph (fig. IV-5.7). Pressure in B at any temperature t°C,

$$P = P_0(1+\alpha_v t)$$

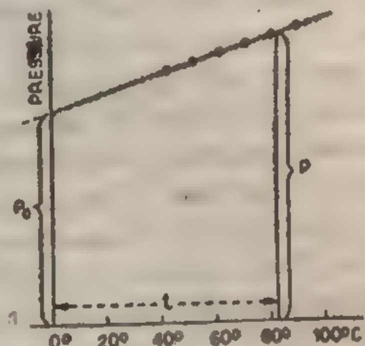


Fig. IV-5.7

from which α_v may be found out. It will be seen that α_v is approximately $\frac{1}{273}$ per °C for nearly all gases.

IV-5.8. A. Ideal or Perfect gas equation. A gas which obeys both Boyle's law and Charles' law strictly is called a *perfect or ideal gas.* Permanent gases (such as hydrogen, helium, oxygen and nitrogen) obey the laws with sufficient accuracy to be treated as perfect gases. These two laws may be combined to give a single relation.

Let the initial pressure, volume and absolute temperature of a given mass of gas be P_1, V_1 and T_1.

(i) Keeping the pressure P_1 contant, let the absolute temperature be changed to T_2, and the new volume to V'. According to Charles' law $V' = V_1 T_2 / T_1$.

(ii) Keeping the temperature T_2 constant, let the pressure be changed to P_2. Then, according to Boyle's law, the new volume V_2 is such that

$$P_2V_2=P_1V'=\frac{P_1V_1}{T_1}T_2$$

$$\text{or}\quad \frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}=\text{a constant} \qquad \text{(IV-5.8.1)}$$

This is the equation of state* for an ideal gas, and is also called the *ideal or perfect gas equation.* It contains both Charles' law and Boyle's law. For if $T_1=T_2$, we get $P_1V_1=P_2V_2$, which is Boyle's law, and if $P_1=P_2$, we get $V_1/T_1=V_2/T_2$, which is Charles' law.

We may write Eq IV-5.8.1 as **PV=RT** (IV-5.8.2) in which P stands for the pressure, V for the volume and T for the absolute temperature of a *given mass* of gas. The value of R will depend on the mass. If the mass is doubled, R will also be doubld. For a gram-molecule or **mole** of *any gas*, R has the same value. This value of R (for a mole) is variously called the **universal gas constant, gram-molecular gas constant** or **molar gas constant.** We shall denote it by R_M.

B. **Value of the molar gas constant.** Let us consider a mole of a gas at S.T.P. (standard temperature and pressure). Then we have

$P_0=1$ standard atmosphere$=1{\cdot}013\times10^6$ dyn/cm^2,

$T_0=$temperature of melting ice$=273$ K

$V_0=$volume at S.T.P. of one mole of a gas$=22{\cdot}4$ litres.

$$\therefore\quad R_M=\frac{P_0V_0}{T_0}=\frac{1.013\times10^6\ \text{dyn/cm}^2\times22400\ \text{cm}^3/\text{mole}}{273\ \text{K}}$$

$$=8{\cdot}31\times10^7\ \text{erg per K per mole (erg K}^{-1}\text{ mole}^{-1})$$

If the mass of the gas taken is n moles, then $PV=nR_MT$.

If m is the mass of the gas in grams and M its molecular weight, then

$$PV=mR_MT/M. \qquad \text{(IV-5.8.3.)}$$

for unit mass, therefore, $R=R_M/M$. (IV-5.8.4)

---

*The relation between the volume, temperature and pressure of a substance is called its Equation of state.

For different masses of the same gas, we get from Eq.IV-5.8.3

$$\frac{P_1V_1}{m_1T_1}=\frac{P_2V_2}{m_2T_2}=\frac{R_M}{M}=R \qquad \text{(IV-5.8.5)}$$

Eqns IV-5.8.1 and 2 enable us to solve problems when any two of the quantities pressure, volume and temperature of a fixed mass of gas change at the same time. Eq. IV-5.8.5 is useful when masses differ (see later Sec. IV-5.10).

Ex. IV-5.8. A litre of air is heated from 27°C to 177°C at constant pressure. Find its volume.

Solution : From Charles' law that $\frac{V_1}{T_1}=\frac{V_2}{T_2}$,

$T_1=273+27=300$ K, $T_2=273+177=450$ K,

$V_1=1$ litre, whence $V_2=1.5$ litres.

Ex IV-5.9. The pressure of a given mass of gas at 33°C is 75 cm of mercury. Find the temperature in °C at which the pressure is doubled, the gas being heated at constant volume. Find also the tempeature at which the pressure is halved.

Solution : Apply equation IV-5.8.1 Since volume remains constant $V_1=V_2$.

(i) $T_1=273+33=306$ K. If $P_2=2P_1$,

$T_2=P_2T_1/P_1=2\times 306$ K$=612$ K$=(612-273)$°C$=339$°C.

(ii) When $P_2=\frac{1}{2}P_1$, $T_2=(P_2/P_1)\times 306$ K$=153$ K

$=(153-273)$°C$=-120$°C.

(Note that since the ratio P_2/P_1 is given, the value of P_1 is unnecessary.)

Ex IV-5.10. The volume of a mass of gas at 47°C and a pressure of 75 cm. of mercury is 640 cm³. Find the volume at S.T.P.

Solution : Here $P_1=75$ cm of mercury, $V_1=640$ cm^3, $T_1=273+47=320$K, $P_2=76$ cm of mercury, $T_2=273$K ; to find V_2.

$\therefore$ From the relation $\dfrac{P_1V_1}{T_1}=\dfrac{P_2V_2}{T_2}$, $V_2=\dfrac{P_1}{P_2}\cdot\dfrac{T_2}{T_1}\cdot V_1$

$$\therefore \quad V_2=\frac{75}{76}\times\frac{273}{320}\times 640 \text{ cm}^3=539 \text{ cm}^3 \text{ (approximately)}$$

Ex. IV-5.11. **Calculate the value of the gas constant for 1g of hydrogen, given that the density of hydrogen at S.T. P.=0·00009 g/cm³.**

Solution : Here $P=76\times 13.6\times 980=1.013\times 10^6$ dyn/cm².

$$V=\frac{1}{0\cdot 00009}\frac{\text{cm}^3}{\text{g}}\;;\; T=273 \text{ K}.$$

$$\therefore \text{ The required value}=\frac{PV}{T}=\frac{1\cdot 013\times 10^6 \text{ dyn}}{273\times 0.00009 \text{ cm}^2}\cdot\frac{\text{cm}^3}{\text{g}}\cdot\frac{1}{\text{K}}$$

$$=4\cdot 12\times 10^7 \text{ erg per K per g (or erg K}^{-1}\text{g}^{-1}).$$

IV-5 9. Conversion of densities from one set of conditions to another. Since the gas equation applies to a given mass, say *m*g of a gas, the density $\rho_1=m/V_1$. Under the new set of conditions $\rho_2=m/V_2$. Then from the relation.

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}, \text{ we get } \frac{P_1m}{\rho_1T_1}=\frac{P_2m}{\rho_2T_2}$$

$$\text{or, } \frac{P_1}{\rho_1T_1}=\frac{P_2}{\rho_1T_2} \text{ i.e., } \frac{P}{\rho T}=\text{Constant} \tag{IV-5.9.1}$$

Ex IV-5.12. A litre of dry air at S.T.P. weighs 1.293g. What would be the mass of 3 litres of dry air at 115°C and a pressure of 4 atmospheres ?

Solution : Here $P_1=1$ atmosphere, $T_1=273$°K, $\rho_1=1.293$ g/litre and $P_2=4$ atmospheres, $T_2=273+115=388$°K. First find ρ_2.

From Eq. IV-5.9.1 $\rho_2=\dfrac{P_2}{P_1}\cdot\dfrac{T_1}{T_2}\cdot\rho_1=4\times\dfrac{273}{388}\times 1.293$ g/litre

$$\therefore \quad \text{mass}=V_2\rho_2=3\times 4\times\frac{273}{388}\times 1.293 \text{ g}=10.9\text{g}.$$

Or, the volume might first be reduced to S.T.P. Let V be the volume of the given mass of air at S.T.P. Since

$$\frac{PV}{T}=\frac{P_0V_0}{T_0}, \text{ we shall have } \frac{1\times V}{273}=\frac{4\times 3}{388} \text{ or } V=\frac{12\times 273}{388} \text{ litre,}$$

whence mass$=V\times 1.293=10.9$ g.

IV-5.10. Gas law for different masses Let M be the molecular weight of a gas and R_M the value of the molar gas constant (IV-5.8.B) Then the value of the gas constant for unit mass will be R_M/M. If V_1 is the volume of a mass m_1 of the gas at pressure P_1 and temperature T_1 and V_2 the volume of a mass m_2 of the same gas at P_2 and T_2, then from Eq. IV-5.8.5 we shall have

$$\frac{P_1V_1}{T_1}=\frac{m_1R_M}{M} \quad \text{or} \quad \frac{P_1V_1}{m_1T_1}=\frac{R_M}{M}$$

$$\text{and} \quad \frac{P_2V_2}{T_2}=\frac{m_2R_M}{M} \quad \text{or} \quad \frac{P_2V_2}{m_2T_2}=\frac{R_M}{M}$$

$$\therefore \quad \frac{P_1V_1}{m_1T_1}=\frac{P_2V_2}{m_2T_2} \qquad \text{(IV-5.10.1)}$$

This equation (the same as Eq. IV-5.8.5) may be applied to cases where the masses the of gas are different. Note that this is also the same as Eq. IV-5.9.1 for $m_1/V_1=\rho_1$ and $m_2/V_2=\rho_2$.

Example IV-5.10.1. A gas cylinder contains 20 kg of air at a pressure of 12 atmospheres. In driving a pneumatic drill for some time, the pressure falls to 10 atmospheres. How much gas has been used?

Solution : We may apply Eq IV-5.10.1. Here $V_1=V_2$, $T_1=T_2$, $P_1=12$ atmos, $P_2=10$ atmos and $m_1=20$ kg. To find m_2.

From Eq. IV-5.10.1. $\frac{12\text{ atmos}}{20\text{ kg}}=\frac{10\text{ atmos}}{m_2}$ or $m_2=\frac{201}{12}=16\frac{2}{3}$ kg

The quantity of gas used$=m_1-m_2=3\frac{1}{3}$ kg.

KINETIC THEORY OF GASES

IV-6.1. **Evidence of molecular structure of matter.** In science, an *evidence* of a theory is rarely direct, e.g. Newton's laws of motion. But when things occur without exception as deduced from the theory, we take it as an evidence of the correctness of the theory. Most evidences in science are of such indirect nature. But science *accepts* such evidence.

Evidence of molecular structure. At least three thousand years ago many Indian and Greek philosophers Kanad and Democritus amongst them, thought that matter was not continuous in structure but made of particles ; but they had no proof to support it. In the 18th century, John Dalton, an English school teacher, successfully explanined the laws of chemical combination assuming that a given substance (such as iron or sulphur) consisted of particles, all of the same kind. He named such particles 'atoms'. * Combinations of atoms of the same or different kinds were later given the name 'molecules. Dalton's laws for the formation of molecules from atoms were fully supported by chemical experiments. This is the first evidence of molecular structure. It came from chemistry and was of an indirect nature.

With progress of science, we could explain various phenomena from different fields on the basis that matter was molecular in structure. We came to know that molecules are so small that we never have any chance of *seeing* them. They have diameters of the order of 10^{-7} cm. The most powerful optical microscope can magnify a small body to about 2000 times. This is not enough to *see* a molecule. Another kind of microscope, called the '*electron microscope*' can magnify upto 10^{5} times. Even this is not

* From *a*—not, *tomos*—I can divide. i.e, atoms are indivisible.

enough. But the pictures of very thin sections of matter that we get from powerful electron microscopes show that matter has a granular (grain-like) structure. Another kind of microscope, known as the '*Field emission microscope*', produces pictures of very sharp metal points. These pictures show that the point is made up of layers of granular particles. These give the nearest visual proof till now that matter has a discontinuons structure and is made up of particles. The granular structures as seen in the pictures mentioned above are not molecules ; they are *groups* of molecules.

We however have enough *indirect evidence* of molecular structure. If alpha particles from radio-active materials shoot through very thin metal foils, some of them are found to be deflected very much from their orginal paths. This is possible only if the metal foils are made up of atoms whose positive charges are confined to very small regions. You will have more of indirect evidences as you learn more of modern physics. Radio-activity will give you much of such evidence. With X-rays we have learnt the sizes of molecules and atoms. The 'mass spectrograph' enables us to find the relative masses of atoms accurately.

IV 6.2. **Evidence of random molcular motion.** (*i*) A gas exerts the *same pressure* all over its containing walls. This supports the view that gas molecules move about at random. Put the same amount of gas in a larger vessel. It will fill that up. This *expansibility* of a gas results from randam molecular motion.

(*ii*) If we drop a crystal of copper sulphate in water, the crystal soon dissolves and its blue colour gradually spreads throughout the water. This can easily be explained, if we assume that copper sulphate crystals are made up of molecules, and that in solution these molecules move about at random. This is the phenomenon of *diffusion*, explicable only by molecular motion.

(*iii*) When water (or any liquid) *evaporates* its molecules are lost in all directions in the atmosphere. They must have random motion ; otherwise evaporation should have occurred in some

special direction only. The same happens for boiling. Evaporated liquid exerts *vapour pressure.*

(*iv*) When a solid melts, the liquid that is formed, takes up the shape of the container. This can happen only if solid is made up of molecules and these molecules move about at random in the liquid state.

(*v*) Suspend very fine particles of diameters about $10^{-3} - 10^{-4}$ cm, in water or air. Look at them through a powerful microscope. They will be found to move about at random with a zig-zag motion (fig. IV-6.1.) [See 'Brownian motion' in the next section.] These particles are not molecules, but are packets of about 10^{6} to 10^{12} molecules. Their zig-zag motion can be explained if we assume that they are being constantly hit by water or air molecules from all sides. When the particles are small, the number of hits from one side may not be equal to those from the opposite side. So we may expect to see resultant motion which may change abruptly now and then, and that is what we actually see in the Brownian motion. Besides, the water or air molecules may not all move with same speed and they may be moving in all possible directions. Such property of the molecules can fully explain the behaviour of the suspended particles as we find it. This is the most direct evidence of the *continuous* and *random* motion of molecules. (The motion of the particles is somewhat similar to the movement of a man who carries a heavy load and is moving in a crowd ; or of a thief chased by a policeman in a startled crowd).

Fig. IV-6.1

IV-6.3. Brownian movement. The phenomenon of Brownian motion was discovered in 1827 by an English botanist, Robert

Brown. He observed that minute pollen grains suspended in a liquid drop showed a constant, irregular motion when observed through a powerful microscope. They danced about madly in the field of view of the microscope without any rhyme and reason. The movements continue indefinitely and never show any sign of stopping or even diminishing. These movements are known as **Brownian motion**. Suspensions of solid particles or droplets (10^{-5}—10^{-4} cm in diameter) in liquids and gases also show these movements, as in fig IV-6,1, for 3 grains in water at 30s intervals.

The **following fact** gradually came to be established about Brownian motion :

(*i*) The motions are continuous, eternal, and completely irregular and random. No two particles in the same locality move in the same direction at the same moment.

(*ii*) The movements are independent of any mechanical vibration of the container.

(*iii*) The lower the viscosity of the liquid the greater the motion.

(*iv*) The smaller the particles the greater the motion.

(*v*) Two particles of the same size move equally fast at the same temperature.

(*vi*) The higher the temperature, more animated the motion.

From these facts the following picture of the processes generating the motions were drawn. The suspended particles are millions of times larger than the molecules of the liquid. The liquid molecules have their heat i.e, thermal motion. So they continuously bombard the particles from all sides. If the particles are large in size, equal number of hit them *on the average* on all sides every moment. But if they are small enough, the molecular impacts may not be balanced. A small particle may therefore be acted upon by an unbalanced force, producing a perceptible motion. (This motion however is opposed by the viscous drag of the liquid.) Since the molecular impacts are at random, the unbalanced force on the particle is randomly directed. This gives

the particle the motion described. Such particles behave like large molecules in the liquid. They take part in the thermal motion of the liquid and provide a visual proof of the basic assumption of continuous and eternal heat motions of molecules. This picture also explains why the motion is more vigorous at high temp and low viscosity.

Estimation of molecular size. How big is a molecule ? A general answer to the question is not possible, because sizes vary widely. Organic molecules may be quite big in size compared with inorganic molecules. In some cases however, it is possible to get an idea of the size of molecules by forming films on water. If a very small quantity of matter forming the film is taken and allowed to spread on a wide enough surface of water, the material forms a film which is only one molecule thick. Such films are called **monomolecular** films.

Some oils and fatty acids have this property. Their molecules are much longer than broad. One end of each molecule has an affinity for water. Such molecules attach the active end to the water and stand erect on it, forming a monomolecular film.

Oleic acid is a suitable material for this purpose. We may prepare a 0·5% solution of it in methyl alcohol by volume i.e. dissolving 0·5 cc of the acid in 100 cc of the alcohol.) We then have 0.0005 cm^3 of the acid in 1 cm^3 of the solution.

We may then take a large glass or plastic dish full of watter and lightly sprinkle lycopodium powder on the surface of water. Using a capillary pipette we may then drop 0·005 cm^3 of the solution in the centre of the water surface. The solution spreads over an area of the suface and pushes the lycopodium powder away from the area. The alcohol soon partly dissolves and partly evaporates, leaving a monomolecular film of oleic acid on water as a nearly circular patch. We then measure the diameter D of the patch.

In the film we have $(0{\cdot}005)^2$ cm^3 of oleic acid. If b is the

length of each molecule, we can treat the film as a thin cylinder of height b and diameter D. Its volume is then $\frac{1}{4}\pi D^2 b$. This is equal $(0{\cdot}005)^2$ cm^3. Measuring D and putting $\frac{1}{4}\pi D^2 b = (0{\cdot}005)^2$ cm^3, we can find b, the length of an oleic acid molecule. The value comes out to be about 10nm (nanometre $= 10^{-9}$m).

We have made a similar calculation in Chap II-2.4

IV-6.4 The Kinetic Theory of Ideal gases. The behaviour of gases as unfolded in the last chapter makes no assumptions as to the structure of gases. They have been treated as amorphous, i.e. continuos entities. Gas laws have been developed from experimental results. This is said to be the *macroscopic* viewpoint for gases, or we might say the *thermodynamic*, depending on the *four variables*, pressure, temperature, volume and mass.

The **Kinetic theory** provides an alternative viewpoint, the *microscopic*, treating of the gas, as an assembly of a very large number of fast molecules in random motion. The theory has succeded remarkably not only qualitatively but also *quantitatively*, in deducing the gas laws, the Avogadro hypothesis, Graham's law of diffusion, coefficients of conductivity and viscosity of gases and a host of others. But its development requires a number of simplifying assumptions. The gases we have seen before, have the simplest structure and the ideal gas is the simplest to treat mathematically. These have been carried out by Clausius, Maxwell and Boltzmann amongst others. Daniel Bernoullie and Robert Hooke. had much earlier initiated the simple calculations.

Basic assumptions. (1) The molecules of a gas are all alike and behave as if they were hard, smooth, *perfectly elastic* spheres. Even in a small volume their number is very large. (In air at S.T.P. 1 cm^3 contains $2{\cdot}7 \times 10^{19}$ molecules).

(2) The molecules are in *continual, random* motion, (fig IV-6.2) colliding with one another and with the walls of the containing vessel. The collisions are taken to be *perfectly elastic*.

(3) Molecules do not exert any appreciable attraction on one another, nor repulsion.

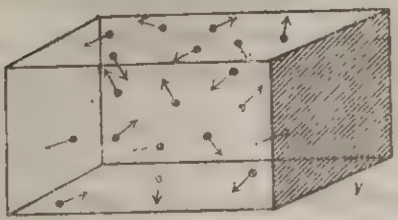


Fig. IV-6.2

(4) The actual volume occupied by the molecules is negligible compared with the total volume of the vessel.

(5) In between two successive collisions the molecules travel in accordance with Newton's laws in straight lines with a constant speed. In the most elementary calculation this speed is taken to be the same for all molecules of a gas at a given temperature. (The speeds are really different.)

(6) The time spent in collision is negligible compared with the time of travel between collisions.

IV-6.5. A. Explanation of pressure of a gas on the Kinetic theory. The molecules of a gas in their random motion constantly collide with and rebound from the walls of the container. A molecule of mass m and speed u colliding perpendicularly with a wall (fig IV-6.3) will rebound from it with speed $-u$. The change in momentum of the molecule is $mu-(-mu)=2mu$. The wall receives at each collision an impulse equal to this change in momentum. If during one second a large number of such impacts occur, the average force on the walls, by Newton's second law, is equal to the total change in momentum during that second. The pressure developed, is the force acting on unit area of the wall.

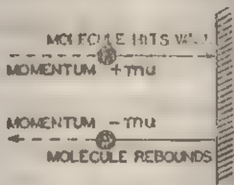


Fig. IV-6.3

B. *Calculation of Pressure. Let us have N molecules in a closed vessel which for sake of convenience, be taken as a cube of side l. Any molecule may have a velocity c in anly random direction

* Not in syllabus

which has components u, v, w along three perpendicular directions (fig IV-6.4) such that $c^2=u^2+v^2+w^2$

Considering the face A_1, we have just now seen that change in momentum for the *normal impact* of a single molecule is $2mu$. Let the molecule now hit the opposite face A_2 assuming that it suffers no collision on the way. From there let it return to A_1 and again suffer an impact; the time interval between its successive hits on A_1 must be $2l/u$. Then the force it applies on A_1 is

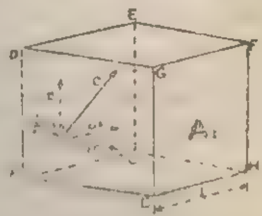


Fig. IV-6.4

$$\frac{\text{change in momentum}}{\text{time-interval}}=\frac{2mu}{2l/u}=\frac{mu^2}{l}$$

and the pressure exerted $=$ Force/Area $=mu^2/l^3$ (IV-6.5.1)

Now each of the N molecules may be considered to have different c's and hence different u-values so that we may write the total pressure on A_1 as

$$p_{ox}=\frac{m}{l^3}\left(u_1{}^2+u_2^2+u_3{}^2+\cdots+u_n^2\right)$$

Let $\bar{u}^2$ reperesent the mean value of all the squares of velocity components along the OX direction i.e.

$$\bar{u}^2=\frac{u_1^2+u_2^2+u_3{}^2+\cdots+u_n^2}{N}=\frac{1}{N}\cdot\sum_{r=1}^{r=n}u_n^2 \qquad \text{(IV-6.5 2)}$$

Then we have $p_{ox}=Nm\bar{u}^2/l^3$

Now with a *very large number* of molecules moving *at random* with velocities ranging from O to ∞ the mean square of the component speed along any of the three perpendicular axes is the same i.e. $u^2=v^2=w^2$

the fact responsible (i) for *equality of gas pressure* in all directions on the container walls as also for (ii) *Pascal's law.*

But as we have $c_1^2 = u^2 + v^2 + w^2$, their mean squares will be

$$\bar{c}^2 = \bar{u}^2 + \bar{v}^2 + \bar{w}^2 \text{ and so } \bar{u}^2 = \tfrac{1}{3}\bar{c}^2 \quad \text{(IV-6.5.3)}$$

$$\therefore \quad p = \tfrac{1}{3} mN\bar{c}^2 / l^3 = \tfrac{1}{3} nm\bar{c}^2 \quad \text{(IV-6.5·4)}$$

for $n = N/l^3$, the no of molecules in unit volume

where m=mass of a molecule.

n=number of molecules per cc of the gas

and $\bar{c}^2$=the mean value of the squares of speeds)

C. Value of c. In the relation $P = \frac{1}{3} mnc^2$, mn is the mass of the gas per unit volume. Hence $mn = \rho_1$ the gas density.

$$\therefore \quad P = \tfrac{1}{3}\rho\bar{c}^2 \quad \text{or } \bar{c} = \sqrt{3P/\rho} \quad \text{(IV-6.5.5)}$$

For hydrogen at S.T.P., $\rho = 0.00009$ g/cm^3, standard atmospheric pressure $P = 76 \times 13.6 \times 980$ dyn/cm^2.

$$\therefore \quad \bar{c} = \sqrt{\frac{3 \times 76 \times 13{\cdot}6 \times 980}{0{\cdot}00009}} = 1.85 \times 10^5 \text{ cm/s}$$ i.e. about a mile a second—much faster than a rifle bullet. $\bar{c}$ is said to be the *r.m.s.* or root mean square speed.

Values of $\bar{c}$ at 0°C for some gases are given below: they differ from gas to gas.

Gas	Speed at 0°C
Hydrogen	18.6×10^4 cm/s
Helium	13.1×10^4 "
Nitrogen	4.93×10^4 "
Oxygen	$4.61 \times 10_4$ "
CO_2	3.92×10^4 "

Problem: Calculate *rms.* speed of N_2 at N. T. P. given ρ for $N_2 = 1.25$ g/litre at N. T. P. and 13.6 g/cc for mercury

(**Ans.** 486 km/s) [H.S. 82]

IV-6.6. Concept of Temperature from Kinetic Theory. In deriving equation IV-6.5.4 note that we have related a *macroscopic*

concept the gas pressure, with a microscopic quantity the *rms.* speed of individual gas molecules. We shall now see that the same microscopic quantity *rms* speed is related to the most important macrothermal concept, that of temperature.

From the relation $p=\frac{1}{3}mn\bar{c}^2$, we get $p/\rho=\frac{1}{3}\bar{c}^2$ where the density ρ of the gas is mn the product of the mass of a single molecule (m) and n the no of molecules in unit volume.

Let us take a *mole* (i. e. gm-molecule) M of a gas at a temp T (absolute) when its volume is V, say. Then we have,

$$\frac{P}{\rho}=\frac{P}{M/V}=\frac{PV}{M}=\frac{1}{3}\bar{c}^2 \text{ or } PV=\frac{1}{3}M\bar{c}^2 \tag{IV-6.6.1}$$

Now from the ideal gas law we have $PV=R_0T$ where R_0 is the molar gas const. Hence we have,

$$R_0T=\frac{1}{3}M\bar{c}^2 \text{ i.e. } \bar{c}^2/T=3R_0/M=\text{const} \tag{IV-6.6.2}$$

i. e. *absolute temperature of a gas is proportional to mean square speed of the gas molecules.* Note that *temperature* is proportional to a square of a quantity showing that (i) it *can never be negative* and also (ii) it *is a scalar quantity*

If again N_A represents the number of molecules in a male of a gas, i.e., N_A=Avogadro's number, we have for one gram-molecule

$$PV=\frac{1}{3}mN_A\bar{c}^2.$$

Comparing it with the perfect gas equation $PV=R_MT$

$$\text{we have } \frac{1}{3}mN_A\bar{c}^2=R_0T \tag{IV-6.6.3}$$

Now, the kinetic energy of a single molecule$=\frac{1}{2}mc^2$.

$$\therefore \quad {}^1 mc^2=\frac{1}{2}\frac{R_0}{N_A}T=\frac{1}{2}kT \tag{IV-6.6.4}$$

where $k=R_M/N_A$ is a constant and is called Boltzmann's constant. *Thus we find that—The kinetic energy of a molecule is proportional to the absolute temperature.*

The statement that the kinetic energy of the centre of mass of a

molecule is proportional to the absolute temperature' is a very important result. We may apply it to liquids and solids as well.

Eq. IV-6.6.4. serves as a definition of temperature. So defined, it is called the **'kinetic temperature'**. Temperature can also be defined in other ways, such as from thermodynamics. But this is not the place to go into them.

As a basic assumption is that in a gas there are no *mutual* forces amongst the molecules they can have no potential energy, all their energy is in the kinetic form and it is this that determines the temp of the gas. *Temperature is hence the manifestation of kinetic energy of gas moleculess.* So from the kinetic idea of temp we may conclude that *all molecular motion ceases at absolute zero.*

From equation IV-6.6.2 we derive two conclusions as to the magnitude of *rms* velocity of a gas—

(*i*) for *a given gas* rms speed $\bar{c} \propto \sqrt{T}$

(*ii*) *at a given temp* rms speed $\bar{c} \propto \sqrt{1/M}$ *i.e.* heavier gas molecules have lower *rms* speeds.

IV-6.7. RMS. Velocity and a few Related Concepts : A. Mean Free Path. The high speed of gas molecules raises a question—why then there occurs a time-lag between the opening of an ammonia bottle at one corner of a room and smelling it the other end of a room. This apparent anomaly comes up, for we have assumed that in their to and fro motion the gas molecules do not collide with each other. That is absurd. Calculations show that in one second in air at N.T.P. there occur *on the average* 5×10^9 collisions. Because of so many collisions the advance of ammonia molecules through air i.e. their speed of diffusion is much smaller compared to molecular speeds. Remember collisions cannot occur regularly i.e. after regular intervals of time or separations. Just as we consider the mean of the squared speeds, here we introduce the *mean free path* (λ) between collisions—the mean of distances covered by the molecules between successive collisions. If *d* be taken as the diameter of a molecule then it can be shown that

the mean free path $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$ (IV-6.7.1)

i e. it diminishes for larger molecules, their number in unit volume and hence the gas pressure.

B. RMS. velocity and the Velocity of Sound. From the above table we observe that in air c is 485 m/s and that of sound 332 m/s; in hydrogen they are 1838 and 1286 m/s; in oxygen 461 and 317 m/s respectively all at N.T.P. however. We picture the propagation of sound waves as due to a directional motion of molecules imposed upon their random thermal motion. So the energy in a sound wave is carried as kinetic energy from one molecule to the next with which it collides but the molecules cannot in spite of their high speeds, move very far during the period of one vibration. An explanation of a macroscopic fact is provided from the microscopic idea.

C. R.M.S. velocity and Escape velocity of gas molecules from Earth : We have seen before (§ II-1.17) that the latter is about 11.2 km/s —30 times as much as velocity of sound and 6 times greater than *rms.* speed of H_2 both at 0°C. Now remember that speeds of individual molecules widely fluctuate from their r.m.s. values and faster molecules would get lost from the atmosphere. These speeds were much higher when the earth was far, far hotter in the remote past and so the lightest of the gases H_2 and He mostly escaped.

D Mean and Root Mean Square Velocity. Though for the most elementary calculations we may take all molecules of a given gas to have the same speed at a given temperature the *molecular speeds are not really equal.* The average value of their speeds at a given temp is the *mean value* corresponding to that temp. It is denoted by C_{av}.

The square root of the average value of the squares of the molecular speeds is called the *root mean square velocity* denoted by $\bar{c}$. That these two are not equal can be readily seen : let us have only 3 molecules with speeds 9, 10 and 11 units. Clearly the mean

velocity (C_{av}) is $\frac{1}{3}$ $(9+10+11)$ units whereas $\bar{c}^2=\frac{1}{3}$ $(9^2+10^2+11^2)=100.67$ or $\bar{c}=10.03$ units.

IV-6.8. Deduction of macroscopic Gas laws from the Kinetic Theory. In spite of the basic assumptions which are not all exact as we shall see, the well-known gas laws can be derived from the basic equation IV-6.5.4 as follows :

(i) **Boyle's Law :** From the equation

$P=\frac{1}{3}m\,N\bar{c}^2/l^3$ we get $PV=\frac{1}{3}mN\bar{c}^2$

where $V=l^3$. Now since *at const temp* $\bar{c}^2$ is const, we have $PV=$ const—the Boyle's Law.

(ii) **Charles' Law :** From above we find

$PV=\frac{1}{3}mN\bar{c}^2=\frac{2}{3}.\ \frac{1}{2}mN\bar{c}^2=\frac{2}{3}$ K.E. of all the molecules

Above we have found that

Total K.E. of all gas molecules $=$const $\times T$.

Hence $V=\frac{2}{3}$ $K.E/P=\frac{2}{3}$ const $\times T/P$.

If P is kept const then

$V=$const $\times T$ or $V\propto T$ which is Charles' law.

(iii) **Avogadro's Law :** Let us have two gases at same temp pressure and volume. Then we must have

$PV=\frac{1}{3}\,m_1N_1\bar{c}_1{}^2=\frac{1}{3}\,m_2N_2\bar{c}_2{}^2$

Since they are at the same temp, the kinetic energy of individual molecules of the two gases are equal i.e. $\frac{1}{2}\,m_1\bar{c}_1{}^2=\frac{1}{2}\ m_2\bar{c}_2{}^2$, So $N_1=N_2$ i e. *molecules in equal volumes of all gases under the same pressure and temp are equal in number* which is Avogadro's Law.

(iv) **Graham's Law of Diffusion :** It states that the *rate of diffusion of a gas through a porous vessel is inversely proportional to the square root of its density.* From the kinetic theory it is obvious that *Rate of diffusion* $\propto$ *R.M.S velocity* $(\bar{c})$

Now from the equation IV-6.5.5 we have

$\bar{c}=\sqrt{3P/\rho}$ $\therefore$ *Rate of diffusion* $\propto$ $1/\sqrt{\rho}$

IV-6.8. Limitations of the ideal gas laws. Ideal gases have been defined as gases which obey both Charles' law and Boyle's law at all temperatures and pressures. No real gas is perfect

because (i) when sufficiently cooled a gas becomes a liquid, and (ii) at high pressures the product PV of all gases increases. Besides, real gases also show other deviations from Charles' law and Boyle's law.

Real gases do not fully satisfy all the basic assumptions of the kinetic theory of ideal gases. It was assumed that (i) molecules do not exert any appreciable attraction on one another and that (ii) the actual volume occupied by the molecules is negligible compared with the total volume of the vessel. When at high pressures the molecules of a gas are brought close together, they not only experience mutual forces of attraction, but the volume they occupy becomes comparable with the total volume in which they are confined. When real gases at high pressures are sufficiently cooled they turn into liquids due to molecular attraction.

Besides, molecules have a structure and all kinds of molecules cannot be treated as hard, perfectly elastic spheres. In this respect, the inert gases (He, Ne, A, etc.) are closest to ideal gases as they are all monatomic. Other gases have two or more atoms to the molecule and gradually cease to behave as perfectly elastic spheres as temperature rises.

As the distance between molecules of a gas increases, the gas behaves more and more like an ideal gas. All the above basic assumptions of the kinetic theory come closer and closer to fulfillment as the pressure of the gas diminishes. At extremely low pressures any real gas behaves practically like an ideal gas unless the temperature is also exteremely low.

Though the ideal gas laws do not strictly apply to real gases under ordinary circumstances, we shall, at this preliminary stage, treat all gases as ideal.

Van der Wals' Equation of State : We have discussed above that real gases do not obey the equtiion $PV=RT$ particularly at high pressure and low temperature. In an attempt to explain this behaviour, Van der walls disregarded the third and fourth of the simplifying assumptions listed in § IV-6.4.

He assumed the existence of *weak attractive forces* between the gas molecules (§ II-2.6D). Because of this, molecules are somewhat slowed down near the container walls thereby lessening the exerted pressure by an amount say p. He also assumed that gas molecules are *not volumeless* and reduces the volume available to the moving molecules by an amount, say b. So he proposed a revised equation of state for real gases which is

$$(P+p)\,(V-b)=RT$$

$$\text{or}\quad (P+a/V^2)\,(V-b)=RT$$

where P is the *observed* pressure exerted and a, a const given by $p=a/V^2$.

The law is fairly successful in explaining the behaviour of real gases, though not fully so.

CHANGE OF STATE

A. SOLID TO LIQUID

IV-7.1 Melting and freezing. When a substance changes its state from solid to liquid, the process is called *melting.* The reverse process *i e.*, a transformation from the liquid to the solid state is called *solidification* or *freezing.*

The temperature at which melting occurs under standard atmospheric pressure is called the **normal melting point**. Similarly, the temperature at which solidification takes place under a standard atmosphere is the *normal temperature of solidification*, or **normal freezing point.** Melting points are slightly affected by change of pressure.

For a *pure crystalline substance* these two temperatures are the same and a sharply defined one. Each pure substance has its own particular temperature at which it melts (unless chemical change occurs before melting). The change from solid to liquid begins only when this temperature is reached. *The substance remains at this temperature until the change is complete*.

Non-crystalline substances such as fat, wax, glass etc, pass through an intermediate soft state before melting. They do not possess a fixed or sharply defined melting point. In some of these substances melting and freezing occur not at the same temperature. Thus butter melts between 28°*C* and 33°*C* but soldifies between 23°*C* and 20°*C*. Impure substances and mixtures do not have sharp melting points. They melt over a range of temperatures. Sharpness of melting point is, therefore, a test of purity, and is used by chemists as such.

The following table gives the melting points of a few substances :

Substance	Melting point	Substance	Melting point
Brass	800°C to 1000°C	Paraffin Hard	52°C to 58°C
Carbon	about 3500°C	Soft	48°C to 52°C
Copper	1083°C	Platinum	1769°C
Gold	1063°C	Silver	960·8°C
Iron (cast)	1100°C to 1300°C	Sulphur	119°C
Lead	327·3°C	Tin	231·9°C
Mercury	− 38·87°C	Tungsten	3380°C
Naphthalene	80 2°C	Zinc	419 5°C

Melting point is generally lowered by impurities Plumber's solder, an alloy of two parts by weight of lead and one part of tin, melts at 180°C, though lead melts at 327°C and tin 232°C By adding bismuth (m p. 271°C to a mixture of 37% lead and 63% tin an alloy called *Rose's metal* melting at 94°C results. *Wood's metal* which contains 50% bismuth, 25% lead, 12·5% antimony (m.p 630 5°C) and 12·5% cadmium m.p 320 9°C), melts at 65 5°C.

Fire-fighting and alarm devices used in factories make use of such low melting alloys. Water pipes are fixed to the ceiling, and at intervals nozzles filled with the alloy are fitted. If fire breaks out the alloy melts and water is sprayed into the room. Another part of the device, *a bimetallic thermostat* rings a warning bell at the same time.

IV-7 2 Latent heat and its origin. When a substance is heated, its temperature rises. *It is supposed* that its molecules then vibrate with incresing amplitude as the temperature rises. The energy required for the larger vibrations comes from the heat supplied. This heat is called ***sensible heat.*** It increases the kinetic energy of motion of the molecules.

But the heat supplied to a solid during melting does not raise the temperature. Where does then the energy go ?

In crystalline solids, molecules are arranged in regular geometrical patterns in space. (see fig. II-2. 1.). They are held in

their equilibrium positions by forces of mutual attraction and repulsion. We have seen that in § II-2-5. Instead of being at rest, the molecules vibrate about their equlibrium positions with frequencies of the order 10^{13}/s. This kinetic energy is proportional to absolute temperature. We have seen that in the last chapter.

The energy supplied to a solid at its melting point is used up in destroying its geometrical arrangement, thus melting the solid into a liquid without change of temperature. The heat which has to be supplied to a solid at its melting point to produce a change in state *without change in temperature* is called latent heat. The latent heat *increases the potential energy of the molecules, and is given out by the liquid when it freezes.*

The specific latent heat of fusion (or melting) is the amount of heat required to convert one gram of solid into liquid without any change of temperature. The same amount of heat will be given out by 1 gram of the liquid when it solidifies at its freezing point without change in temperature.

Use of the word *specific* to mean *per unit mass* is of late origin. Older texts write 'latent heat' to mean 'specific latent heat', that is, latent heat per unit mass. The statement that "Latent heat of ice is 80 caloris" is a loose statement and means 80 calories of heat will be required to convert 1 gram of ice at 0°C to 1 gram of water at 0°C Latent heat is expressed in heat units per unit of mass, such as *calories per gram* (or *Btu per lb* etc.).

The following table shows the melting points and latent heats of a few substances :—

Substance	Melting point (in°C)	Latent heat (in cal/g)
Acetic acid	16·7	44·7
Ammonia	−77·7	108
Benzene	5 5	30 1
Glycerine	20	48
Ice	0	79 7
Lead	327	6
Mercury	−39	2·8
Sulphuric acid	10·5	24
Tin	232	14

The large ***latent heat of water*** may be utilised to prevent damage to meat, fruits and vegetables in storage. Extreme cold may freeze their sap or otherwise reduce their food value. To prevent such freezing, a few buckets of water are kept in the store room. When the temerature of the room goes below 0°C the water in the buckets gradually freezes and releases its latent heat. This raises the temperature of the room and it may not go down far enough to freeze the sap. Freezing releases the latent heat; *freezing may thus be considered as a heating process.*

No freezing occurs unless latent heat is extracted. A liquid will not freeze by merely reaching its freezing point. Latent heat of fusion must be extracted from it. This can be easily proved by keeping water in a small phial fully surrounded by ice. The water will soon reach the freezing point (0°C), but will not freeze. It must be cooled below 0°C, even though slightly, so that the latent heat may be extracted. (Loss of heat lowers the temperature of a body. On this view, ***latent heat is not heat***, ***but internal potential energy*** of the molecules of the liquid.

The same applies to melting. A solid kept at its melting point will not melt unless the latent heat is supplied.

IV-7. 3. **Supercooling.** Sometimes, a pure liquid, cooled undisturbed, does not freeze even though it has been cooled below the normal freezing point, This phenomenon is known as **supercooling.** Sodium thiosulphate (popularly known as 'hypo'), thymol, naphtalene, etc, can be melted and then cooled carefully, much below their normal freezing points without freezings. Supercooled liquids are very unstable; a slight disturbance or the addition of a crystal of it may be enough to start solidificiation of the whole mass. The temperature rises to the normal freezing point at the same time.

Sublimation. There are some solids which, when heated, directly pass on to the gaseous state without passing through the liquid one. This phenomenon is called sublimation. Camphor, iodine, dry ice (solid CO_2) sublime even at the room temperature. Sublimation also requires latent heat.

IV-7. 4. Determination of the latent heat of ice. Take a clean dry calorimeter with a *wire gauze* stirrer and weigh it empty. Fill it partly with water and weigh again The difference gives the weight of water taken (m g). The wire-gauze is necessary to keep the lighter ice pieces submerged in water.

Put a thermometer in it and record the initial temperature. Let it be t_1°C.

Take one or two *small* pieces of ice and soak away all water from them by means of a piece of blotting paper. Holding the ice pieces in the blotting paper, drop them into the calorimeter. Stir the water *keeping the ice immersed under water below the stirrer*. Note the lowest temperaturature reached. Let it be t°C.

Wait till the calorimeter and its contents are again at the room temperature. Weigh them again. The excess in weight gives the amount of ice added. Let the mass of ice added be m' g.

Let W be the water equivalent of the calorimeter, and L, the latent heat of ice per gram.

Then, heat lost by the calorimeter $= W(t_1-t)$;

heat lost by the water $= m(t_1-t)$,

heat gained by ice on melting $= m'L$,

heat gained by water formed of ice $= m't$.

Since heat lost = heat gained

we have $(W+m)(t_1-t) = m'L + m't$

$$\therefore \quad L = \frac{(W+m)(t_1-t)}{m'} - t \qquad \text{(IV-7.4.1)}$$

Note : To guard against such a fall in temperature of calorimeter as will deposit water from the atmosphere on its sides, only one or two small pieces of ice should be added.

The method is that of mixtures. It can alternatively be found by Black's Ice calorimeter, § IV-2. Uesing a ball of known sp. heat, specific latent heat can be found from eqn $S=mL/Mt$. But the method is not so accurate.

Example. IV-7.1 100 g of ice at $-10^\circ C$ are heated till fully converted into water at $30^\circ C$, Find the amount of heat necessary. (Given, latent heat of ice = 80 cal/g and specific heat of ice = 0.5 cal $g^{-1} C^{\circ -1}$.)

Solution : Heat required to raise 100 g of ice from $-10^\circ C$ to $0^\circ C = 100 \times 0.5 \times 10 = 500$ cal.

Heat required to melt 100 g of ice at $0^\circ C$ into water at $0^\circ C = 8000$ cal $= 3000$ cal.

Heat required to raise 100 g of water from $0^\circ C$ to $30^\circ C = 100 \times 30$ cal.

$\therefore$ Total heat required $= (500 + 8000 + 3000)$ cal $= 11500$ cal.

Ex. IV-7.2. 75 grams of water at $100^\circ C$ are added to 20 grams of ice at $-15^\circ C$. Find the resulting temperature. (Given, specific heat of ice = 0.5 and latent heat of ice = 80 cal/g.)

Solution : Let t be the final temperature.

Heat lost by the warm water $= 75 \times 1 \times (100 - t)$ cal.

Heat gained by ice for being raised from $-15^\circ C$ to $0^\circ C = 20 \times 0.5 \times 15$ cal

Heat required for melting of ice $= 20 \times 80$ cal,

Heat gained by water (formed of the ice) for being raised from $0^\circ C$ to $t^\circ C = 20 \times 1 \times t$ cal.

$\therefore \quad 75 \times (100 - t) = 150 + 1600 + 20t$ whence $t = 60.52^\circ C$

Ex. IV-7.3 20 grams of water at $100^\circ C$ are added to 75 grams of ice at $-15^\circ C$. Find the result.

N. B. *Note that this problem is of the same nature as the previous one. If treated in the same way it will give a common final temperature lower than that of the ice. Obviously this is absurd as by adding hot water the temperature of ice cannot be lowered.*

The *fallacy* lies in assuming that all ice has melted. This is true in the first case, but does not hold for the second.

The following method of treatment is recommended.

Heat required for raising 75 g of ice from $-15°C$ to $0°C = 75 \times 0.5 \times 15 = 592.5$ cal

Heat required for melting 75 g of ice at 0°C to water at $0°C = 75 \times 80 = 6000$ cal.

Now if the hot water cools upto 0°C the heat given out will be $20 \times 1 \times 100 = 2000$ cal. Of this heat 592.5 cal will be required to bring the ice upto 0°C. The remaining 1437.5 cal will be available for melting ice.

$\therefore$ The quantity of ice melted would be $1437.5/80 = 17.97$ g. The result will be 57.03 g of ice and $20 + 17.97$ g of water at 0°C.

Ex. IV 7.4. 900 g of iron at 500°C are transferred into a hole in a block of ice and 680 cm^3 of water collected from it. If the specific heat of iron is 0·12, find the latent heat of ice.

Solution : The mass of 680 cm^3 of water = 680 g. Iron cooled from 500°C to 0°C. Hence heat lost by iron $= 900 \times 0.12 \times 500$ cal. The heat gained by ice $= 680 \times L$.

$$\therefore \quad 680L = 900 \times 0.12 \times 500 \quad \text{or} \quad L = 79.4 \text{ cal/g}.$$

Ex. IV 7.5 A calorimeter at 30°C contains 220 g of paraffin oil. The water equivalent of the calorimeter is 40g. When 15 g of ice at 0°C are added to it the temperature falls to 20°C. Find the specific heat of paraffin.

Solution : Heat lost by paraffin $= 220 \times s \times (30-20) = 2200s$.

Heat lost by the calorimeter $= 40 \times 10 = 400$ cal.

Heat gained by ice in melting $= 15 \times 80 = 1200$ cal.

Heat gained by water so formed in rising from 0°C to 20°C

$= 15 \times 20 = 300$ cal.

$$2200s + 400 = 1500, \text{ whence } s = 0.4$$

IV 7.5. A. Change of volume on melting. The volume of most solids increases on melting, but there are a few substances, *viz.* ice, cast iron, bismuth, antimony and typemetal, which show a decrease in volume on melting. This property of cast iron and type metal is taken advantage of in making castings of various

designs from iron and in the manufacture of types for printing. The fact that ice floats in water is of great consequence in keeping aquatic animals safe in the seas of the arctic regions or in waters which freeze in winter.

In expanding on solidification water exerts a considerable force. The following examples testefy to that.

1. In cold countries water pipes are sometimes found to burst in winter, hot water pipes bursting more often than cold water pipes. Hot water contains much less dissolved air than cold water. In cold water pipes the dissolved air remains as air pockets when water cools and finally freezes. The increase of volume of water on solidification compresses these air pockets. The cold water pipes may thus be saved from bursting.

2. Boulders which have retained some water in their cracks may fall into pieces when, in cold winter nights, the water in the cracks freezes and expands. This triggers off deadly avalanches on high mountains.

B. Calorimeters based on Decrease of Volume of Ice on Melting.

Bunsen's Ice Calorimeter. This is the most accurate of all ice calorimeters. Its action depends on the change of volume that ice undergoes on melting.

Description. It consists of a test tube *P* sealed to the wider limb *Q* of a U tube *SQ* (fig. IV 7.1). The upper part of *Q* is filled with water from which air has been expelled and the remaining portion of the U tube contains mercury up to the mouth of *S*. The mouth is closed by a rubber stopper through which passes a bent glass tube *R* of narrow bore. A scale is placed behind *R*. The screw *T* through the stopper is used for adjusting the position of the free end of mercury in the tube *R*.

Procedure, action and theory. To work with the apparatus the whole of it is placed in melting ice for about 12 hours so that the temperature of the contents reaches 0°C. Some ether, previously cooled to 0°C, is then poured into the test tube *P*. Air is blown

through the ether, which evaporates and produces local cooling. As a result some water in the limb Q of the U tube deposits as ice on the sides of the test tube P. When all the ether has been evaporated a small quantity of water at 0°C is put in the test tube. The position of the free end of the mercury thread in the tube R is then suitably adjusted by working T and its reading on the scale taken.

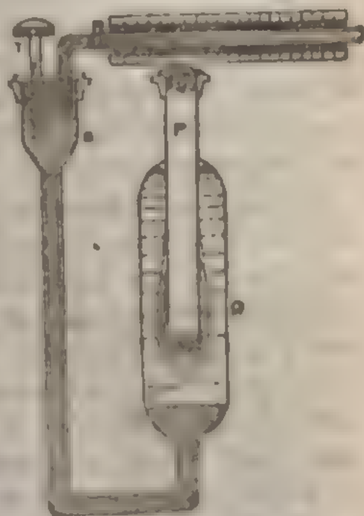


Fig. IV-7.4

A weighed piece of the experimental solid at a known initial temperature is introduced into the test tube and the open mouth of the test tube plugged with cotton. The solid loses heat, which melts some of the ice around P. As the volume of ice diminishes on melting, the mercury thread in R recedes towards S. When the mercury thread attains a steady position the solid must have fallen in temperature to 0°C.

If we could know the mass m of the ice melted, we could apply the same formula as in Black's ice calorimeter, viz $s = mL/M$. But m cannot be determined directly. The shift of the mercury thread is known and the inner cross section of the tube R is determined. Thus the change in volume of ice v on melting is found.

Since we know that a diminution of 0.0907 cc. takes place when 1 gram of ice melts, a diminution of v c.c must be due to the melting of $\frac{v}{.0907}$ gm. Substituting this value of m in the eq. $MS(t-0) = mL$ we have

$$s = \frac{v \times L}{.0907\,Mt} \qquad \text{(IV 7.5.1)}$$

Advantages. The apparatus can be conveniently used to measure the specific heat of a substance or the latent heat of ice accurately. (i) It can measure the specific heats of small bodies, such as gems, pearls etc., or of rare materials which can only be obtained in small quantities ; there is (ii) no loss due to radiation, the greatest source of error in the method of mixtures, (iii) the water equivalent of the calorimeter need not be known (iv) nor are thermometers necessary.

IV 7.6. Effect of pressure on melting point. Melting point of substances are slightly affected by pressure. Most liquids contract on cooling and this continues till they turn solid. In these cases increase of pressure assists the change from liquid to solid. Such a liquid will therefore, freeze more easily, that is, at a higher temperature when under pressure. Liquids like water which expand on freezing, will freeze at a lower temperature when subjected to pressure. Here the increase in pressure opposes the change ; hence it becomes necessary to reach a lower temperature before freezing can take place. Ice melts at $-1°C$ under a pressure of about 134 atmospheres.

Reduction of pressure on ice will raise its melting point. In a vacuum it melts at about 0.072°C.

A. **Regelation.** The familiar phenomenon of fusion of two pieces of ice to form a single piece by pressure is an example of lowering of melting point under pressure When two pieces of ice are pressed together the points of contact between them are subjected to high pressure Hence the melting point at these places is lowered. The actual temperature at these points, *i. e.*, 0°C, is higher than the melting point under the condition of high pressure. Hence ice melts at these points taking the latent heat from their immediate surroundings. When the pressure is released the water around the points of contact freezes as it is lower than 0°C due to the abstraction of latent heat from it during melting. The two pieces are thus cemented together. The melting of ice

by pressure followed by solidification of the water so formed on removal of pressure, is known as **regelation.**

Skating on ice is another example of regelation. The steel edges of the skate exerts a high pressure on the ice. The ice below the steel edge of the skate immediately melts. This reduces friction to the motion of the skate. As soon as the skate moves away, the water solidies as the pressure on it has been reduced.

B. Bottomley's experiment. A single turn of *bare, thin copper wire* is wound round a large block of ice supported on two wooden blocks as shown in fig. IV-7 2 and is loaded by a weight of several kilograms The copper wire will be seen to cut through the block of ice and pass out of it, while the block remains intact. This is no magic.

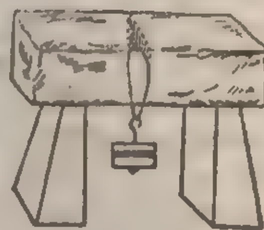


Fig. IV-7 2

Due to the load, the thin wire exerts a rather high pressure on the ice just below it and lowers the melting point at that place. The ice at that place, therefore melts, taking the latent heat from the surroundings through the copper wire. Copper is a very good conductor of heat. The water so formed flows round the wire which sinks. This water freezes immediately due to the removal of this pressure on it. The latent heat it gives out passes through the wire and supplies the latent heat of fusion for the ice below the wire. This process is repeated and each layer is cemented as soon as the wire cuts through it.

For the success of the experiment it is necessary that heat be allowed to flow from the solidifying water to be melted below the wire. The wire must, therefore, be made of a material which conducts heat well. The experiment will not succeed if the wire is a bad conductor of heat (a string), nor if the wire is thick. (Why ?)

C. Glacier and lava. High up on the mountains there are heavy deposits of ice. At a particular depth from the top the pressure may be enough to push the melting point below the

actual temperature at that depth. The ice at that place, therefore, melts and detaches the portion above it from the general mass. The detached portion moves slowly down the slope. These large moving ice-masses, which may be looked upon as solid rivers of ice, are called **glaciers.** The lowest layer of a glacier is always melting. Our Himalayan rivers rise from such glaciers.

A volcano occasionally gives out a stream of **lava**. This may be explained as follows. The basaltic igneous rocks below the earth's crust containing sedimentary rocks etc., are at such a temperature that they would remain in the molten state under normal pressure. But due to the pressure of the earth's crust their melting point is raised (for basalt increases in volume on melting). These rocks therefore exist in a semi-solid state. When for any reason the pressure over the hot semi-solid substance is reduced, as may occur during a volcanic outburst, the basaltic rock becomes fluid due to the removal of pressure and the molten mass gushes forth through the crater as lava.

While flowing along the mountain slope the lower layers of the lava stream will solidify earlier. The lower layers have a higher freezing point as they are subjected to the pressure of the upper layers. Liquid lava flows over the solidified lower layers.

Heavy Ice. It has been shown by Prof. P. W. Bridgeman of the Harvard University, U. S. A., that when water is frozen under a pressure of nearly 20,000 atmospheres, the ice formed is heavier than water. This heavy ice formed under a pressure of 20,000 atmospheres melts at a temperature of 76°*C*.

IV-7.7. A. Freezing Point of a Solution : A pure solvent may have a definite freezing point, but if a solute is dissolved in it the freezing point is lowered. Thus pure water freezes at 0°C, but a solution of common salt in water freezes at a temperture lower than 0°*C*. the actual temperature of freezing depending on the concentration of the salt in solution. The higher the concentration the lower the freezing point.

When a solution of common salt is gradually cooled below 0°C some water freezes below a certain temperature depending on the concentration. The ice so formed does not contain salt. With gradual cooling more and more water freezes and the percentage of salt in the remaining solution increases. This continues till at a temperature of $-21.2°C$ the solution freezes as a whole. In this conditon the solution contains about 22.4% of common salt. If originally more salt than this is dissolved in water and the solution is gradually cooled, the excess of salt is deposited as crystals. The temperature at which a solution freezes as a whole (without separation of solute or solvent) is called the **eutectic point** and the mixture is called a **eutectic mixture.**

B. Freezing mixture : When salt is mixed with icethe heat of the salt melts some ice anda saturated solution of salt at 0°Cis formed. But ice cannot remain in equilibrium with such a solution, just as it cannot do so with pure water above 0°C. So the ice melts drawing the latent heat from the system, which cools. If enough salt is present this cooling continues until the eutectic point is reached, at which the ice and the solution can remain in equilibrium. Cooling is aided by the heat absorbed when a salt dissolves in water

If there is not enough salt the eutectic point will not be reached. It there is more salt than is necessary to reach the eutectic point when all the ice has melted, the extra salt will remain undissolved. Its heat will prevent the temperature from going down to the eutectic point. So will leakage of heat from the surroundings.

When powdered ice and salt are mixed in the proportion 4 : 1 by weight the temperature falls to $-18°C$. If ice and $CaCl_2$ are mixed in the proportion 7 : 3 the temperature falls to $-50°C$. A mixture of powdered ice and KOH in the same proportion will make a better freezing mixture.

Freezing mixtures are used to freeze milk-preparations the well-known ice-creams.

IV-7.8. Laws of Fusion : Summarising what has gone before we have (i) All substances begins to fuse at a characteristic temp, constant for a crystalline substance provided the pressure on it is unchanged. So long as the whole mass does not melt the temp does not change. For non-crystalline substances, fusion occurs over a small range of temp.

(2) If a substance expands on solidification (like ice) the m. p. is lowered with rise in pressure ; most substances contract on melting and for them a rise in pressere leads to rise in m.p.

(3) For an alloy its m.p is lower than that of any of its constituents

(4) For a crystalline solid, specfic latent heat fo fusion is a constant characteristic of the substance.

(5) Freezing point of a solution is always lower than the freezing-point of the pure solvent.

B. CHANGE OF STATE (LIQUID TO VAPOUR)

IV-7.9.A. Vaporization. When a liquid is heated it ultimately changes into vapour. Such a change is called *vaporization.* It can take place in two ways viz., (i) by *evaporation*, and (ii) by *boiling*. The reverse process of change of state from vapour to liquid is called *condensation* or *liquefaction*. *Vapour exerts pressure like a gas.* This may be seen as follows.

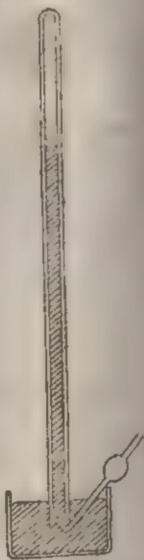


Fig. IV-7.3.

Take a barometer tube filled with mercury and invert it over a mercury trough. Introduce a few drops of a liquid, such as water, alcohol or ether, into the tube with the help of a bent pipette (fig. IV-7.3). As the first drops rise through the mercury column and reach the vacuous space above, they turn completely into vapour. The vapour so formed depresses the mercury column, showing that it exerts pressure like a gas.

As you introduce more and more of the liquid into the

tube, a state will be reached when it no longer evaporates, but accumulates on the mercury surface. When this condition is reached, the pressure exerted by the vapour increases no more.

These observations show that (i) *a vapour exerts pressure*, and (*ii*) *there is a limit to the amount of vapour that a space* (here the Torricellian vacuum) *can hold*, If we increase the temperature of the Torricellian space by surrounding it with a suitable bath, it will be found that the maximum vapour exerted by the liquid increases with rise of temperature and the space can hold more vapour at higher temperature.

By using different liquids, we can see that the maximum vapour pressure a liquid can exert at a given temperature depends on the nature of the liquid.

B. Vapour pressure. When a liquid is confined in a closed space, the escaping molecules move about at random in the available space. They constantly collide with the walls limiting the space. Thus they exert a pressure at every point of the boundary surfaces including the liquid surface. Some of the molecules impinging on the liquid return to it while some others leave the liquid. Soon a state of equilibrium is reached in which as many molecules leave the surface as enter it in a given time. When this condition has been reached the pressure exerted by the vapour is a maximum. This pressure is variously known as the *equilibrium vapour pressure, maximum vapour pressure, saturated vapour pressure* (abbreviated SVP) or simply, *vapour pressure* of the liquid. For a *given* liquid SVP is detemined only by the temperature. Its value increases with rise of temperature, and depends on the nature of the liquid. The nearer a liquid is to its normal boiling point the greater will be its vapour pressure.

IV-7.10 Evaporation. Evaporation is the slow process of conversion of a liquid into the vaporous state. It is easy to confirm the following facts about evaporation. Many of them are part of our daily experience.

(i) Evaporation takes place at all temperatures and under all pressures.

(ii) The rate of evaporation increases with rise of temperature.

(iii) The rate of evaporation also depends on the pressure. The lower the pressure on a liquid the faster will it evaporate. Evaporation is quickest in a vacuum.

(iv) The greater the area of the exposed surface the greater the evaporation.

(v) Evaporation is speeded up by a current of air over the liquid.

(vi) The rate of evaporation depends on the amount of the of the liquid present over the liquid surface. The higher it is the slower is the evaporation. A moist cloth dries much slower on a wet day than on a dry day because there is more moisture in a wet day.

(vii) Of two liquids the one which has a lower boiling point will evaporate more quickly. Ether evaporates much more quickly than alcohol, and alcohol quicker than water. The boiling points of ether, alcohol and water are 35°C, 78°C and 100°C respectively.

(viii) *Heat is required for evaporation. If it is not supplied the liquid cools itself or its surroundings to get the heat.*

All liquids have a tendency to turn into vapour at all temperatures. This tendency is measured by the maximum vapour pressure a liquild can exert at the temperature at which it is. Vapour pressure rises with increasing temperature. A liquid begins to boil when its vapour pressure equals the pressure on it.

The average kinetic energy of the molecules of a liquid depends on its temperature, and diminishes with lowering of temperature. In evaporation, some of the faster molecules near the liquid surface overcome the attraction of the rest of the liquid, and leave the liquid. When a liquid loses its faster molecules, the average molecular kinetic energy is lowered. This means a lower temperature, that is, cooling. Unless heat enters the liquid

from the surroundings, its temperature will be lowered due to evaporation.

In rapid evaporation, there can be much cooling because the faster molecules are carried away very fast.

IV-7.11. **Boiling.** Take some water in a flask fitted with a rubber stopper. Pass through it a thermometer, an open-tube manometer, and a delivery tube provided with an india-rubber tubing and a pinch cock (fig. IV-7.4)

Heat the flask slowly over a Bunsen flame. As the temperature gradually increases you will find more and more steam rising *from the surface*. (This is not boiling). Also, the dissolved air in the water forms into small bubbles, rises to the surface and escapes. At about 70°– 80°C, small bubbles of water vapour will be seen to form at the bottom of the flask. They rise and collapse as they reach the upper layers of colder water. At this stage a simmering sound is heard. In the final stage the bubbles rise to the top and burst at the surface. When this happens we say that the liquid is boiling. *During boiling, vapour rises to the surface from all points throughout the liquid. The temperature of the liquid remains consant so long as the boiling continues.*

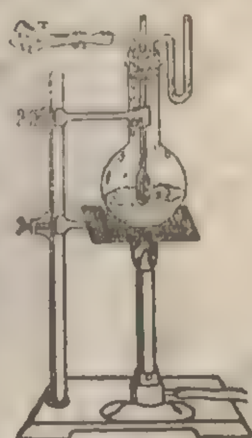


Fig. IV, 7.4

The vapour within the flask is invisible like air. As it comes out of the delivery tube it condenses into small water drops which look like a white cloud. This cloud soon vanishes as the droplets evaporate in the open air.

During boiling, the manometer registers a constant pressure. When the delivery tube is wide open, mercury in both limbs of the manometer stands at the same height, showing that the pressure inside the flask is equal to that of the atmosphere. If boiling has continued long enough we may safely conclude

that air inside the flask hes been completely replaced by water vapour. So the pressure inside the flask is that exerted by the water vapour only. This shows that **when there is a free communication with the atmosphere a liquid boils when its vapour pressure is equal to the atmospheric pressure.**

The constant temperature at which a liquid boils under normal atmospheric pressure is called its **normal boiling point.** The boiling point of water under a pressure of 76 cm of mercury is 100°C.

IV-7.12 Effect of pressure on boiling point.

(1) Increase of pressure on a liquid raises its boling point. During brisk boiling, close the pinch cock partially (fig IV-7.4). If it is closed enough so that rate of outflow of steam through the tube becomes less than the rate of generation, the manometer will show a higher pressure in the flask. The thermometer also shows a higher temperature when the liquid boils under this condition of increased pressure on its surface.

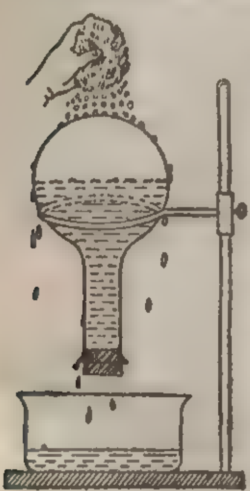


Fig. IV-7. 5.

We, therefore, conclude that an increase in pressure on the liquid surface results in an increase in the boiling point of a liquid.

(2) Lowering of pressure on a liquid lowers its boiling pont.

(a) Franklin's experiment. Take a flask half-full of water and boil it briskly. After boiling has proceeded for some time so the air inside has been driven out by water vapour, remove the flask from the flame and quickly close it with a rubber stopper. Note that boiling stops immediately. Now invert the flask on a retort holder and pour cold water on it (fig IV-7.5). The water within the flask will be seen to boil again. This boiling lasts for a short time. The experiment can be repeated several times after the first boiling.

The explanation is simple. Part of the water vapour above the surface of the liquid condenses into water due to the cooling of the surface of the flask. This results in a diminution of pressure on the liquid surface. If the pressure is lowered below the vapour pressure of the water in the flask, the liquid boils.

This shows that a liquid may be made to boil at a lower temperature than its normal boiling point. This happens when the pressure on it is reduced below the normal atmospheric pressure.

Boiling under reduced pressure has its application in industry. There are chemicals, such as H_2O_2, which decompose before reaching the normal boiling point. In manufacture, they are separated by distillation at a reduced pressure.

In the sugar industry, sugar is separated by crystallization from solution. The syrupy solution has a high boiling point. It is made to boil at a lower temperature by reducing the pressure on it. This economizes fuel consumption.

(b) **Boiling of water at room temperature.** Water may be made to boil at room temperature without heating. For this purpose a high speed exhaust pump and towers containing absorbents of water vapour, such as anhydrous $CaCl_2$, are necessary.

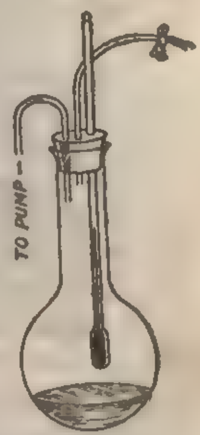


Fig. IV-7. 6.

Take a rather small quantity of water in a flask and close it with a rubber stopper. Through the stopper pass a thermometer and two glass tubes, one of which is connected to the pump and the other to a closed-tube manometer (fig. IV-7.6). The pump is connected to the flask through two absorbing towers containing fused calcium chloride. See that the system is leak tight. Now work the pump. When the pressure falls considerably the water in the flask begins to boil at the room temperature. As the boiling is continued the thermometer may register a temperature a little lower than the room temperature.

(c) **For a liquid to boil, its vapour pressure must be equal to the pressure on it.** A liquid is said to boil when bubbles of vapour formed within it rise to the top and burst on the surface. The pressure inside a bubble is due to the vapour it contains. To form this bubble the vapour must push the liquid away at the place where it is formed. For this purpose the pressure of the vapour must at least be equal to the pressure at that place of the liquid. Hence the vapour pressure of the liquid must at least be equal to the pressure on the liquid.

At lower depths in a liquid the temperature must be a little higher than at the top so that bubbles may form. Hence in measuring a boiling point, the thermometer should be placed in the vapour, and *not* in the liquid.

Boiling point of a solution. Make an arrangement as in fig. IV-7.4. Add some salt to the water in the flask and put the thermometer bulb *within the liquid*. Note that boiling sets in at a higher temperature when the salt is added.

Slowly raise the thermometer *above* the liquid in the flask. Note that the temperature recorded gradually diminishes until it is the same as we would get if we had pure water boiling inside. This shows that the boiling point of a solution is higher than that of the pure solvent under the same pressure. But the temperature of the vapour is ultimately the same as for the pure solvent boiling under the same pressure.

Boiling point of a solution rises with its concentration.

IV-7.13. Characteristics of boiling and factors affecting it.

Summarising, we may say as follows :

(i) Boiling takes place at a definite temperature under a definite pressure. The temperature at which a liquid boils under normal atmospheric pressure is called the **normal boiling point.**

(ii) For pure liquids the temperature at which boiling takes place at a given pressure depends on the nature of the liquid.

(iii) *A liquid boils when its vapour pressure is equal to the pressure on its surface. Hence the boiling point of a liquid rises or falls with the pressure on it.*

(iv) At a given pressure, the boiling point of a solution is higher than that of the pure solvent. But the vapour formed has finally the same temperature in both cases.

(v) Boiling is accompanied by absorption of heat. The rate of boiling depends on the rate of heat supply.

(vi) Boiling is accompanied by a large change in volume. 1 cm^3 of water, on boiling at 100°C, forms about 1650 cm^3 of steam.

Table. Normal boiling points of some substances in °C

Substance	Boiling point	Substance	Boiling point	Substance	Boiling point
Sulphur	444°	Water	100°	Ammonia	− 34°
Mercury	357°	Alcohol	78°	Oxygen	−183°
Paraffin	350°-530°	Chloroform	61°	Hydrogen	−253°
Glycerine	290°	Ether	35°	Helium	−269°

Difference between evaporation and boiling. From the characteristics of the two processes already discussed, we may say that the fundamental differences between them are the following :—

(i) Boiling takes place at a definite temperature under a definite pressure, but evaporation takes place at all tempertures under any pressure.

(ii) Boiling takes place throughout the bulk of the liquid while evaporation takes place only from the exposed surface.

IV-7.14. Hypsometry. Boiling at high altitudes. Atmospheric pressure is the greatest at the sea level and diminishes as we go up. Since the boiling point of a liquid diminishes with lowering of pressure, we should expect that in hill stations water will boil at a lower temperature then on the plains. This is actually so. On Mt. Blanc (15, 782 ft) water boils at 83°C; on Mt.Everest (29,028 ft) it would boil at 70°C. It has been estimated that at a height of 65,000 ft water would boil at 37°C the human body temp. At this height the water in our body, *if exposed*, would start boiling.

The following table gives the approximate temperatures at which water boils at the heights mentioned.

Height above sea-level (in ft)	Boiling point (in °C)
2000 (≈ 600 m)	98
4000	96
7000	93
10,000	90
15,000	85
20,000	80
25,000	74

It would be seen that the boiling point of water diminishes roughly by 1°C per thousand feet (960 ft to be more precise) or 300 metres of ascent above the sea-level upto about 20,000 ft. A knowledge of the boiling point of water can, therefore, give us an idea of the height of the place above sea-level.

IV-7.15. Boiling under increased pressure. Cooking at temperatures less than 100°C is of great inconvenience. For cooking at high altitudes or in a shorter time, a special kind of cooker, called the **pressure cooker** is used. It is now practically a common household equipment. In some industries, such as the paper industry, it is necessary to soften materials with water boiling at a high temperature, Such devices go by the general name of 'digesters'.

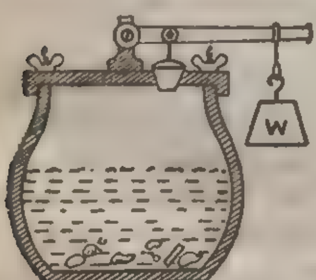
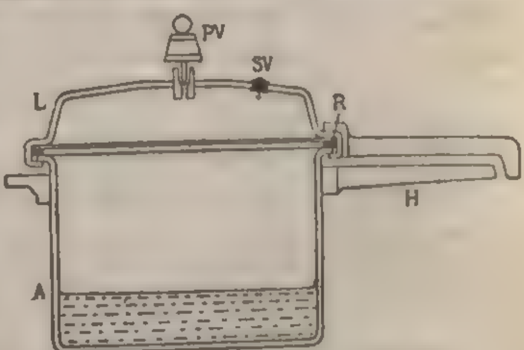


Fig. IV. 7. 7a.

The principle of the cooker or the digester may be explained with the aid of fig 14 IV-7.7a. The vessel is made of thick sheet metal. The lid is attached to it airtight. A valve closes an opening in the lid. It is kept pressed in position by a suitable weight W (which, in industry, may act through a lever). The weight and the position of W determine the pressure of the steam in the vessel which will open the valve. If the steam

generated within the vessel attains a greater pressure, the valve will be forced open and the excess of pressure released. By suitably choosing W the water inside may be made to boil at any desired temperature. If W exerts an extra pressure of about one atmosphere the boiling point of water on the plains will be about 120°C. Meat can be cooked in 15 minutes with such a cooker. The external force on the valve can also be applied by an adjustable spring. Modern pressure cookers (IV-7.7 b) have a valve with a fixed weight on it. The temperature inside is about 110°–115°C.

Papin's digester is used in manufacturing paper pulp by boiling sawdust and caustic soda under pressure, in the manufacture of artificial silk and for extracting gelatine from bones. In boilers of steam engines steam is generated under high pressure by making water boil at a temperature higher than 100°C. Safety valves open when a pre-determined pressure is reached. Boilers of locomotive engines may have a steam pressure of about 250 lb per sq. inch (*i.e.* over 15 atmospheres) the water boiling at about 200°C.



[Pressure Cooker. A = Aluminium container L = Lid ; R = Rubber sealing ring ; PV = Pin valve for escape of Steam ; SV = Safety valve, H = Handle]

Fig. IV. 7. 7b.

In autoclaves and hospital sterilizers water may be made to boil at about 135°C corresponding to a pressure at 30 lb per sq. inch above atmosphere pressure. At this temperature bacteria and bacterial spores are killed. Dressings, towels etc. are placed in the chambers of the sterilizer. High pressure steam circulates

through an outer jacket. Canning food is cooking under pressure Sealed cans are heated by high pressure steam to about 150°C to kill bacteria and also to cook. But it destroys the vitamins.

IV-7. 16. Latent heat of vaporization. We have seen that there is no change of temperature when a liquid boils under a constant pressure. Nevertheless, heat must be supplied even at the boiling point in order that the liquid may boil. Heat is also necessary to evaporate a liquid. This heat, which does not raise the temperature of the liquid, but brings about a change of state from liquid to vapour, is called *latent heat of vaporization.*

There is some attraction between the molecules of a liquid, but in a vapour the mutual attraction between molecules is negligible. To convert a molecule of the liquid into a molecule of the vapour, it must be removed beyond the range of attraction of other liquid molecules. In other words, some energy is necessary to effect the change of state and this energy will not, as in the general case, go to raise the temperature of the liquid. Latent heat of vaporization supplies this energy. It may be looked upon as being stored up as potential energy of the vapour molecules, and is released as heat when the vapour condenses.

Quantitatively, the latent heat of vaporization is the amount of heat required to convert unit mass of a liquid into vapour at the same temperature. It is measured in *heat units per unit mass,* generally in *calories per gram.* Latent heat is independent of the process by which the change of state is brought about. Whether the liquid changes into the vapour by evaporation or by boiling, it requires the same latent heat at a given temperature.

The statement that *the latent heat of steam is 539 calories per gram* then means that, water at 100°C is converted into steam at 100°C, each gram will require 539 calories for the change of state only. When 1 g of steam at 100°C condenses into water at 100°C, 539 calories of heat are released.

More heat is released by condensation of steam than by cooling of boiling water. 1 g of water at 100°C will give out 100 cal of heat if cooled to 0°C. But 1 g of steam at 100°C gives out 539 cal of heat on condensing. Thus the heat available by condenstion of steam is always much greater than the heat available by cooling water. This is why in cold countries steam is led through pipes into condensers in a room which it is required to heat. As the steam condenses it gives out its latent heat and the condensed water returns through another pipe to the boiler.

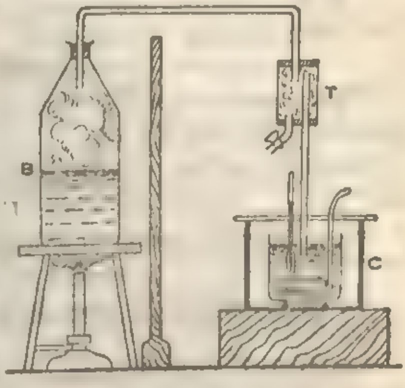


Fig IV-7. 8

The reason why a scald due to steam is more severe than one due to hot water lies in the larger amount of heat released by the condensation of steam.

IV-7. 17 Determination of the latent heat of steam. Take a clean dry calorimeter and weigh it. Fill it more than half with water and weigh again. Note its initial temperature.

Boil water in a vessel *B* (fig. IV-7. 8) provided with a delivery tube leading to a steam trap *T*. From the trap only steam can enter into the calorimeter *C* through another tube. The condensed water will be arrested in the trap and can be drained off from time to time. A wooden screen separates the boiler *B* from the calorimeter *C*. Allow the steam to condense in the calorimeter for some time. Stir the water well and read off the final temperature after the supply of steam has been cut off.

Wait till the calorimeter and its contents reach the room temperature. Weigh them again. The difference from the second weight gives the mass of the steam condensed.

Let m g $=$ initial mass of water taken.

m' g $=$ mass of steam condensed.

W g $=$ water equivalent of the calorimeter.

$t_1\,°C$ $=$ initial temperature of water,

$t\,°C$ $=$ final temperature,

L $=$ latent heat of steam in calories per gram.

Then, the heat lost by the steam at $100°C$ to condense into water at $100°C = m' \times L$ cal.

Heat lost by the water so formed to cool through $(100-t)°C$

$$= m'(100-t) \text{ cal.}$$

Heat gained by water and the calorimeter

$$= (m+W)(t-t_1) \text{ cal.}$$

$\therefore\ m'L + m'(100-t) = (m+W)(t-t_1)$, from which L can be calculated.

Ex. IV-7. 7 11.5 g of steam at 100°C were passed into 480 g of water at 11°C. The temperature rose to 25°C. If the mass of the container is 190 g and the specific heat of its material, 0.1, find the latent heat

Solution Let L be the latent heat. Heat lost by 11.5 g of steam at 100°C to condense into water at $100°C = 11.5 \times L$ cal.

This water at 100°C falls through $(100-25)C°$ and the heat lost $= 11.5 \times 75$ cal $= 862.5$ cal.

Heat gained by 480 g of water

$= 480 \times 1 \times (25-11)$ cal $= 6720$ cal.

Heat gained by the vessel $= 190 \times 0{\cdot}1 \times (25-11)$ cal

Equating the heat lost to the heat gained we have

$11.5L + 862.5 = 6720 + 266$ whence $L = 532.5$ cal/g.

Ex. IV-7 8 20 g of ice at $-15°C$ are heated till fully turned into steam at 100°C. The steam is further heated to 110°C. Find the amount of heat necessary. Given, latent heat of ice $= 80$ cal/g ; latent heat of steam $= 538$ cal/g ; sp. heat of ice $= 0.5$; sp. heat of steam $= 0.48$.

Solution : Heat gained by ice in being heated from $-15°C$ to $0°C = 20 \times 0.5 \times 15 = 150$ cal.

Heat gained by ice in melting at $0°C = 20 \times 80 = 1600$ cal.

Heat gained by water to rise in temperature from 0°C to 100°C

$$= 20 \times 1 \times 100 = 2000 \text{ cal.}$$

Heat gained by water at 100°C for conversion into steam at

$100°C = 90 \times 538 = 10760$ cal.

Heat gained by steam in being heated from 100°C to 110°C

$= 20 \times 0.48 \times 10 = 96$ cal.

$\therefore$ Total heat required = 14606 cal.

Ex. IV 7.9 Steam at 100°C is injected into an ice calorimeter and 100 grams of water at 0°C were collected. Find the amount of ice melted, given, latent heat of steam = 537 cal/g. and latent heat of ice = 80 cal/g

Solution : Let x g of ice melt. Then steam condensed $= (100 - x)$ g.

Heat given out by the steam at 100°C in condensing into water at $100°C = 537(100 - x)$ cal.

Heat given out by water at 100°C in cooling down to 0°C $= 100(100 - x)$ cal.

Heat gained by ice at 0°C in melting into water at 0°C $= x \times 80$ cal

$\therefore \quad 537(100 - x) + 100(100 - x) = x \times 80$, whence $x = 88{\cdot}84$g.

IV-7.18 Cold caused by evaporation. A liquid cannot change to vapour unless the requisite amount of latent heat is supplied. Hence when a liquid evaporates, the portion that evaporates draws its latent heat from its neighbourhood including the rest of the liquid if there is no external source of heat. As a result the liquid and its surroundings are cooled.

This explains a number of phenomena which we come across in everyday life.

(i) When spirit is sprinkled on our skin, the spirit evaporates and takes the latent heat from the skin, which feels cold.

(ii) If after a bath we expose ourselves to a wind, or after perspiration sit below a fan the water on our body evaporates taking the latent heat from our body. We therefore feel cold.

In tropical countries an adult may perspire to the extent of a litre or more of water per day. A considerable part of it evaporates taking from the body the necessary latent heat, which is about 580 000 calories per litre. This is one of the ways our body gets rid of the superfluous heat.

(iii) Water in an earthenware pot is cooled in supplying the latent heat of evaporation to the water that comes out through the pores in the pot and evaporates.

(iv) The dentist sprays the gum with a liquid known as ethyl

chloride before he makes an injection. The rapid evaporation of ethyl chloride cools and 'freezes' the gum so as to make it less sensitive.

IV-7.19 Freezing by evaporation. (i) Pour a little water on a wooden block and place on it a thin tin can containing ether. Bubble air rapidly through the ether by means of a glass tube and hand bellows. The rapid evaporation of ether cools the water and causes it to freeze. The wooden block can then be lifted by the tin can.

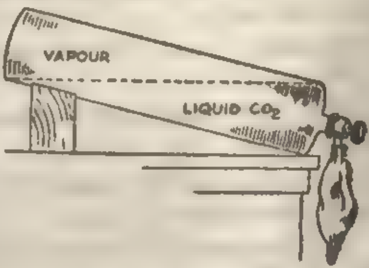


Fig. IV-7. 9

(ii) Preparation of dry ice, which is solid carbon dioxide, is an example of freezing by evaporation. Liquid carbon dioxide contained under pressure in a cylinder (fig. IV-7.9) issues in a jet through a valve. The rapid evaporation of the liquid cools it and a part freezes. This is collected in a muslin bag attached to the nozzle.

Dry ice has a temperature of $-78°C$. It is used for cooling. The ice cream vendor uses dry ice to keep his goods from melting.

Refrigeration. Use of refrigerators is now very common. You often see them in a druggist store, a sweetmeat shop or a household. They provide cold chambers, around 0°C or less, in which you keep food, sera, vaccines etc. for a long time without deterioration. Large rooms can be kept cool in the same way as a household refrigerator and used for large scale storage of meat, fruit and vegetables for months. Ships which transport such items of food from one country to another are provided with cooling plants in their holds as in refrigerators.

Cooling in these devices is produced by the evaporation of a liquid under reduced pressure in a set of coils called the *evaporator coils*. The coolant may be ammonia, sulphur dioxide, ethyl chloride or freon (CF_2Cl_2). The coolant is called the *refrigerant*.

An *ice machine* is in principle a refrigerator which cools brine (*i. e* a solution of salt in water). Tin cans containing pure water are embedded in the brine. Water in these cans is frozen by the cold brine.

HYGROMETRY

IV-8.1. Water vapour in the atmosphere. Evaporation is constantly taking place from the surface of water which covers more than two-thirds of the face of the earth. Many millions of tons of water are drawn up into the air every hour. Air must, therefore, always contain some water vapour. Though the quantity of water vapour is small compared with the other constituents of air, such natural phenomena as rain, dew, fog etc. are due to the condensation of moisture in air.

We feel comfortable when the moisture in air lies within certain limits *Good living conditions*, therefore, *require a controlled amount of moisture in the air.* A similar control is necessary for carrying out some industrial processes connected with cotton and wool industries, artifical seasoning of timber, manufacture of artificial silk etc. Besides, forecasting of weather requires a knowledge of the amount of moisture in the air. It, therefore, becomes necessary for us to determine the moisture content in air at different places and times. *Hygrometry* is concerned with the measurement of the amount of moisture in the air.

IV-8.2. Saturated and unsaturated vapour. Before proceeding further it will be worth while to go back to Sec. IV-7.9 where an experiment has been described to show that

(i) water vapour exerts pressure ;

(ii) there is a limit to the amount of water vapour that a given space can hold.

(iii) the vapour pressure as well as the amount of vapour in the given space increases with rise of temperature.

When a space contains the maximum amount of water vapour it can hold at a given temperature, it is said to be *saturated with the vapour.* Otherwise, it is *unsaturated.* The terms saturated and unsaturated are also applied to the vapour itself. The

maximum pressure which a vapour can exert at given temperature is variously called **saturated vapour** pressure (abbreviated SVP in this book). *maximum vapour pressure*, or simply, *vapour pressure* of the liquid at that temperature. It easily follows that at a given temperature, a saturated vapour exerts the maximum vapour pressure. If the pressure exerted by a vapour is less than the maximum it can exert at that temperature, the vapour is unsaturated.

If a closed space contains both a liquid and its vapour, the **vapour is saturated.**

IV-8.3 Dew **point.** Ordinarily air does not contain enough moisture to saturate it. If the air at any place is gradually cooled, it will tend towards saturation. At a certain temperature a given volume of air will be saturated by the moisture it contains. This temperature is called the *dew point*. Further cooling causes dew to appear. When you add ice to water in a tumbler, dew forms on the outside of the tumbler due to the cooling of the surrounding air below the dew point.

As the total pressure continues to be the local atmospheric pressure, the pressure due to the vapour actually present in the air is unaffected by this cooling. Since at the dew point the vapour is saturated, ***the saturation vapour pressure corresponding to the dew point is also the acual pressure due to the vapour present.***

The SVP of water vapour at different temperatures has been accurately determined and compiled into a table. If the dew point at any place at a given time be determined, the actual pressure of vapour in the air could be found by reference to the table.

Table *SVP of Water*

Temperature (°C)	Pressure (cm of Hg)	Temperature (°C)	Pressure (cm of Hg)	Temperature (°C)	Pressure (cm of Hg)
0	0.46	40	5.53	80	35.51
10	0.92	50	9.25	90	52 59
20	1.75	60	14.94	100	76 00
30	3.18	70	23.37	120	178.9

IV-8.4. Humidity of air. We often use the terms 'dry' or 'moist' with respect to air. The use of these terms is generally dictated by our *feeling* as to whether the air contains little or much moisture. In reality this feeling depends on two factors, *viz.*, (a) the actual amount of moisture present in the air and (b) the moisture necessary to saturate the air under the existing conditions. Strictly speaking, *our sensation of dryness or dampness of air depends more on the ratio of (a) to (b) than on the value of (a)* The humidity (or dampness) of air may be expressed in either of two ways *viz.*, (*i*) by the absolute humidity and (*ii*) by the relative humidity.

The **absolute humidity** is defined as the mass of water vapour present in unit volume of air. It is expressed in grams per cubic metre of air

Problem Air at 30°C and 90% humidity is drawn into an air conditioning unit and cooled to 20°C, relative humidity being reduced to 50%. How many grams of water vapour have been removed by the air conditioner from a cubic metre of air? (Absolute humidity at 30°C = 30g/m³ and at 20°C = 17 g/cm³.)

[Ans. 18·5g/m³.]

The **relative humidity** is the ratio of the actual mass of water vapour present in a certain volume of the air to the mass of water vapour required to saturate the same volume of the same temperature. It is usually expressed as a percentage. If 1 cu metre of air at a particular temperature contains 18 g of water vapour and if 30 g of vapour are required to saturate it at the same temperature, the relative humidity is 60%, while the absolute humidity is $18 g/m^3$.

Assuming that water vapour obeys Boyle's law up the point of saturation, the density of the vapour, and hence the mass present in a given volume, is proportionol to the pressure it exerts. This assumption is not quite correct, but the inaccuracy is not important for the purpose. This leads to another definition of relative humidity, *viz.*,

Relative humidity

$$=\frac{\text{actual pressure of water vapour present in the air}}{\text{SVP of water at the temperature of the air}}\times 100.$$

Since the actual pressure of the vapour present is equal to the saturation pressure at the dew point, we may also define relative humidity as

Relative humidity

$$=\frac{\textit{Saturation pressure of water vapour at the dew point}}{\textit{Saturation pressure at the temperature of the air}}\times 100$$

If the temperature of the air $=35^\circ$C. dew point $=26^\circ$C. S V P at $35^\circ\text{C}=42.14$ mm of mercury and S V P at $26^\circ\text{C}=25{\cdot}18$ mm of mercury we have

$$\text{R.H.}=\frac{25.18}{42.14}=0\ 598=59\ 8\%.$$

The rate of evaporation of water is determined by the relative humidity of air, and not by its absolute humidity. For comfort the relative humidity of the air in a room should be kept between 50 and 60%.

IV 8.5 **Measurement of humidity : Hygrometers**. They are instruments for measuring the humidity of air. There are different types, namely, (i) dew point hygrometers, (ii) absorption or chemical hygrometers and (iii) empirical hygrometers. Of fundamental importance in dew point hygrometry are tables of SVP of water and ice. We shall consider only one type of dew point hygrometer.

Dew point hygrometer : Regnault's hygrometer :

The action of dew point hygrometers is based on the definition of relative humidity as the ratio of two pressures. They are used with vapour pressure tables which give the SVP of water at different temperatures. The hygrometer determtnes the point.

In Regnault's dew point hygrometer, which is the best of the kind, a glass tube A (fig IV-8. 1) closed at its upper end by a stopper carrying a thermometer T, is fitted at its lower end with a silver cup B. Ether is poured into the cup and fills it completely. Air is sucked through the ether via the tubes CD. As the ether evaporates the silver cup is cooled until it reaches the dew point. Dew then deposits on it.

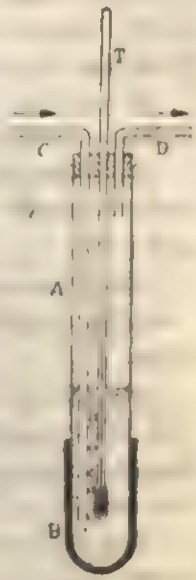


Fig. IV-8.1

As soon as dew is detected T is read. Another reading is taken when the dew disappears as the apparatus is allowed to warm up. The mean of these two values give the dew point.

For convenience of comparison, another similar tube with the silver cup only, is mounted on the same stand. Apperance of dew dulls the silver surface. Hence its appearance and disappearance can be well judged by comparison with the other silver cup. Observations are made from a distance with a telescope.

Properly handled, it gives accurate values for dew point.

If the dew point thus determined is $T''C$ and the room temperatue is $T^{\circ}C$, then.

$$\text{Relative humidity} = \frac{\text{Saturation pressure of water at } T''^{\circ}C}{\text{Saturation pressure of water at } T^{\circ}C} \times 100.$$

IV-8.6. Dew, fog and cloud.

Dew. During the day objects on the surface of the earth are heated by direct radiation from the sun, and the air in contact with them is also heated. During the night the bodies lose heat by radiation and those which radiate well, quickly cool below the the temperature of the surrounding air. Air in contact with them

cools too If in this way the air cools below the dew point, a portion of the vapour is deposited as dew on the surfaces of adjacent bodies.

It is evident that the following conditions are necessary for the deposition of dew :

(*i*) *A clear sky.* If there is cloud in the sky at night, cooling of bodies by radiation is impeded.

(*ii*) *Absence of wind.* If there is wind, air near a cold object will not remain in contact with it long enough to cool below the dew point.

(*iii*) *Presence of copious moisture in the air.* If the initial humidity is high it does not require much cooling to reach dew point.

(*iv*) *The presence of good radiators close to the earth.* Such objects cool rapidly and bring the air below its dew point. They should be close to the earth as otherwise the cooled air will sink downwards and be replaced by warmer air from above.

Dew deposited on grass and leaves of plants is formed by water vapour given out by the leaves when the air around them has been been cooled to the dew point.

Hoar-frost. When the dew point is below 0°C and the temperature of bodies lower still, water vapour in the air condenses directly as solid without passing through the liquid state. The solid is depsited on grass, etc. and is known as *hoar-frot.*

Fog and Mist. In a windless night the air at regions near the surface of the earth may be cooled to the dew point, when condensation of moisture occurs throughout the mass of air. The result is a *fog* or *mist.* There is no fundamental difference between two. A mist in which one can no longer see objects at a distance of one kilometre, is usually spoken as a fog.

The cooling is ordinarily due to radiation from the earth's surface. Hygroscopic particles play an important role in the formation of fog and mist.

Mountain mists or fogs are due to the contact of the cold air coming down from the mountain sides with the warm saturated air rising from the valleys.

Clouds. When warm, moist air rises from a large surface of water, it gradually cools as it rises higher into the atmosphere. When the temperature of the air as a whole is reduced below the dew point, a cloud is formed by the condensation of vapour in the form of small droplets. The nuclei of condensation are minute salt particles originating from the sea and present in the air in sufficient numbers.

A cloud is thus a fog or a mist formed high up in the atmosphere. For a fog or a mist the masses of air involved are often at rest or in very slow motion, but not so in a cloud.

The cooling which leads to the formation of a cloud may occur in a variety of ways. The chief causes are (i) the mixing of warm saturated air with a current of cold air and (ii) cooling by expasion as the air in its ascent moves to regions of lower pressure and increases in volume.

The formation of cloud due to the cooling effect of expansion of air can be shown by the following experiment. A bell jar as in fig. IV·8.2 is connected to an exhaust pump through a tube passing through its air tight stopper. A thermometer is also inserted. Partial vacuum is created within the jar by working the pump. When the inside and the outside temperatures become the same, some air is admitted from outside. The air expands suddenly and falls in temperature. If the admitted air is nearly saturated and contains dust particles, a cloud will be formed as soon as the air enters the jar. After a while the cloud disappears.

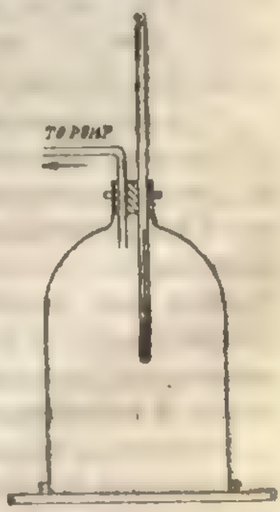


Fig I-V 8. 2

When air saturated with moisture cools on sudden expansion, water molecules preferentially condense on any ions that may be present. Wilson, an atomic phyisicist, constructed his 'cloud chamber' on this principle. This chamber enables us to see or photograph the track of even a single ion passing through it. The **Wilson cloud chamber** is a very important instrument it atomic physics. Wilson was awarded the Nobel prize for this invention.

Rain. The drops of water in a cloud coalesce and increase in size. Whether they will fall to the ground as rain, depends on the condition of the air below the cloud, the upward speed of the cloud and various other factors.

The speed with which a rain drop moves towards the earth depends on its size. It is not a free fall under gravity, as the resistance due to air increases with velocity. A drop actually falls with a limiting speed, which depends on its size. Rain drops cannot be larger than 5.5 mm in diameter. Larger drops break up into smaller ones as they fall. Drops of this size fall with a speed of 8 metres per second or about 20 miles per hour. Other drops fall more slowly.

For a rain drop to fall to the ground, (i) the upward velocity of the cloud must be lower than the limiting velocity of fall at the drop, (ii) it must not encounter an upward moving mass of air of higher speed and (iii) must survive complete evaporation during the fall.

Water drops in a cloud are constantly going through the process of forming into large drops, breaking up into smaller ones and coalescing once again *Every time a drop breaks there is a seperation of electricity.* This is perhaps the principal source of electricity in a thunderstorm. Consequently thunderstorms are associated with heavy rainfall.

IV-9

TRANSFER OF HEAT

IV-9.1 The three ways of transfer of Heat. There are three different ways in which heat can be transferred from one place to another, *viz.*,

(1) *Conduction.* When one end of an iron rod is put in a fire and the other end held in hand, it will be noticed that this end gradually becomes warmer. Heat (vibrational energy) has passed from molecule to molecule along the rod from the warmer to the cooler end without any sensible movement of the molecules. This process of transfer of heat is called **conduction.**

(2) *Convection.* When the heated rod is removed from the fire and the hand is placed a few inches *above* the hot end, it feels warm. Air, heated by contact with the iron carries the heat to the hand. Here the heat is actually carried by the particles of air, the movement of which is due to a change in density, warmer air being lighter than colder air. Transfer of heat due to the bodily motion of the warmer portions of a medium, brought about by a difference of density, is called **convection.** It should be clear that convection is impossible in solids, but occurs in liquids and gases.

(3) *Radiation.* If instead of holding the hand above the hot end of the rod, it is held a few inches *below* the rod, the hand feels warm, but not to the same extent as before. Heat could not have been transferred by conduction or convection. The process by which heat is transmitted under this condition is called radiation. The heat that reaches the earth from the sun is transferred by radiation. We may say that **radiation** *is the process by which heat passes from one body to another across a space without heating the space between the two.*

IV-9.2 Conduction of Heat, Transfer of heat by conduction requires a material medium. The particles of the medium take up heat energy from their neighbours on one side and hand it over to their neighbours on the other side. None of them leave their positions.

Substances differ in their ability to conduct heat. One end of a glass rod may be put in the fire and brought to red heat while the other end is held in the hand, But if instead of glass you take an iron rod it will not be possible to hold it for long. This shows that heat is more easily transferred by conduction through iron than glass. We say that conductivity of iron is higher than that of glass. You can easily hold a faggot burning at the other end.

IV-9 3 Comparing conductivities of different solids— Ingenhausz's experiment Conductivities of solids may be compared by Ingenhausz's method. The experimental materials (copper, iron, brass, glass etc.) are taken in the form of long thin rod, of *equal length, identical in area of crossesection* and in *surface finish* They are coated with wax and arranged as in fig. IV-9 1 with one end projecting into a tank in which water is maintained at the boling point.

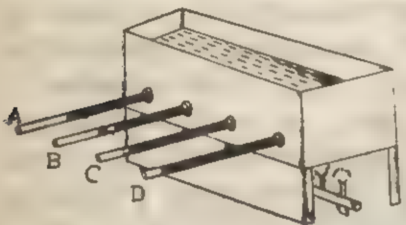


Fig IV-9 1

Heat is conducted from the water through the rods and melts the wax. After a steady state has been reached, the wax is observed to have melted to different distances along the rods. Conductivity is proportional to the square of the distance over which the wax melts.

Metals are the best of conductors, silver heading the list with copper as a close second. Solids are in general better conductors than liquids and liquids better than gases.

The low conductivity of water may be demonstrated in the following way. Load a piece of ice so that it sinks in water. Put it in a test-tube about two-thirds full of water (fig IV-9.2). Heat the tube near the top. The water at the top will be seen to boil before much of the ice has melted.


Fig IV-9 2

Conductivity of gases is very low. Copper conducts heat 700 times better than water, and water, 25 times better than air. Wood, cotton wool, felt etc. are bad conductors of heat and are used as thermal insulators. *They owe their conductivity to the innumerable air pockets that they enclose.*

IV-9.4 Thermal conductivity. Suppose heat is flowing in parallel straight lines through a medium. Consider a rectangular slab of material of cross-sectional area A perpendicular to the lines of flow and of thickness x in the direction of flow (fig. IV-9.3). The quantity of heat Q that passes through the slab in time t is

(*i*) proportional to the area A

(*ii*) proportional to the temperature difference $T_1 - T_2$ between the opposite faces

(*iii*) inversely proportional to the thickness x, and

(*iv*) proportional to t, the time of flow.

In symbols, we may write $Q \propto A \dfrac{T_1 - T_2}{x} t$ or $Q = KA \dfrac{T_1 - T_2}{x} t$,

where K is a constant for the material and is called its *thermal conductivity* (also *coefficient of thermal conductivity*) $(T_1 - T_2)/x$ is called the *temperature gradient*. If we put A, $(T_1 - T_2)/x$ and t each equal to unity in the equation for Q above, we get $K = Q$. In this way, we can define K by saying that thermal conductivity of a substance is the heat transferred in unit time by conduction through a plane of unit area under a temperature gradient of unity, the lines of heat flow being perpendicular to the plane.

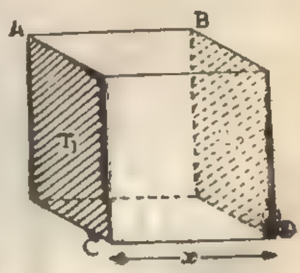


Fig IV-9 3

Units. The definition helps us in finding the units in which thermal conductivity can be expressed in a given system. In the cgs system Q will be in calories, A in cm^2, $T_1 - T_2$ in °C, x in cm and t in seconds. Thus

$$K = \frac{Q\,(\text{cal})\; x\,(\text{cm})}{A\,(\text{cm}^2)\; t\,(\text{s})\,(T_1 - T_2)\;(^\circ\text{C})} = \frac{Qx}{At\,(T_1 - T_2)}\,\frac{\text{cal}}{\text{cm s}^\circ\text{C}}$$

Thus the cgs unit of K is 1 cal/cm s °C or cal $cm^{-1}s^{-1}C^{-1}$. In the sps system, it is Btu $ft^{-1}s^{-1}{}^\circ F^{-1}$ In SI units, it is $Jm^{-1}\,s^{-1}$ $^\circ K^{-1}$.

If we say thermal conductivity of copper is 0·92 in cgs units, we mean that 0.92 cal of heat will pass perpendicularly through a plane of unit area in copper in one second it the temperature gradient at the position of the plane is 1°C per cm. *Temperature gradient* is the temperature difference between two planes unit distance apart, the planes being perpendicular to the direction of heat flow. It is also the rate at which temperature changes with distance.

Searle's method of determining thermal conductivity.

The method is applicable only to good conductors. A diagram of the apparatus is given below.

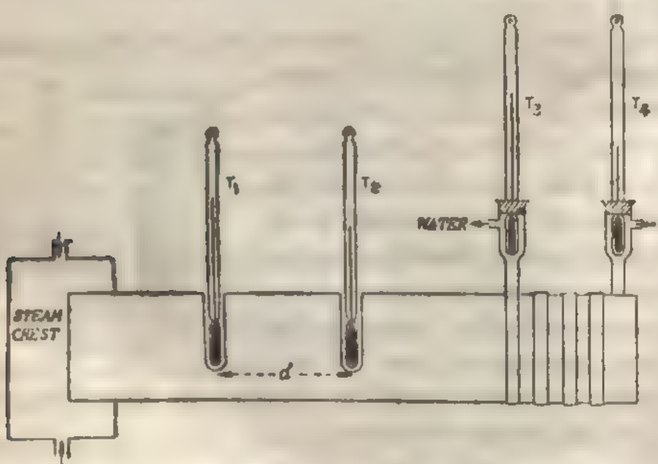


Fig IV-9. 3. b

A copper bar about 5 cm in diameter is polished, surrounded by dry felt and enclosed in a wooden box. Steam circulating

through a chamber attached to one end of the bar maintains this end at 100°C. A steady flow of cold water is kept passing through a copper spiral soldered round the other end. It keeps this end at a steady lower temperature. This enables the rate of flow of heat through the bar to be measured. Thermometers T_1 and T_2 are placed in holes in the bar at a known distance d apart. The holes contain mercury to ensure good thermal contact. Two more thermometers T_4 and T_3 record the inlet and outlet temperatures of flowing water.

When all thermometer readings are steady, the number m of grams of water flowing through the spiral in a known time t seconds is noted. The temperatures are all recorded.

Let θ_1 and θ_2 be the steady temperatures indicated by the thermometers T_1 and T_2, and θ_4 and θ_3, the inlet and outlet temperatures respectively. Then the heat flowing through the bar in t seconds is $m(\theta_3-\theta_4)$. If r is the radius of the bar, its area of cross-section is πr^2. Therefore, we shall have from the definition of thermal conductivity

$$m(\theta_3-\theta_4) = K\pi r^2 (\theta_1-\theta_2)t/d.$$

Since all values except K are known, it can be calculated. Using cgs units with t in °C, we get K in cgs units.

(iii) It may be noted that ***good thermal conductors, such as metals, are also good electrical conductors.*** In fact, the ratio of the thermal conductivity to the electrical conductivity is the same for all metals at a given temperature. This ratio is proportional to the absolute temperature. (The result is known as Wiedemann-Franz law.) Most pure metals obey this law with reasonable accuracy at ordinary temperatures.

The sameness of the ratio shows that the same process must be responsible for conduction of heat and electricity in metals. In metals, free electrons conduct electricity. Therefore, they must also be responsible for thermal conduction in metals.

Table of thermal conductivities in cgs units
(cal cm^{-1} s^{-1} °c^{-1})

Aluminium	0.50	Lead	0.083	Cork	0.00011
Brass	0.26	Mercury	0.020	Asbestos	0.00030
Copper	0.92	Water	0.0014	Brick	0.00003
Silver	0.97	Glass	0.0025	Felt	0.00006
Gold	0.72	Oak	0.0006	Air	0.00006
Iron	0.115	Sand	0.00013		

Example IV-9.1 Find the quantity of heat that will be transmitted in 10 minutes across a plate of copper 1 sq. metre in area and 5 cm thick, the difference between the temperatures of its faces being 10°C, (K of copper$=0.9$ cgs unit).

Solution : $Q=KA(T_1-T_2)t/x$.

Here $K=0.9$, $A=100^2$ or 10^4 cm^2. $T_1-T_2=10°C$, $t=600s$, $x=5cm$.

$\therefore \quad Q=0.9\times10^4\times10\times600/5=1.08\times10^7$ cal.

Note that the temperature gradient here is $(T_1-T_2)/x$ $=2°C/cm$.

EX IV-9. A kettle of well-thickness 1 mm and of total outside area 1000 cm^2 is filled with water temperature of which is maintained at 100°C. It is found that 120 kg of ice melt in 1 min. when the kettle is fully immersed in a tub containing melting ice. Calculate the thermal conductivity of the material of the kettle, given that latent heat of ice$=80$ cal/g

Solution : Here the temperature difference between the two faces of the walls of the kettle is 100°C. Since 120 kg of ice melt in 1 min. the heat transmitted through the kettle in this time is 120,000 × 80 cal. If K is the required conductivity we have

$$120{,}000\times80 \text{ cal}=K\times1000 \text{ cm}^2\times100°C\times60 \text{ s}/0.1 \text{ cm}.$$

$$\therefore \quad K=\frac{10{,}000\times80 \text{ cal}\times0.1 \text{ cm}}{1000 \text{ cm}^2\times100°C\times60s}=0.16 \text{ cal cm}^{-1} \text{ s}^{-1} °C^{-1}$$

IV-9.5. Some consequences of thermal conduction. (*i*) When we alternately touch a piece of iron and a piece of stone, both lying in the sun, the iron feels warmer than the stone though their temperatures are the same. This is due to the better conducti-

vity of iron. Heat flows more quickly from it and produces an impression of greater warmth. When their tempeatures are lower than that of our body, the iron gives the impression of being cooler due to the quickness with which heat is transferred from the body to it. If a piece of metal in arctic cold be touched with a finger, the finger sticks to the metal due to the large and quick transference of heat.

(*ii*) **Woollen and cotton garments**. The 'warmth' of woollen garments is due to the innumerable air pockets they enclose. Because of the very low thermal conductivity of air. the garment as a whole has a low conductivity and retards the flow of heat through it. Two garments worn together will be more effective in preventing heat transference than only one of the same total thickness. The layer of air between the two further resists the flow of heat.

Woollen fibres are rough and crooked. So they can enclose innumerable air pockets. Cotton fibres are straight and smoother than wool. So they cannot form such air pockets. Hence it is easier for heat to pass through cotton garments than woollen garments.

(*iii*) **Water can boil in a paper bag.** The temperature at which paper catches fire is much higher than that of boiling water. This makes it possible to boil a small quantity of water in a thin paper bag. Much of the heat supplied to the paper passes through it and while this heat raises the water to its boiling point, the temperature on the lower side of the paper remains below that at which the paper ignites.

(*iv*) **Copper gauze and flame.** When a clean copper gauze or spiral is lowered in the flame of a burner, it will be found that the flame does not get through the gauze (fig. IV-9.4). The gauze conducts the heat away from the flame so rapidly that the temperature on the other side of the gauze is lowered below that at which the gas burns. Combustion is therefore confined to the

part of the flame below the gauze. *When the gauze itself is heated to the ignition temperature, it ceases to be effective in preventing the combustion of the gas above it.*

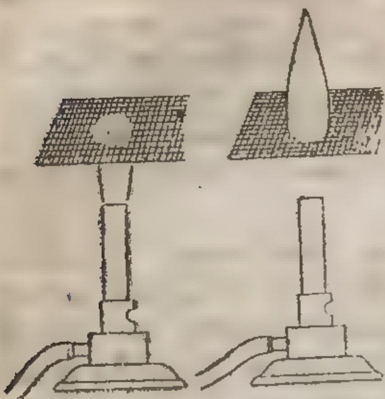


Fig. IV-9.4

When the gauze is held about an inch above Bunsen burner and the flame lighted from top, it burns above the gauze (fig. IV-9.4). The conductivity of the gauze prevents the flame from extending below gauze.

(*v*) The high thermal conductivity of copper makes it the material of choice in the construction of cooking utensils or small boilers, but its cost is relatively high. Aluminium being lighter and a better thermal conductor than iron is preferred for the construction of piston heads and engine cylinders.

(*vi*) **Davy's safety lamp.** The principle illustrated in the flame-and-gauze experiment (fig IV-9.4) was utilized by Sir Humphrey Davy for the construction of a safety lamp for coal miners.

It can be used in mines without a risk of explosion even when combustible gases are present. It is an oil lamp with a wire gauze *B* (Fig IV-9.5) arranged above the flame. The flame itself is surrounded by a glass cylinder *A* to increase the illuminating power of the lamp. Air enters the lamp at *C* and burnt gases leave the lamp at *D*.

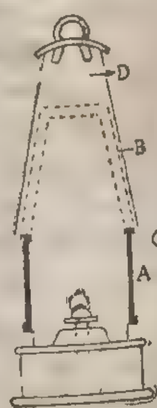


Fig IV-9.5

In an explosive atmosphere the gases which penetrate to the flame are ignited and burn inside the gauze with a peculiar blue flickering. But the flame produced is unable to extend outwards through the gauze. The gauze conducts the heat away from the burnt gases so quickly that the explosive gas outside the lamp never becomes hot enough to explode.

Now-a-days miners use electric lamps. But the leader carries a Davy lamp to detect the presence of 'fire damp' (an explosive gas). If present in strong concentration, the gas burns with a blue flame.

(*vii*) Asbestos, a fibrous mineral is a bad conductor and is non-inflammable. It is extensively used for lining the walls of cookers and refrigerators. Boilers and steam pipes are often covered with an asbestos cement (asbestos fibre and plaster) to reduce leakage of heat. But it carries health hazard.

(*viii*) Hard chalk deposits called ***scales*** are found on the inside of boilers and kettles in areas where hard water is used. The deposit is a bad conductor. Unless it is removed from time to time much wastage of fuel is inevitable.

(*ix*) An ice box or a refrigerator is constructed with a double wall. The space inside may be filled with sawdust or cork powder Ice kept in sawdust or a piece of blanket melts slowly because of the bad conductivity of the innumerable air pockets enclosed between particles of saw dust and blanket pones.

(*x*) Cotton-wool, asbestos-wool, cork, felt, cellular glass etc. are some of the best non-conductnrs of heat. They owe this property to the innumerable air pockets that they enclose. Their conductivities are a little higher than those of air. Such materials. used for heat insulation, are known as ***thermal lagging***.

IV.9 6 **Convection.** Convection is possible only in liquids and gases but not in solids. When a fluid is heated it becomes lighter and ascends, while its place is occupied by cooler portions of the fluid. The current so set up within the fluid is called **convection current.** The ascending hot current carries the heat along with it. This process of transfer of heat is called **convection.** It differs from conduction, since in the latter the medium through which heat flows, is not displacd.

Simple experiments on convection. (*i*) Drop a tiny crystal of

potatsium permanganate into a beaker of water wh·ch is being heated by a burner from below. Coloured water will be seen to move from the bottom up the middle and down the sides (fig. IV-9.6). This shows that the water particles themselves carry heat from the bottom to the top while cooler particles come down to be heated and then move upward again.

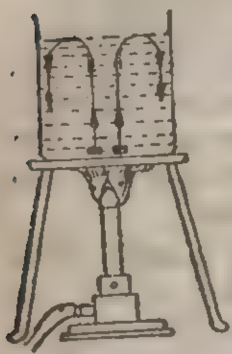


Fig IV-9. 6

(*ii*) When a candle burns strong, a convection current developes in the air around the flame (fig.IV. 9. 7.) The air in contact with the flame is heated and ascends, while the surrounding cold air moves up to the flame. Thus the flame gets a continuous supply of oxygen.

Fig IV-9.7

(*iii*) In hurricane lamps it is also the convection current that feeds the flame continuously with oxygen. Fig. IV-9. 8 shows the path of the air current .The burnt gases leave at the top.

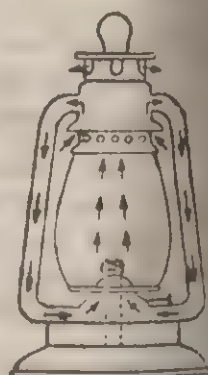


Fig IV-9.8

(*iv*) **Cooling system of an automobile engine.** The water, cooling the engine of a motor car is kept in circulation by convection. The water flows in pipes surrounding the engine (fig. IV-9. 9) As this water is heated by taking heat from the engine it rises to a tank at the front of the bonnet. Before returning to the engine it flows through small pipes cooled by air from a fan driven by the engine.

(*v*) When a burning candle is placed inside a chimney in a way that does not allow air to enter through the bottom (fig. IV.

9.10) the flame soon goes out owing to lack of oxygen. If a T-shaped card-board or metal sheet is placed as shown in the figure

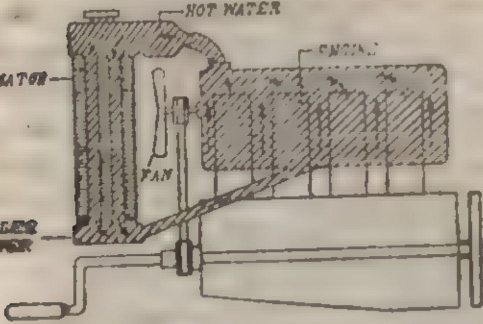


Fig. IV-9.9

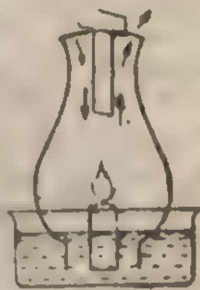


Fig IV-9.10

the candle continues to burn. The insertion of the sheet makes possible the flow of convection currents as shown.

(*vi*) *Ocean currents* are convection currents due to unequal heating of the water masses in the oceans by heat from the sun. Winds are also huge convection current due to the same cause.

(*vii*) **Land and sea breezes.** Specific heats of rocks and soils are smaller than that of water. So when the sun rises and shines on neighbouring masses of land and sea, the land warms up more quickly than the sea. The air above the land becomes warmer than that above the sea. This warm air rises and makes room for the flow of cooler air from the sea. The latter is known as *sea-breeze*.

As the sun sets, both land and sea cool down, but the land cools quicker than the sea. The air above the sea becomes warmer than that above the land. The warm air rises and makes room for the flow of cooler air from the land to the sea. This is known as *land-breeze*.

Note that these breezes are due to the specific heat of water being greater than that of rocks and soil.

(*viii*) **Ventilation.** Ventilation is the process by which polluted air is replaced by fresh air without draughts Convection currents produce ventilation. The principle of ventilation is illustrated in fig. IV-0.11. Two glass chimneys are fitted into the top of a wooden box provided with a glass front. In the box, just below one chimney, there is a lighted candle. The heat from it creates a convection eurrent. To demonstrate it hold a burning cigarette over the top of the otner chimney. The path of the smoke will show you the path of the convection current.

Formerly, coal mines were ventilated on this principle. The mine had two shafts At the bottom of one a fire was lighted. This caused the air to rise up the shaft. Air travelled down the other shaft and through the mine to maintain the updraught.

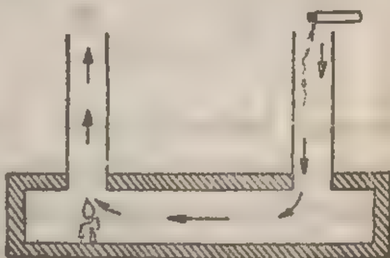


Fig. IV-9. 11

Breathed out air is generally warmer than the air in a living room. It therefore rises and escapes through the window or the ventilator Windows open at the top are intended to allow the warm air to escape. Circulation in a room is maintained by air flowing in at the bottom of open doors and windows. and leaving at the top.

As you learn more, you will come to understand that conduction and radiation are the two basic modes of heat transfer. They obey different laws. Convection is a combination of conduction, radiation and fluid flow.

IV-9-7. **Radiation.** In radiation heat passes from one body to another without the help of any material medium. The vast distanee of 93 million miles from the sun to the earth is a void.

(The density of matter in this region does not exceed one or two molecules per cm^3.) Heat cannot therefore come from the sun to the earth by conduction or convection. It reaches the earth by radiation. In reaching the earth, heat from the sun passes through the earth's atmosphere, but this does not heat the air. Maxwell defined radiation as the *transfer of heat from a hot body to a cooler body without appreciable heating of the intervening space.*

Experiment to show transference of heat by radiation ; A heating coil is suspended inside a bell jar through its lid (fig IV-9. 12). Air from inside the jar can be pumped out by an exhaust pump. A thermometer is placed outside the jar. As the pump works a vacuum is created within the jar. The thermometer still shows a rise in temperature. There is no medium in the bell jar and as such the transference of heat by condnction or convection cannot take place. So there must be a third process by which heat is transferred. This process is radiation.

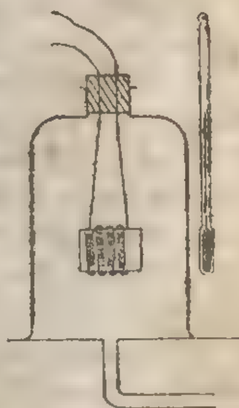


Fig IV-9.12

The form of energy when heat is transferred from one body to another by radiation is called **radiant heat, heat radiation** or **thermal radiation.** We now know that it is of the *same nature as light* and has all properties of the latter except that it does not produce the sensation of sight.

IV-9.8. **Radiant heat obeys the same laws as light.** That radiant heat has the same nature as light will be clear from the similarity of their behav our as stated below.

(*i*) Both radiant heat and light are transmitted through vacum with the same speed, During a solar eclipse light and heat from the sun are cut off at the same time. When the eclipse is over they reappear at the same time. Clearly they move with the same speed across the vast void separating the earth from the sun.

(*ii*) **Radiant heat travels in strright lines as light does.** When an opaque body (an umbrella) is held in the sun, it casts a shadow, where no heat is felt. As shadows are due to the rectilinear propagation of light and no heat is felt in the shadow, heat must have traveled in straight lines.

(*iii*) The intensity of light varies inversely as the square of the distance. Radiant heat obeys the same law. When the distance between a Leslie's cube and a thermopile is doubled the deflection of the galvanometer reduces to a quarter of its value. (Leslie's cube is a simple device for producing thermal radiation at about 100°C. A thermopile is a sensitive device for detecting thermal radiation. It works in conjunction with a galvanometer.)

(*iv*) When radiant heat is reflected from a polished metal surface, its angle of reflection is equal to the angle of incidence.

(*v*) Heat radiation may be focussed at a point by a concave mirror or a lens in the same manner as light.

IV 9.9. Absorption and emission of radiation. The amount of heat lost from a body by radiation depends on.

(*a*) its temperature and that of the surroundings, being proportional to the difference when it is not large ; (*b*) the nature of its surface ; (*c*) the area of the surface ; and (*d*) the time.

When radiation falls on matter, a part is absorbed, a part reflected and a part transmitted. If a, r and t respectively denote the fractions of the incident radiation absorbed, reflected and transmitted, we shall obviously have.

$$a+r+t=1.$$

It will be seen from this equation that *a good absorber*, such as lamp black ($a=0.98$) *is a poor reflector*, and that a *good reflector*, such as a polished metal surface, *is a poor absorber*. Absorbed radiation raises the temperature of the absorber.

It has been found that **good absorbers of radiation are also good emitters.** Black is a better emitter or absorber than white. There is thus good reason for wearing white clothes in the tropics ; it absorbs little heat. A rough surface absorbs or emits radiation

more than a polished surface. Calorimeters, therefore, have polished surfaces so that loss or gain of heat by radiation may be a minimum.

Cloud and radiation. Water droplets in the air absorb much of the solar heat. On a cloudy day in summer the temperature of the earth is comparatively low. On a clear day the temperature rises high. Clouds have a screening effect on the earth. Presence of clouds at night prevents radiation loss from the earth.

Greenhouse. Glass absorbs longer heat rays, but transmits shorter rays. Long heat rays radiated by plants in green housee are absorbed by glass and re-radiated into the greenhouse. A greenhouse thus acts as a *heat trap.*

IV-9.10 Prevost's theory of exchanges. The energy that a body radiates by virtue of its temperature is called **temperature radiation** or **thermal radiation.** [Part or whole of the energy that a body receives from its surroundings (or any other object) which goes only to raise its temperature, but produces no other change, is given out by the body as thermal radiation] It will be wrong to assume that a body does not radiate unless it is heated. Suspend a slab of ice in a vacuum so as to reduce to a minimum the gain of heat by conduction and convection. It receives heat by radiation from the surroundings (if at a higher temperature than that of ice) and melts. If dry-ice (solid carbon di-oxide) is similarly kept surrounded by ice, it evaporates. It cannot be logical to assume that ice radiates in the latter case, but not in the former. It is more reasonable to think that radiant energy is given out by ice in both cases, Melting takes place in the first case because the ice receives more heat by radiation than it gives out. In the second case, it gives out more heat by radiation to the dry-ice than it receives from the latter.

Prevost applied the idea of dynamic equilibrium to the thermal radiation emitted or received by a body. If the body is at a higher temperature than its surroundings, it will radiate more

energy than it receives and will be cooled until temperature equilibrium is established. If the body is at a lower temperature it will receive more energy than it radiates and will rise in temperature. If it receives as much energy as it radiates, there will be temperature equilibrium. **The difference of the radiant energy absorbed and the radiant energy emitted is 'heat'.**

Consideration of such exchange of radiation led Prevost (1751-1839) to propose that.

"A body radiates heat at a rate which depends only upon its surface and temperature, and that it absorbs heat at a rate which depends only upon its surface and the temperature of its surroundings. When the temperatures are the same the rates of absorption and emission will be equal."

Put in other words, Prevost's theory states that

The amount of energy emitted as radiation from a body depends solely upon what takes place inside the body and is not influenced by its surronndings.

Prevost's theory proved very valuable in establishing the laws of thermal radiation. One of the earliest of these laws is Stefan's law. It is discussed below.

IV-9.11 **Stefan's law of radiation.** By virtue of its temperature every body radiates some energy in all directions in the form of electromagnetic waves. Such radiation is called 'temperature radiation' or 'thermal radiation'. The rate of radiation per second per unit area depends upon (*i*) the temperature and (*ii*) the nature of the surface.

Tyndall was the first to carry out some experiments on the amount of energy radiated by a hot platinum wire to its surroundings kept at a fixed temperature. From these experiments Stefan (1879) concluded that ***the net energy interchange by radiation between a hot body at temperature T_1 K and a colder body at temperature T_2 K is proportional to $T_1^4 - T_2^4$.*** This is known as Stefan's law of radiation.

IV-9.12. Black body. Later Boltzmann (1844-1906) deduced Stefan's law on theoretical grounds. To eliminate the effect of the nature of the surface, an ideal radiator had to the imagined. It is called a 'black body' and has the property that the thermal radiation it emits per unit time per unit area is greater than that emitted by any other body at the same temperature. Theoretically, it was found that energy radiated by a black body per unit area per second is given by $E=\sigma T^4$ where T is the absolute temperature of a black body, and σ is a constant of value 5.67×10^{-5} erg/cm^2 s. K^4. It is called the Stefan-Boltzmann constant.

An actual body radiates less than an ideal black body at the same temperature. But the radiation from a black surface or a nearly closed space (such as an oven or a furnace) is close to that from the ideal black body. This enables us to calculate the temperature of an incandescent body with the help of Stefan's law. The sun's temperature has been so measured. The temperature of the visible surface of the sun was found with the help of Stefan's law to be about 6160 K. Instruments called 'total radiation pyrometers' can measure temperatures above 800 C. They work on the basis of Stefan's law of radiation, and are used in industries for measuring temperatures of furnaces and other very hot bodies. Such temp. measurement is called *Pyro-metry* (*Pyros*-fire, *metry*—measurement).

HEAT AND WORK

IV-10. 1. **Heat is a form of energy.** To the question 'What is heat ?' we have long ago answered by saying that 'Heat is a form of energy , it is not matter'. A body weighs the same whether it is hot or cold. If heat were a form of matter, the body would have weighed more when hot.

It is more difficult to show that heat is a form of energy. We have, however been led to this conclusion by a variety of phenomena. When we hammer a piece of metal or bend a wire back and forth, it gets hot. Pressing a piece of steel on a grinder produces a crop of sparks. Examples of this kind where heat is produced by motion can be multiplied without limit. Steam engines (and other engines too) produce motion from heat. All these show that kinetic energy and heat are intimately related and one may be converted into the other. Heat must, therefore, be a form of energy.

We now *associate* temperature with the mean kinetic energy of motion of molecules, absolute temperature being proportional to the mean kinetic energy of molecules according to the kinetic theory. As the temerature rises, the kinetic energy of a molecule —whether in a solid, liquid or gas—also increases. Increase in the pressure of a gas with increase in temperature, heating of a gas by sudden compression and cooling by sudden expansion, cooling of a liquid by evaporation, etc. support this view.

But what is the form of heat energy ? We now define heat as being *energy in transit due to temperature difference.* The molecules which make up a piece of matter have both kinetic and potential energies. This total energy is called the **internal energy** of the body. If there are two bodies whose average molecular kinetic energies are different, they are at different temperatures.

If these bodies are put in contact, some of the kinetic energy from the stock of internal energy of the warmer body passes on to the cooler body and increases the internal energy of the latter. This goes on until the average molecular kinetic energies of both bodies are equalized. The flow of energy then ceases. The energy that has been so moving, is heat. When the flow is over the energy *ceases to be heat*, it becomes part of the internal energy of the cooler body. You may compare 'heat' with rain drops falling into a lake. So long as they fall in the form of drops they are rain drops. As soon as they reach the lake, they become part of the water of the lake.

Remember the case of kinetic energy. It is kinetic energy so long as the body is *moving*. As soon as the body stops, kinetic energy is transformed into some other form, and ceases to be kinetic energy. Similar consideration applies to heat.

We say mechanical energy is converted into heat, as in compressing a gas. When work is done on a gas, that is, it is compressed, its volume diminishes and pressure increases. This increases the mean kinetic energy of the gas molecules. So the temperature of the gas rises. When a body increases in temperature, we say heat has been added to it, the amount of heat being given by the relation $Q=mst$ So, a temperature rise of t is equivalent to the addition of an amount of heat Q. Really there has been an *increase of internal energy* of amount Q

IV-10-2. The mechanical equivalent of heat. When we rub our hands together, heat is produced : friction always produces heat. This shows that mechanical work can be converted into heat. The equivalence between heat and mechanical work was established by Joule whose experiments show that (*i*) **work can be fully converted into heat** and (*ii*) **whenever mechanical work is fully converted into heat, the heat generated is directly proportional to the work done.** The equivalence is independent of the way in which the work is derived or the means by which the transformation from work to heat is brought about.

If W units of mechanical work, on *full* conversion into heat produce Q units of heat energy, then

$$\frac{W}{Q} = \text{constant} = J \qquad (17.2.1)$$

where the constant J is called lhe **mechanical equivalent of heat** or **Joule's equivalent**. The numerical value of J however depends on the units in which W and Q are expressed. If W is in ergs and Q is in calories.

$$J = 4.2 \times 10^7 \text{ ergs per calorie.}$$

(More accurasely $J = 4.1852$ joules/calorie)

This means that when 4.2×10^7 ergs of work are *fully* converted into heat, the heat developed is one calorie, which can raise the temperature of 1 g of water by 1°C.

If W is in joules and Q in calories, we have

$$J = 4.2 \text{ joules per calorie.}$$

If W is in foot-pounds and Q in British thermal units

$$J \doteq 778 \text{ ft-lb per Btu.}$$

If Q is measured in lb. °C (centigrade heat unit)

$$J = 1400 \text{ ft-lb per lb °C.}$$

Thus if 1400 ft-lb of mechanical work, derived in any way, be *fully* converted into heat, it will raise the temperature of 1 lb of water through 1°C.

Note that using the joule as the unit of heat makes, use of the term mechanical equivalent of the heat unnecessary. In SI units, the joule is the unit of heat, heat being a form of energy. So, when SI units are used, $J = 1$.

Example IV-10. A mass of 10 g moving with a speed of 300 m. per sec is suddenly brought to rest. If the whole of its kinetic energy is converted into heat, find (i) the heat developed and (ii) the rise in temperature of the body, assuming that the heat developed is fully retained by it. (Sp. heat = 0,03).

Solution : Speed of the body = 30,000 cm/s :

$$\text{kinetic energy} \quad = \tfrac{1}{2} \times 10 \times (30{,}000)^2 = 4.5 \times 10^9 \text{ erg.}$$

$$Q = \frac{W}{J} = \frac{4.5 \times 10^9 \text{ erg}}{4.2 \times 10^7 \text{ erg/cal}} = 107.1 \text{ cal}$$

Since heat gained = mass × sp. heat × temp. rise,

$$\text{the rise in temp. of the body} = \frac{107.1}{10 \times 0.03} = 357°\text{C.}$$

Ex. IV-10.2 From what height must a gramme of ice at 0°C fall in order to melt itself by the heat generated in the impact ?

Solution : In the absence of any statement to the contrary it it assumed that the whole of the heat developed is retained by the body.

Let *h cm* be the required height. The potential energy of the body at that height is $mgh = 980\ h$ ergs. When it reaches the ground this energy is fully kinetic. On impact, it is converted into heat. The heat developed should be 80 calories, the latent heat of 1 gramme of ice.

$$\therefore \quad 80 = \frac{980h}{4.2 \times 10^7} \text{ or } h = 3.43 \times 10^6 \text{ cm } (= 34.3 \text{ km }).$$

Ex IV-10. 3. A lead bullet moving at 400 m/s strikes a target. Find its rise of temperature assuming that the heat produced is shared equally by the target and the bullet. Sp. heat of lead = 0.03.

Solution : Kinetic energy $= \frac{1}{2} m \times (40{,}000)^2$ ergs, where m is the mass of the bullet in g.

$$\text{Heat developed} = \frac{\frac{1}{2} \times m \times (40\,000)^2}{4.2 \times 10^7} \text{ cal}$$

Since the heat is equally shared by the target and the bullet half of this heat is available for raising the temperature of the bullet.

$$\text{Rise in temperature} = \frac{\text{Heat}}{\text{mass} \times \text{sp. heat}}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2} \times m \times 16 \times 10^8}{4.2 \times 10^7 \times m \times 0.03} = 317.5°\text{C.}$$

Ex IV-10. 4. The Victoria Falls are 343 ft in height. Calculate the differnce in temperature of the waters at the foot and at the top, assuming a value of 778 ft. lb per Btu for J.

Solution: Consider a mass m lb of water The potential energy of this mass at the top of the falls is 343 m ft.lb. When converted into heat this will give $(343m/779)$ Btu. If t is the temperature rise of water, then, since the sp. heat of water is unity, we have

$$mt = 343m/778 \text{ or } t = 343/778°F = 0.44°F.$$

IV-10.3. Joule's experiment for the determination of J. The first exact determination of the mechanical equivalent of heat was made by Joule with the apparatus shown in fig. IV-10.1, C is a cylindrical copper vessel containing water. Four radial partitions or vanes are fixed vertically inside the cylinder. A paddle can be rotated in the water about a vertical spindle S. It carries eight sets of vanes which pass through spaces cut in the radial partitions as the paddle rotates. In the figure the radial partitions are reprsented by V_1 and the movable vanes by V_2. To turn the spindle (S) it is attached by a pin (P) to a rod (R). Two threads wound round the rod (R) pass over two pulleys (P_1, P_2) and may be loaded at the other ends with known weights.

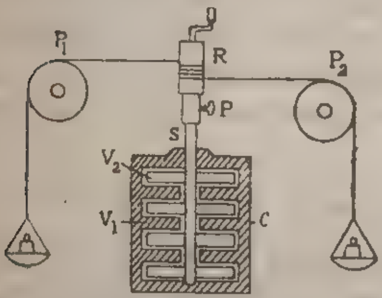


Fig IV 10. 1

When the weights are allowed to fall under gravity, the rod rotates and so does the paddle. As the water is churned its kinetic energy is destroyed at the radial vanes(V_1) and converted into heat. When the weights reach the ground, the pin (P) is detached and the weights are wound up again by turning a handle attached to the rod (R). As the churning continues watnr is gradually heated.

The elevated weights possess potential energy. When they reach the ground they have some kinetic energy.* The difference of the two energies gives the kinetic energy imparted to the water. This is converted into heat.

Let the mass of each load attached to a string $= m$ g,
distance through which each weight falls $= h$ cm,
acceleration due to gravity $= g$ cm/s^2
total number of times each weight is made to fall $= n$,
speed of a weight when it reaches the ground $= v$ cm/s,
water-equivalent of the calorimeter with its contents $= W$ g,
initial temperature of the water $= t_1$°C,
final temperature $= t_2$°C.

Then, the potential energy of the weights $= 2mgh$,
their kinetic energy at the end of the fall $= 2 \times \frac{1}{2}mv^2$,
the mechanical energy converted into heat at each fall $= m(2hg - v^2)$,
the total energy converted into heat in n falls $= mn(2gh - v^2)$,
the total heat developed $= W(t_2 - t_1)$ cal.

$$\therefore \quad J = \frac{\text{mechanical work transformed}}{\text{heat generated}} = \frac{mn(2gh - v^2)}{W(t_2 - t_1)}$$

Corrections were made for the frictional losses and the loss due to radiation from the calorimeter. As in all calorimetric work, conduction and convection were minimzed.

In one of Joule's experiments the weights were 30 lb each and fell 20 times in all, each time through an effective height of $5\frac{1}{3}$ ft. The water equivalent of the calorimeter and its contents was 14 lb and the rise in temperature, 0.50°F.

$\therefore$ Heat generated $= 14 \times 0{\cdot}50 = 8{\cdot}26$ Btu.
Work done $= 2 \times 20 \times 30$ lb $\times 5\frac{1}{3}$ ft $= 6400$ ft-lb.
whence $J = 6400$ ft-lb $\div 8.26$ Btu.
$= 775$ ft-lb/Btu.

*Due to resistance offered by water to the motion of the paddles, they fall with a terminal (constant) velocity v. v can easily be measured during the fall.

More accurate experiments to find the value of J were performed by Callendar, Rowland and others.

Since heat is energy, it can well be measured in terms of joules. In that case there would be no necessity of having any quantity named 'mechanical equivalent of heat' nor any separate heat unit such as the calorie. Many modern writers use 'joule' instead of 'calorie', 1 calorie being put equal to 4.2 joules (or more accurately, 4.1855 joules).

IV 1.4 **First law of thermodynamics.** When heat is added to a body, the internal energy of the body increases. The body can also do some work if conditions are appropriate. Take for example a gas contained in a cylinder and fitted with a gas-tight piston If a quantity of heat is supplied to the gas its temperature will increase. The pressure will also rise. The increased pressure will push the piston outward against the atmospheric pressure until the gas pressure equals the pressure outside. Thus, on addition of heat the gas increases in internal energy and also does some work.

Let Q be the heat added to a body, U' the *increase* of internal energy of the body and W the work done by the body. The relation between these these three quantities is given by

$$Q = U' + W \qquad \text{(IV-10 4.1)}$$

The quantities must all be expressed in the same units. They may be either heat units or work units namely, calories or joules.

The relation is known as the *first law of thermodynamics.* It covers all cases of conversion of heat into work and vice versa. Q is positive when heat is added, U' is positive when the internal energy increases and W is positive when work is done by the body.

Note that in the worked examples in Sec. IV-2 or in Sec. IV-3, there is no heat flow, that is, flow of energy due to temperature difference. Still we speak of heat. This can be easily reconciled with Eq. IV10-4.1. Since there is no addition of heat $Q=0$. W is negative as work has been done *on* the body. Therefore $U-W=0$ or $W=U'$, This means that *work done on the body increases its internal eneagy* by U'. We have equated U' to an amount of heat $Q=mst$ The temperature rise t corresponds to an increase U'

of internal energy. The same increase could have been brought about by adding $Q=mst$ amount of heat.

We can represent the transformations of energy as $W=U'=Q$ ($=mst$). W/Q is J, the mechanical equivalent of heat if W is expressed in work units and Q in heat units. If Q is also expressed in work units, the need for J does not arise.

(In the language of present day physics, the cases are those of *transformation of work into internal energy.*)

IV-10.5. Isothermal and adiabatic expansion of gases. If the temperature of a gas remains constant during expansion, the expansion is said to be **isothermal**. Imagine a quantity of gas contained in a cylider of uniform cross-section A (fig. IV-10.2) fitted with a frictionless gas-tight piston. The gas presses on the piston with a pressure P and force PA. An equal pressure P is applied from outside on the piston to keep it in equilibrium.

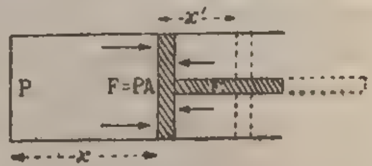


Fig. IV 10.2

If now the external pressure on the piston is decreased even slightly, the gas will expand. During expansion the gas does work as it pushes the piston outward. Let the difference of pressure on the two sides of the piston be infinitely small. If the piston moves outward by a very small distance x', the work done by the gas will be $PA \times x' = PV'$ where V' ($=Ax'$) is the volume swept out by the piston, that is, the increase in volume of the gas.

Unless energy is supplied to the gas, the energy for doing this work PV' must come form the internal energy of the gas. As a resul the gas will be cooled. In order to keep the temperature of the gas constant during expansion, an amount of heat $Q=PV'$ must be supplied to it. (From the first law of thermodynamics it follows that if U' is to be 0, we must have $Q=W=PV'$.)

We therefore find that during an isothermal expansion of a gas, the gas has to be supplied with an amount of heat equivalent to the work it does. If W is the work done, we must supply to the gas an amount of heat Q equal to W/J ($J=$Joule's equivalent).

Conversely, if the external pressure on the piston is increased, work will be done on the gas and the gas will be compressed. The work done on the gas will increase its internal energy. The temperature of the gas will therefore rise. To keep the temperature constant an amount of heat equivalent to the work done on the gas must be withdrawn from the gas.

Adiabatic expansion. If during expansion of a gas no heat is allowed to enter or leave the gas, the expansion is said to be **adiabatic.** A gas does work while expanding. So, in an adiabatic expansion a gas will be cooled (since no heat can enter it). For the same reason, a gas will be heated in an adiabatic compression. The rise in temperature will correspond to a supply of heat equivalent to the work done on the gas. During expansion the fall in temperature is similarly considered.

If we want to apply the first law of thermodynamics to an adiabatic expansion (or compression), we must put $Q = 0$ in Eq. IV (10.4). For expansion, W is positive. Hence $W = -U$. For compression, W is negative. Therefore $W = U$. Loss of internal energy ($-U$) means cooling; gain of internal energy ($+U$) means heating.

An adiabatic expansion may be brought about in either of two ways. The gas may be contained in a cylinder made of thermally nonconducting material. (But no material is a perfect nonconductor of heat. So, in such a case, the expansion is very nearly adiabatic.)

Another way is to make the expansion so sudden that heat does not get time enough to enter the gas.

For an isothermal expansion the cylinder should be made of good conducting material. It will then be able to pass on heat to the gas from outside.

IV. 6. Specific heats of gases. When heat is added to a gas the resulting change of temperature depends on the manner in which the gas is allowed to expand. During expansion the gas does work against the external pressure. The energy required for this purpose is derived from the heat supplied to the gas. Thus,

of the total heat supplied to a gas, a part may go to do external work and the rest raises its temperature.

Hence if we are to define the specific heat of a gas the conditions under which the heating takes place must be stated. To each possible mode of expansion there is a corresponding specific heat of the gas. The two specific heats of a gas generally considered are the *specific heat at constant volume* (c_v) and that at *constant pressure* (c_p). If unit mass of a gas is heated through one degree under the condition that its volume remains constant, the heat it requires, is called its **specific heat at constant volume**. For a mass m undergoing a change of temperature t under this condition the heat involved is mc_vt. When unit mass of a gas is heated through one degree under the condition that its pressure remains constant the heat required is called its **specific heat at constant pressure**.

To understand the difference between these two specific heats consider one gram of a gas contained in a vertical cylinder fitted with a movable frictionless piston (fig. IV [illegible]). Weights may be

Fig. IV [illegible]

placed on the piston to adjust the pressure on the gas. [illegible] sufficient heat to raise the temperature of the gas by 1°C at the same time keeping the volume constant [illegible] the piston (figs. IV [illegible] and IV [illegible]). The amount of heat supplied is the specific heat at constant volume. The whole of this heat goes to increase the thermal energy of the molecules while no work is done by the gas against the external pressure on the piston [illegible].

with the same initial conditions. Supply sufficient heat to raise the temperature of the gas by 1°C, but this time without altering the weights on the piston. The quantity of heat supplied is the specific heat at constant pressure.

In the second case, as the gas expands it raises the weights and so does work (Fig. IV-10. 3*c*). Let V_1 and V_2 be the initial and final volumes of the gas, A the area of the piston, x the distance through which the piston moves and P the pressure on the piston. The force on the piston is PA and the work done against the external pressure is $PA \times x = P(V_2 - V_1)$. The heat which is equivalent to this work is $P(V_2 - V_1)/J$, where J is the mechanical equivalent of heat. Hence the specific heat at constant pressure

= heat required to increase the thermal energy corresponding to a rise of temperature of 1°C + heat equivalent of the work done.

= specific heat at constant volume + heat equivalent of the work done.

In symbols $$c_p = c_v + P(V_2 - V_1)/J \qquad \text{(IV-10.6.1)}$$

It follows that c_p is greater than c_v. In the case of solids and liquids the difference is generally small enough to be ignored for most purposes. In the case of copper, the difference is less than 3%. For solids and liquids the specific heat determined is that at constant pressure.

A distinction is sometimes made in the symbols for the specific heat of 1 gram and that of 1 gram-molecule of a gas. The former is written with a small c as done above and the latter with a capital C. Thus C stands for gram-molecular (or molar) speific heat at constant volume, and C_p for gram-molecular specific heat at constant pressure. But the distinction is not always observed.

Eq. IV-10.6.1 enables us to determine J when the other quantites are known. Note that $V_2 - V_1 = V_0 \alpha_p$ where V_0 is the volume of the gas at 0°C and α_p its coefficient of volume expansion at contant pressure.

Ex IV-10.5. Find the value of J, the mechanical equivalent of heat, from the following data for air :

$C_p = 0.2375$ cal per gm per °C, $C_v = 0.1688$ cal per gm per °C, density of air at S.T.P. = 0.001293 gm/cm^3, coefficient of expansion = 1/273 per °C, normal atmospheric pressure = 1.013 $\times 10^6$ dynes/cm^2.

Solution. From Eq IV-10.6.1 we have $J = \dfrac{p(v_2 - v_1)}{c_p - c_v}$

Here $v_2 - v_1$ is the change in volume of 1 gm of air due to 1°C temperature, the pressure being $p = 1$ standard atmosphere. The volume of 1 gm of air at S.T.P.

$$= \frac{1}{0.001293 \text{ gm/cm}^3} = 773.4 \text{ cm}^3/\text{gm}.$$

$\therefore$ $v_2 - v_1 =$ its expansion due to 1°C rise $= 773.4 \times 1/273 =$ 2 832 cm^3 per gm per °C.

$$\therefore J = \frac{1{,}013 \times 10^6 \text{ dyne. cm}^{-2} \times 2\,832 \text{ cm}^3 \text{ per gm per °C.}}{(0.2375 - 0.1688) \text{ cal per gm per °C.}}$$

$$= \frac{1\,013 \times 10^6 \times 2.832}{0.0687} \frac{\text{dyne. cm}}{\text{cal}}$$

$$= 4\,177 \times 10^7 \frac{\text{dyne. cm}}{\text{cal}} = 4.177 \times 10^7 \text{ ergs/cal.}$$

Gram-molecular specific heats. *The difference between the gram-molecular specific heats at constant pressure and at constant volume is equal to the gram-molecular gas constant.*

Refer to equation IV-10.6.1 that $c_p - c_v = p(v_2 - v_1)/J$. Let us multiply both sides of the equation by the molecular weight M of the gas. The corresponding specific heats are then called the *gram molecular specific heats.* Denote them by C_p and C_v. Since Mv_1 and Mv_2 are the gram-molecular volumes at temperatures t°C and $(t+1)$°C, we have

$$pMv_1 = R_m(t+273) \text{ and } pMv_2 = R_m(t+1+273),$$

where R_m is the gram-molecular gas constant.

$$\therefore \quad C_p - C_v = R_m/J. \qquad \text{(IV-10 6.2)}$$

Often the subscript m is dropped and simply R written instead. The meaning of R is to be understood from the context. It may refer to one gram or to one gram-molecule.

Problem. Calculate the difference between the two principal specific heats of hydrogen and nitrogen form the following data :

Density of mercury = 13.6 gm/cc., molecular weight of hydrogen = 2, and of nitrogen = 28, 1 mole of a gas at S.T.P. occupies 22.4 litres. [Ans : 0.990 ; 0.0707.]

The value of J_m is 8.31×10^7 ergs per kelvin per gm molecule. This is approximately equal to 2 calories/K/mole.

The ratio C_p/C_v, denoted usually by the greak letter γ, is of great importance in connection with adiabatic expansion of gases. For a perfect gas, the relation between P and V for isothermal expansion is PV = constant. But for an adiabatic expansion, this relation is

$$PV^{\gamma} = \text{constant}. \qquad \text{(IV-10.6.4)}$$

EXERCISES

IV-1. (Heat and temperature)

[A] **Essay type questions :**

1. Define temperature. Distinguish between heat and temperature.
   What are the different forms of heat. Mention some effect of heat.
2. What is a thermometer ? What physical properties are taken into account in the design of thermometers ? Describe a mercury-in-glass thermometer. Compare mercury and alcohol as thermometric liquids.
3. How many thermometric scales are in use ? Define upper and lower fixed points. What is meant by fundamental interval ? Find a relation between different thermometric scales.

[B] **Short answer type questions :**

4. Distinguish between 20°C and 20C°.
5. Temperature is not determined by the amount of heat in a body—Explain the statement.
6. Why the column of mercury first descends and then rises when a mercury-in-glass thermometer is put in a flame ?
7. State some objections to using water-in-glass as a thermometer.
8. How a thermometer can be made (i) quick acting (ii) Sensitive ?
9. How can temperature of a human body be measured with a thermometer if the temperature of the surrounding air is 110°F ?
10. Can a clinical thermometer be employed to find (i) maximum and minimum temperature of a day and (ii) the beiling point of water. Give reasons for your answer. (J.E.'73).

11. How could a thermometer be used to find whether the atmospheric pressure was above or below the normal ? (H.S. comp. '60).

12. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which one will respond quickly to changes ? (I.I.T. '73).

13. To take body temperature of a man with a thermometer 2-3 minutes are necessary. But when a jerk is given after taken out from the body, the mercury fall down in few seconds—Explain.

14. To measure the temperature of a humum body two accurate thermometers are used. One reads 35° and the other 95°. Why is this difference ?

[C] **Numerical problems :**

15. The temperature of the surface of the sun is about 6000°C Express this on the Fahrenheit scale. [10832°F]

16. Express normal human body temperature, 98·6°F, on the celsius scale. [37°C]

17. Express the normal boiling point of oxygen,—183°C, on the Fahrenheit scale. [—297·4°F]

18. At what temperature do the Fahrenheit and celsius scales give the same reading ? [—40°]

19. The temperature of a body rises by 25°C. How much is the increase in Fahrenheit scale ? [45°F]

20. Find out the temperature when the degrees of the Fahrenheit thermometer will be 5 times the corresponding degrees of the centigrade thermometer. [10°C, 50°F]

21. A thermometer reads 5° in melting ice and 99° in dry steam at normal pressure. Find the temperature on the Fahrenheit scale when the thermometer reads 52°. (J.E. '68). [120°F]

22. Two thermometers—one reading in celsius and the other in Fahrenheit, were successively dipped into two baths.

In both cases, the difference in readings between the two thermometers was found to be 20. If the baths were at different temperatures, find their temperatures in celsius scale. [—65° and—15°]

23. The freezing point of Fahrenheit thermometer is correctly marked and the bore of the tube is uniform but it indicates 103.5° when a standard centigrade thermometer reads 40°C. What is the reading of boiling point on the Fahrenheit thermometer ? (Bihar) [210·75°]

24. A faulty thermometer reads—1.5°C in melting ice and 98·5°C in dry steam at normal pressure. Find the correct temperature in Fahrenheit scale when this faulty thermometer reads 40°C. [106·7F]

25. A faulty thermometer reads—0·5°C in melting ice and 98·5 C in dry steam at normal pressure. What correction to be applied to get correct reading when this thermometer reads 49°C ? [+1°]

26. A faulty celsius thermometer reads 1·5°C in pure melting ice and 98·5°C in dry steam at a pressure of 747 mm of mercury. What is the correct temperature in Fahrenheit scale when this thermometer reads 20°C ? The steam point at 748 mm of mercury pressure is 99 C (Utkal.) [66·16°F]

IV 2. (Calorimetry)

[A] Essay type questions :

1. Define a caloric. In what other unit (used in mechanics) can heat be expressed ? Express the caloric in this unit. Why is it necessary to mention the temperature in the definition of calorie ?

2. Define specific heat and show that the heat lost or gained by a substance is equal to the product of its mass, specific heat and change in temperature.

3. What is the fundamental principle of calorimetry ? What precautions are necessary in a calorimetric experiment by the method of mixture ?
4. Define thermal capacity and water equivalent. What are the units in which they are expressed ?
5. Explain what is meant by the 'specific heat' of a material. How would you proceed to determine the specific heat of a metal ? Point out sources of error and mention the precautions necessary to avoid them.

[B] **Short answer type questions :**

6. Will the temperature be different if equal quantities of heat are supplied to different substances of equal mass ?
7. Two exactly similar vessels—one containing water and the other containing an equal mass of milk are placed side by side on oven. The rise of temperature of milk is found to take place at a quicker rate than that of water. Explain.
8. Why water is used in hot water bags ?
9. The specific heat of a substance can be defined as thermal capacity per unit mass—Explain the statement.
10. The principle of calorimetry is—"heat lost by warmer body = heat gained by cooler body." Will then the relation hold if (i) the calorimeter contains water and solid is sugar ? (ii) The solid and the liquid in the calorimeter react chemically ? (iii) The calorimeter is kept on the table and is exposed to the air ?
11. Why are calorimeters made of copper ?
12. Suppose you are given a thermometer reading only from 50°C to 100°C and some water of which the temperature is below 20°C. Describe an experiment to determine roughly the temperature of the water, without using any other thermometer.
13. How would you find the temperature of a red-hot iron ball if only mercury thermometers were available ?

14. What is meant by the statement —"specific heat of copper is 0·089".
15. Why water is not suitable as a calorimetric liquid ?
16. What physical properties are taken into account in choosing calorimetric liquid ?
17. What is the difference between specific heat and specific gravity ?
18. Explain the fact that the presence of a large body of water nearby, such as a sea or ocean, tends to moderate the temperature extremes of the climate on adjacent land.
19. A sphere, a cube, a thin circular plate, all made of the same material and having the same mass are initially heated to temperature of 200°C. Which of them cools faster and which slowest ?
20. Two friends ordered tea in a restaurant and waited for another friend to arrive. One of them poured tea in his cup and mixed cold milk with it and the other poured his tea and mixed the milk after the friend arrived. Whose cup of tea will be hotter ?
21. Two liquids taken in identical vessels are allowed to cool through the same temperature range. If the masses are same, which of them cools faster ?
22 What is meant by calorific value of fuels ?

[C] **Numerical problems :**

22 Which of the two following cases requires the greater quantity of heat (i) 500g of water heated from 35°C to 95°C or (ii) 4 lbs of water heated from 100°F to 212°F.

[The 2nd case.]

23. What volumes of iron, lead and aluminium have the same heat capacity as a litre of water ? The specific heat of iron, lead, aluminium and water are respectively 500, 130, 920 and 4200 J/kg°C and their densities are respectively 7500, 11400, 2700 and 1000 kg m^{-3}.

[$1{\cdot}12\times10^{-3}$, $2{\cdot}83\times10^{-3}$, $1{\cdot}69\times10^{-3}$ m^3.]

24. How much heat is required to raise the temperature of a copper ball, weighing 180 g. from 25°C to 95°C ? Sp. heat of copper = 0.09. Find also the thermal capacity and water equivalent of the ball.

[1134 cal, 16·2 cal/°C 16·2 g.]

25. Compare the thermal capacities of equal volumes of water and mercury. (Sp. heat and Sp. gr. of mercury are respectively 1/30 and 13·6 g cm^{-3}.) [1 : 0·453]

26. An iron ball weighing 50 g was placed in a furnace for some time and then quickly transferred to a calorimeter containing 1 kg of water at 35°C. It raised the temperature of the calorimeter and its contents to 40°C. The water equivalent of the calorimeter is 125 g and the Sp. heat of iron 0·12. calculate the temperature of the furnace.

[977·5°C]

27. A metal ball at 130°C and weighing 20 g is dropped into a calorimeter of water equivalent 10 g, containing 50g of a liquid of Sp. heat 0·5, at a temperature of 40°C. The final temperature is 50°C. Calculate the Sp. heat of the metal.

[0.22]

28. A calorimeter of water equivalent 25g contains 100g of an oil at 40°C. A solid of Sp. heat 0·1, weighing 50g is heated to 120°C and quickly dropped into the calorimeter. The resulting temperature is 45°C. Calculate the Sp. heat of the oil.

[0·5]

29. A calorimeter contains 500g of water at 30°C. 200g of water at 90°C are poured into it. If the water equivalent of the calorimeter be 10g what is the resulting temperature ?

[46·9°C]

30. 200g of an oil of Sp. heat 0·42 are dropped at a temperature of 60°C into a calorimeter containing 36g of water at 23°C. The resulting common temperature is 30°C. What is the water equivalent of the calorimeter ? If the

mass of the calorimeter be 100g what is the specific heat of the material ? [10g, 0·1]

31. A vessel contains 60 litre of air (Sp. heat 0·24) at 30°C weighing 1·3 gl^{-1}. How much heat is required to raise temperature of the air to 40°C. [187·2 cal.]

32. On pouring 1 kg of water at 90°C into a hot water bottle at 35°C, the temperature of the water fa ls to 85°C. Calculate the weight of the bottle. The Sp. heat of the material of the bottle is 0·2. [500g.]

33. A vessel placed on a flame asorbs heat at the rate of 200 calories per second. How long will it take to raise the the temperature of 3 kg of water from 30°C to the boiling point assuming that the vessel alone absorbs 3000 calories of heat. [17 min. 45s.]

34. A cylinder of metal weighing 450 g at 150°C is dropped into 200 g of water at 15°C contained in a brass calorimter weighing 200 g and the temperature rises to 38°C. What is the thermal capacity of the metal ? (Sp. heat of brass=0·092) [44·85 cal/°C]

35. A calorimeter of water equivalent 15 g contains 60 g of water at 40°C. Due to the loss of heat by radiations, its temperature falls to 30°C in 5 minutes. If the same calorimeter is filled with 50 g of another liquid, the time in which the same drop of temperature takes place is found to be 2 minutes. Calculate the Sp. heat of the liquid assuming that the rate of loss of heat due to radiation is same in both cases [0·3]

36. A celsius thermometer (water equivalent 10g) when immersed in 10g of water rises by 15°C and show a temperature of 35°C. What is the temperature of water before insertion of thermometer ? [50°C]

37. A thermometer of mass 55 g and Sp. heat 0·2 gives 15°C as the room temperture. When dipped in some water, kept in a calorimeter, it reads 44·4°C. If the water

equivalent of the calorimeter be 50 g and mass of water taken be 250 g, find the temperature of water before the insertion of the thermometer. (J E. '74) [45·5°C]

38. A piece of iron and piece of copper, each of mass 100 g are joined together. The combined mass is heated to 100°C. It is then dropped into water in a calorimeter of mass 100 g containing 132·4 g of water at 20°C. The resulting temperature is 30°C. The experiment repeated with a mass of water 294 g taken in the calorimeter results in a temperature of 25°C. Calculate the Sp. heats of iron and copper. [0·112, 0·09]

39. A body initially at 80°C cools to 64°C in 5 minutes and to 52°C in 10 minutes. What will be its temperature after 15 minutes and what is the surrounding temperature ? [43°C, 16°C]

40. 200 g of water and equal volume of another liquid of mass 250 g are placed in turn in the same calorimeter of mass 100 g and Sp. heat 0·1. The water and the liquid, which are constantly stirred are found to cool from 60°C to 40°C in 180 s and 140 s respectively. Find the Sp. heat of the liquid. [0·613]

IV-3. (Expansion of solids)

[A] Essay type questions :

1. Define coefficient of linear expansion. What is the unit in which it has to be expressed ? How does its value change if the temperature scales is changed from celsius to fahrenheit ?
2. "Different materials expand differently"—describe a suitable experiment in support of this statement.
   Give three examples where expansion on heating is made to serve a useful purpose.
3. Give three examples where expansion on heating causes inconvenience, and say how it is remedied.

4. Prove that volume coefficient of a solid is three times and the coefficient of surface expansion twice than the coefficient of its linear expansion.

5. Describe a laboratory method for determining the coefficient of linear expansion of a solid. In this method which quantity should be measured most accurately to get correct result and why ?

6. Describe an experiment to show the existance of force that brought into play by expansion or contraction of solids. Calculate its magnitude for a bar in terms of its cross-section A and Young's modulus Y.

7. How can pendulum be constructed to have an invariable length ?

[B] **Short answer type questions :**

8. A metal ball can pass through a metal ring. When the ball is heated, however, it gets stuck in the ring. What would happen if the ring, rather than the ball, were heated ?

9. Does coefficient of linear expansion depend on the (i) unit of length, (ii) unit of temperature ?

10. Explain what is meant by 'coefficient of linear expansion of iron is ·000012 per degree celsius' ?

11. Why does the glass stopper of a bottle become loose when heated ?

12. Why in railway lines small gaps are left in between two successive rails ?

13. Why the holes of fish-plate are oval-shaped ?

14. Thick glass vessels crack move often than a thin glass vessels—why ?

15. A platinum wire can easily be fused in a glass rod but an iron wire can not be—why ?

16. Does the change in volume of a body, when its temperature is raised, depend on whether the body has cavities inside ?

17. A bimetallic strip is made of copper and steel. What happen when its (i) temperature is raised, (ii) cooled ? α for copper and steel are respectively 1.7×10^{-5} and 1.1×10^{-5}/°C.

18. Can a scale made of metal measure lengths accurately at all temperatures ?

19. An isosceles triangle is formed with three zinc rods. Is there be any change of the base angle when the triangle is heated ?

20. Explain why a clock goes fast in winter and slow in summer.

21. Is there be any change in (i) internal diameter, (ii) mass, (iii) volume, (iv) density of a metal ring when heated ?

22. Two rods of different materials differ by constant length at all temperature. Show that the length of the rods at initial temperature are inversely proportional to their respective coefficients of expansion.

23. For reinforcement of concrete structure iron or steel rods are used. Why other metals are not used ?

24. A copper disc fits tightly in a hole in a steel plate. Should you heat or cool the system to loosen the disc from the hole ? α for copper and steel are as given in Ex. 17.

25. α for steel is 12×10^{-6} per °C. What is its value in °F.

[C] Numerical problems :

26. There is a gap of 0·5 inch between each two rail-lines of 66 ft long at 10°C. At what temperature will the gap be filled up ? ($\alpha = 12 \times 10^{-6}$/°C) (H.S. '79) [66.5°C]

27. An aluminium rod is 240 cm long at 35°C. To what temperature must it be heated to expand 0·1 cm ? α for aluminium $= 26 \times 10^{-6}$ per °C. [51°C]

28. Find the temperature to which a Zinc rod 50 cm long at 15°C must be heated in order that its length may be 50·05 cm. α for Zinc $= 25 \times 10^{-6}$ per °C. [55°C]

29. α for steel is 12×10^{-6} per °C. If the length of a steel rail at 20°C is 5 m, what interval must be left between the rails to allow for a rise in temperature to 45°C ? [0·27 cm]

30. The old Howrah bridge was 656 m long. Between a winter night and a summer noon the temperature changes from 8°C to 50°C. Find how much it expanded if $\alpha = 12 \times 10^{-6}$ per °C. [33 cm]

31. A 5 m long girder increases by 0·15 cm when temperature changes from 40°C to 55°C. Find the coefficient of linear expansion of the material of the girder. [20×10^{-6}/°C]

32. The diameter of a sphere at 35°C is 2·5 cm. Find the temperature to which it should be raised so that it will just fail to pass through a circular hole of diameter 2·501 cm. $\gamma = 75 \times 10^{-6}$ per °C. [51°C]

33. A brass scale is correct at 30°C. A length measured by it at 55°C is 40 cm. What is the true length ? α for brass $= 20 \times 10^{-6}$ per °C. [40·02 cm]

34. A wheel has a diameter of 75 cm. A steel tyre of inner diameter 74·8 cm is to be fitted on it. Calculate the range of temperature in degrees celsius through which the tyre must be heated so that it may be mounted on the wheel. α for steel $= 12 \times 10^{-6}$ per °C. [222·8°C

35. Two bars of brass and steel, standing side by side, have one end of each rigidly attached to the other. The other ends are free to expand. The steel bar is 1 m long. What should be length of the brass bar so that the distance between the free ends of the bars remains the same at all temperatures ? α for brass $= 20 \times 10^{-6}$ per °C and α for steel $= 12 \times 10^{-6}$ per °C. [60 cm]

36. A pyrex flask has a capacity of 1 litre at 15°C. Find its capacity at 35°C. $\alpha = 4 \times 10^{-6}$ per °C. [1·00018 litre]

37. A steel sphere has to be passed through a brass ring. At 20°C, the diameter of the sphere is 25·0 cm and the internal diameter of the ring is 24.9 cm. If the sphere and the

ring are heated togather what is the temperature rise necessary? α for steel $= 12 \times 10^{-6}$ per °C and α for brass $= 20 \times 10^{-6}$ per °C. [About 500°C.]

38. The density of alluminium at 20°C is 2.55 g cm^{-3}. Compare its densities at 0°C, and 100°C, given α for aluminium $= 25 \times 10^{-6}$ per °C. [1·005 : 0·994.]

39. The time period of a simple pendulum is 2 s. It is made of iron. For a change of 25°C, what will be the change in the length of the pendulum. α for iron $= 12 \times 10^{-6}$ per 0°C. [0.0296 cm.]

40. A brass scale correct at 0°C measures a Zinc rod at 10°C and the length is found to be 1.0001 m. Find the correct length at 0°C and 10°C. α for brass $= 19 \times 10^{-6}$ per °C and that for Zinc $= 29 \times 10^{-6}$ per °C. [1m, 1.00029 m.]

41. An iron pendulum makes 86405 oscillations in a day; at the end of the next day the clock has lost 10 s. Find the change in temperature. α for steel $= 12 \times 10^{-6}$ per °C. [19.6°C.]

42. The ends of steel rod exactly 1·5 cm^2 in cross-Sectional area are held rigidly between two fixed points at a temperature of 33°C. Determine the pull in the rod when the temperature drops to 23°C. Y for steel $= 2 \times 10^{12}$ dy cm^{-2} and $\alpha = 11 \times 10^{-6}$ per °C. [3.3×10^{6} dy.]

43. The base BC of an equilateral triangle is made of an aluminium wire and the other two sides AB and BC are made of copper. A wire of iron connects the vertex A to the mid-point of the base BC. If for a small rise of temperature, the arms of the triangle do not buckle, find the coefficient of linear expansion of iron. α for copper $= 16 \times 10^{-6}$ and that for aluminium $= 26 \times 10^{-6}$ per °C. [12.7×10^{-6} per °C]

44. Three rods form an equilateral triangle at 0°C. Two of the rods are of same material and the third is of inver (whose themal expansion may be neglected). If after heat-

ing to the steam point, the angle between the identical rods be$\left(\frac{\pi}{3}-\theta\right)$, prove that the coefficient of linear expansion of the material of the rod is $\frac{\sqrt{3}\theta}{200}$ per °C (J.E. '76)

45. A 'thermal tap' used in certain apparatus consists of a silica rod which fits tightly inside an aluminium tube whose internal diameter is 8 mm at 0°C. When the temperature is raised, the fit is no longer exact. Calculate what change in temperature is necessary to produce a channel whose cross-section is equal to that of a tube of 1 mm internal diameter. α for silica $=8\times10^{-6}$ per °C and that for aluminium $=26\times10^{-6}$ per °C. (oxf & camb.) [161°C]

46. A steel cylinder has an aluminium alloy piston and, at a temperature of 20°C when the internal diameter of the cylinder is exactly 10 cm, there is an all-round clearance of 0.05 mm between the piston and the cylinder wall. At what temperature will the fit be perfect? α for steel $=12\times10^{-6}$ and that for aluminium alloy $=16\times10^{-6}$ per °C (oxf & camb.) [270°C]

47. The area A of a rectangular plate is ab. It's coefficient of linear expansion is α. After a temperature rise $\triangle T$, side a is longer by $\triangle a$ and side b is longer by $\triangle b$. Show that it we neglect the small area $\triangle a.\triangle b$, then $\triangle A=2\alpha A\triangle T$.

48. Prove that, if we neglect extremely small quantities, the change in volume of a solid on expansion through a temperature rise $\triangle T$ is given by $\triangle V=3\alpha V\triangle T$ where α is the coefficient of linear expansion.

49. Two rods of length l_2 and coefficient of linear expansion α_2 are connected to a third rod l_1 of coefficient of linear expansion α_1 to form an isosecles triangle. The arrangement is supported an a knife-edge at the mid point of l_1 which is horizontal. Find the relation between l_1 and

l_2 so that apex of the triangle is to remain at a constant distance from the knife edge as the temperature changes.

[illegible]

50. A bimetallic strip consists of two tapes 0.5×10^{-3} m thick, one of steel whose coefficient of linear expansion is $10 \times 10^{-6}/°K$ and the other of german silver of expansion coefficient $18 \times 10^{-6}/°K$. The plates are welded togather face to face. A 0.10m length of the strip is firmly fixed at one end to a support. How much does the other end move when the temperature is changed by 100°K ?

$[7.99 \times 10^{-3}$ m$]$

51. Two equal bars, 50 cm long, one of brass, the other of iron, are joined togather at one end and a needle of [illegible] mm in diameter and carrying a pointer is clipped between their free ends. When the bars are heated, the needle rotates through 10°. What is the temperature interval through which they were heated ? [illegible]

52. A uniform pressure P is exerted on all sides of a solid cube at temperature t°C. By what amount should the temperature of the cube be raised in order to bring its volume back to the volume it had before the pressure was applied. The coefficient of volume expansion of the cube is [illegible] and the bulk modulus of elasticity is K. [illegible]

$\left[\frac{P}{K\gamma}\right]$

53. A [illegible] cylinder of mass [illegible] g and radius [illegible] cm is placed on [illegible] bearings are rotated with an angular velocity [illegible] rad s^{-1} about the cylinder axis at [illegible]. If now the temperature of the cylinder is increased to [illegible]°C without [illegible] contact will there be any change in the angular velocity, angular momentum and kinetic energy of rotations ? Explain your answer. Calculate the percentages of the above quantities, if any.

assuming the mean coefficient of linear expansion of brass to be 2×10^{-5} per °C. [illegible]

[illegible]

54. A piece of metal weighs [illegible] g in air. When it is immersed in liquid of sp. gravity 1.24 at [illegible]°C, it weighs [illegible] g. When the temperature is raised to [illegible]°C the metal piece weighs [illegible] g; sp. gravity of liquid at [illegible]°C is [illegible]. Calculate the coefficient of linear expansion of the metal. [illegible]

[2.31×10^{-5}/°C]

55. A composite rod is made by joining a copper rod, end to end, with a second rod of different material but of the same cross-section. At [illegible]°C the composite rod is [illegible] in length, of which the length of the copper rod is [illegible] cm. At 125°C the length of the composite rod increases by 1.91 mm.

When the composite rod is [illegible] by keeping it between two rigid walls, [illegible] that the lengths of the two constituents do not change with the rise of temperature. Find the Young's modulus and coefficient of linear expansion of the second rod. [illegible] [illegible]

[illegible]

IV 4 (Expansion of liquids)

I A. Essay type questions :

1. Distinguish between [illegible] and [illegible] expansion of a liquid, and [illegible] between the corresponding coefficients.

2. How does the density of a liquid change with temperature? What is the peculiar [illegible] behaviour of water [illegible] in this respect?

3. How would you show that water at 4°C [illegible] [illegible]

What is the consequence of this anomolus behaviour of water on marine life ?

4. Which coefficient of expansion of a liquid is determined by a weight thermometer ? Describe the method in detail.

5. Describe Dulong and Pelit's method of determining the coefficient of real expansion of a liquid.

6. (a) Find the relation between the densities of a liquid at different temperatures,

(b) Show how the apparent weight of a solid immersed in a liquid changes with rise in temperature.

[B] **Short answer type questions :**

7. A long cylindrical vessel having a linear coefficient of expansion α is filled with a liquid up to a certain level. On heating, it is found that the level of the liquid in the cylinder remains the same. What is the volume coefficient of expansion of the liquid ?

8. What is the difference between a weight thermometer and Dulong and Pelit's method. (J.E. '78)

9. A block of ice floats in a beaker filled with water to the brim. When ice melts completely, what happens to the level of water if the temperature of water is (i) at 4°C. (ii) at 0°C, (iii) at higher temperature.

10. Explain why lakes freeze first at the surface.

11. What difficulties would arise if you define temperature in terms of the density of water ?

12. A sp. gravity bottle is marked '50 c.c., 20°C'—say what it means. How can you test its accuracy ?

13. A block of wood is floating on water at 0°C with certain volume V above water-level. The temperature of the water is slowly raised from 0°C to 20°C. How does the volume V change with the rise of temperature ? (I.I.T. '74)

14. The top of a lake is frozen. Air in contact is at $-15°C$. What do you expect to be the maximum temperature of water—(i) in contact with lower surface of ice, (ii) at the bottom of the lake ? (I.I.T. '73)

15. A beaker filled with water at 4°C overflows if temperature is increased or decreased explain. (I.I.T. '69)

16. A piece of ice is floating in a beaker completely filled with a liquid of density 1.5 g cm^{-3}. Will there be any overflow of liquid when the ice melts ? Explain. (I.I.T. '83)

17. Which coefficient of expansion is determined by a weight thermometer ? Why is the instrument so named ?

18. A big lump of ice with a piece of lead forzen into it floats in water. What will be the change of level of water when ice melts completely ? What will happen if the lead is replaced by equal volume of cork ?

19. How will the level of mercury in a thermometer change when the bulb heat (i) quickly (ii) suddenly ?

20. 'A barometer reading needs correction'—Explain.

[C] **Numerical problems :**

21. A mercury-in-glass thermometer has a bulb 1 cm^3 in capacity. It is desired to make each celsius degree on the scale 5 mm -long. Calculate the cross-sectional area of the capillary tube. The coefficient of apparent expansion of mercury in glass is 16×10^{-5} per °C. [0·0032 cm^2]

22. What should be the volume of mercury to be put in a glass flask of capacity 720 cm^3 so that the volume above the mercury remains the same at all temperatures ? The coefficient of real expansion of mercury is 18×10^{-5}/°C and the coefficient of cubical expansion of glass is 25×10^{-6}/°C. [100 cm^3]

23. The coefficient of expansion of water between 4°C and 20°C is 0.00015. Calculate the weight of 1 litre of water at 20°C. [997.6g]

24. A thin copper sphere of diameter 10 cm has a uniform tube of diameter 0.5 cm attached to it. Water fills the sphere and stands at a height of 10 cm in the tube. What will be the change in the level of water in tube if the sphere is (i) suddenly, (ii) slowly heated from 30°C to 60°C ? α for cu $= 16.7\times10^{-6}$/°C, coeff. of absolute expansion of water $= 45\times10^{-6}$/°C.

[will drop 4 cm.
will rise 32 cm.]

25. A column of mercury at 100°C is balanced by a column at 0°C. Their heights are 50·90 cm and 50·00 cm respectively. Calculate the coefficient of real expansion of mercury. [18×10^{-5}/°C.]

26. How much mercury must be placed inside a glass flask, having an internal volume of 300 cm^3, so that the volume of the remaining space inside the flask may be constant at all temperatures ? coefficient of cubical expansion of mercury $= 18\times10^{-5}$/°C and coefficient of linear expansion of glass $= 9\times10^{-6}$/°C. [45 cm^3]

27. A mercury barometer has a brass scale which is correct at 0°C. The barometer reads 750mm at 20°C. Calculate the correct barometric height. Given α for brass $= 18\times10^{-6}$/°C and γ for mercury $= 18\times10^{-5}$/°C.

[747.57 mm of Hg.]

28. The volume of the bulb of a mercury thermometer at 0°C is V_0 cm^3 and the cross-section of the capillary is A_0 cm^2. The coefficient of linear expansion of glass is α_g per °C and the coefficient of cubical expansion of mercury is γ_m per 0°C. If the mercury fills the bulb at 0°C, what is the length of mercury column in the capillary at t°C ? (AM.I.E.) [$V_0 t(\gamma_m - 3\alpha_g)/A_0 + 2\alpha_g t$.]

29. A glass bottle full of mercury at 12°C contains 506.732g of Hg. When heated to 100°C, the bottle contains 500.00g

of Hg. Find the coefficient of real expansion of Hg if the coefficient of linear expansion of glass is 9×10^{-6} °C^{-1}]

[0·00018°C^{-1}]

30. A vertical glass tube, closed at the bottom, contains Hg to a height of 0.5 m at a temperature of 10°C. Find the height of mercury column at 30°C. γ for Hg$=18 \times 10^{-5}$/°C and α for glass$=8 \times 10^{-6}$/°C. [0·5016m]

31. If the density of mercury at 0°C is 13596 kgm^{-3}, what is its value at 60°C if the coeff. of cubical expansion of Hg is 0.000182 $degC^{-1}$. [13447 kg m^{-3}.]

32. It is desired to use glass in the construction of a celsius mercury thermometer reading from -10°C to 100°C in which degree divisions shall be 0.002m apart. What must be the capacity of the bulb ? The coefficient of cubical expansion of mercury and glass are 0·000182 and 0·000025 $degC^{-1}$ respectively. [3.99×10^{-7}m.]

33. The mass of Hg overflowed from a weight-thermometer is 5.4 g when heated from ice to steam-point. The thermometer is placed in an oil bath at 20°C. On heating the bath, 8.64 g of mercury flows out. Determine the temperature of the bath. [180°C.]

34. Aniline is a liquid which does not mix with water, and when a small quantity of it is poured into a beaker of water at 20°C if sinks to the bottom. The densities of the two liquids at 20°C being 1021 and 998 kg m^{-3} respectively. To what temperatures must the beaker and its contents be uniformly heated so that the aniline will form a globule which just floats in the water ? The coefficient of absolute expansion of aniline and water are 0·00085 and 0·00045 K^{-1} respectively. (London) [79°C]

35. Using the following data, determine the temperature at which wood will just sink in benzene. Density of benzene and wood at 0°C are 9×10^{2} kg m^{-3} and 8.8×10^{2} kg m^{-3}

respectively ; Coefficient of cubical expansion of bengene and wood are $1.2\times10^{-3}K^{-1}$ and $1.5\times10^{-4}\ K^{-1}$ respectively. (London.) [21·7°C]

36. A glass tube nearly filled with mercury is attached to the bottom of an iron pendulum rod 100 cm long. How high must the mercury be in the glass tube so that the centre of mass of this pendulum will not rise or fall with change of temperatures ? α for iron $=11\times10^{-6}/°C$ and γ for $Hg=18\times10^{-5}/°C$. (46·4 cm.)

37. In a mercury-in-glass thermometer if the V_0 is the volume of mercury that just-fills the bulb at 0°C and the cross-section of the capillary is A_0, show that the length of the mercury column in the capillary at a temperature t°C is $V_0\ (\gamma-3\alpha)\ t/A_0$ where γ is the volume coefficient of mercury and α is the linear coefficient of expansion of glass. Assume that the cross-section of the capillary is constant.

38. A sphere of diameter 7 cm and weighing 266.5 g is floating in a liquid. When the temperature of the liquid is raised, it is found that the sphere just immersed at 35°C. If the density of the liquid at 0°C be 1.527 g cm^{-3}, calculate the coefficient of volume expansion of the liquid. Neglect the expansion of the sphere. (I.I.T. '62) [$8\times10^{-4}/°C$]

39. Two thermometers A and B are made of same kind of glass and contain the same liquied. The bulbs of both the thermometer are spherical. The internal diameter of the bulb in A is 7.5 mm and the radius of cross-section of the tube is 1.25 mm, the corresponding figures in B being 6.2 mm and 0.9 mm. Compare approximately the length of a degree in A with that in B. (J.E. '70) [0·8 approx.]

40. A Sinker of weigh W_0 has an apparent weigh W_1 When weighed in a liquid at temperature t_1 and W_2 when

weighed in the same liquid at temperature t_2. The coefficient of cubical expansion of the matrial of the sinker is β. What is the coefficient of volume expansion of the liquid ? (I. I. T. '78) $\left[\frac{W_1 - W_2}{(W_0 - W_1) \times (t_2 - t_1)} + \beta\right]$

IV-5. (Expansion of Gases)

[A] Essay type questions :

1. Describe a experiment to show that gases expand when heated. How does expansion of a gas differ from that of a solid and liquid ?
2. State Boyle's law and charles' law. How would you verify the laws experimentally ?
3. State the two fundamental laws of change of pressure, volume and temperature of gases and show that they may be expressed in the form of a single equation.
4. State charles' law and show how it gives rise to the idea of an absolute zero of temperature. What is an absolute scale of temperature ?
5. Define volume and pressure coefficient for a gas. Why is it necessary to define two coefficients for a gas ? Show that for a perfect gas, the volume coefficient of expansion is equal to the pressure coefficient.
6. Describe Regnault's constant pressure air thermometer. How can you determine the volume coefficient of a gas ?

[B] Short answer type questions :

7. The expansion of a gas due to application of heat depends upon its condition—Explain.
8. Explain what you mean by a perfect gas.
9. Calculate the value of absolute zero in both celsius and Fehrenheit scales.
10. Why the absolute zero is treated as more fundamental and more universal than the celsius zero ?

11. How is the celsius scale related to absolute scale of temperature ?

12. What do you mean by 'universal gas constant' ? Why is it so called ?

13. A liquid has an apparent coefficient of expansion but gases have none—why ?

14. Why do we always take the initial volume and pressure at 0°C which in defining coefficient of expansion of a gas in contrast to the cases of solid and liquid ?

15. Given samples of 1 cm^3 of hydrogen and 1 cm^3 of oxygen both at N.T.P. Which sample has a larger number of molecule ?

[C] **Numerical problems :**

16. A certain mass of a gas is at 40°C Calculate the temperature at which the volume will be doubled, pressure remaining the same. If the volume is kept constant, what will be the temperature if the pressure increases to three times its initial value ? [353°C, 666°C]

17. A gas occupies 50 cm^3 at 67°C and pressure 70 cm of mercury. If the volume becomes 40 cm^3 at 17°C, what is the new pressure ? [74·6 cm of Hg.]

18. A gas occupies 125 cm^3 at 60°C and 75 cm of mercury. If the pressure be increased to 80 cm at temperature 30°C what is the new volume ? [106·5 cm^3.]

19. A thin glass bulb is sealed at 27°C, the inside pressure being 1 atmosphere. The maximum internal pressure the bulb can withstand is 95 cm of mercury. At what temperature will the bulb burst ? [102°C]

20. A bicycle tyre contains air at a pressure of 2 atmosphere at 30°C. If the temperature rises 40°C, what should be the pressure of air in the tyre, the volume remaining constant. [2·07 atmos.]

21. At constant pressure 1 litre of a certain gas expands by 128 cm^3 when heated from 0°C to 35°C. Calculate from these data the value of the absolute zero [–273°C]

22. Compare the density of air at 10°C and 750 mm pressure with that at 15°C and 760 mm pressure. [1 : 1·004]

23. A litre of gas weighs 1.562 g at 0°C under a pressure of 76 cm of mercury. The temperature rises to 250°C, and the pressure to 78 cm of mercury. What is the weight of one litre of gas under the new conditions? [1·47 gl^{-1}]

24. A 10 litre vessel containing air at one atmosphere is heated from 0°C to 100°C. What will the pressure be if the vessel was a closed one? What fraction of the mass of air will escape if the vessel was open to the atmosphere? [1·37 atmos, 100/373]

25. A glass globe of capacity 1 litre is heated at sea-level from 0°C to 100°C with its mouth open. Find the mass of air expelled thereby. Density of air at N. T. P. = 0.001293 g cm^{-3}. [0·3466 g.]

26. At constant pressure the volume of a certain gas increases in the ratio 1 : 1·035 between 15°C and 25°C. Calculate from these data the value of the absolute zero. [–270 C]

27. A glass vessel contains air at 60 C. To what temperature must it be heated to expel one-third of the air, the pressure remaining constant? [226·5 C]

28. The density of oxygen at N. T. P. is 1·429 gl^{-1}. A certain mass of the gas is enclosed in a cylinder, whose volume is 2·5 litre, under a pressure of 780 mm at a temperature 27°C. What is the mass of the gas in the cylinder? [3·336 g]

29. A gas at 1 °C has its temperature raised at constant pressure so that its volume is doubled. What is the final temperature? [299°C.]

30. The volume of an air-bubble increases five-fold in raising from the bottom of a lake to the surface If the barometric

height be 30 inches, find the depth of the lake. Density of mercury $=13.6$ gcm^{-3}. [136 ft.]

31. A vertical cylinder of total length 100 cm is closed at the lower end and is fitted with a movable frictionless gas-tight disc at the other end. An ideal gas is trapped under the disc. Initially the height of the gas column is 90 cm, when the disc is in equilibrium between the gas and the atmosphere. Mercury is then slowly poured on top of the disc and it just starts overflowing when the disc has descended through 32 cm. Find the atmosphric pressure. Assume the temperature of the gas to remain constant and neglect the thickness and weight of the disc.

[76·1 cm of Hg.]

32. Two equal bulbs are joined by a very narrow tube and the whole is initially filled with a gas at N. T. P. and sealed. What will be the pressure of the gas when one of the bulbs is immersed in boiling water and the other in ice ?

(Bihar Univ.) [87·76 cm of Hg]

33. 1 litre of hydrogen at N. T. P. weighs 0·0896 g. Find the value of R considering one gramme—molecule of the gas. [$8{\cdot}28\times10^7$ erg/°C]

34. A barometer reads 760 mm when the temperature is 27°C. One milligram of water is then let into the Torricellian vacuum and the whole of water evaporates. The barometer reading now is 730 mm. Calculate the volume occupied by the water vapour. Gas constant $R=8{\cdot}3\times10^7$ erg per g-mol per °C. [34·6 cm^3]

35. A gas is collected over water in a 100 cm^3 tube and measures 72·8 cm^3, the temperature and pressure of the atmosphere at the time are 25°C and 74·39 cm of Hg respectively. Calculate the volume of dry gas at N. T. P. The vapour pressure of water at 25°C is 23·45 mm of Hg. [64·42 cm^3.]

36. An air filled cylindrical vessel at atmopheric pressure is lowered into water with its open month downwards until water occupies $\frac{1}{3}$rd part of the whole volume of the cylinder. How much further it has to be dipped into water so that water occupies $\frac{2}{3}$rd of its volume ? Atmopheric pressure $=76$ cm of Hg and density of Hg$=13.6$ gcm^{-3}. [15·5 m]

37. Two vessel of volumes V_1 and V_2 are filled with a gas and pressures in them are P_1 and P_2 respectively. The vessel are then connected by a narrow tube with a stop cock. What will be the final pressure of the gas if the stop cock is opened. Assume the temperature to remain unchanged. [$(P_1V_1+P_2V_2)/V_1+V_2$]

38. Two vessel of volumes 5 and 3 litre contain air at pressures of 3 and 7 atmospheric respectively. What will be the resultant pressure when they are connected togather with a small tube. Assume the temperature to remain unchanged. [4·5 atmos.]

39. A vessel contains an ideal gas at a pressure of 5 atmosphere and a temperature of 300°K. It is connected by a thin tube to another vessel with a connecting tap, whose volume is 4 times the volume of the first vessel. The second vessel contains the same ideal gas at a pressure of 1 atmosphere and temperature 500 K, Now the connecting tap is opened, equilibrium sets in. Assuming temperature to be constant, find the final pressure of the system. [2 atmos.]

40. A capillary glass tube with fine bore has both the ends sealed and a small mercury pellet inside. The pellet divides the tube into two parts whose lengths are in the ratio of 3:1. If the initial temperature of the whole arrangement is 0°C and if it is heated afterwards to 273°C, what will be the change in the air-pressure inside the tube ? [In each part the pressure will be doubled.]

41. A glass capillary tube, sealed at both ends is 100 cm long. It lies horizontally with the middle 10 cm occupied by mercury. The two ends of tube (which are equal in length) contain air at 27°C and at a pressure of 76 cm of mercury. Keeping the tube in a horizontal position, the air-column at one end is now kept at 0°C and the other end is maintained at 127°C. Calculate the length of the air column which is at 0°C and also its pressure. Neglect the change in volume of mercury and glass. (I. I. T. '75).
[36·5 cm, 85·2 cm of Hg]

42. A uniform narrow tube closed at one end contains some air confined by a mercury 15 cm long. When the tube is held vertically with its closed end downwards, the air column is 20 cm long but when it is held horizontally the air column is 24 cm long. Calculate the atmospheric pressure. [75 cm of Hg.]

43. An air bubble starts rising from the bottom of a lake. Its diameter is 3·6 mm at the bottom and 4 mm at the surface. The depth of the lake is 2·5 m and the temperature at the surface is 40°C. What is the temperature at the bottom of the lake ? Assume that the variation of density of water with depth is negligible. Atmospheric pressure is 76 cm of mercury and acceleration due to gravity is 980 cm s^{-2}) (I.I.T.'66). [10.2°C]

44. A cylinder closed at both ends is divided into two halves by a thermally insulated piston. Both halves of the cylinder contain equal masses of gas at a temperature of 27°C and pressure 76 cm of Hg. What distance from the centre of the cylinder will the piston move if the gas at one section is heated to 57°C ? What will be the final pressure in each section ? The initial length of each section is 42 cm. [2 cm, 79·8 bm of Hg.]

45. A flask contain 1 g of oxygen at a pressure of 10 atmosphere and at a temperature of 47°C. At a latter time it

was found that because of a leak the pressure has dropped to 5/8 of its original value and the temperature has decreased to 27°C. Find the volume of the flask, and how many grams of oxygen leaked out. [82 cm^3, 0 33 g.]

IV-6. (Kinetic theory of Gases)

[A] **Essay type questions :**

1. What evidence can you offer to support that matter has a molecular structure ?
2. What evidence can you offer to support that in liquid and gases molcules have a random motions ?
3. What is Brownian motions ? What is its importance in connetion with the Kinetic theory of gases ?
4. What are the fundamental assumptions of the Kinetic theory of gases ?

   State briefly how the pressure of a gas is explained on the Kinetic theory of gases.
5. How do you derive the concept of temperature from the Kinetic theory of gases ?

   How can Boyle's law, Avogadro's hypothesis and Dalton's law of partial pressure be accounted for by the Kinetic they of gases ?
6. If the pressure and density of a gas are known, how can the velocity of a gas molecule be determined from them ? What relation is there between temperature and velocity of a gas molecule ?
7. How does a real gas differ from a perfect gas ? Under what condition will a real gas behave closest to a perfect gas ?
8. What are 'Avogadro's number', the 'universal gas constant' and Boltzmann's constant. How they are related ?

B) Short answer type questions :

9. Explain what is meant by mean velocity and root-mean square velocity. Are they identical ? Which one is more important in Kinetic theory ?

10. When heat is supplied to a body what physical process goes on inside it ? (J. E. E. 79)

11. What are the limitations of ideal gas equation PV=RT ?

12. Prove that the average kinetic energy of a molecule is the same for all gases at the same temperature.

13. 'At same temperature molecules of different gases have the same R. M. S. velocity'. Is it true ? Justify your answer. (I. I. T. '81)

14. How does the R. M. S. velocity of the molecules of an ideal gas vary, when (i) temperature is increased and (ii) density is increased.

15. In Brownian motion, do we observe the motion of the molecules of a liquid or a gas or of some other particles ?

16. Show from kinetic theory of gases that the average Kinetic energy of a gas molecule is proportional to the absolute temperature of the gas.

17. Discuss the limitations of ideal gas equations. (H S '79)

18. The average velocity of the molecules of a gas is zero if the gas as a whole and the container are not in translational motion. How is it can be that the average speed is not zero ?

19. Is ideal gas equation applicable to real gases ?

20. Write down Van-der-waal's equation of state with proper explanation.

21. How Kinetic theory explains (i) thermal expansion (ii) change of state and (iii) evaporation and vapour pressure.

22. Account for the diffusion phenomenon from kinetic theory.

23. "If a pungent gas, like ammonia, is let free at one end of the room, it will not be sensed by a person at the other end of the room at once" —explain the statement.

24. What do you mean by "mean free path"?

C) **Numerial problems :**

25. If the R. M. S. velocity of hydrogen at N. T. P. is 1.84 km s^{-1}, calculate the R. M. S. velocity of the oxygen molecules at N. T. P. Molecular weight of hydrogen and oxygen are 2 and 32 respectively. (Gorakhpur) [0.46 km s^{-1}]

26. At what temperature, pressure remaining constant, will the R. M. S. velocity of a gas be half of its value at 0°C? (Agra) [−204.75°C]

27. Calculate the mean kinetic energy of a molecule of a gas in electron-volt at a temperature of [illegible] K. Given R = 8.31 × 10^7 erg deg^{-1} $mole^{-1}$, N = [illegible] (Bombay) [illegible]

28. Calculate the number of molecules in 1 cm^3 of a perfect gas at [illegible] °C and at pressure of [illegible] mm of mercury. Mean kinetic energy of a molecule at [illegible] °C = 4 × [illegible] erg, density of mercury = 13.6 g [illegible] [illegible]

29. Find the temperature at which the root mean square velocity of nitrogen molecules in earth's atmosphere equals the velocity of escape from the earth's gravitational field. Mass of nitrogen atom = [illegible] g, radius of earth = [illegible] km, Boltzmann's constant = 1.38 × 10^{-16} erg [illegible], g = [illegible] [illegible]

30. Calculate the [illegible] kinetic energy of 1 g of helium (molecular weight = 4) at [illegible]. R = 8.3 × 10^7 erg per g mole per degree. [illegible] [illegible] erg

31. Calculate the [illegible] at N. T. P. [illegible] at N. T. P. [illegible]

32. The mean kinetic energy of a molecule of hydrogen at 0°C is $5{\cdot}64 \times 10^{-14}$ erg and moler gas constant is $8{\cdot}32 \times 10^{7}$ erg mole^{-1} 0$_c^{-1}$. Calculate Avogadro's number. (Banaras).
[$6{\cdot}04 \times 10^{23}$]

33. At what temperature will the r. m. s. velocity of hydrogen be double of its value at N. T. P., Pressure remaining constant ? [819°C]

34. Calculate the r m. s. velocity of hydrogen at N. T. P. and also at 2184°C. One litre of hydrogen weighs 0·08987 g at N. T. P. [1·84 km s^{-1}, 5·52 km s^{-1}]

35. The diameter of the molecule of gas is 3×10^{-8} cm. Calculate the mean free path and N. T. P. Boltzmann's constant$=1{\cdot}38 \times 10^{-16}$ erg 0$_c^{-1}$. [9.3×10^{-6} cm]

36. With what speed would one g mole of oxygen at 300°K be moving in order that the translation kinetic energy of its centre of mass is equal to the total random K. E. of all its molecules, molecular weight of oxygen being 32 ?
[$4{\cdot}8 \times 10^{4}$ cm s^{-1}]

37. Calcuiate the density of an ideal gas at S. T. P. if the r. m. s. velocity of the molecules be 0·5 km s^{-1}. What will be density of the gas at 21°C, the pressure remaining the same ? (Assume 1 standard atmosphere$=10^{5}$ N m^{-2})
(J. E. E. '83) [1·2 kg m^{-3}, 1·114 kg m^{-3}]

38. One g-mole of helium at 60°C is mixed with one g-mole of argon at 30°C. What is the temperature of the mixture ?
[45°C]

IV-7. (Change of State)

[A] **Essay type questions :**

1. What is latent heat ? What does it do ? What do you understand by the statement that latent heat of ice is 80 ? Is it a proper statement to say only 80 without any mention of unit ? What should be a correct statement ?

Describe an experiment for measuring the specific latent heat of ice what precantions are necessary for two experiment ?

2. How is melting point affected by pressure ? What is regelation ? Explain how a loaded copper wire can pass through a block of ice. Will the experiment succeed if a silk thread replaced the copper wire ? Explain your answer,

3. How would you show that a vapour exert pressure and that the maximum pressure a vapour can exert depends on the tempera'ure.

4. State the principal facts about evaporation and boiling. What is the fundamental difference between them ?

5. How would you show that a liquid boils when its vapour pressure is equal to the pressure on it ?

6 What is the effect of change of pressure on boiling point ? What is meant by normal boiling point ?
Describe one experiment each to show the effect of (a) an increased pressure, (b) a reduced pressure on boiling point.
Mention some applications of boiling under (a) increased, (b) reduced pressure.

7. What do you understand by the statement that the latent heat of steam is 540 calories per gram ?
Describe an experiment to measure this latent heat. What precautions are necessary for the experiment ?

8. Give some examples of cold produced by evaporation. Mention some practical uses of it.

[B] **Short answers type questions :**

9. Suppose that the latent heat of ice were suddenly reduced to half its value. What may be the effect of such a change on the snows of the Himalayas ?

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10. Do all substances have sharp meling points? Give examples.
11 In what sense is freezing a heating process? How does the heat thus produced protect plant and temper the climate?
12. Brass is not suitable for type making why?
13. In cold countries hot water pipes burst more often than cold water pipes running side by side. Why?
14. Explain is a small tube filled with water will freeze if kept surrounded by melting ice?
15. Explain why two pieces of ice can be joined by pressing them togather for a few seconds and then releasing the pressure.
16. What is meant by 'supercooling'?
17. 'In expanding in solidification water exerts a considerable force'—illustrate it with suitable example.
18. A loaded copper wire cuts through a piece of ice without dividing the piece into two halves. State with reason what will happen if (i) the copper wire be replaced by an ordinary thread and (ii) temperature of air goes below 0°C.
19. Why do cracks found in rocks?
20. What are 'Sensible heat' and 'latent heat'?
21. Why is ice at 0°C a better cooling agent than water at 0°C?
22. When can heat be supplied to a substance without cauing a change of temperature? How can this heat be recovered?
23. Why electrical fuses are made of alloys?
24. When common salt is mixed with ice, the temperature of the mixture goes much below 0°C. Why?
25. In cold countries, why water of the radiator of automobiles is mixed with glycol or glycerine?
26. What is eutectic temperature and eutectic mixture?

27. Two metal sphere one of aluminium and other of lead, each having the same weight, are suspended with threads. A vessel containing molten napthalene in equilibrium with solid napthalene is taken and the two spheres at room temperature are simultaneously plunged into the clear molten liquid napthalene at the top of the vessel, kept immersed for about a minute and then taken out. Each gets coated with some frozen napthalene and on weighing the napthalene coated aluminium sphere appears heavier than the napthalene coated lead sphere.

Briefly explans the reason for the difference and state your conclusion about the relevent properties. (J.E.E. '74).

28. Water boils (i) on a hill station, (ii) at sea-level, (iii) at the bottom of a deep mine. How do the boiling points differ at the three places ?

29. A flask containing water is heated. After the water has been boiling briskly for some time, the flask is quickly stoppered and removed from the flame. Explain why, when the flask is dipped into cold water, the water inside begins to boil.

30. To cook food at the top of a high mountain, you employ a method different from that used on the plains. Explain why.

31. How do earthenware pots keep water cool in Summer ?

32. How is dry ice formed ?

33. Can water in a beaker kept on a table be made to boil by passing steam at atmospheric pressure through it ? Why ? (H.S. '65).

34. A piece of boiling paper is placed in a beaker of ether with part of the paper projection over the edge. Soon frost is noticed on the projecting portion of the paper. Why ?

35. The temperature recorded by a thermometer decreases

when its bulb is covered with a piece of cloth soaked in alcohol. Explain.

36. Can water be frozen by its own evaporation ?
37. Why does evaporation cause cooling ?
38. What is the difference between 'latent heat of vaporisation' and 'latent heat of evaporation' ?
39. How can you show that vapour exerts pressure ?
40. Dintinguish between a gas and a vapour.
41. Why is hydrozen called a parmanent gas ?
42. What is the advantage of taking water as the hot substance in a hot water bottle ?
43. A person gets a more severe burn by steam at 100°C, than by boiling water. Why ? (H. S. '82.)
44. Some astronaut on the surface of the moon took some water at about 20°C out of his thermosflask and poured over into a glass beaker. Briefly explain what you would expect to happen to water. (J.E.E. '76).
45. It is found that at ordinary temperature some gases like ammonia can be liquified with suitable pressure, where as other like oxygen cannot be so liquified whatever be the pressure applied. Explain why it is so. (J.E.E. '7:)
46. Why much longer time is required to boil away a quantity of water than that necessary to bring it to the boiling point, heat being supplied at constant rate ?
47. Why the reading of a thermometer is lowered where its bulb is wrapped with a wet rag ? What difference will be observed when the rag is wetted with (i) ether, and (ii) water ? (J.E.E. '74)
48. 'At Darjeeling water boils at much lower temperature than 100°C'—Explain.
49. Why the temperature of a boiling liquid cannot be increased further on application of heat ?
50. It a few drops of ether is poured on the bulb of a thermometer an immediate lowering of temperature is produced.

But when the thermometer is dipped in a bottle of ether, no such lowering of temperature is observed. Explain with reasons.

51. Why does one kg of iron melt more ice than one kg of lead, both being at the same temperature of 100°C ?
52. Why it is harmful to dry wet clothes on the body ?
53. Why does water keep cool in an earthen pitcher but not so in a brass one ?
54. Why does tea cool rapidly when poured in a flat dish ?
55. Why 'Khaskhas' screens are used on the windows in summer ?
56. Why is a thermometer bulb not kept immersed in the liquid in determining boiling poin ?
57. Why are some pieces of broken glass put in a vessel while boiling water in the vessel ?
58. While cooking rice, if a lid is placed on the pan, rice is well-cooked. Why ?
59. Why do we feel comfort under an electric fan during summer ?
60. Why a cold sensation is feel when few drops of ether is poured on the hand ?
61. Why increased pressure lowers the melting point of ice but increases that of wax ?
62. Why does a strong jet of steam burst out from a boiling Kettle when the gas burner is switehed off, while no steam was visible before ?
63. Why vegetables etc are not boiled well on the top of the hill ?

[C] **Numerical problems :**

64. 3 kg of copper heated to 72°C are placed on a block of ice. How much of ice will melt ? Sp. heat of copper is 0·1 and lalent heat of fusion of ice is 80 cal/g. [270g]

65. A piece of ice at 0°C and weighing 15 g is dropped into 85 g of water at 20°C in a copper calorimeter. When all the ice melts the temperature falls to 10°C. If the latent heat of fusion of ice is 80 cal/g, Calculate the water equivalent of the calorimeter. [50 g]

66. 40 g of ice at – 15°C (sp. heat 0·5) are mixed with 150g of water at 90°C. What is the resulting temperature? [45·25°C]

67. 250 g of copper at 90°C are dropped into a copper calorimeter weighing 100 g and containing 10g of ice and 25g of water at 0°C. The specific heat of copper is 0·1 and latent heat of fusion of ice 80 cal/g. Find the resulting temperature. [20·7°C]

68. A 100 g brass ball is cooled to the temperature of liquid air i.e., – 190°C and dropped into a calorimeter containing water at 0°C. Calculate the mass of ice formed (neglecting absorption of heat from the calorimeter). The sp heat of brass in this range is 0·08 and latent heat of fusion of ice is 80 cal/g. [19g]

69. A quantity of ice at 0°C is added to 50g water at 30°C in a calorimeter of water equivalent 10 g. The final common temperature is 10°C. Calculate the mass of ice added. Latent heat of fusion of ice is 80 cal/g. [$13\frac{1}{3}$g]

70. A brass ball weighing 10 g and heated to 250°C is put inside a cavity in a block of ice, of which 2·7 g melt. If the latent heat of fusion of ice is 80 cal/g, caculate the specific heat of brass. [0·09 cal g^{-1}°C^{-1}]

71. A metal vessel containing 250 g of water at 30°C is placed in a refrigerator which abstracts heat at the rate of 275 calories per minute. Calculate the time taken by the water to be converted into ice at 0°C. The latent heat of fusion of ice is 80 cal/g. The weight of the vessel may be neglected. [1 hr. 40 min]

72. 100 g of ice at 0°C are added to 200 g of water at 30°C. What will be the final temperature ? [0°C]

73. The latent heat of fusion of sulphur which melts at 113°C is 9 cal/g and the sp heat of solid sulphur is 0.17 cal/g°C. Find the rise is temperature when 35 g of liquid sulphur at its melting point is poured into a copper calorimeter weighing 40 g and containing 100 g of water at 14°C. (sp. heat of copper = 0·1 cal/g°C). [8·2°C]

74. A pitcher holds 1 litre of water. We wish to cool it from 35°C to 10°C. How many 20 g ice-cubes must be added ? [10]

75. A can of water equivalent 100 g contains 500 g of water at 40°C. Steam at 100°C is allowed to condense in the vessel. What quantity of steam is required to raise the water to its boiling point ? Latent heat of steam is 540 cal/g. [66·7g.]

76. What quantity of steam at 100°C should be condensed in a mixture of 300 g of water and 30 g of ice at 0°C so as to make the final temperature 30°C ? Latent heat of fusion of ice is 80 cal/g and latent heat of steam 540 cal/g. [11·7 g]

77. A calorimeter of which the heat capacity may be neglected, contains 1000 g of water at 30°C. It is placed on an electric stove, when the temperature of water rises to 100°C in 10 minutes. Calculate the amount of heat absorbed by the water per minute. How long will the water at 100°C take to boil away completely ? [7000 cal/min. 77·1 min]

78. By the evaporation of 25g of ammonia 85g of ice at 0°C are formed from water at 20°C. If the latent heat of fusion of ice is 80 cal/g, Calculate the latent heat of vaporization of ammonia. [340 cal/g.]

79. If 70,000 cal of heat are extracted from 100 g of steam at 100°C, what will be the result? (Latent heat of steam= 540 cal/g, Latent heat of ice=80 cal/g.]

[75 g of ice and 25 g of water at 0°C.]

80. A copper vessel of water equivalent 60 g contains 600 g of water at 30°C. A bunsen burner, adjusted to supply 100 calories per second is used to heat the vessel. Neglecting all losses calculate (i) the time required to raise the water to boiling point and (i) the time required to boil away 50 g of water. (Latent heat of steam=540 cal/g.)

[462 s, 270 s after (i)]

81. A mass of ice, 5g in weight, initially at – 20°C is heated till it is all converted in to steam at 100°C.
Calculate the total amount of heat required, assuming the sp. heat of ice to be 0·5, the latent heat of fusion of ice 80 cal/g and that of steam 540 cal/g. [3650 cal]

82. When steam at 100°C is passed into a mixture of ice and water contained in a vessel it is found that 1·5g is condensed before all the ice is melted and the mixture rises to 4°C. How much ice was there to begin with? (water equivalent of the vessel and water = 50g, Latent heat of water=80 cal/g, latent heat of steam=`40 cal/g.)

[9g nearly.]

83. 10g of water at 0°C is rapidly evaporated until the remaining water freezes to ice. Assuming that no heat is absorbed from the surroundings, calculate how much water evaporates away when the remaining water is all frozen. (Latent heat of evaporation of water is 600 cal/g, latent heat of ice=80 cal/g.) [12g nearly.]

84. Steam at 100°C is allowed to flow over a piece of ice at 0°C. After sometime it is found that the amount of water formed is 225g. The initial and final mass of ice

are 850g and 650g respectively. Find the latent heat of vaporisation of steam. (Latent heat of ice=80 cal/g).

[540 cal/g.]

85. A porous vessel having 100g of water in its pores contains 500g of water at 25°C. How much water must evaporate so that the remaining water should be at 20°C ? The latent heat of vaporisation of water at 25°C is 580 cal/g. [5·17g]

86. A mixture of 250g of water and 200g of ice at 0°C is kept in a calorimeter which has a water equivalent of 50g. If 200g of steam at 100°C is passed through this mixture, calculate the final temperature and weight of the contents at the calorimeter. (I.I.T,'74) [$72·22g of water at 100°C]

8.. Equal quantities of hot water and ice were mixed. When ice melted the temperature of the mixture was found to be 0°C. What was the temperature of the hot water ?

[80°C]

88. 10g of ice at −80°C are dropped into some water at 20°C. When all ice melts, the temperature of the mixture is 15°C. Again 15·6g of ice at 0°C added to the mixture. After melting, the temperature of the mixture is brought down to 10°C. Find the latent heat of fusion of ice. Find also the initial mass of water, sp. heat of ice = 0·5 cal/g.

[79·4 cal/g. 268·8g.]

89. Density of water and ice at 0°C are $1 gcm^{-3}$ and $0{\cdot}916 gm^{-3}$ respectively. A piece of metal, weighing 10g is heated to 90°C and is then dropped into mixture of water and ice. Some ice melts and the volume of the mixture contracts by $0{\cdot}1 cm^3$ without any change of temperature. Find the sp. heat of the metal, if the latent heat of fusion of ice=80 cal/g. (I.I.T, '64) [0·087]

90. 10g of a substance was taken in the solid state at −10°C. 64 calories were required to heat it to 2°C (still in the solid state) and 880 and 900 calories were required to heat it to the liquid state at 1°C and 3°C respectively.

Assuming that the sp. heat of the meterial in the solid and liquid state has values S_1 and S_2 ($S_1 = S_2$) respectively, find their values. Show that the latent heat of fusions L is related to the melting point temperature t_m by $L = 79 + 0 \cdot 2\, t_m$. (J.E.E. '82) [0·8, 1]

91. The boiling point of water at a lower station is 100°C and that at the upper stations is 96°C, the temperature of air at the two stations being 14°C and 10°C respectively. Density of air at N. T. P. is $1{\cdot}293 \times 10^{-4}$ g cm^{-3}. Find the height between the stations. [1·277 km.]

92. 100g of copper nails are heated to 100°C and are then dropped into a calcrimeter whose mass is 100g. The calorimeter contains 40g of a mixiure of ice and water. If the final temperature is 10°C, find the mass of ice in the mixture. Sp. heat of copper = 0·09 and latent heat of fusion of ice = 80 calg^{-1}. (I.I.T. '62) [4g]

93. 5g of water at 30°C and 5g of ice at – 20°C are mixed togather in a calorimeter. Find the final temperature of the mixture. Water equivalent of the calorimeter is negligible. Sp. heat of ice = 0·5 ; Latent heal of ice = 80 cal g^{-1}. (I.I.T. '77). [0°C]

94. An aluminium container of mass 100g contains 200g of ice at – 20°C. Heat is added to the system at the rate of 100 calories per second. What is the temperature of the system after 4 minutes ? Sp. heat of aluminium = 0·2 ; Sp. heat of ice = 0·5 ; Latent heat of fusion of ice = 80 cal g^{-1}. (I.I.T.'73) [25·5°C]

95. An earthen pitcher loses 2g of water per minute due to evaporation. If the water equivalent of the pitcher is 0·5kg and the pitcher contains 9.5 kg of water, calculate the time required for the water in the pitcher to cool to 28°C form its original temperature of 30°C. Neglect radiations effects. Average latent heat of vaporisation of

water in this range of temperature = 58 cal g^{-1}. (I.I.T.'70) [34.4 min.]

96. 2 g of steam at 100°C is allowed to pass through 55g of water at 10°C, kept in a copper calorimeter of mass 50g. The final temperature of the mixture is 30°C. Find the latent heat of vaporisations of water. Sp. heat of copper = 0·1. (J.E.E. '72) [530 cal g^{-1}]

97. How should 1kg of water at 5°C be so divided that one part of it when turned into ice at 0°C would by this change of state, gives out a quantity of heat just sufficient to vapories the other part ? (J.E.E. '80) [881·96g, 118·04g.]

98. When a piece of metal weighing 48·3g at 10·7°C was immersed in a current of steam at 100°C, 0·762g of steam was found to condense. Calculate the sp. heat of metal. Assume latent heat of vaporisations of water as 540 cal g^{-1}. (I.I.T. '63) [0·095]

99. 7.5g of copper at 97°C were dropped into liquid oxygen at its boiling point (−183°C) and the oxygen evaporated occupied 1·89 litres at 20°C and 750mm pressure. Calculate the latent heat of vaporization of oxygen. Sp. heat of copper = 0·08 ; Density of oxygen at N.T.P. = 1·429 gl^{-1}. (Cambridge) [50·74 cal g^{-1}.]

100. A copper calorimeter weighing 100g contains 150g of water at 30°C. Pieces of ice which have not been dried are dropped in and the final temperature after stirring is 5°C. The weight of the calorimeter and its contents is then 300g. How much water was put in with the pieces of ice ? (Take latent heat of ice as 80 cal g^{-1} and the specific heat of copper as 0 1). (Oxford.) [3·125g.]

101. In an industrial process 10kg of water per hour is to be heated from 20°C to 80°C. To do this, steam at 150°C is passed from a boiler into a copper coil inmersed in water. The steam condenses in the coil and is returned to the boiler as water at 90°C. How many kg of steam

are required per hour ? (Sp. heat of steam = 1 cal/g°C ; Latent heat of Steam = 540 cal g^{-1}.) (I.I.T. '72) [1 kg]

102. A sample of steam initially at 110°C and a piece of ice originally at −15°C come to thermal equilibrium with water at a temperature of 40°C at normal atmospheric pressure. Find the ratio of the mass of ice to the mass of steam used. Given latent heat of steam = 540 cal g^{-1}, that of ice = 80 cal g^{-1} ; Sp. heat of steam = 0·53 and that of ice = 0·48. [4·73]

103. 15g of ice at 0°C are put into a calorimeter of mass 100g containing 200g of water and the final temperature reached is observed. The experiment is then repeated with the same masses and calorimeter and with the water at the same initial temperature but with the ice initially cooled to −180°C. The final temperature is then 6 degree celsius lower than before. Neglecting heat exchange between the calorimeter and its surroundings, calculate a value for the mean sp. heat of ice between −180°C to 0°C. The sp. heat of copper may be taken as 0.10 cal^{-1}°Cg^{-1}. (London). [0·5 cal g^{-1} °C^{-1}.]

104. In one method for storing solar energy, Glauber's salt can be allowed to warm up to 45°C in the sun's rays during the day and the stored energy is used during the night, the salt cooling down to 25°C. Glauber's salt melts at 32°C. Calculate the mass of the salt needed to store 10^6 joules. Sp. heat of solid salt = 110 $Jkg^{-1}k^{-1}$, sp, heat of molten salt = 160 J $kg^{-1}k^{-1}$, latent heat of fusion $= 1{\cdot}4 \times 10^4$ Jkg^{-1}. [59·5kg.]

105. 20 minuts were required to heat a certain quantity of water from 0°C to the boiling point with an electric heater. A further 1 hr. 48 min. were needed to turn all the water into steam under the same condition. Determine from these data the latent heat of vaporisation of water. [540 cal g^{-1}.]

106. Heat is supplied to 50 g of a solid at the rate of 5 cal/s and the temperature of the solid rises 11°C/min. After a time the temperature remains steady for 13 min and then begins to rise 6°C/min. Calculate the sp heat of the substance in the solid and liquid state and its latent heat of fusion. [0 545, 1, 78 cal g^{-1}.]

107. A certain amount of ice is supplied heat at a constant rate for 7 minutes. For the first 1 minute, the temperature rises uniformly with time, then if remains constant for the next 4 minutes and again rises at a uniform rate for the last 2 minutes. Explain physically these observations and calculate the final temperature. (J.E.E. '75) [40°C]

108. 800 g of ice is found to melt when 100 g of steam at 100°C is condensed in an ice cavity at 0 C. Further it is found that when a copper ball of 150 g at 500 C is dropped in the ice cavity it melts 95 g of ice. If the sp heat of copper =420 Jkg^{-1}, calculate the sp. latent heat of ice and steam. [$33{\cdot}2\times10^4$ Jkg^{-1} $223{\cdot}6\times10^4$ Jkg^{-1}.]

109. 20 g of ice at 0 C are dropped into a mixture of oil and water at 30 C and are just melted in cooling the mixture to 0°C. If the sp. heat of oil is 0·6 and the total weight of the liquid at the end of the experiment is 85 g, find the quantities of oil and water in the original mixture. [29·2g oil and 35·8 g water.]

110. A test tube containing 3 g of ether is immersed in a beaker of water which is surrounded by melting ice. When whole is at 0°C, a stream of air at 0 C is blown through the ether until it has all evaporated. The cap of ice formed round the test tube has a mass of 3·14 g. Find the latent heat of vaporisation of ether. Latent heat of fusion of ice= 80 cal g^{-1}. (Oxford) [83·73 cal g^{-1}]

IV-8 (Hygrometry)

[A] **Essay type questions :**

1. Define dew point and relative humidity. How is the later expressed ? Distinguish between relative and absolute humidity.
2. Describe Regnaul.s hygrometer for measuring dew point. How would you determine humidity with its help ?
3. What is vapour pressure ? How would you show that (i) a liquid can exert a maximum vapour pressure at a given temperature, (ii) this maximum value increases with rise of temperature, and (iii) different liquids have different vapour pressures at the same temperature ?
4. Explain how dew and frog are formed.

[B] **Short answers type questions :**

5. The bulb of a thermometer is wrapped round with cotton, which is wetted in turn with (i) water, (ii) ether, (iii) oil. How will the readings differ and why ?
6. Explain why on a hot summer day immediately after a rain, a block of ice on a cart appear to steam copiously.
7. What becomes of the steam which a boiling kettle discharges into a room ?
8. A piece of glass is dimmed when you blow on it with your month on a winter morning, but not on a summer noon. Explain.
9. Why is summer heat often oppressive before a shower ?
10. What kind of weather would you expect to find when the dew point and the air temperature are the same ?
11. Why does warm moist air cause more discomfort than warmer dry air ?
12. What would be the effect on the readings of a barometer if (i) little air, (ii) a little water, were left in the Torricellian vacuum ? How would you detect whether it was air or water ?

13. What is meant by the statement—The aquous tension at 20°C is 17.5 mm.
14. The relative humi'ity of an atmosphere is 60%—Explain the statement.
15. 'The dew-point on a certain day is 19.5°C'—Explain.
16. Explain what effect, if any, there will be on the dew point and humidity if (i) a quantiiy of water is gradually sprinkled in the room, (iii) the temperature of the atmosphere in the room is raised.
17. Mention the difference between a saturated and unsaturated vapour.
18. Is it possible for the dew point to go below 0°C ? Explain wi'h reasons.
19. Under what condition, will the room temperature be equal to the dew point ?
20. Dews are found to deposit sometimes on the outer surface and sometimes on the inner surface of a window glass pane. Under what conditions such formation of dews may occur ?
21. When is dew point not found ?
22. Delhi is more comfortable than puri on an equally hot day. Explain why ?
23. Why is wet clothes usually dry up quicker in winter than in rainy season although the temperature in winter much less ?
24. Which one is necessary for weather forecasting—relative humidity or absolute humidity ?
25. Cloudiess nights help better deposition of dews than cloudy night—Explain.
26. The temperature of the two rooms are equal but the relative humidity are different. Which room will appear more comfortable ?
27. Why do dews accumulate on the outer surface of a glass vessel when ice water is poured in it ?

28. Why do dews form on some substance than other ?
29. What are the factors responsible for copions deposition of dews ?
30. What is the saturated vapour pressure of water at 100°C ?
31. Why is dew formed more on the grass than on the tree leaves at night ?
32. When humidity lies within a definite range we fell comfortable—Explain why ?
33. Why we apply glycerine on the lips during winter ?
34. In a closed room if the temprature is increased what happens to (i) the dew point, (ii) relative humidity ?

[C] **Numerical problems :**

35. Find the relative humidity when the room temperature is 40°C and the dew point 30°C. S.V P. of water at 40°C and 30°C are respectively 5.53 cm and 3.18 cm. [54%]
36. The temperature in a closed room is observed to be 15°C and the dew point is 8°C. If the temperature falls to 10°C how will the dew point be affected ? (S.V.P. at 7°C and 8°C are respectively 7.49 mm and 8.02 mm of Hg. [Lower by 0.25°C]
37. Calculate the mass of 7.5 litre of most air at 27°C, given that the dew point is 15°C and the barometric height 76.275 cm. Calculate also the humidity of air. S.V.P. of water at 27°C and 15°C are 25.5 mm and 12·75 mm respeetively. [8.9g, 0·5]
38. Calculate the dew point when the air is 2/3 saturated with water vapour, the temperature being 15°C, given that for the pressure of 7,9,11,13 mm Hg, the corresponding boiling points of water are 6°C, 10°C, 13°C and 15°C respectively. [9.3°C]
39. Calculate the temperature at which dew will be deposited when the hygrometric state of the air is 20°C and relative humidity is 53%. The S.V.P. at 20°C, 10°C and 9°C are repectively 17·5, 9·2 and 8 6 mm of Hg. [10.1°C]

40. The pressure of a saturated vapour at 21°C, 22°C and 30°C are respectively 18.6 mm, 19.1 mm and 31.7 mm of Hg. If the temperature of air is 30°C and the dew point is 21.5°C, find the relative humidity of air. [60.6%]

41. The temperature of air on a certain day is 23°C and the relative humidity is 50%. What fraction of the mass of water vapour in air would condense, if the temperature falls to 10°C ? (S.V.P. at 23°C=21.1 mm and at 10°C =9.2 mm.) [0.23]

42. If 200 g of water are collected to evaporate in a room containing 50 m^3 of dry air at 30°C and 760 mm of Hg pressure, what will be the relative humidity of the air in the room ? (S.V.P. at 30°C is 31.6 mm.) [0.13]

43. The relative humidity in a closed room at 15°C is 60%. If the temperature rises to 20°C, what will be the relative humidity then ? On what assumptions is your calculation based ? S.V.P. at 15°C=12·67 mm Hg and at 20°C= 17.36 mm Hg. [45%]

44. A closed room of volume 125 m^3 is filled with air at 27°C. If the relative humidity falls from 90% to 40% find the mass of water vapour condensed. (S.V.P at 27°C=26.7 mm Hg and $R=8{\cdot}3\times10^7$ erg/deg/mole, Molecular weight=18.) (I.I.T. '70) [1286 g]

45. A vessel of water is put in a dry sealed room of volume 76m^3 at a temperature of 17°C. The saturated vapour pressure of water at 17°C is 15 mm of Hg. How much water will evaporate before the water is in equilibrium with its vapour ? (I. I. T. '78) [1.135 kg]

46. A flask is completely filled with 1 g of saturated steam at 100°C. How much water will condense into water if the temperature of the flask is reduced to 25°C ? (Aqueous tension at 25°C=30 mm Hg.) (J.E.E. '81) [0.96 g]

47. Calculate the mass of 1 litre of moist air at 27°C when the barometer reads 753·6 mm of Hg and the dew point is 16,1°C. (S.V.P. at 16·1°C = 13·6 mm Hg ; the density of air at N.T.P = 0.001293 g cm^{-3}, that of saturated water vapour at N.T.P. = 0.000808 g cm^{-3}.) (I.I.T. '77) [1.159 g]

48. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm Hg. The temperature of the jar is reduced by 1%. The saturated vapour pressures of of water at the two temperatures are 30 and 25 mm Hg. Calculate the new pressure in the jar.

(I.I.T. '81) [817 mm Hg]

49. Air at 30°C and 90% relative humidity is drown into an air conditioning unit and cooled to 20°C, the relative humidity being reduced to 50%. How many gram of water vapour must be removed by the air conditioner from a cubic metre of air ? Neglect the change of volume of air. Given density of saturated vapour in air at 30°C is 30 g m^{-3} and at 20°C is 17 g m^{-3}. (J E.E. '72) [18.79 g]

50. A capillary tube of uniform bore has some air entrapped by a water index. When the atmospheric pressure is 76.25 cm. and the temperature is 20°C, the air-column is 15·6 cm long. With the tube immersed in water at 50°C, it was 19.1 cm long the atmospheric pressure remaining the same. If the S V.P. of water at 20°C is 17.5 mm, deduce its value at 50°C. [9.17 cm Hg]

51. A closed vessel of constant volume contains a mixture of air and water vapour. The air and water vapour exert pressure of 5 KN/m^2 and 1.50 KN/m^2 respectively at 20°C. Determine the temperature at which the water vapour becomes saturated as the temperature is gradually reduced to 0°C. S.V.P. of water at 0°, 5°, 10°, 15° and 20°C are respectively 0.61, 0.86. 1.21, 1.70 and 2.33 KN/m^2 respectively. (12 5°C).

IV-9 (Transfer of heat)

[A] **Essay type questions :**

1. Illustrate with suitable examples the different methods of transfer of heat. Bring out clearly the difference between them.

   How would you show that different substances have diffierent conductivities ?

2. Describe briefly and explain the actions of (a) a practical appliance where use is made of the good conducting power of a material, (b) an appliance where a bad thermal conductor is employed.

3. Define thermal conductivity of a substance. Explain whether the establishment of steady state is essential for this definition to be applicable. What is the unit of thermal conductivity in C.G.S. and F.P.S. systems ?

4. Draw a sketch of Davy's safety lamp. Explain the use of wire gauze surrounding the flame.

5. What is a convection current ? Give three examples of convection currents where they are made to serve some useful purpose.

   Distinguish between conduction and convection.

6. State Stefan's law. Write a short note on it, stating any use that you know of it.

   When Stefan's law is used for measuring the temperature of a molten metal is an open crucible, do we get the correct temperature of the metal ? Give reasons for your answer.

[B] **Short answer type questions :**

7. Hot water is placed in two identical jugs, one with a polished white surface, and the other with black surface. Which one will cool more quickly and why ?

8. Clouds prevent loss of heat by radiation from the earth's

surface. How does presence of night clouds affect formation of mist and fog ?

9. The squirrel wraps its brushy tail round its body during its winter sleep. Explain why ?

10. Explain why do bird puff their feathers out on a cold day.

11. A hen, wishing to sit on eggs hatch out chicks, casts feathers from her breast. Why ?

12. A house with a straw roof keeps cool in summer and warm in winter—why ?

13. The bulb of a delicate thermometer is immersed at a slight depth below the surface of water. The upper surface of the water is heated ; but the thermometer hardly shows any temperature rise. On heating the water from below, the thermometer shows an immediate rise. Explain with reasons.

14. A gas-filled electric lamp feels hotter than an evacuated lamp when lit. Why ?

15. To minimise transfer of heat between a calorimeter and its surrounding, it is placed on cork tips (or suspended by silk or cotton threads) inside an enclosure. Also, its surfacc is polished and silvered.

    Briefly explain how all these mininise the tranference of heat.

16. Explain why a thermometer with its bulb covered by soot, when kept in the sun, records a higher temperature than an ordinary one.

17. Why the flame of a bunsen burner cannot get through a piece of wire gauge placed upon it ?

18. (i) Why the space between the double walls of a themos-flask is evacuated ? Why the inner surface of these walls is silvered and (iii) Why the mouth of the flask is closed by a cork stopper ?

19. What is meant by the statement that the thermal conductivity of glass is 0.002 C.G.S. unit ?
20. What are your arguments in favour of the conclusion that radiant heat is but invisible light ?
21. When a piece of metal cooled much lower than 0°C is touched, a sensation of scorching is produced. Why ?
22. Why are cooking utensils usually made of metal ?
23. Why are metal fins attached to a motor car radiator ?
24. On a hot day the surface water of a pond is warmer than the water below, but on a day when it is nearly freezing, surface of the water is colder, Why ?
25. Why does human body remains warm on cold day and a lump of ice remains cold on a hot day if each is wrapped in a blanket ?
26. A stone floor feels cold in the winter, but when a carpet is laid we feel warm. Why ?
27. Why does a greenhouse keep hot in winter ?
28. If you touch a piece of iron and a piece of wood lying exposed to the heat of sun, which feels hotter and why ?
29. Why are woollen garments called warm ?
30. Why is the handle of a kattle wrapped with cane ?
31. Which will give you comfort in winter—one thick shirt or two of half the thickness, the material being the same ?
32. What should be the nature of the bottom of a cooking vessel ?
33. A wooden rod can be held at one end for a pretty long time with the other end pushed into a fire but an iron rod similarly placed cannot be held for a long time. Why ?
34. You have taken a very fine powder of chalk in one hand and a course powder in other. If you exchange hot metal ball from one hand to other, which hand will feel warmer ?
35. Why is it hotter the same distance over the top of a fire than it is in front ?

36. Black clothes are prefered in winter. Why ?
37. Why is the umbrella cloth usually black ?
38. Two rods of different material are of equal length and of crosssection A_1 and A_2. If the temperatures at the two ends of each rod are θ_1 and θ_2, find the condition so that the rate of heat flow be equal.
39. At what temperature would a block of wood and a block of matal feel equally cold or equally hot when touched ? (I.I.T. '76)
40. Explain why felt rather than air is used for thermal insulation even though the thermal conductivity of air is less than that of felt ? (I.I.T. '78).
41. Heat is generated continuously in an electric heater but its temperature becomes constant after some time. Why ?
42. In a desert, it is too hot during day time and too cold during night. Explain.
43. Two thermometers are constructed in the same way except that one has a spherical bulb and the other an elongated cylindrical bulb. Which one will respond quickly to temperature changes ? (I.I.T. '75)
44. Why do we feel maximum hot inside a room a few hours after the maximum temperature outside ?
45. 'Good emitters one good absorbers and bad emitters are bad absorbers'—Discuss the statement.

[C] Numerical Problems :

46. An Aluminium pan is placed on a fire for water to boil in it. If the bottom is 1 mm thick what is the temperature of the side facing the fire, given that 1 g of water vaporizes in 2 minutes per cm² area of the bottom. (K for Al=0.5 cgs unit, Latent heat of steam = 540 cal g^{-1}.)

[101°C nearly]

47. The metal plate of a boiler is 1.5 cm thick. Find the difference in temperature between its faces if 32 kg of

water are evaporated from the boiler per m^2 per hour. (Latent heat of steam$=540$ cal g^{-1}, k of boiler plate$=0\cdot15$ cgs unit). [4·8°C]

48. The inside of a glass window, 2 mm thick and $1m^2$ in area, is at a temperature of 15°C and the outside is at 5°C. Calculate the rate at which heat is escaping per hour from the room by conduction through the glass. [$7\cdot2\times10^8$ cal]

49. A rod of metal of thermal conductivity 0·9 is 31·41 cm long and 4 cm in diameter. One of its ends is kept exposed to steam at 100°C and the other in contact with a block of ice at 0°C. How much ice will melt per minute in a steady state ? [27 g.]

50. An iron plate is 1 mm broad and its area is 150 cm^2. The two opposite surfaces of the plate are at temperature 100°C and 30°C and in 1 second 3940 calories of heat flow from one surface to the other. What is the thermal conductivity of iron ? (H.S. '83) [0·15 cgs unit.]

51. A composite slab is made of two material x and y of eqal area. The thickness and the thermal conductivity of x are d_1, k_1 and that of y are d_2, k_2. If θ_1 and θ_2 be the temperature of the free surfaces of the composite slab, then show that the composite slab may be replaced by a single material whose thermal conductivity

$k=\dfrac{k_1k_2(d_1+d_2)}{k_2d_1+k_1d_2}$ and the temperature of the interface is

$$\theta=\frac{k_1\theta_2d_2+k_2\theta_1d_1}{k_2d_1+k_1d_2}$$

52. A bar of copper of length 75 cm and a bar of steel of length 125 cm are joined togather end to end. Both are circular cross-section with a diameter 2 cm. The free end of copper and steel are maintained at 100°C and 0°C respetively. The surface of the bars are thermally insula-

ted. What is the temperature of the copper-steel junction ? Thermal conductivity of copper $=9{\cdot}2\times10^{-2}$ k cal/m/°C/S and that of steel $=1{\cdot}1\times10^{-2}$ k cal/m/°C/S. (I.I.T. '77)

[93·3°C, 0·258 cal s^{-1}]

53. A uniform copper bar, 50 cm long, is lagged and has its ends exposed to ice and steam respectively. If there is a layer of water 0·1 mm thick at each end, calculate the temperature gradient in the bar and the temperature of the two ends of the bar. Thermal conductivities of copper and water are 1·04 and 0·0014 cgs units respectively. (Combridge) [1·542°C/cm, 11·46°C, 88·54°C]

54. Three rods of equal length L cm and equal areas of cross-section S cm^2 are joined in series. The thermal conductivities of the materials are k, 2 k and 1.5 k cgs unit. If the open end of the first and last rods are at 200°C and 18°C, calculate the temperature at the two junctions and rate of flow of heat through the system neglecting radiation losses. (Rajasthan '75). [74°C, 11°6C, 84 ks/L]

55. Calculate approximately the heat passing per hour through the walls and windows of a room 5 by 5 by 5 metre, if the walls are of bricks of thickness 30 cm and have windows of glass 3 mm thick and total area 5 m^2. The temperature of the room is 30°C below that of the outside and the thermal conductivity of the bricks and of the glass are respectively 12×10^{-4} and 25×10^{-4} cgs unit. (Agra)

[$49{\cdot}104\times10^{-6}$ cal]

56. A boiler is made of a copper plate 2·4 mm thick coated inside with a layer of tin 0·2 mm thick. Surface area exposed to hot gases at 700°C is 100 cm^2. Calculate the maximum amount of steam that could be raised per hour at atmospheric pressure. Conductivities of copper and tin are 0.9 and 0.15 cgs unit respectively ; Latent heat of steam at normal pressure is 540 cal g^{-1}. (A-M.I.E.)

[10^6 g]

57. The temperature gradient in the earth's crust is 42°C per km and the mean conductivity of the rocks is 0·008 cgs. unit. Taking the radius of the earth as 6000 km, calculate the daily loss of heat by the earth. (Gujrat Univ.) [$1{\cdot}009 \times 10^{20}$ cal.]

58. A Slab of compressed cork 5 cm thick and 2 m^2 area has a heating coil at one face. A current of 1·18 amp at 20 volt passes in the coil. The face of the slab attain steady temperature of 12·5°C and 0°C. Assuming that the whole of the heat developed in the coil is conducted through the slab, calculate the conductivity of cork. (Punjab Univ.) [$1{\cdot}12 \times 10^{-4}$ cgs unit.]

59. An ice box built of wood 1,75 cm thick, lined with cork 3 cm thick. If the temperature of the inner surface of the cork is 0°C and that of the outer surface of wood 12°C, what is the temperature of the interface ? Thermal conductivity of wood=0·0006 and that of cork=0·000012 cgs unit. [10·75°C]

60. A person has covered his body with a flannel shirt 4 mm thick. If the outside temperature and body temperature be 27°C and 98·6°F, how much heat will he lose per hour per m^2 of his body ? Thermal conductivity of flannel= 0·00012 cgs unit. (J.E.E. '74) [101×10^{3} cal.]

61. Assume that the thermal conductivity of copper is four times that of brass. Two rods of copper and brass having the same length and cross-section are joined end to end. The free end of the copper rod is kept at 0°C and free end of the brass rod is kept at 100°C. Calculate the temperature of the junction of the two rods at equilibrium. Neglect radiation loss. (I.I.T. '70) [20°C]

62. A closed cubical box made of a perfectly insulating material has walls of thickness 8 cm and the only way for heat to enter or leave the box is through two solid cylindrical metallic plugs, each of cross-sectional area 12

cm^2 and length 8 cm fixed in the opposite walls of the box. The outer surface A of one plug is kept at 100°C while the outer surface B of the other plug is maintained at 4°C. The conductivity of A and B is 0·5 cgs unit. A source of energy generating 36 cal s^{-1} is enclosed inside the box. Find the equilibrium temperature of the inner surface of the box assuming that it is the same at all points on the inner surface. (I.I.T. '72) [76°C]

63. A copper block consisting a resistance coil is suspended in vacuum by two wires 20 cm long and 1·0 mm in diameter. To maintain the temperature of the block at 40°C above the other end of the suspension, a power of 0·084 watt must be dissipated in the coil. Calculate the co-efficient of thermal conductivity of the material of the suspension wire. Neglect the energy transferred by radiations. (Cambridge). [0·6366 cgs unit]

64. A copper calorimeter with a copper heating element fitted inside weighs 300 g. It is filled to a certain mark with 200 g of a liquid. With 41 watt electrical input to the heater, the temperature rises from 20°C to 45°C in 10 minutes. When 140g of the liquid is replaced by 1250 g of copper rivets such that the liquid level remains the same and the calorimeter heater is supplied with the same electrical input as before, the temperature rises from 20°C to 45°C in 9 minute 5 second. It is found steady temperature of 45°C can be maintained in either case with an electrical input of 2 watt. The room temperature remaining constant at 20°C throughout the experiments. Calculate the specific heats of (i) copper and (ii) the liquid. (J.E.E. '80) [0·095, 1·0]

65. Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 10 cm in diameter at a temperature of 227°C, when placed in an

enclosure at 27°C. (Stefan's constant $\sigma = 5{\cdot}7 \times 10^{-12}$ watt cm^{-2} $°C^{-4}$). (Allahabad). [52 02 cal s^{-1}.]

66. How much faster does a cup of tea cool one degree from 100°C than from 30°C in a room at 20°C ? [11·2 times]

67. Luminosity of Rigel Star in Orion consteuation is 17,000 times that of our sun. If the surface temperature of the sun is 6000°K, calculate the temperature of the Star. (Punjab Univ.) [68520°K]

68. If each cm^2 of the sun's surface radiates energy at the rate of $1{\cdot}5 \times 10^3$ cal s^{-1} cm^{-2} and Stefan's constant is $5{\cdot}7 \times 10^{-5}$ erg s^{-1} cm^{-2} degree absolute^{-4}, calculate the temperature of the sun's surface in degree celsius, assuming that Stefan's law applies to the radiation.. (London Inter) [5392°C]

69. A black body with an initial temperature of 300°C is allowed to cool inside an evacuated enclosure surrounded by melting ice at the rate of 0·35°C per second. If the mass, sp. heat and surface area of the body are 32g, 0·10 and 8 cm^2 respectively, calculate the Stefan's constant. [$5{\cdot}7 \times 10^{-5}$ erg cm^{-2} s^{-1} degree^{-4}]

70. Estimate the value of the Stefan's constant if the temperature of the filament of a 40 watt tungsten lamp is 2170°C and the effective surface area of the filament is 0·66 cm^2. You are to assume that the energy radiated is 0.31 of that from a black body in similar conditions and that any effect due to radiation from the glass envelope is negligible. (London Univ.) [$5{\cdot}497 \times 10^{-5}$ erg cm^{-2} s^{-1} k^{-4}]

IV-10 (Heat and work)

[A] **Essay type questions :**

1. What arguments would you offer to show that heat is a form of energy ? What is your idea of the nature of heat ?

2. Explain the term mechanical equivalent of heat. Describe Joule's experiment for the determination of this quantity. How much is a calorie equivalent to in erg ?
3. When heat is added to a body a part of it will increase the internal energy of the body and a part may do external work. Express the result in mathematical symbols, explaining what the positive or negative sign of a symbol would mean. Illustrate the equation with an example.
4. Explain what is meant by the first law of thermodynamics.
5. Explain what you understand by (i) isothermal expansions, (ii) adiabatic expansion of a gas. How can adiabatic expansion be practically realized ?
6. Define the specific heat of a gas at constant pressure (c_p) and that at constant volume (c_v). Explain why they are different. Which is bigger and why ?

[B] Short answer type questions :

7. 'Heat is energy in transit due to temperature difference'—comment.
8. 'The joule can serve as the unit of heat ; it is unnecessary to introduce the calorie'—comment.
9. What do you mean by the statement that the mechanical equivalent of heat is 4·2 joule/cal ?
10. Sparks are produced when cutleries are sharpened. Where does the heat come from ?
11. Give two examples where mechanical energy is converted into heat.
12. What is meant by the internal energy of a gas ?
13. Why there are two sp. heats of gases ?
14. What is the importance of ratio of two sp. heats of a gas ?
15. Isothermal change is essentially a very slow process but adiabatic change is essentially a very fast one—Explain.
16. Why rubbing generate heat ?

17. If you hammer a body, it gets heated—Why ?

18. Why the rushing air from a bursting tyre cold ?

19. There is a difference of temperature of water at the top and at the bottom of a waterfall. Why this differnce ?

20. When air is pumped into a tyre, the pump is heated. Why ?

[C] **Numerical Problems :**

21. In a determination of Joule's constant two 10 kg weight are allowed to fall 20 times through a height of 3 m. This increases the temperature of the calorimeter by 0·4°C, the water equivalent of the calorimeter and its contents being 7 kg. Calculate the value of J from these data.
[$4{\cdot}2\times10^7$ erg cal^{-1}]

22. From what height must 100 g of copper (sp. heat=0·1) fall so that its temperature may rise by 1°C ? Assume that all the energy is retained by the copper. J=4·2 J cal^{-1}
[About 43 m]

23. A glass tube, 1 mm long, is closed at both ends. It contains 250 g of lead shots and 1 litre of water. The tube is set up vertically and is then suddenly inverted. How many times must this be done so that the temperature of the water may rise by 1°C. Ignore effects due to buoyancy of lead shots and the heat that goes into the lead shots and the glass tube.
[1723 times]

24. A lead shot hits a rigid target and increases in temperature by 200°C. It the entire energy is retained by the shot what was its velocity ? sp. heat of lead=0.03.
[224·5 m s^{-1}]

25. A body weighing 10 kg. drops from a height of 1 km. If its potential energy is fully converted into heat, how many calories will be generated ? J=4·2 J cal^{-1}.
[$2{\cdot}33\times10^4$cal].

26. A lead shot moving with a speed of 336 ms^{-1} hits a target. If 75% of the kinetic energy is converted into heat in the shot, what will be its temperature rise? $J = 4{\cdot}2 \times 10^7$ erg cal^{-1}, sp. heat of lead $= 0{\cdot}03$. [336 C°]

27. In an experiment for determining J, a card board tube, 1 m long and containing 800 g of lead shots, is inverted 50 times. The temperature of the shots is found to have increased by 28.84 C°. sp. heat of lead is 0·031 and $g = 980$ cm s^{-2}. Assuming that the heat generated has been fully retained by the shots, calculate the value of J.

[4.12×10^7 erg cal^{-1}]

28. A water fall is 300 m high. If 50% of the potential energy is converted into heat and retained by the water, what is the temperature difference between the water at the top and the bottom. $J = 4{\cdot}2$ J cal^{-1} [0·58°C]

29. Calculate the change in the internal energy when 5 g of air are heated from 0°C to 2°C, the sp. heat of air at constant volume being 0·172 cal g^{-1} °C^{-1}. (Agra) [$7{\cdot}19 \times 10^7$ erg.]

30. One gram of water (1 cm^3) becomes 1671 cm^3 of steam when boiled at a pressure of one atmosphere. The latent heat of vaporization at this pressure is 540 cal g^{-1}. Compute the external work and the increase in internal energy. [41 cal, 499 cal]

31. An electric drill rated at 240 V, 1·5 A is used to drill a hole in a piece of iron weighing 200 g. Assuming that 75% of the total energy heats the iron, calculate the rise in temperature in 15 second. sp. heat capacity of iron $= 0{\cdot}47 \times 10^3$ J kg^{-1}deg C^{-1} [44°C]

32. A fire hose with an outlet diameter of 4 cm delivers one cubic meter of water in 16 seconds. The jet is directed against a rigid wall. Calculate the rise in temperature of the water assuming that all the kinetic energy of the jet is converted into heat which is retained in the water. Density

of water$=1000$kg m^{-3}, sp. heat capacity of water$=4{\cdot}2 \times 10^3$J/kg deg C. [0·29°C]

33. A man rapidly ascends a flight of stairs spending half of his energy in the ascent, the remaining half transformed into heat energy. If the man ascends through 60·96 m, find the increase of his body temperature. The sp-heat of the man may be taken to be equal to that of water. [0.071°C]

34. A 200 ton train has its speed reduced from 30 to 20 mph in 30 second. It the work done is completely transformed into heat, find the amount of heat produced. [9677·23 B Th U]

35. Calculute the value of J, if the sp. heat capacity of air at constant pressure is 0·239 and the density of air at N. T. P. is 0·0013 g l^{-1}. Given $\gamma=1.40$. [$4{\cdot}198 \times 10^7$ erg cal^{-1}]

36. Calculate the sp. heat capacity of air at constant volume given that the sp. heat capacity at constant pressure is 0·23, density of air at N. T. P. is 1·293 g l^{-1} and J$=4{\cdot}2 \times 10^7$ erg cal^{-1}.

37. If the two sp. heat capacities of a gas C_p and C_v are respectively 0·2375 and 0·1690 cal, calculate the value of J. Given that 1 cm^3 of the gas at N. T. P. weighs 0·00129 g and the atmospheric pressure$=1.013 \times 10^6$ dyn cm^{-2}. [$4{\cdot}19 \times 10^7$erg cal^{-1}]

38. The sp. heat of argon at constant pressure is 0·12 and the sp. heat at constant volume is 0·08. Find the density of argon at N. T. P. Given J$=4{\cdot}2 \times 10^7$ erg cal^{-1}and normal atmospheric pressure$=1.01 \times 10^6$dyn cm^{-2} [$2{\cdot}2 \times 10^{-3}$g cm^{-3}]

39. The volume of a certain gas, having a density of 0·00125 g cm^{-3} under a pressure of 1 atmosphere and at a

temperature of 0°C, is 8 litre. 30 calories of heat are required to raise the temperature of this gas to 15°C at constant pressure. Find the sp. heat of the gas at constant pressure and also at constant volume. (R = 2cal/°C mole) (J.E.E.'81) [5.6 and 3·6 cal/°C mole]

40. A gas is compressed from a volume of 20 litre to 10 litre by a constant pressure of 10^6 dyn per cm^2. Calculate the heat developed. (J.E.E'82) [$20 \cdot 4 \times 10^2$ cal]

Questions of H. S. Exam.

from 1978 to 1986

1978

PHYSICS (First Paper)

Answer any *seven* questions, of which at least *three* must be from each group.

Group—A

1. (a) State Newton's second law of motion. Show how this law will give us a unit of force when the unit of mass is 1 kg and the unit of acceleration is 1 m/s^2. What is the name of this unit of force ?

(b) Express the weight of 1 kg in the MKS unit of force.

(c) A train is moving along a straight track. Explain what would be the nature of its velocity time graph in the following cases—

(i) It is moving with a uniform acceleration.

(ii) Its acceleration is increasing. 6+2+2

2. (a) Define kinetic energy. Obtain an expression for it in the case of a particle of mass m moving with a velocity v.

(b) A body of mass 100g is let loose from a tower 100m high. Calculate the kinetic energy of the body (i) one second after release and (ii) when at the bottom of the tower.

(c) A man rises in a lift carrying a bucket of water. Explain :—

(i) if any work is done by the man on the bucket of water

(ii) if the energy of the bucket of water would remain unaltered. 4+4+2

3. (a) Explain what is meant by centripetal and centrifugal forces.

(b) Find an expression for the centripetal force.

(c) A cyclist is describing a circle of 20m radius at a speed of 18 km. per hour. What is his inclination to the vertical ? (Assume the rider and the cycle to be in one plane) 2+4+4

Or

(a) What is meant by elasticity of matter and elastic limit ? Define and explain the different types of modulus of elasticity.

(b) A mass of 5 kg is suspended from a vertical wire 1 m long and of 1 mm radius, If the Young's Modulus of the material of the wire be $2{\cdot}0\times10^{12}$ dynes/cm^2, find the length of the wire without load. 6+4

4. (a) What do you mean by a second's pendulum ? Calculate its length in cm. at a place where $g=980$cm/sec^2

(b) State and explain whether the time period of the pendulum will change in the following cases :—

(i) If a hollow bob is taken instead of a solid bob.

(ii) If the hollow bob is partially filled with water.

(iii) If the pendulum is taken to the top of a mountain.

(c) An artificial satellite revolves around the earth in a circular orbit at a height of 400 km above earth's surface. Assuming radius of earth to be 6000 km and value of g at earth's surface to be 980 cm/sec^2, calculate the velocity of the satellite. 3+3+4

5. (a) Explain why a lump of iron (sp. gr. 7 8) sinks in water but floats in mercury (sp. gr. 13.6) ? State the principle you will need for your explanation.

(b) Calculate the fraction of the total volume of the iron lump which will be under the mercury surface when floating in the latter.

(c) State Pascal's law in relation to the transmission of pressure in a liquid. 4+4+2

6. (a) Show with the help of an experiment that the atmosphere exerts pressure.

(b) 'At a cetain place the atmospheric pressure is equal to 760 mm. of mercury.' What do you understand by the above statement ? Calculate the atmospheric pressure in c g s. units. (Given $g=980$ c. g.s. units and density of mercury=13.6 gm/c.c.)

(c) Explain briefly the working of a siphon. 3+4+3

Group—B

7. (a) Define 'co-efficient of linear expansion' of a solid.

(b) Obtain the relation between the co-efficient of linear expansion and volume expansion of a solid.

(c) A steel scale is correct at 60°F. The length of a brass rod measured by it at 50°C is found to be 1·5 m. Find out the true length of the rod at 50°C (linear expansion co-efficient of steel$=11{\cdot}2\times10^{-6}$ per°C) 2+4+4

8. (a) Explain what is meant by apparent and real expansion of a liquid. Which co-efficient of a liquid is determined by a weight thermometer ?

(b) Describe the method.

(c) How does fish live in a frozen lake ? 4+5+1

Or, (a) What do you mean by 'natural vibration' and 'forced vibration' ? (b) Distinguish between 'forced vibration' and 'resonance'. (c) Why orders are given to soldiers to break steps while crossing a suspension bridge ? (d) What is the utility of the hollow box of a violin ? 2+4+2+2

9. (a) What do you understand by the thermal capacity of a body ? How does it differ from the water equivalent of the body ?

(b) An alloy contains 60% copper and 40% nickel. A piece of the alloy weighing 50g is heated to 80°C and is dropped into a calorimeter of water equivalent 10g. The calorimeter contained 90g of water at 10°C. Find the final temperature of the mixture.

(Sp. ht. of copper=0·09 and sp. ht. of nickel=0·11)

(c) What part does the specific heat of water play in causing the sea-breeze ? 3+5+2

10. (a) Explain why the specific heat of a gas at constant pressure is greater than that at constant volume.

(b) A tube of heat insulating material, closed at both ends, contains 800g of lead shots. The tube is 1 m long and is held vertically. It is then suddenly inverted so that the lead shots fall to the other end. After 50 such inversion the temperature of the lead shots is found to rise by 3·89°C. Assuming that the whole of the heat generated remains within the lead shots, calculate the mechanical equivalent of heat. (Sp. ht. of lead=0·03)

(c) What is the first law of thermodynamics ? 3+5+3

11. (a) State the fundamental assumptions in kinetic theory of gases.

(b) Discuss the pressure of a gas and the concept of temperature according to kinetic theory.

Or, (a) What is Laplace's correction of Newton's formula for the velocity of sound in a gas? Why was the correction necessary?

(b) Discuss the characteristics of a musical sound. On what factors does the quality of a musical sound depend? 5+5

12. (a) Define the following terms in connection with wave motion :—

(i) Wave-length.

(ii) Frequency.

(ii) Amplitude.

(b) What is a stationary wave? Describe a simple experiment to demonstrate the formation of stationary waves.

(c) Discuss the formation of beats. 3+4+3

1979

FIRST PAPER

(Answer any *seven* questions of which at least *three* must be from each group.)

Group—A

1. (a) What is meant by relative velocity ? Two particles moving with velocities u and $2u$ are inclined at an angle 60°. Find the relative velocity of one with respect to the other.

(b) During the rains, the rain drops falling vertically appear to come down obliquely to a person sitting in a running train. Explain. 2+6+2

2. (a) Define momentum and impulse and state the principle of conservation of linear momentum.

(b) State and explain Newton's third law of motion.

(c) A 8 gm bullet is shot from a 5 kg gun with a speed of 400 m/sec. Find the velocity of recoil of the gun. 3+4+3

3. Explain the terms 'kinetic energy' and 'potential energy'. State the principle of conservation of energy and prove it for bodies falling freely under gravity.

Water is being raised from a well to a height of 25 ft by means of a 5 H. P. motor-pump. If the efficiency of the pump be 80%, how many gallons of water will be raised per minute (1 gallon of water weighs 10 lb, $g=32.2$ ft/sec^2). 2+5+3

4. State and explain Newton's law of Gravitation.

What is meant by acceleration due to gravity ? How does it vary with the alitude above sea-level and latitude of a place ?

The mass of the earth is 80 times that of the moon while its radius is four times that of moon. By what ratio will the weight of a body be reduced on the moon ? 2+5+3

5. *Either*

Define 'angular velocity', 'angular acceleration', 'angular momentum' and 'torque'. How is the torque related to the angular acceleration ? Explain the principle of conservation of angular momentum with suitable examples. 7+3

Or

(a) Describe a laboratory method for the determination of Young's modulus of elasticity of the material in the form of a wire.

(b) Explain and define surface tension of a liquid. How does it depend on temperature ?

Explain the following :—

(i) Small drops of mercury are spherical in shape.

(ii) Kerosene oil rises along the wick of a lamp. 5+5

6. State and explain Archemedes' principle. Does it hold in a satellite moving in a circular orbit around the earth? Explain.

How can you use Archemedes' principle to determine the proportion of two pure metals in a piece of their alloy ?

A hollow spherical ball of a material of density 3.1 gm/c. c. is found to float in a liquid of density 1.5 gm/c. c. just fully immersed. If the external diameter of the ball be 10 cm., find the internal diameter of the ball. 4+3+3

Group—B

7. *Either*

(a) What is the co-efficient of real expansion of a liquid ? The heights of the mercury columns in equilibrium in the two vertical limbs of a U-tube kept at 0°C and 100°C respectively are 76·35 cms and and 75 cms respectively. Calculate the coefficient of real expansion of mercury.

(b) How will you demonstrate the anomalous expansion of water ? 6+4

Or

(a) What is meant by simple harmonic motion (S. H. M.) ? State the conditions under which a particle will execute S. H. M.

(b) Explain the terms 'amplitude', 'phase' and 'period of an S. H. M.

(c) Describe an arrangement for measuring the velocity of sound in air by producing resonance between a tuning fork and an air column. 3+3+4

8. Describe a laboratory method of determining the specific heat of a solid insolube in water. Does the relation "heat lost= heat gained" hold if the solid and the liquid in the calorimeter react chemically ? Explain.

How much heat is required to convert 5 gms of ice at $-10°C$ to steam at $100°C$? (Specific heat of ice=0·5, Latent heat of ice =80 cals/gm ; Latent heat of steam=540 cals/gm.) 5+2+3

9. State 'The first law of Thermodynamics'. What is meant by mechanical equivalent of heat ? Describe a method for the determination of mechanical equivalent of heat.

The mechanical equivalent of heat is 4·2 Joules per calorie, Express the same in F. P. S. system. 2+1+5+2

[1 kg=2·2 lb and 1 inch=2·54 cm]

10. Define co-efficient of thermal conductivity. Describe an experiment to compare the thermal conductivities of copper, iron and brass given in the form of rods of equal cross-sections.

Why do we feel warmer at night if the sky is cloudy ?

2+6+2

11. Define 'Dew point' and Relative humidity' and describe a method by which 'Dew point' may be determined.

(i) What will be effect on dew point if a quantity of water is sprinkled inside a closed room ? Explain.

(ii) In determining the 'boiling point of a liquid, the bulb of a thermometer is not kept immersed in the liquid. Explain why ?

2+4+2+2

12. What is meant by Brownian motion ? Describe the characteristics of such motion.

What do you understand by R. M. S. velocity ? Discuss the limitation of ideal gas laws. 5+5

Or

What is the difference between a sound wave and a light-wave ? Discuss the nature of sound wave with illustrations. A stone is dropped from the top of a tower and the sound is heard 4·4 sec later. Find the height of the tower. [Velocity of sound in air= 1000 ft/sec and acceleration due to gravity=32 ft/sec^2.] 2+5+3

1980

PHYSICS (Paper I)

Answer any *seven* questions of which at least *three* must be from each Group.

Group A

1. (a) Obtain the equation $s = ut + \frac{1}{2}ft^2$ graphically, where u, f, s and t have usual meanings.

(b) A train, travelling with a speed of 36 ft per sec. is subjected to a uniform retardation of 2 ft/sec^2. How far would the train move before coming to a stop?

(c) State the principle of conservation of linear momentum and obtain it from Newton's third law of motion.

2. (a) State the laws of static friction. Define co-efficient of static friction. Obtain a relation between the angle of repose and the co-efficient of friction.

(b) A body of mass 50 gms. rests on a rough horizontal table. Minimum horizontal force of 20 gm-wt. is necessary to set the body in motion along the surface of the table. What is the value of co-efficient of static friction between the body and the table?

(c) Can you walk freely on a frozen lake? Justify your answer.

3. (a) Define work and power. State their units in C. G. S. and F. P. S. systems.

(b) Deduce the expression for the kinetic energy of a body of mass m moving with the velocity u

(c) "The power of an engine is 10 H. P."—What is the meaning of the statement?

(d) A man weighing 120 lbs. can climb the top of a tower 220 ft. high in 4 mins. Calculate the power of the man in horse-power.

4. (a) What do you mean by normal atmospheric pressure in C. G. S. unit, when $g = 980$ cm/sec^2 and density of mercury is 13·6 gms/c.c.

(c) What will you conclude if you observe :

(i) that the height of mercury in the barometer rises steadily ?

(ii) that the height of mercury in the barometer drops suddenly ?

(d) It is required to transfer a liquid of sp gravity 0·8 over an obstacle by means of a siphon. What must be the limiting height of the obstacle which will render siphoning possible when the atmospheric pressure is 30 inches of mercury of sp. gr. 13·6 ?

5. (a) State and explain Pascal's Law.

(b) Explain the principle of action of a hydraulic press.

(c) The diameter of the smaller piston of a hydraulic press is 2″ and of the larger piston is 18″. If a thrust of 16,200 lbs. is to be developed on the larger piston, how much force should be applied to the smaller piston ?

6 (a) What do you mean by tensile stress and tensile strain ?

(b) State Hooke's law of elasticity. Define Young's modulus of elasticity and Poisson's ratio. What is elastic limit ?

(c) Which is more elastic—steel or diamond ? Justify your answer.

(d) If a 10 kgm. wt. is suspended at one end of a steel wire of length 5 metres, the elongation of the wire becomes 5 mm. If the Young's modulus of steel be $9 \cdot 8 \times 10^{11}$ dynes/cm^2, calculate the cross-section of the wire [g=980 cm/sec^2].

Or, Centripetal force is a real force and centrifugal force is a 'pseudo' force. Justify the statement.

How does the earth's rotation affect the weight of a body at the equator ?

A body weighing 70 gms. is tied at the end of a string 20 cm. long. The other end of the string is held by hand and rotated in a circle 10 times per sec. with uniform angular velocity. Calculate the centrifugal force on the body.

Group—B

7. (a) Distinguish between heat and temperature.

(b) Obtain the temperature for which readings for both Celsius and Fahrenheit thermometers will be the same.

(c) What do you mean by Absolute Zero of temperature ? What is its value in Celsius and Fahrenheit scales of temperature ?

(d) The volume of a given mass of gas at 47°C is 640 c.c. when the pressure is 75 cm. of mercury. At what temperature will the pressure of the gas be doubled if the gas be heated keeping the volume constant ?

8. (a) Define : Calorie, water equivalent, sp. heat and thermal capcity.

(b) You are provided with a thermometer which can read from 0°C to 100°C. Explain how with the help of this thermometer you can measure the temperature of a furnace whose temperature is 2000°C.

(c) 20 gms. of ice at 10°C is added to 270 gms. of water at 30°C contained in a copper calorimeter of mass 300 gms. Calculate the final temperature.

Sp. heat of copper=0·1 ; Sp. heat of ice=0·5.
Latent heat of fusion of ice=80 cals/gm.

9. (a) What are the fundamental assumptions of kinetic theory of gas ?

(b) Explain the pressure and temperature of an ideal gas in the light of kinetic theory.

Or, (a) Assuming Newton's Formula for the velocity of sound obtain Laplace's correction.

(b) Discuss the characteristics of musical sound. On what factors do they depend ?

(c) What is interference of light ? State the conditions under which it can be produced.

10. (a) What is mechanical equivalent of heat ? Give two examples of the conversion of mechanical work into heat.

(d) The height of a water-fall is 200 metres. What will be the difference of temperature of water at the top and bottom of the water-fall ?

[$g=980$ cm/sec^2, $J=4{\cdot}2$ joules/cal].

(c) Define thermal coductivity. If a block of copper of cross-section 100 sq cm. and thickness 5 cm. be heated such that the difference of temperatures between the two oppposite faces is 5°C, calculate the amount of heat flowing between the opposite faces in 5 mins. if the conductivity of copper be 0·9 C. G. S. unit.

11. (a) What do you mean by absolute humidity, relative humidity and dew point ?

(b) What would be the condition of the atmosphere if the room temperature is equal to the dew-point ?

(c) The temperature of a room is increased. What will be its effect on the dew-point and the relative humidity ?

(d) Although the room temperature in the winter season is much less, wet clothes dry up much quicker in the winter than in the rainy season. Why ?

(e) On a certain day the temperature of the room is 25°C and the dew-point is 12°C. The aqueous tensions at 12°C, 25°C and 26°C are respectively 10·50 mm, 23·73 mm. and 25·1 mm of mercury. Calculate the relative humidity.

12. (a) Distinguish between transverse and longitudinal vibrations, giving examples.

(b) Explain what is meant by free and forced vibrations. When is resonance observed ? Give an example of it.

(c) Draw a diagram showing the pattern of the first three modes of vibration of a stretched string fixed at both ends, showing the positions of the nodes and the antinodes.

Or, Distinguish between real and apparent expansion of a liquid and obtain a relation between them.

Discuss the effect anomalous expansion of water on marine life.

If a glass flask has a volume of 2 litres at 30°C, calculate the increase in volume when the temperature rises to 50°C. The coefficient of linear expansion of glass is 8×10^{-6} per degree C.

PHYSICS—First Paper (1981)

Group—A

Answer the following questions

1. (a) State the laws of static friction.

(b) Friction is useful in some ways but it is wasteful in other ways,—Discuss.

(c) A man holds a book weighing 2 lb wt between his hands and keeps it from falling by pressing both hands against the book with force of 5 lb wt each. Find the coefficient of friction between the book end the hand.

Or, (a) Prove that for a uniformly accelerated particle $v^2=u^2+2fs$, where the symbols have their usual significance.

(b) A particle with initial velocity 10 cm/sec attains a velocity of 20 cm/sec after travelling a distance of 50 cm with constant acceleration. Find its accceleration.

(c) A train with initial velocity 30 miles per hour applies brakes and creates a retardation of 2ft/sec^2. When will the train stop ?

2. (a) Define kinetic energy.

(b) A body of mass 1 kg is let fall from a tower 100 metres high. Calculate the kinetic energy of the body—(i) one second after its release and (ii) when at the bottom of the tower.

(c) A man rises in a lift carrying a box. Explain if any work is done by the man on the box.

Or, (a) Define centre of mass and centre of gravity.

(b) State the conditions of equilibrium of three forces.

(c) A body of mass 2 kg and another of 5 kg are placed at the ends of a rod of length 10 cm. Find the position of balance neglecting the weight of the rod.

Group—B

Answer *any two* questions

3. (a) State Newton's law of universal gravitation.

(b) What is the unit of universal gravitational constant in C.G.S. system ?

(c) An atificial satellite revolves round the centre of the earth in a csrcular orbit at a height of 400 km above the surface of the earth. Assuming the radius of the earth to be 6000 km and value of 'g' at the earth surface to be 980 cm/sec^2 calculate the velocity of the satellite.

4. (a) Explain what is meant by elasticity of matter.

(b) Define rigidity modulus and Poisson's ratio.

(c) A mass of 10 kg is suspended from a verrical wire of radius 1 mm. The length of the wire becomes 1 metre. If Young's modulus of the material of the wire 2.0×10^{12} dynes/cm^2, find the length of the wire without load.

5. (a) State the laws of simple pendulum.

(b) A pendulum of lenght 'l' loses 5 seconds a day. By how much should its length be shortened to keep correct time? (Given $g=981$ cm/sec^2)

(c) A hollow sphere is filled with water and is hung by a long thread. A small hole is made at the bottom of the sphere and water flows out slowly through the hole. It is observed that the period of oscillation of the sphere first increases then decreases. Explain.

6. (a) Describe an experiment to show that liquids exert pressure in all directions.

(b) 'A floating body apparently weighs nothing'—Explain.

(c) A faulty barometer reads 28 inches and 30 inches when a true barometer reads 28.5 inches and 31 inches respectively. Find the reading of the true barometer when the faulty barometer reads 29 inches.

Group—C

Answer *any two* questions

7. (a) Establish the relation connecting the different thermometric scales that are in general use.

(b) A faulty thermometer reads 5°C in melting ice and 99°C in dry steam. Find the correct temperature in Centigrade scale when the faulty thrmometer reads 52°C.

(c) What are the advantages of using mercury as a thermometric substance ?

8. (a) Define coefficient of linear expansion. Show that it does not depend on the unit of length but depends upon the scale of temperature.

(b) Coefficient of linear expansion of brass is $19 \times 10^{-6}/°C$. What is its value in Fahrenheit scale ?

(c) Two metal rods, A and B, differ in length by 25 cm. whatever be the change in temperature. If the coefficients of linear expansion are $1{\cdot}28 \times 10^{-5}/°C$ and $1{\cdot}92 \times 10^{-5}/°C$. respectively, find the lengths of A and B.

9. (a) Define latent heat of fusion of a solid and latent heat of vaporisation of a liquid.

(b) What is regelation ? Describe an experiment to demonstrate regelation.

(c) 100 gms of ice at 0°C is added to 100 gms of water at 40°C. Find the common temperature and the final contents at the common temperature.

10, (a) State the first law of Thermodynamics.

(b) What is meant by mechanical equivalent of heat ?

(c) Describe a method for the determination of mechanical equivalent of heat.

(d) A mass of 1 kg. falls from a height of 1 km to the ground. If all the energy is converted into heat, find the quantity of heat developed. $J = 4{\cdot}1 \times 10^{7}$ erg/cal.

Group—D

Answer *any one* question

11. (a) Define simple harmonic motion.

(b) Establish the equation of a simple harmonic motion.

(c) Find the resultant of superposition of two co-linear simple harmonic motions of same period and phase but of different amplitudes.

(d) A particle is moving in a straight line with s.h.m. Its velocities are 4 ft/sec. and 3 ft/sec. when its distances are 2 ft and

3 ft. respectively from its mean position of motion. Find its amplitude and time period.

Or, (a) Explain progressive and stationary waves.

(b) State the characteristics of progressive and stationary waves.

(c) The resonant length obtained in an experiment with resonance column tube is 27 cm. with a tuning fork of frequency 300. If velocity of sound in air is 330 metres per sec, find the second resonant length.

(d) Two tuning forks, when sounded together, produce 4 beats par second. Is the frequecy of one of them be 256, find the possible frequency of the other.

12. (a) State the laws of transverse vibration of a stretched string.

(b) Describe how you would verify the above laws.

(c) A sonometer wire emits a note of frequency 150. What will be the frequency of the note emitted by the same wire, if the tension is increased in the ratio of 9 : 16 and the length is doubled ?

Or, (a) What is Laplace's correction of Newton's formula for velocity of sound in a gas ? Why was the correction necessary ?

(b) How does the velocity of sound in a gas depend on pressure and temperature ? Give reasons in support of your statements.

(c) Define—fundamental tone, overtone and harmonics.

(d) A gun is fired from an aeroplane travelling horizontally at 120 miles per hour and the echo from the ground is heard after 3 seconds. Find the height of the aeroplane, if velocity of sound be 1120 ft./sec.

(c) A person gets a more severe burn by steam at 100°C than by boiling water. Why ?

(d) A vessel of water requivalent 5 gm. contains 55 gm. of water at 20°C. 25 gm. of ice at 0°C is dropped into it. What will be the composition of the contents of the calorimeter and the temperature in the final mixture ? [Latent heat of ice=80 cals/gm.]

9. (a) Define thermal conductivity and state its unit in the c.g.s. system.

(b) The area of the glass window of a room is 2 sq. metres. The temperatures inside and outside the room are respectively 20°C and 5°C. The thickness of glass is 3 millimetres. How much heat will pass through the closed window per second? [Thermal conductivity of glass = ·0012 c.g.s. unit]

(c) An iron and a wooden chair are kept in the sun. Why does the iron chair appear hotter?

(d) State with reasons what should be the nature of the bottom of a cooking vessel—smooth, rough, black or white.

10. (a) What are the basic assumptions underlying the kinetic theory of ideal gases? Calculate the r. m. s. velocity of nitrogen molecules at 0°C. Density of nitrogen at S.T.P. 1·25 gm/litre. Density of mercury = 13·6 gm/c.c.

(b) The water of a waterfall drops from a height of 50 metres. If 75% of its energy is converted to heat and absorbed by the water by how much does the water rise in temperature? [$J = 4{\cdot}2 \times 10^7$ ergs per calorie ; $g = 9{\cdot}8$ metres/sec^2.]

Group—D

Answer *any one* question from this group.

11. (a) What is meant by simple harmonic motion? Define amplitude and period of a simple harmonic motion.

(b) The equation of motion of a body executing simple harmonic motion is given by $y = 2 \sin 40\pi t$ cm. What are its amplitude, period of oscillation and acceleration at the maximum displacement of the oscillation.

(c) Prove that for a body executing a simple harmonic motion, the sum of the potential and kinetic energies at any instant is a constant.

(d) Explain with examples what you mean by forced and resonant vibrations.

12. (a) How are stationary waves formed? Give suitable examples, one each, of stationary longitudinal and transverse waves.

(b) Explain with a diagram what you mean by nodes and antinodes.

(c) Deduce the relation between velocity and wavelength of a wave.

(d) A wind pipe 50 centimetres long and open at both ends is vibrating at resonance with a tuning-fork. If the velocity of sound in air is 350 metres/sec, what is the frequency of the tuning-fork ?

(e) Two wires of the same sample (having same mass per unit length) are stretched on a sonometer. One is twice as long as the other. But they are resonating with each other. Compare their tensions.

PHYSICS—(First Paper) 1982

Group—A

Answer *any two* questions.

1. (a) State Newton's Second Law of motion. 2

(b) From this law deduce the formula $P=mf$, the symbols having their usual meaning. 3

(c) Define the M.K.S unit of force. 1

(d) A car weighing 100 kg is moving with a velocity of 2 metres per sec. What force must be applied to stop it within a distance of 10 metres ? 2

Or, (a) State the law of conservation of linear momentum for perfectly elastic bodies. 2

(b) Why does a gun exert a backward push when a bullet leaves the gun ? 2

(c) A gun carriage weighs 500 kg. A shot of mass 10 kg leaves it with a velocity of 20 metres per sec. If the frictional force of the earth is one-tenth of the weight of the carriage, then how far will the carriage recoil ? 4

2. (a) Why is a centripetal force necessary to rotate a body in a uniform circular motion ?

(b) Show that when a cyclist is going round a curved path of radius r with a velocity v, his body must become inclined to the vertical by an angle θ where ten $\theta=v^2/rg$, g being the acceleration due to gravity. 2

(c) A ball of mass 15 gm. tied to a peg with a 9 cm. long thread is rotating uniformly round the peg on a horizontal smooth plane at the rate of 10 rotations per minute. The thread is always in a state of taut. What is the tension of the thread ?

[Take $\pi^2=9{\cdot}8$] 3

Or, (a) Define watt and horse-power. Show that 1 H.P$=746{\cdot}4$ watts. 2+2

(b) A boy of weight 20 kg carrying a box of weight $2\frac{1}{2}$ kg. climbed the roof of a house 18 metres high in $1\frac{1}{2}$ minutes. At what rate did he do work ?

Group—B

Answer *any two* questians from this group.

3. (a) What is acceleration due to gravity ? How is it related to the universal gravitational constant ?

(b) Calculate the mass of the earth, given that its radius is 6400 km., $g=9{\cdot}8$ m/sec^2 and $G=6{\cdot}7\times10^{-8}$ c.g.s. unit.

(c) Explain with diagram why and how the acceleration due to gravity varies from place to place on the earth's surface.

(d) A piece of stone let fall from the top of a tower touches the ground after 5 seconds. What is the height of the tower ? [$g=9{\cdot}8$ m/sec^2]

4. (a) Define stress and strain of an elastic body. What are their units in the c. g. s. system ?

(b) Justify the statement—Steel is more elastic than rubber.

(c) Show graphically the nature of the load-extension curve of a solid.

(d) A source of 10 Newtons is required to elongate (within elastic limit) a rope of radius 3·5 millimetres by $\frac{1}{20}$ th of its length. What is the Yaung's modulus for the material of the rope ?

5. (a) Why is it necessary to make the dam of a water reservoir thicker at the bottom than at the top ?

(b) Things can be cut by the sharp edge of a knife easily but cannot be cut by she blunt edge. Explain the reason.

(c) What is the pressure at the bottom of a clear lake of water 10 metres deep ? [Atmospheric pressure=76 cm. of mercury and density of mercury—13·6 gm. per. c.c.]

(d) Does the rate of flow of a liquid through a syphon change if the barometric pressure is changed ? Explain your answer.

6. (a) State Archimedes' principle.

(b) State the conditions of flotation of a body.

(c) A steamer weighs 10 metric tonne. When it enters into a sweet water lake from the sea, it displaces 50 litres more water. What is the density of the sea-water ? [1 litre of water weighs 1 kg]

(d) A string supports a piece of iron weighing 160 gm. totally immersed in water. If the specific gravity of iron be 8, calculate the tension in the string. [$g=980$ cm/sec^2.]

Group—C

Answer *any two* questions from this group.

7. (a) Explain with a diagram why two kinds of expansion namely, real and apparent expansions are mentioned in case of liquids.

(b) Define the coefficients of real and apparent expansion of a liquid and deduce a relation between the two. Which of these is a characteristic of the liquid only ?

(c) Coefficients of volume expansion of glass and mercury are respectively $2{\cdot}4\times10^{-5}$ per °C and $1{\cdot}8\times10^{-4}$ per °C. What fraction of the inner volume of a glass vessel should be kept filled with mercury so that the volume of the remaining inner space will be the same at all temperatures ?

8. (a) Define (i) specific heat. (ii) latent heat of fusion.

(b) What is the advantage of taking water as the hot substance in a hot water bottle ?

1983

PHYSICS (1st Paper)

Group—A

Answer any two questions

1. (a) State Newton's laws of motion. Define the unit of force from the second law of motion. 3+2

(b) A mass of 10 kg is suspended from a string whose other end is held in hand ; find the tension in the string when (i) the hand is stationary, (ii) the hand is moved up with a uniform acceleration of 5 m/sec^2, (iii) the hand is moved down with a uniform acceleration of 5 m/sec^2 [$g=9{\cdot}8$ m/sec^2] 3

Or, Prove that a uniformly accelerated particle moves through a distance equal to $u+\frac{1}{2}f\ (2t-1)$ in the t-th second, where the symbols have their usual meanings. 3

(b) A body moving with uniform acceleration covers 65 ft. in the 5th second and 105 ft. in the 9th second. Find the distance it will cover in 20 secs. 3

(c) Show by vector method that the line joining the middle points of two adjacent sides of a triangle is parallel and half the third side. 2

2. (a) What is the difference between—'work done by a force' and 'work done against a force' ? 1

A man on a boat rowing against the current is stationary with reference to shore. Is he doing any work ? 1

(b) Find an expression for the kinetic energy of a particle of mass m moving with velocity v. 2

(c) Prove that the sum of kinetic energy and potential energy is everywhere constant for a freely falling body. 3

Or, (a) What do you mean by centripetal and centrifugal forces ? Why is centrifugal force called a fictitious force ? 2+1

(b) A stone of mass 4 lb is rotated in a horizontal circle being tied at the end of a string 10 m long which can stand a maximum tension of 5 lb-wt. Calculate the maximum velocity of rotation

that can be given to the stone and the greatest number of revolutions per minute which the stone can make without breaking the string. 2

(c) What do you mean by angular momentum? Explain the principle of conservation of angular momentum.

Group—B

Answer any two questions.

3. (a) State Newton's law of gravitation.

What do you mean by acceleration due to gravity?

Which one is greater—the attraction of the earth on 1 lb of lead or the attraction of 1 lb of lead on the earth? 2+1+1

(b) What should be the velocity in km/hour of an earth satellite revolving at a height of 1600 km. above the surface of the earth in a circular path with the centre of the earth, assumed spherical?

Given, radius of the earth $=6400$ km, mass of the earth $=6\times10^{27}$ gm, and $G=6{\cdot}7\times10^{-8}$ C.G.S. units. 1+1

(c) The point of suspension of a simple pendulum moves with a uniform acceleration in the horizontal direction. How is the period of the pendulum affected? 1

(d) A person will feel weightless on an artificial satellite of the earth—explain. 2

4. (a) Define: stress, strain, Poisson's ratio. 3

(b) State Hooke's law. 2

What do you mean by Young's modulus?

What are the units of Young's modulus in C.G.S and MKS systems? 1+1

(c) Two wires of different materials are of equal length. One is of diameter 1 mm and other 3 mm. They are each stretched by the same tension. The elongation of the first is thrice that of the other. Compare their Young's moduli. 3

5. (a) State Pascal's law for transmission of pressure through liquid. 2

Explain the principle of working of a hydraulic press.

(b) A solid wooden cylinder is placed in a container in contact with the base in such a manner, that when water is poured into the container, no water can go beneath the solid. Will the cylinder float up? Give reason for your answer.

(c) At what depth under water will the pressure be three times the atmospheric pressure? Assume the atmospheric to be 10^6 dynes / sq. cm. 3

6. (a) State the conditions of equilibrium of floating bodies. 2

A rectangular parallelopiped of wood 5 cm long, 4 cm broad and 3 cm high floats with 2·5 cm of its height immersed in water. Find the density and weight of piece of wood. 3

(b) A solid weight 23.75 gms. in air and 12.5 gms. when totally immersed in a liquid of specific gravity 0·9. Calculate the specific gravity of the solid. Also calculate the specific gravity of the liquid in which the solid would float with three quarter of its volume immersed in the liquid. 5

Group—C

Answer any two questions

7. (a) What are the fixed points of a thermometer?

What do you mean by the fundamental interval of a thermometer? 1+1

(b) A faulty thermometer reads 0·5° at the ice point and 105·5° at the steam point. Find the correct temperature in centigrade scale when this faulty thermometer reads 52·5°. 2

(c) State the advantages of using mercury as a thermometric substance. 3

(d) Define co-efficient of linear expansion of a solid. 1

Show that it does not depend on the unit of length but depends upon scale of temperature. 2

Or, (a) Establish relation connecting the co-efficients of linear and volume expansions of a solid. 3

(b) If the gap between two consecutive rails of a railway line, each 66 ft long, is 0·5 inch at 10°C, at what temperature will the two rails just touch? (α for iron $= 11 \times 10^{-6}/°C$) 2

(c) Describe the method of determining the co-efficient of apparent expansion of a liquid by weight thermometer. 3

(d) An isoceles triangle is formed with three zinc rods. What will be the change of the base angle if there is change of temperature ? Give reasons for your answer. 2

8. (a) Define : Calorie, British Thermal Unit, Therm, 15°-Calorie.

(b) Specific heat of copper is 0·09—explain.

Distinguish between thermal capacity ard water equivalent.

(c) The densities of two substances are in the ratio 2 : 3 and their specific heats are in the ratio 0·12 : 0·09. Compare their thermal capacities per unit volume. 2

9. (a) Find the result of mixing 50 gm of water at 10°C and 15 gm of ice at 0°C. Latent heat of fusion of ice=80 cal/gm. 3

(b) Give two natural examples of cooling by evaporation. 2

(c) State the first law of thermodynamics. What is meant by the mechanical equivalent of heat ?

(d) The temperature of a piece of metal rises br 1·4°C when it falls from rest through 90 m on the ground. If $\frac{2}{3}$ of the heat generated goes to raise the temperature of the metal, then what is the value of the mechanical equivalent of heat ? (Specific heat of the metal=0·1 : g=980 cm/sec^2). 3

10. (a) State the fundamental assumptions of the kinetic theory of a perfect gas.

(b) What do you understand by the root mean square velocity ? 1

(c) The thickness of an iron plate is 4 mm and the area of its face is 150 sq. cm. If the temperatures of the two opposite faces of the plate be 100°C and 30°C respectively and the heat conducted from one face to the other in 1 second is 3940 calories, find the coefficient of thermal conductivity of iron. 4

(d) Explain : A blackened vessel is preferred to a polished one for cooking ;

Birds puff up their feathers on a cold day. Why ? 2

Group—D

Answer any *one* question

11. (a) Prove that the motion of a simple pendulum is simple harmonic. 3

(b) Write down the equation of a S. H. M. with an amplitude 2 cm, initial phase 0° if 150 oscillations are performed in one minute. 2

(c) State Newton's formula for the velocity of sound in a gas. What is Laplace's correction ? 2

(d) The frequency of a tuning fork is 400 per sec. and velocity of sound is 320 metres per sec. Find how far the sound will travel in air when the fork executes 30 complete vibrations. 2

Explain : Formation of beats ; 3+3

Doppler effect.

12. (a) State the laws of transverse vibrations of a stretched string. 3

(b) Describe the resonance column method of finding the velocity of sound wave. 3

(c) Two wires whose lengths are in the ratio of 3 : 2 are stretched with equal tension and they emit the same note. If the wires are of different materials and if their radii be in the ratio of 1 : 3, compare their densities 5

(d) Explain the formation of echo. 2

A man fires a gun standing between two parallel cliffs. He hears the first and the second echoes after $1\frac{1}{2}$ and $2\frac{1}{2}$ seconds respectively. Find the distance between the cliffs. Given, velocity of sound in air=1200 ft/sec. 2

1984

PHYSICS (1st Paper)

Group—A

Answer any Two questions

1 (a) Establish graphically the equation $s = ut + \frac{1}{2}ft^2$ where u, f, s and t have usual meaning. 4

(b) Uniform retardation is produced by applying brakes when a train is moving with uniform speed of 45 miles per hour and the train comes to a stop after one minute. Calculate the rate of retardation and find also the distance traversed by the train after the application of the brakes. 4

2. (a) Write down the laws of static friction. What do you mean by the angle of repose of an inclined plane? Obtain the relation between the coefficient of friction and the angle of repose. 6

(b) A body, moving on the ground with a speed of 15 miles per hour, comes to rest. The coefficient of friction between the ground is 0.22. Calculate the distance traversed by the body on the ground before it comes to a stop. 2

Or, Give an example of the disadvantage of friction. How is it minimised? 2

3. (a) Define work and power. Write down their units in the M. K. S system.

(b) Obtain the relation between Horse Power and Watt. 3

(c) A train moves with the speed of 50 miles/hour against a force of 1200 lb. wt. Calculate the power of the engine in H. P. 2

4. (a) What do you mean by fictitious force? Why is the centrifugal force known as a fictitious force? 2

(b) How is the weight of a terrestrial body affected by the earth's diurnal rotation? 2

(c) A stone of mass 500 gms is tied at one end of a thread and is rotated along a circular path of diameter 200 cm in a vertical plane with the speed of 4 metre/sec. Calculate the tension of the thread when the stone is at its highest and lowest positions. ($g = 980$ cm/sec^2.) 4

Group—B

Answer any Four questions

5. (a) Discuss how the acceleration due to gravity changes as one goes above the earth's surface and also within the earth. What would be the weight of a body, placed at the centre os the earth? A straight tunnel is dug passing from the northpole to the south-pole of the earth. What would be the motion of a body if it is dropped into the tunnel? 6

(b) A body is dropped from a balloon when it is at a height of 390 metres from the earth's surface during its ascent. It is found that the body reaches the ground after 10 seconds. If the acceleration due to gravity be 980 cm/sec^2, calculate the speed of ascent of the balloon at the time whan the body is dropped from it. 4

6. (a) State Archimedes' Principle and verify the same experimentally. 6

(b) The weights of a body in air and water are respectively 50 gms and 30 gms. What would be its weight in a liquid of sp. gr. 0.8? Calculate the sp. gr. of the body in relation to the liquid. 4

7 (a) What do you mean by the standard atmospheric pressure? What will be your conclusion if there is sudden drop of height of mercury in a barometer? Calculate the value of the standard atmospheric pressure in the C. G. S. unit ($g=980$ cm/sec^2) 5

Density of mercury $=13.6$ gm/c.c.

(b) Explain the principle of action of a siphon. Explain under what conditions a siphon does not work. 5

8. (a) Define the coefficient of linear expansion of a solid. Establish the relation between the coefficient of linear expansion and the coefficient of superficial expansion of a solid. 4

(b) The density of brass at 0°C is 7.8 gm/c.c. Calculate its density at 100°C.

Coefficient of linear expansion of brass

$=18.0\times10^{-6}/°C$ 2

(c) Two plates of copper and iron of identical shape and size are rivetted together. What will happen to the combined system if the temperature be raised ?

Explain your answer with a diagram. Give an example of the practical application of such a bimetallic strip. 4

9. (a) Define the pressure and volume coefficients of a gas. Prove that in the case of an ideal gas these two coeffcients are equal. 5

(b) Define the sp. heat and the latent heat of fusion. A calorimeter of water equivalent 30 gms contains 270 gms of water at 30°C. A piece of ice of mass 10 gms at -10°C is dropped into the calorimeter. Calculate the final temperature of the mixture. (Sp. heat of ice=0·5) 5

(Latent heat of fusion of ice=80 cal/gm)

10. (a) Define dew point and relative humidity. How are they affected by the temperature of a room ?

The temperature and the dew point in a room are respectively 26.5°C and 14°C on a certain day. The aqueous tensions at 14°C, 26°C and 27°C are respectively 12·00 mm, 25·00 mm and 26·60 mm of mercury. Calculate the relative humidity in the room.

(b) What do you mean by mechanical equivalent of heat ?

The temperature of water at the top of a waterfall is less than that its bottom by 0·47°C. If the work done by water in falling through the height of waterfall is fully converted into heat, calculate the height of the waterfall. 4

$g=980$ cm/sec^2, $J=4{\cdot}2\times10^7$ ergs/cal

Group—C

Answer any Two questions

11. (a) What do you understand by simple harmonic motion ? What is meant by time period ?

(b) What do you mean by forced vibration and resonance ? Explain their difference by giving examples. 4

12. (a) Show that the sum of the potential energy and kinetic energy of a particle executing S. H. M. is a constant. 4

(b) The time period and amplitude of a particle, executing S. H. M., are respectively 12 secs and 13 cm. Calculate the velocity of the particle when it is 5 cm away from the mean position of rest. 3

13. (a) How is a stationery Wave formed? What are its characteristics? 3

(b) Show that only the odd harmonics are produced in the case of vibration of an air column in a tube closed at one end. 4

1985

PHYSICS (1st Paper)

Group—A

Answer any *two* questions

1. (a) Establish graphically the equation $v^2=u^2+2fs$ where u, v, f and s have usual meaning. 4

(b) The initial velocity of a train is 30 miles per hour and the velocity of the train becomes 15 miles per hour after moving through 363 ft with uniform retardation. How far will it move further before stopping if it moves with the same uniform retardation ? 4

2. (a) Define power and write down it units in C. G. S. and M. K. S. systems. 3

(b) What do you understand by the law of conservation of energy ? Give a few examples. 2

(c) If a body of mass 100 gms be falling freely from a height in a downward direction calculate its kinetic energy 5 seconds after the beginning of downward falling. [$g=980$ cm/sec^2] 3

3. (a) What do you mean by relative velocity ? Two particles are moving with velocities u and v respectively making angle θ between them. Obtain the relative velocity of one particle with respect to the other. 4

(b) One ship is moving due east with a velocity of 9 kms per hour and another is moving due south with a velocity of 12 kms per hour. What would be the velocity and direction of motion of the first ship with reference to the second ? 4

Group—B

Answer any *four* questions

4. (a) State the laws of gravitational attraction and explain what you mean by universality of the law of gravitation 2

(b) Write down and explain the laws of a freely falling body. 3

(c) A body is thrown vertically upward with a velocity of 80 ft per sec. If the acceleration due to gravity be 32 ft/sec^2 find the time of ascent. How far will it ascend and what will be its velocity midway? 6

5. (a) Write down Hooke's law of elasticity. Explain how you can verify the law experimentally. What do you understand by elastic limit? Define Young's modulus of elasticity and Poisson's ratio. 6

(b) An 8 kg weight is suspended at the end of an iron wire 2 metres log and 1 mm in diameter. Calculate the elongation of the wire if Young's modulus of iron be 2×10^{12} dynes/cm^2.

[$g = 980$ cm/sec^2] 4

(c) The diameters of two pistons of a hydraulic press are 4 inches and 40 inches respectively. The arms of the lever of the press are respectively 6 inches and 3 ft. A force of 50 lb is applied at the end of the longer arm of the lever Calculate the thrust on the larger piston. 5

7. (a) What is the distinction between heat and temperature? Establish the relation between the Celsius and Fahrenheit scacles.

(b) Obtain the temperature at which the readings of Celsius and Fahrenheit Thermometers will be the same. 2

(c) The lower and upper fixed points of a thermometer are 0·5° and 101° respectively. What will be the reading of this thermometer at 60°C? 4

8. (a) Define real and apparent coefficients of expansion of a liquid and establish the relation between them 4

(b) The internal volume of a glass flask is v cc. Find the volume of mercury that must be kept within the flask so that the volume above the mercury remains the same at all temperatures. 3

Coeff. of vol. expansion of mercury = 0·00018/°C

Coeff. of linear expansion of glass = 0·000009/°C.

(c) An alloy contains 60% copper and 40% nickel. A piece of this alloy of mass 50 gms is heated to 50°C and then dropped into

140 gms of water at 20°C kept in a calorimeter of water-equivalent of 10 gm. Calculate the final temperature of the mixture. 3

Sp. ht of copper = 0.09 ;

Sp. ht, of Nickel = 0.11.

9. (a) Write down the different methods for transmission heat and explain the difference between them. Explain why ventilators are placed near the ceiling of a room. 5

(b) There are five glass windows in a room, the area of the glass af each window and the thickness of the glass being 2 sq. m and 2 mm respectively. The temperatures of the inner and outer surfaces of the glass are 20°C and −5°C respectively. Calculate the amount of heat flowing out from the room per min. through the windows by conduction.

Conductivity of glass = 0·002 C.G.S. Unit. 5

Or (a) Write down the basic assumptions of the kinetic theory of an ideal gas. Explain the concept of pressure and temperature of an ideal gas with the help of the kinetic theory. 5

(b) The R. M. S. velocity of a hydrogen molecule at N.T.P. is 1·85 km/sec. Calculate the density of hydrogen gas. 3

Group—C

Answer and *two* questians

10. (a) What do you understand by beats? Explain the formation of beats. 3

(b) Two tuning forks produce 6 beats per sec. The frequency of one of the forks is 312/sec. If the mass of a prong of the fork of unknown frequency be increased gradually by fixing small amounts of wax on it, at a time the beating frequency gradually decreases to 3/sec. Calculate the frequency of this fork. Explain whether it is possible to produce 6 beats per sec again by increasing the mass of the prong further. 4

11. (a) Write down and explain the laws of transvesre vibration of a string. 3

(b) A sonometer wire of length 75 cm is in unison with a tuning fork. If the length of the wire be decreased by 3 cm then it produces 6 beats per sec with the fork. Calculate the frequency of the fork. 4

12 (a) A person sets his watch at 27°C by hearing the sound of a gun-fire from the fort at a fixed hour and finds that the watch runs behind correct time by 10 seconds Explain the reason If the velocity of sound at 0°C be 330 metres/sec. calculate the distance between the person and the fort. 3

(b) An engine is advancing toward a hill with a velocity of 30 miles/hour by whistling sound of frequency 224/sec. Calculate the frequency of sound which is heard by the driver of the engine after the reflection of the sound from the hill. Assume the velocity of sound in air to be 1080 ft/sec. 4

1986

PHYSICS (First Paper)

Group—A

Answer any two Questions

1. (a) Establish graphically the equation $S = ut + \frac{1}{2} ft^2$, where u, f, s, and t have their usual meaning. 4

(b) A particle moving along a straight line with uniform acceleration describes 14.5 cm and 1.5 cm in the sixth and the tenth second respectively. What distance will it cover in the sixteenth second? 3

(c) A particle moves 'a' metres along a straight line and then it moves 'b' metres along a direction making angle θ with the original direction. Obtain the resultant displacement of the particle. 3

2. (a) Define centripetal force and establish an expression for the same. Why is centrifugal force called a pseudo force? 5

(b) Explain the effect of diurnal motion of the earth on the weight of a body on the surface of the earth. 2

(c) Establish the relation between linear velocity and angular velocity. 3

3. (a) What is work? Define joule, erg and watt. 4

(b) What is meant by potential energy and kinetic energy? 2

(c) Deduce an expression for the kinetic energy of a body moving with linear velocity u. 4

Group—B

Answer *any two* Questions

4. (a) Write down the laws of simple pendulum. 4

(b) Explain how the height of a hill can be determined by determining the time periods of a pendulum at the foot and top of the hill. 3

(c) The length of a simple pendulum is four times the length of another simple pendulum. If the time period of the pendulum of longer length is 6 secs., calculate the time period of the other pendulum. 3

5. (a) Define Young's modulus of elasticity. Same weight is suspended at the lower ends of two wires of same length but of different material. The diameters of the wires are 0·4 mm and 0·6 mm respectively. If the elongation of the first wire is three times the elongation of the second wire, compare the Young's modulii of the materials of the two wires. 1+3

(b) What should be the velocity of an artificial satellite of the earth so that it may revolve round the earth along a circular orbit 300 km above the earth's surface? Explain weightlessness of a body on an artificial satellite. What is meant by escape-velocity?

$g=980$ cm/sec^2; Radius of the earth$=6400$ km. 3+2+1

6. (a) State Archimedes' Principle. How would you verify the same experimentally? 4

(b) A body is floating on water with a portion of the body immersed. Will the body float up or down if the entire air above the body be removed? Give reason for your answer. 3

(c) The internal and external diameter of a hollow sphere is 6 cm and 8 cm respectively. If it floats just fully immersed in a

liquid of sp. gr. 1·2, find out the density of the material or the sphere. 3

7. (a) What do you mean by the statement that normal atmospheric pressure is 76 cm of mercury column? 2

(b) What is a siphon? Explain its principle of action. Mention the conditions for the working of a siphon. 6

(c) Calculate the limiting height of obstacle for siphoning kerosene oil of sp. gr. 0·80 from one vessel to another when the atmospheric pressure is 76 cm of mercury. Density of Hg = 13·5 gm/cc. 2

Group—C

Answer *any two* Questions

8. (a) Define coefficients of linear expansion, surface expansion and volume expansion of a solid. Establish the relation between them. 6

(b) A metre scale, made of steel, is correct at 10°C. What would be the correct distance between two consecutive cm-markings on this scale at 30°C and 60° F? 1+2

A brass rod measured with the help of the above scale at 30°C is found to be 5 metre long. What would be the correct length of the rod at 10°C? Coeff. of linear expansion of steel $= 12 \times 10^{-6}/°C$. Coeff. of linear expansion of brass $= 18 \times 10^{-6}/°C$. 3

9. (a) State Boyles' law and Charles' law. Establish the equation of the state for a perfect gas. How can you obtain the absolute scale of temperature from Charles' law? 7

(b) Deduce the relation between the volume and pressure coefficient of an ideal gas. 5

10. (a) Define water-equivalent and thermal capacity. How do they differ? A block of platinum of mass 200 gms is heated in a furnace and then dropped into 650 gms of water at 10°C, kept in a vessel of water equivalent 50 gm. If the final temperature of the mixture be 25°C, calculate the temperature of the furnace. Sp. heat of platinum = 0·030. 2+1+3

(b) Distinguish between vapour and gas. Define dew point and relative humidity.

The dew point and the temperature on a certain day were 10°C and 16·5°C respectively. The saturation vapour pressures at 10°C, 16°C and 17°C are respectively 9.10 mm, 13·50 mm and 14·40 mm of mercury column. Calculate the relative humidity of the day.

2+2+2

11. Write notes on : (a) Effect of pressure on melting point and boiling point. (b) First law of thermodynamics. 6+6

Group—D

Answer *any two* Question

12. (a) What is simple harmonic motion ? Mention its characteristics. Show that the principle of conservation of mechanical energy holds good in such motion. 1+3+2

(b) The amplitude of a particle of mass 4 gm executing simple harmonic motion is 5 cm. The acceleration of the particle is 12 cm/sec² when it is 3 cm away from the mean position. Calulate the velocity of the particle and the force acting on the particle when it is 4 cm away from the mean position. 2

13. (a) What are forced vibration and resonance ? 4

(b) A vibrating tuning fork of frequency 256 vibrations per sec is held near the upper open end of a vertical tube 200 cm long filled with water and water is allowed to run out slowly from the lower end. Neglecting end correction, calculate the positions of water surface in the tube for first and second resonances. Velocity of sound in air=320 m/sec. 4

14. (a) Write down Newton's formula for the velocity of sound in air. What is its defect ? How was the formula modified by Laplace ? 1+1+2

(b) Write down the difference between longitudinal and transverse wave. How would you establish that light is a transverse wave ? 2+2

1987

PHYSICS

(New Syllabus)

Group A

Answer *any two* questions

1. (a) Draw and explain the velocity—time graph of a particle moving with uniform acceleration. Explain the nature of the graph when the acceleration decreases with time.

   (b) A train begins to move with an acceleration of 2metre/sec^2 when a man is 9 metres away from a door of the train. The man begins to run and catches it in 3 sec. What is his acceleration ?

   (c) Two particles are dropped from the top of a tower at an interval of 2 seconds. Find their relative velocity and relative acceleration at a moment when both of them are falling.

2. (a) State Newton's first and second laws of motion, and define force from the first law.

   (b) Mass is the measure of inertia—explain.

   (c) What do you mean by static friction and kinetic friction ?

3. (a) Determine the value of potential energy of a body of mass m, situated at a height h.

   (b) Show that the law of conservation of energy is obeyed by a body falling freely under gravity.

   (c) Show that the momentum of a body having mass m and kinetic energy E is $\sqrt{2mE}$.

Group B

Answer *any two* questions

4. (a) State Newton's law of gravitation. What is universal gravitational constant ? Find the relation between the said constant and the acceleration due to gravity.

   (b) The mass and the radius of the earth are respectively 81 times and 4 times the values for the moon. Compare the accelerations on their surfaces.

   (c) Why does a body feel weightless in an artificial satellite ?

5. (a) State and explain Hooke's law.

   (b) Draw and explain the loan-elongation characteristic of an elastic solid material.

   (c) If the strain be 1% of 0·1, find the change in length of a wire 5 metre long. If the cross-section is 1 sq.mm. and load 10 kg.wt., what is the ratio of stress and strain ?

   (d) Define Poisson's ratio and bulk modulus.

6. (a) Compare specific gravity and density.

   (b) A body of density d_1 is placed slowly on the surface of a liquid of density d_2 and depth h. After time t, the body reaches the bottom of the liquid. Determine the value of t.

   (c) State Pascal's law and with its help, explain the principle of operation of a hydraulic press.

7. (a) Explain with the help of diagram the working principle of a common pump used for lifting of water. Discuss if there is any upper limit of the height to which water can be raised by such a pump.

   (b) If water is used in place of mercury in a barometer, what will be the height of the water column at normal atmospheric pressure ? Density of mercury 13·6 gm/c.c.

Group C

Answer *any two* questions

8. (a) Define the real and apparent coefficients of expansion of a liquid and establish the relation between them.

   (b) What do you understand by anomalous expansion of water ? Explain with the help of a diagram the variation of volume of a certain amount of water, if temperature changes from 0°C to 10°C.

   (c) Give three practical examples where expansion of solids has been made use of.

9. (a) Define latent heat of ice, dew point and relative humidity.

   (b) What is the difference between evaporation and boiling ?

   (c) When two pieces of ice are pressed together, they form a single piece. Explain why.

   (d) If 64,800 calorie of heat is extracted from 100 gm of steam at 100°C, what will be the result ? Latent heat of steam = 540 cal/gm ; latent heat of ice = 80 cal/gm.

10. (a) Write down the postulates of the kinetic theory.

    (b) What is mechanical equivalent of heat ?

    (c) Explain isothermal and adiabatic processes.

    (d) A meteorite of mass 42 kg has its velocity reduced from 15 km/sec to 5 km/sec while crossing the atmosphere of the earth. What will be the amount of heat produced for this change of velocity ? $J = 4.2 \times 10^7$ ergs/cal.

11. (a) Define thermal conductivity.

    (b) Write the differences between conduction and convection of heat.

    (c) Wind blows from sea to land during daytime but from land to sea at night.—Explain.

    (d) A metallic vessel having surface area of 1 sq. metre and thickness 0·5 cm is filled with ice and then placed inside

water at 100°C. How much ice will melt in 36 seconds ? Thermal conductivity of metal = 0.02 c.g.s. units, latent heat of ice = 80 cal/gm.

Group D

Answer *any two* questions

12. (a) What do you mean by time period ?
 (b) If the angular amplitude and the length of a simple pendulum are respectively θ and l, find the velocity of the bob at the lowest position.
 (c) Show graphically the resultant motion of a particle on which two simple harmonic motions of the same frequency are superposed (i) in phase and (ii) out of phase.

13. (a) Explain the difference between progressive and stationary waves.
 (b) An aeroplane is moving parallel to the ground with a velocity v. A sound from the plane reaches it again by reflection from the ground after t sec. Find the height of the plane, if the velocity of sound be V.
 (c) What are beats ?

14. (a) State the laws of transverse vibration of a string.
 (b) Determine the relation between the length of a closed pipe and the wavelength of the vibrating air column inside the pipe.
 (c) What are the characteristics of musical sound ?

1988

PHYSICS

First Paper

[New Syllabus for 1987 and 1988 Candidates]

Group—A

Answer any *two* questions.

1. (a) Show the subtraction of two vectors by a figure. What do you mean by the resolution of a vector ? How do you find out the rectangular components of a vector ?

(b) A man can reach just the opposite point of the bank by swimming in time t_1 and can swim the same distance down the current in time t_2. If the speed of the man in still water be u and that of current be v, then find the ratio of t_1 and t_2.

(c) A man is walking at the rate of 2 km per hour. The rain appears to him to fall vertically with a speed of 2 km/hour. Find out the real value and direction of velocity of rainfall.

2. (a) State Newton's second law of motion. How can you establish the first law from his second law ?

(b) Show that if a weight is hung from the midpoint of a wire, it cannot be kept horizontal.

(c) Action and reaction are equal and opposite—describe an easy experiment to show this.

(d) A mass of 2·0 kg is hanging from the hook of a spring balance fixed to the roof of a lift. What will be the reading of spring balance when the lift is (i) ascending with an acceleration of 0·2 m/s, (ii) descending with an acceleration of 0·1 m/s, (iii) moving up uniformly with a velocity of 0·15 m/s ?

3. (a) Find tne relation between angular velocity and linear velocity. Obtain the expression for centripetal acceleration.

(b) The weight of a body on the surface is less due to diurnal motion of the earth—explain.

(c) Define Joule and Horse-power.

(d) A cycle rider is moving with a speed of 18 km/hour along a curved path of radius 20 meter. Find the angle of his inclination with the vertical. ($g=980$ cm/sec^2).

Group—B

Answer any *two* questions.

4. (a) Find out how acceleration due to gravity at a place varice with altitude.

(b) What do you mean by a geo-stationary satellite ?

(c) Derive an expression for the velocity of an artificial satellita moving in a circular orbit.

(d) A satellite is moving round the earth in a circular path of 300 km from the surface. Find out its velocity. Radius of the earth $=6400$ km , $g=980$ cm/sec^2.

5. (a) Define Young's modulus and Poisson's ratio.

(b) Find out the work done per unit volume in case of extension of a metal wire.

(c) What is a spring balance ?

(d) Two ends of a steel wire of diameter 4 mm are fixed by supports at 300°C. What will be the force exerted on the supports if temperature falls down to 30°C ? Coefficient of linear expansion of steel $=10\times10^{-4}$/°C , Young's modulus of steel $=1{\cdot}1\times10^{12}$ dynes/cm^2.

6. (a) Explain the principle of action of a siphon, mentioning clearly the conditions under which it will be in action.

(b) Explain whether the rate of flow of liquid through a siphon will change if the atmospheric pressure changes.

(c) How can you determine the height of a mountain with the help of a barometer, if the average density of air is known ?

(d) Kerosine oil has to be brought over an obstacle by the action of a siphon. What should be maximum height of the obstacle for this purpose ? Sp. gr. of kerosine $=0{\cdot}8$, atmospheric pressure $=$ 76 cm of Hg.

7. (a) Stat. and explain Archimedes' principle.

(b) What is buoyancy ? S .ow how and on what factors its value depends.

(c) A floating body loses its weight—explain.

(d) The weight of a body in air is 0·4 gm. I ..eight along with an immerser in water is 3·37 gm. The weight of immerser in air is 4·0 gm. Find the sp. gravity of the body. Sp. gr. of immerser =8 0.

Group—C

Answer any *two* questions.

8. (a) Derive the relation between the coefficient of cubical expansion and coefficient of linear expansion of a solid.

(b) A metal scale does not give correct value at all temperatures —explain.

(c) The density of glass at 10°C and 60° C is respectively 2·6 gm/cc and 2·596 gm/cc. Find out the average value of the coefficient of linear expansion of glass in this temperature range.

(d) Explain whether the base angle of an isosceles triangle made by copper rods will change by heating.

9. (a) State Boyle's law and Charles' law. Derive the ideal gas equation from the same. What is the universal gas constant ?

(b) How can you get Boyle's law and Avogadro's law from the kinetic theory of gases ?

(c) How do you get the idea of the absolute zero of temperature from Charles' law ?

10. (a) What are the factors on which the rate of evaportaion depends ?

(b) State and explain the fundamental principle of calorimetry ; mention the conditions under which it is applicable.

(c) State the influence of pressure on the melting point of a substance.

(d) A piece of metal of mass 10 gm was heated to 100°C and then dropped in a mixture of water and ice. The volume of mixture

decreased by 0·1 cc though the temperature remained the same. Find the specific heat of the metal. The latent heat of ice = 80 Cal gm. Densities of ice and water at 0°C are 0·916 and 1·0 gm/cc respectively.

11. (a) State and explain the first law of thermodynamics.

(b) Describe a method for determining the mechanical equivalent of heat

(c) At which temperature a piece of wood and a piece of metal will appear equally hot on touching and why ?

(d) The velocity of a meteorite of mass 42 kg changed from 15 km/sec to 5 km/sec while passing through the atmosphere. Calculate the amount of heat produced during the process $J = 4·2 \times 10^7$ ergs/cal.

Group—D

Answer any *two* questions.

12. (a) what is resonance ? Explain with an example.

(b) Show that the equation $x = a \sin wt$ represents a simple harmonic motion.

(c) Explain the terms amplitude, Phase and time period of a simple harmonic motion.

13. (a) Discuss the correction made by Laplace to Newton's formula for the velocity of sound in gas.

(b) Explain what information is obtained about its nature from polarisation of light. Why there is no polarisation of sound waves ?

(c) Find out the velocity of sound in hydrogen gas at N T. P. The density of hydrogen at N.T.P. = 0·090 gm/ litre , $\gamma = 1·4$.

14. (a) What is interference of waves ? Describe an experiment to show the interference of sound wave.

(b) Derive the relation between the fundamentals produced in two tubes of same length—one open, the other is closed.

(c) 5 beats are produced when two tuning forks *A* and *B* are sounded together. Find out the frequencies of the forks, if they produce resonance with 36 cm and 37 cm respectively of air column of a tube closed at one end.

Dr. D.P. Ray Chaudhuri needs no introduction to the scientific community. A lifetime spent in teaching Science to millions of students had enabled him to keenly appreciate the problems faced by them in understanding the intricacies of the subject.

A Text Book of Physics has been written as per the latest Higher Secondary syllabus for those students who are to appear for Higher Secondary examinations as also students who are preparing for Joint Entrance, IIT and other competitive examinations. This book is Dr. Ray Chaudhuri's parting gift to young Physics students and will surely be appreciated by student and teacher alike.

Dr. D.P. Ray Chaudhuri has been ably assisted by Prof. J.K. Mukherjee of Scottish Church College, Calcutta and Prof. Dipak Roy of New Alipore College, Calcutta in the preparation of this book.